Low mass stars seismology with WhoSGlAd and EGGMiMoSA

Good vibrations seminars

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Study stars ?

- Heavy elements factory,
- Stellar ages → galactic history,
- Exoplanetary masses, radii and ages,
- ...

Credits: NASA

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Stellar models limitations

- 'Classical' methods : mainly superficial information (T_{eff}, $[Fe/H]$,..)
- Model dependency on inferred quantities: e.g. chemical composition (e.g. Lebreton & Goupil 2014), age,..

But stellar models need improvement :

- Convective layers and their extension;
- Transport processes : Chemical elements, angular momentum (Eggenberger et al. 2012);
- ...

$\rightarrow$ Information about internal structure needed

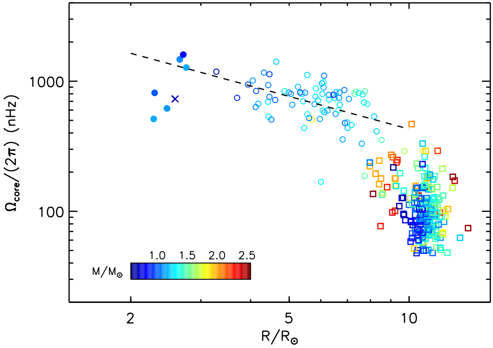
Asteroseimology in a Nutshell

Asteroseismology accurately probes stellar interiors

- Stellar structure may oscillate around an equilibrium state
- Stellar oscillation frequencies directly linked to stellar internal structure
  - $\rightarrow$ $c(r)$, internal rotation, chemical composition profiles,...
- Many successes : helioseismology, constraints about stellar structure, asteroseismology of red giants,...
- But also highlights models limitations

An example

Unexpectedly slow giants core rotation



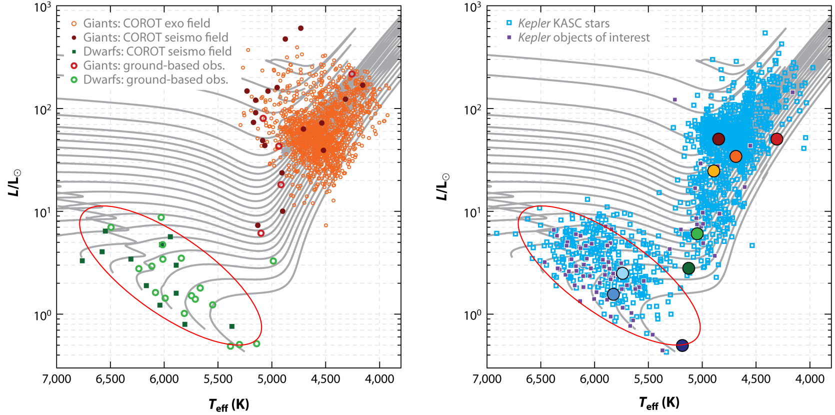
Credits: Deheuvels et al. (2014)

Asteroseimology and data

- Very precise data
  → CoRoT (Baglin et al. 2009), Kepler (Borucki et al. 2010), TESS (Ricker et al. 2014), PLATO (Rauer et al. 2014)

- And precise methods
  ① **WhoSGlAd**: Main sequence stars (Farnir et al. 2019,2020)
  ② **EGGMiMoSA**: Sub- and red-giants (Farnir et al. in prep.)

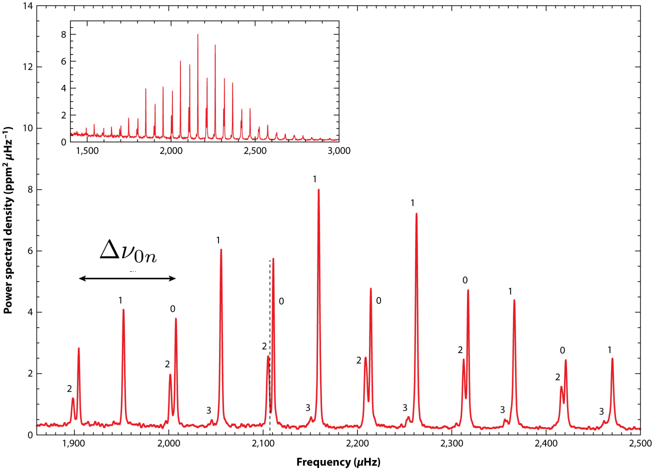
Part I: Main sequence & WhoSGlAd



Chaplin WJ, Miglio A. 2013.
Annu. Rev. Astron. Astrophys. 51:353–92

Oscillation spectra

$$\nu_{n,l} \simeq \left(n + \frac{l}{2} + \epsilon\right) \Delta\nu$$ Tassoul (1980), Gough (1986)



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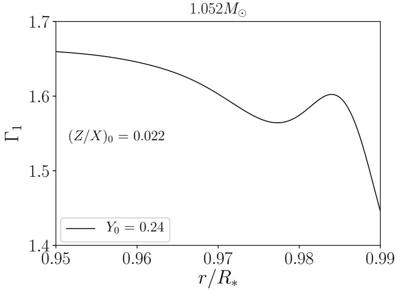
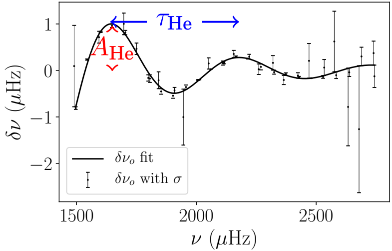
Acoustic glitches

- Oscillation spectrum → smooth and glitch parts
- Glitches : due to sharp features in the stellar structure
- Provide local information





$$\delta\nu = \nu - \nu_{\mathrm{smooth}}$$

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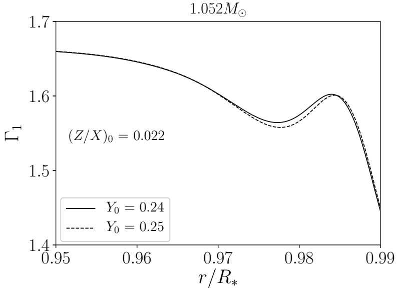

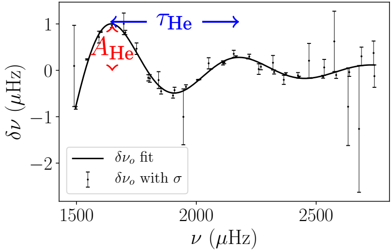

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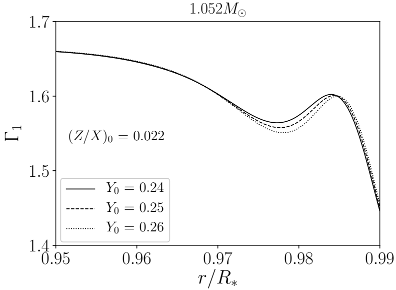

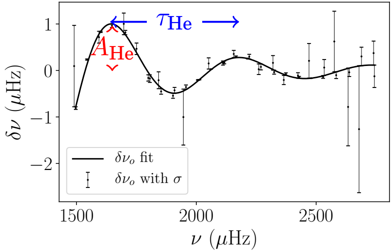

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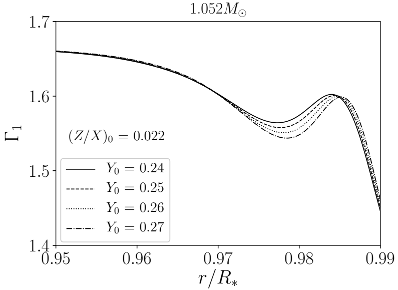
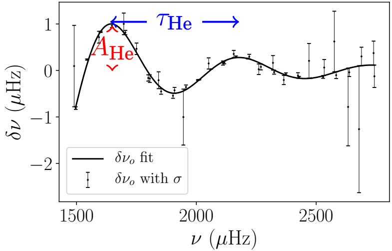




$$\delta\nu = \nu - \nu_{\mathrm{smooth}}$$

Glitch fitting

Several techniques:
Monteiro et al. (2000), Basu et al. (2004), Mazumdar et al. (2014), Verma et al. (2014)

Limitations :

- Non linear formulation,
- Smooth part regarded as dispensable,
- Correlated indicators,
- Regularisation term needed.

$$f(n,l) = \overbrace{\sum_{k=0}^{4} A_{k,l} n^k}^{\text{Smooth}} + \overbrace{\mathcal{A}_{He} \nu e^{-c_2 \nu^2} \sin(4\pi\tau_{He}\nu + \phi_{He})}^{\text{He Glitch}} + \underbrace{\frac{\mathcal{A}_{CZ}}{\nu^2} \sin(4\pi\tau_{CZ}\nu + \phi_{CZ})}_{\text{CZ Glitch}} \quad (1)$$

Verma et al. (2014)

Principle

WhoSGlAd - **Whol**e **S**pectrum and **Gl**itches **Ad**justment (Farnir et al. 2019,2020)

Analyses oscillations spectrum as a whole
⇒ proper correlations are derived;

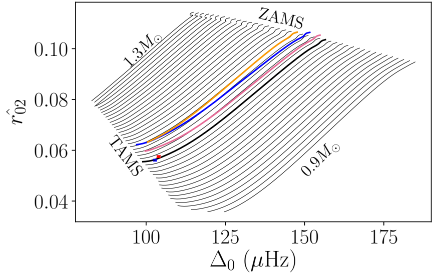
Consider the frequencies vector space:

1. Build orthonormal basis of functions (Gram-Schmidt);
2. Project the frequencies on the basis → get independent coefficients;
3. Combine the coefficients into indicators as uncorrelated as possible;
4. Use the indicators to obtain best fit stellar models.

Seismic indicators

Smooth:

- $\Delta \rightarrow \sim \Delta\nu$, Mean density (Tassoul 1980, Ulrich 1986)
- $\hat{r}_{0l} = \frac{\overline{\nu_0}-\overline{\nu_l}}{\Delta_0} + \overline{n_l} - \overline{n_0} + \frac{l}{2} \rightarrow \sim$ Roxburgh & Vorontsov (2003), Composition and evolution



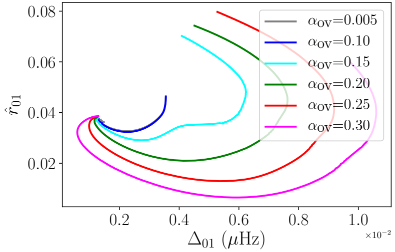
Farnir et al. 2019

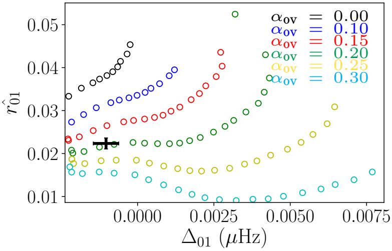
- Linear combination of ν
- $\sigma(\hat{r}_{02})/\hat{r}_{02} \simeq 0.6\%$

Seismic indicators

Smooth:

- $\Delta_{0l} = \frac{\Delta_l}{\Delta_0} - 1 \rightarrow$ Overshooting





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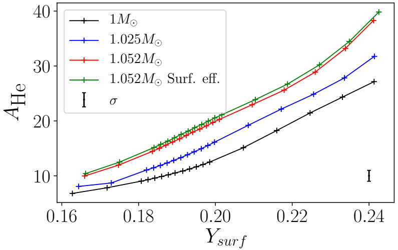
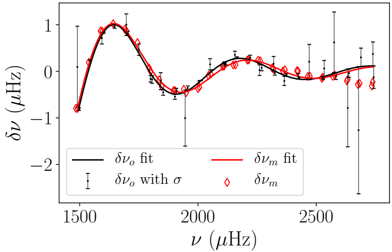
Seismic indicators

Glitch:

- $A_{\mathrm{He}} \rightarrow$ Helium content





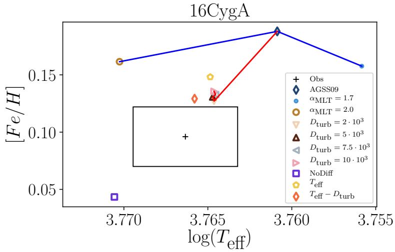
Farnir et al. 2019

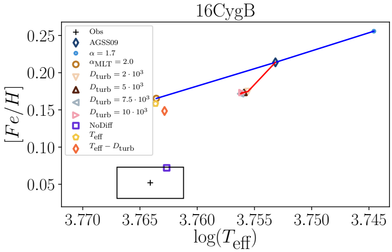
Independent of smooth indicators

Application to 16 Cygni

Fitting Δ, $\hat{r}_{01}$, $\hat{r}_{01}$ and A_{He}:

$$t_A\ (Gyr) : \quad 7.1 \pm 0.5 \qquad X_{0,A} : \quad 0.72 \pm 0.05$$
$$M_A\ (M_\odot) : \quad 1.08 \pm 0.04 \qquad Y_{s,A} : \quad 0.23 \pm 0.02$$





Farnir et al. 2020

Necessity of non-standard processes

Part I: Conclusions & Perspectives

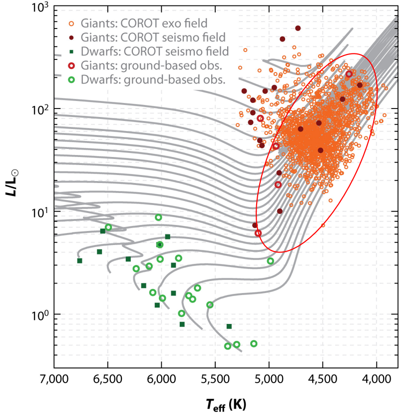
Conclusions:

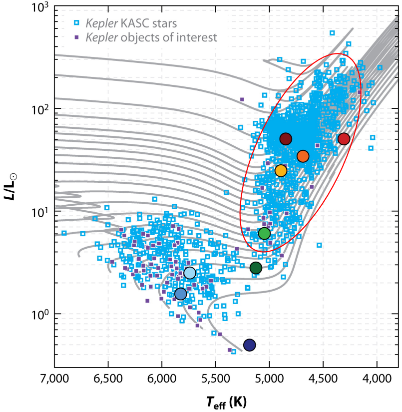
- Linear formulation:
  - → Reduced correlations,
  - → Fast computations,
  - → Stringent constraints on the structure
- Constraint on Y_s
- Thorough adjustment of the 16 Cygni system (Farnir et al. 2020) → show models shortcomings (e.g. constrain mixing)

Perspectives:

- Taking advantage of AIMS (Rendle et al. 2019), adjust the Kepler LEGACY sample (Lund et al. 2017),
- Automated treatment of PLATO data

Part II: Giants & EGGMiMoSA





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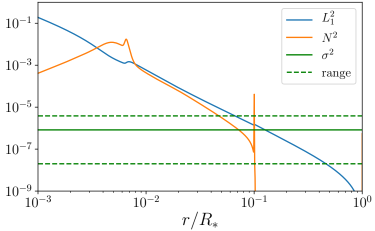
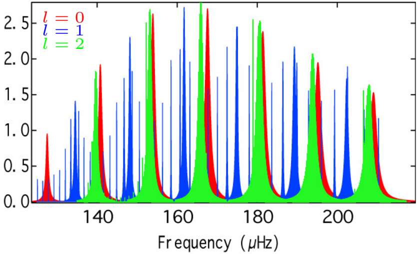
Mixed-modes

- Modes of mixed p and g character

→ pressure and gravity cavities coupled via evanescent region





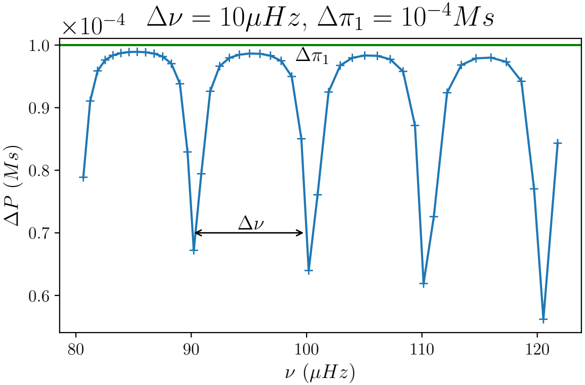
Credits: Grosjean et al. (2014)

A typical spectrum

2 characteristic quantities:

① $\Delta\nu = \left(2\int_0^R \frac{dr}{c}\right)^{-1} \sim (\bar{\rho})^{-1/2}$ (Tassoul 1980, Ulrich 1986)

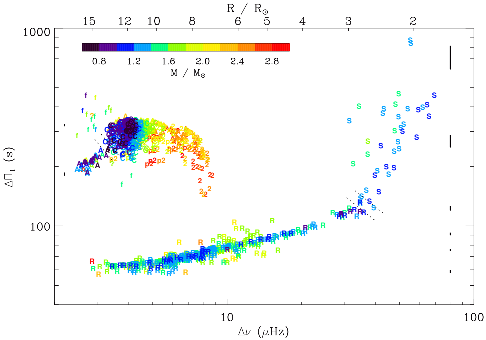
② $\Delta\pi_1 = 2\pi^2 \left(\int_g \frac{N}{r} dr\right)^{-1} \sim M_{\mathbf{core}}$ (Tassoul 1980, Montalbàn et al. 2013)



Mixed-Modes

- H-shell vs core-He burning (Bedding et al. 2011)

- Core mass (e.g. Montalbàn et al. 2013)
- Core rotation (e.g. Gehan et al. 2018)



Credits: Mosser et al. 2014

EGGMiMoSA

EGGMiMoSA:
Extracting **G**uesses about **G**iants via **Mi**xed-**Mo**des **S**pectrum **A**djustment (Farnir et al. in prep.)

- **Goals**:
  - → Provide a seismic adjustment of mixed modes spectra (e.g. Hekker et al. 2018),
  - → Define relevant seismic indicators,
  - → Study the evolution of seismic indicators along a grid of models,
  - → Future implementation in AIMS (Rendle et al. 2019)

Developed in collaboration with M.-A. Dupret and C. Pinçon

Formalism

Asymptotic formulation coupling between p and g cavity:

$$\tan\theta_p = q\tan\theta_g \qquad (2)$$

Shibahashi 1979, Unno et al. 1989,

where:

$$\theta_p = \pi\left[\frac{\nu}{\Delta\nu} - \epsilon_p\right] \qquad (3)$$

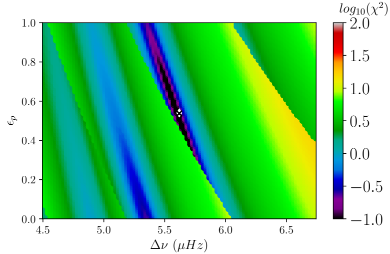
$$\theta_g = \pi\left[\frac{1}{\nu\Delta\pi_1} - \epsilon_g + \frac{1}{2}\right] \qquad (4)$$

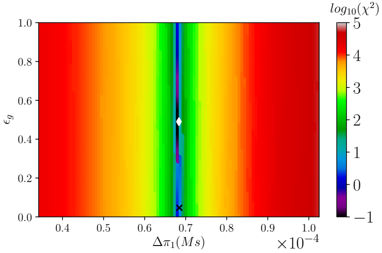
Adapted from Mosser et al. 2015.
See also Pinçon et al. 2019

5 parameters L-M minimisation: $\Delta\nu$, $\Delta\pi_1$, ϵ_p, ϵ_g, q
No further simplifications $\Rightarrow$ adapted to red and subgiants

Parameters estimation

Local method ⇒ need of proper estimates

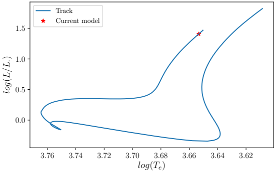


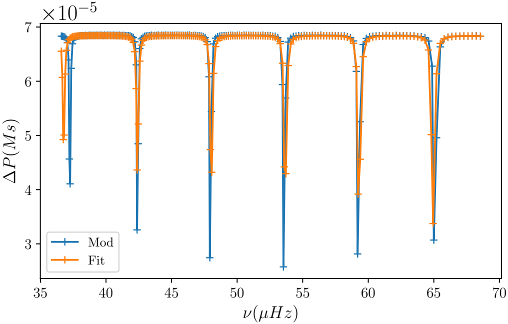


→ Guess within 10% of target value

Evolved giant: g-dominated

$$\mathcal{N} = 29.6$$



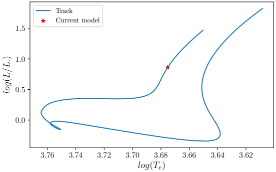


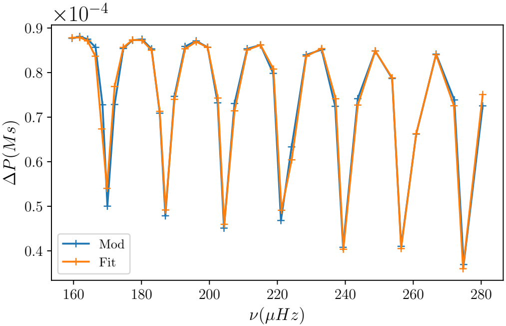
- $M = 1M_{\odot}$,
- $X_0 = 0.72$,
- $Z_0 = 0.015$

$$\mathcal{N} = \frac{\Delta\nu}{\Delta\pi_1 \nu_{max}^2}$$ Mosser et al. (2015)

Giant: g-dominated

$$\mathcal{N} = 3.8$$





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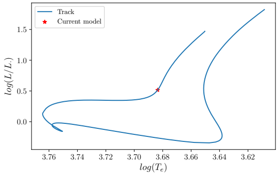
$\mathcal{N} \simeq 1$: transition

$$\mathcal{N} = 1.01$$



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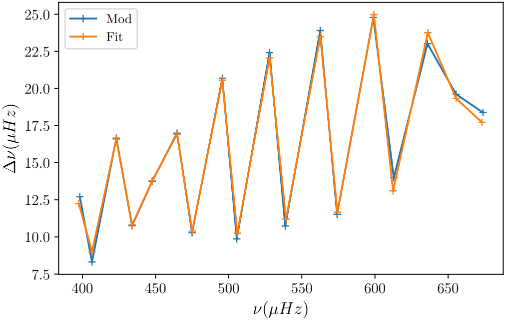
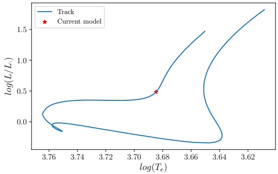
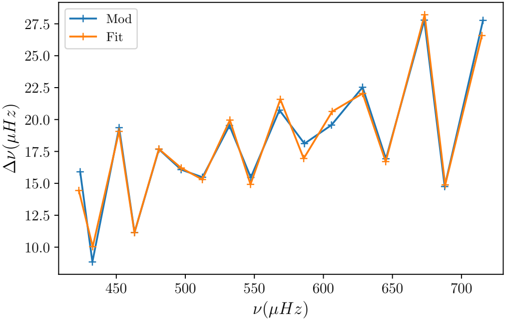


Late subgiant: p-dominated

$$\mathcal{N} = 0.89$$





- $M = 1M_{\odot}$,
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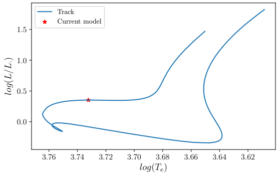
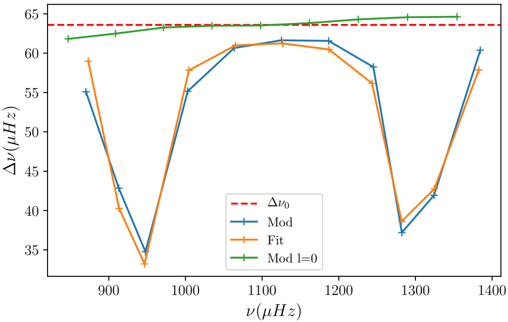
- $Z_0 = 0.015$

Subgiant: p-dominated

$$\mathcal{N} = 0.16$$





- $M = 1M_{\odot}$,
- $X_0 = 0.72$,
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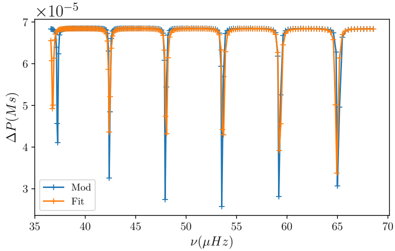
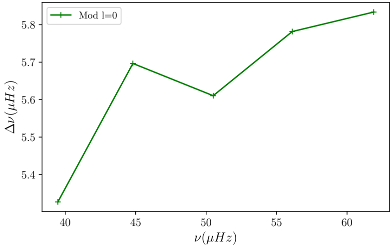
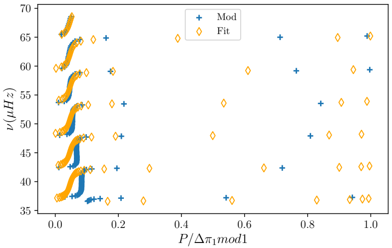
Evolved giant: p-modes drift

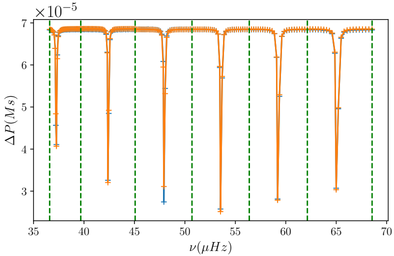
$$\mathcal{N} = 29.6$$

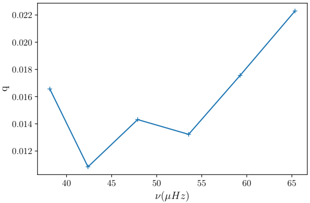




Evolved giant: p-modes drift and q





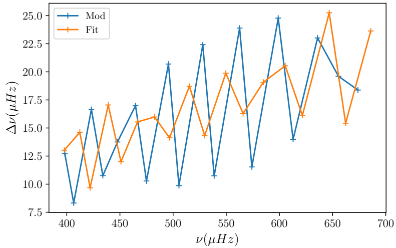


q may vary with ν (Cunha et al. 2019)

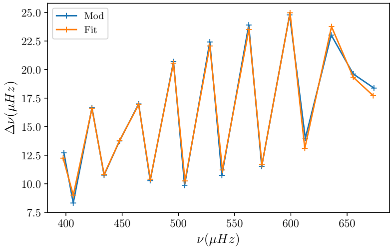
$\mathcal{N} \simeq 1$: transition

$$\mathcal{N} = 1.01$$

With bad guesses

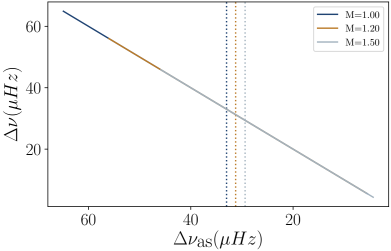


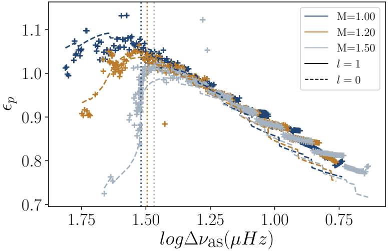
With an improved guess



Parameters evolution

Evolution along subgiant and red giant phases





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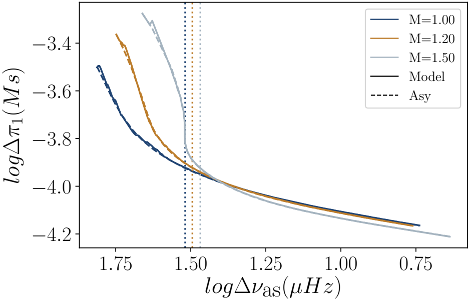

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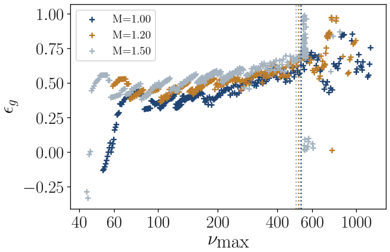

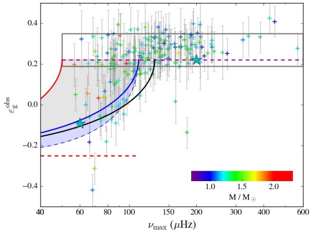

Credits: Pinçon et al. (2019)

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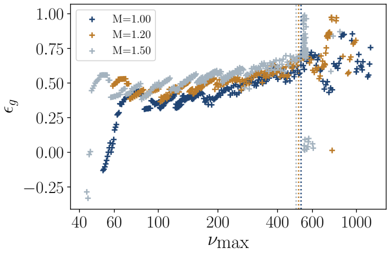

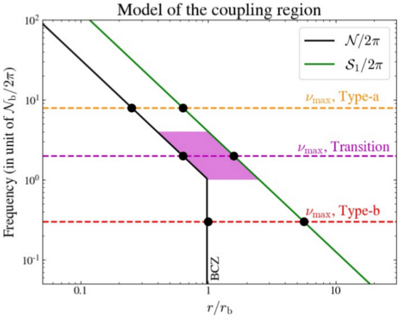

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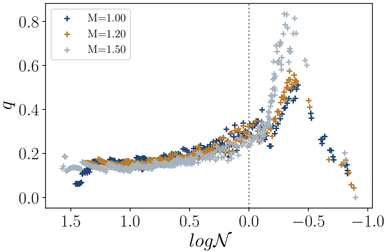
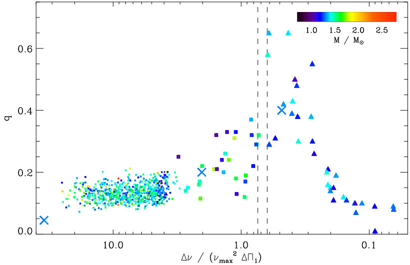
Parameters evolution

Evolution along subgiant and red giant phases





Credits: Mosser et al. (2017)

Part II: Conclusions

- Efficient, fast and automated adjustment;
  - → Both for sub- and red-giants
  - → Adapted for large surveys
- Relevant constraints on the structure;
  - → Agrees with observation and asymptotic studies.

Perspectives

Short term:

- Continue to explore seismic indicators representative of stellar structure,
- Comparison with asymptotic indicators,
- Comparison with observed indicators (e.g. Mosser et al. 2015),
- Coupling with AIMS (Rendle et al. 2019),
- Model Kepler stars

Perspectives

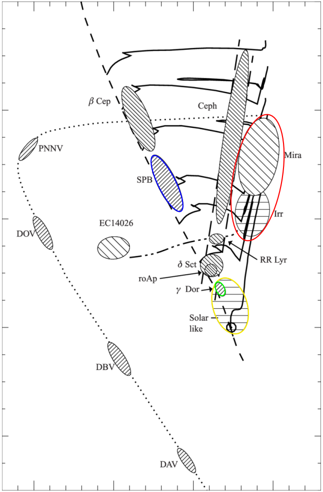
Long term:

- Include second order in θ_p for subgiants,
- $q(\nu)$ relation for evolved giants (Cunha et al. 2019),
- Adjust for the signature of glitches:
  - $\rightarrow$ Helium glitch: WhoSGlAd (Farnir et al. 2019,2020)
  - $\rightarrow$ Buoyancy glitch: see Miglio et al. 2008, Mosser et al. 2015, Cunha et al. 2015

Appendices

Pulsating Stars

- Solar-like ($P \sim 2 - 15min$),
- γ Dor ($P \sim 0.5 - 3days$),
- SPB ($P \sim 0.8 - 3days$),
- Red giants and subgiants ($P \sim 3 - 30days$),
- ...



Credits: Christensen-Dalsgaard J.

WhoSGlAd basis Elements

We selected the basis functions:

- Smooth

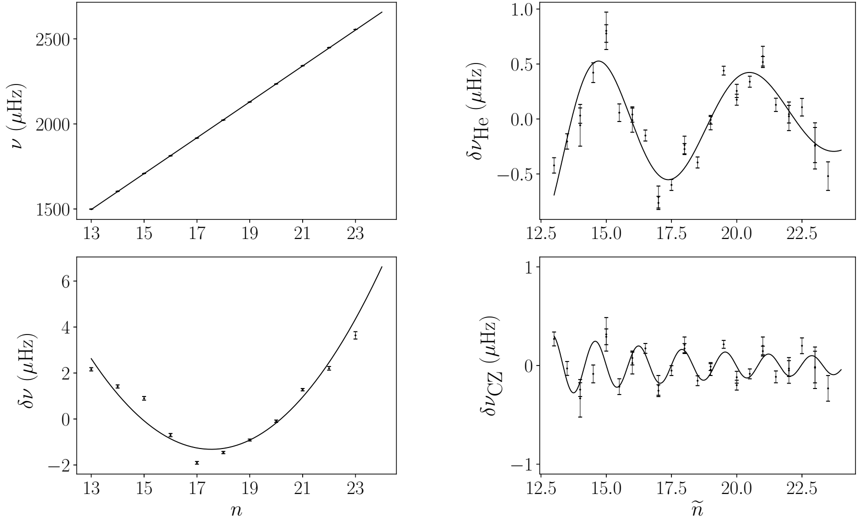
① $p_0(n) = 1$
② $p_1(n) = n$
③ $p_2(n) = n^2$

- Glitch

He
$$p_{\mathrm{He}Ck}(\widetilde{n}) = \cos\left(4\pi T_{\mathrm{He}}\widetilde{n}\right)\widetilde{n}^{-k}$$
$$p_{\mathrm{He}Sk}(\widetilde{n}) = \sin\left(4\pi T_{\mathrm{He}}\widetilde{n}\right)\widetilde{n}^{-k}$$
with $k = 5, 4$, $\widetilde{n} = n + l/2$

CZ
$$p_{\mathrm{C}C}(\widetilde{n}) = \cos\left(4\pi T_{\mathrm{CZ}}\widetilde{n}\right)\widetilde{n}^{-2}$$
$$p_{\mathrm{C}S}(\widetilde{n}) = \sin\left(4\pi T_{\mathrm{CZ}}\widetilde{n}\right)\widetilde{n}^{-2}$$

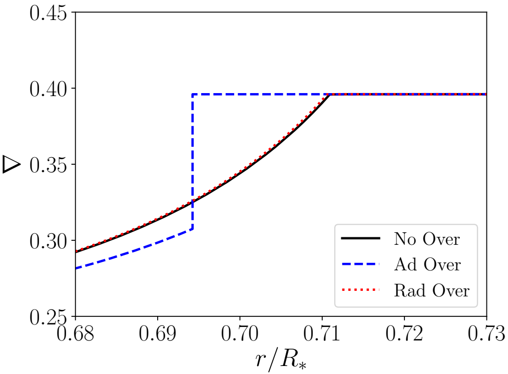
An Illustrative Example



Convection Zone Glitches

Mixing processes badly constrained

→ Convection zone glitch : radiative - convective regions transition
⇒ Transition localisation



g-dominated

$\mathcal{N} > 1$: g-dominated

$\rightarrow$ θ_p varies slowly

$$\begin{aligned}(2) \Leftrightarrow \theta_g &= \phi\left(\theta_p\right) + n_g \pi \\ &= \pi \left[\frac{1}{\nu \Delta\pi_1} - \epsilon_g + \frac{1}{2}\right] \\ \Rightarrow P_i &= \left\{\phi_i/\pi + \epsilon_g + n_{g,i} - 1/2\right\} \Delta\pi_1\end{aligned}$$

with $\phi\left(\theta_p\right) \equiv \phi_i = \arctan\left[q^{-1} \tan \theta_p\right]$

Period spacing fitting

To be rid of ϵ_g we fit:

$$\Delta P_i = P_{i+1} - P_i = \{\overbrace{(n_{g,i+1} - n_{g,i})}^{1} + (\phi_{i+1} - \phi_i)/\pi\}\Delta\pi_1 \quad (5)$$

Levenberg-Marquardt adjustment of ΔP_i with $\Delta\nu$, $\Delta\pi_1$, ϵ_p and q as parameters.

Periods fitting

To find ϵ_g, we have to minimise $\chi^2 = \sum_i \left(\frac{P_{\mathrm{obs,i}} - P_{\mathrm{th,i}}}{\sigma_i}\right)^2$

Taking advantage of the previous step, we get:

$$P_{\mathrm{th,i}} = P_1 + \sum_{j=1}^{i-1} \Delta P_j$$

with $P_1 = \phi_1/\pi + \epsilon_g + n_{g,1} - 1/2$

Periods fitting

We have to solve $\frac{\partial \chi^2}{\partial \epsilon_g} = 0$
This yields:

$$\epsilon_g = \frac{\sum_i \left[P_{\mathrm{obs,i}} - \sum_{j=1}^{i-1} \Delta P_j \right] / \sigma_i}{\Delta \pi_1 \sum_i 1/\sigma_i^2} - (n_{g,1} + \phi_1/\pi - 1/2)$$

Parameters estimation

Mosser et al. 2015 define:

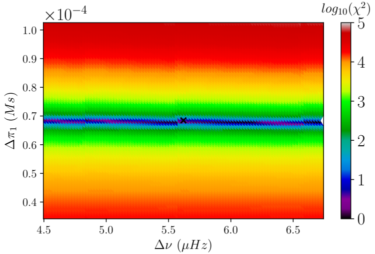
$$\zeta = \left[1 + \frac{q}{\mathcal{N}} \frac{1}{q^2 \cos^2\theta_p + sin^2\theta_p}\right]^{-1} \quad (6)$$

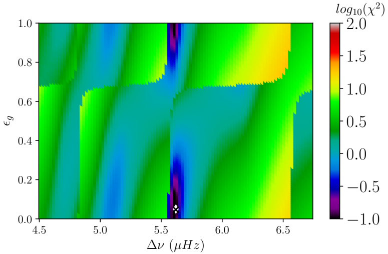
such that $\frac{dP}{dn} = \zeta \Delta\pi_1$

- q: Estimated from ratio $Z = \frac{\zeta_{\max}}{\zeta_{\min}}$
- $\Delta\pi_1$: $\Delta\pi_{1,0} \simeq \max\left(\Delta P_{\text{obs}}\right)$
  $\rightarrow \ \Delta\pi_1 \simeq \Delta\pi_{1,0}/\zeta_{\max}$
- $\Delta\nu$, ϵ_p: radial modes fitting with WhoSGlAd (Farnir et al. 2019,2020)

Parameters estimation

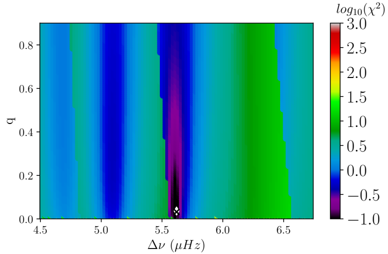
Local method ⇒ need of proper estimates

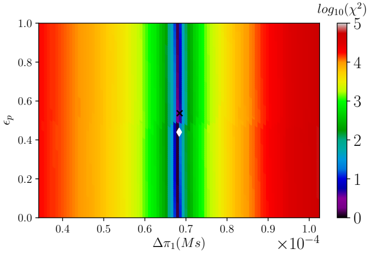




Parameters estimation

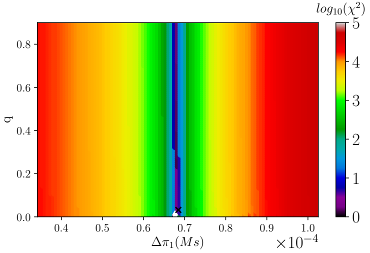
Local method ⇒ need of proper estimates

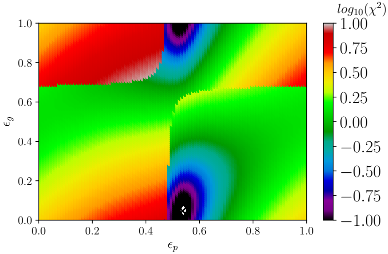




Parameters estimation

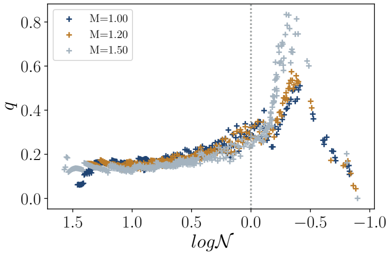
Local method ⇒ need of proper estimates

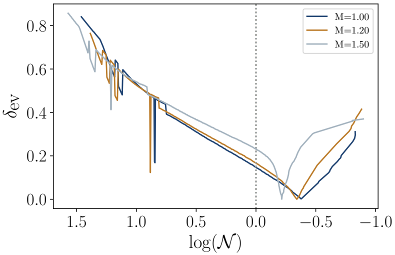




Parameters evolution

q and evanescent region size





Stellar Modelling