TYPES OF STELLAR INSTABILITIES

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Aside from violent phenomena, regular forms of motions originate often in instabilities and the linear theory with terms $\propto \exp(st)$ yields already significant information. The system, here a spherical star, will be the seat of an instability if $R(s)>0$. In general, s will be complex as both conservative (adiabatic) and non-conservative (non-adiabatic) factors are present. However if the latter (small) are neglected, the eigen-values s^2 often denoted $-\sigma^2$ are real. If at least one $\sigma^2<0$, then the star is dynamically unstable.

Radial perturbations. If an appropriate average value $\bar{\Gamma}_1>4/3$, then all σ^2 are positive. If $\bar{\Gamma}_1<4/3$ (formation phase: ionization; late evolution: nuclear equilibrium; degeneracy in white dwarfs and neutron stars or radiation in very large masses plus general relativistic effects) the fundamental eigenvalue σ_0^2 only becomes negative.

The positive eigenvalues $\sigma_i^2= 4\pi^2/\tau_i^2$ are inversely proportional to the sound travel time across the radius or a fraction thereof for higher modes. The fundamental period varies from a few hundred days for red supergiants through a few days for cepheids, one hour for the sun, a few seconds for a white dwarf to a few thousandths of a second for a neutron star. The qualitative behaviour of the fundamental eigenfunction is illustrated in Fig. 1.

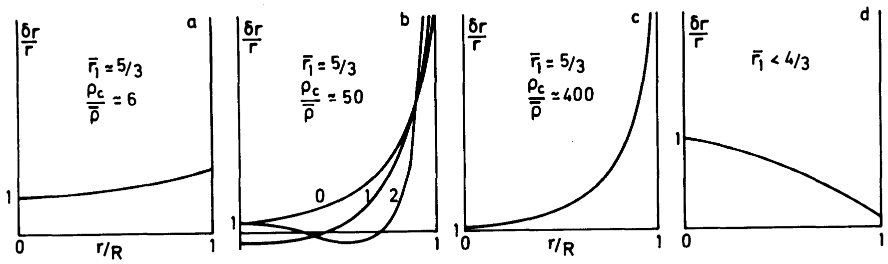

Fig. 1 - Radial eigenfunctions for various central condensations $\rho_c/\bar{\rho}$ and values of $\bar{\Gamma}_1$ both $>4/3$ and $<4/3$.

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Dynamical instability may result in violent collapse or explosion but it may also lead to relaxation oscillations or to a new equilibrium state.

Non-radial perturbations $\propto P_\ell^m(\cos\theta)e^{im\theta}$, $-\ell<m<\ell$. In the adiabatic approximation, the problem is also self-adjoint, but σ^2 entering non-linearly, the spectrum breaks into two parts, the acoustic p-modes with the $\sigma_p^2\to\infty$ and the gravity or g-modes with $\sigma_g^2\to 0$, often separated by an external surface f-mode with no nodes inside. Dynamical instability never enters through the f- and p-modes : σ_f^2 and $\sigma_p^2>0$, always.

If $A=\left((1/\rho)d\rho/dr-(1/\Gamma p)dp/dr\right)$ is <0 everywhere then also all the σ_g^2 become negative simultaneously (cf. Fig. 2).

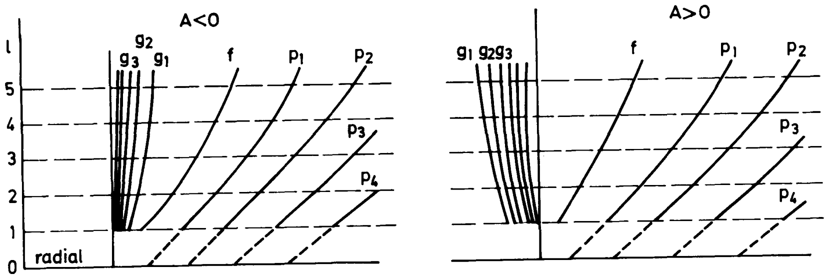

Fig. 2 - Eigenvalue spectra for non-radial oscillations.

As far as the p-modes are concerned, the corresponding periods are in the same range as those of radial oscillations and their behaviour is very much alike too. However, contrarily to the radial case, all the amplitudes here tend towards zero at the centre.

The behaviour of g-modes depends strongly on the distribution of A, sign included, inside the star. If $A<0$ everywhere, the periods are practically always longer than those of p-modes, the longer, the higher, the mode. The amplitude do not increase so rapidly towards the surface.

For high ℓ values, the amplitude of the p-modes is appreciable only close to the surface (skin effect) while that of the g-modes can be largely dominant in the interior.

If A changes sign once along the radius (one turning point in $A\neq 0$), the g spectrum splits into two : one positive (stable) oscillating with appreciable amplitudes only in the region $A<0$ and decreasing exponentially in the other and one negative (unstable) with exactly the opposite behaviour (cf. Fig. 3). Note that, however small the region with $A>0$, an unstable g spectrum must result.

If two or more negative (positive) A zones exist, one could expect as many positive (negative) σ_g^2 spectra. However, the latter are not completely independent as interresonance between regions with the same sign of A may give rise to a unique eigenfunction with large amplitude simultaneously in

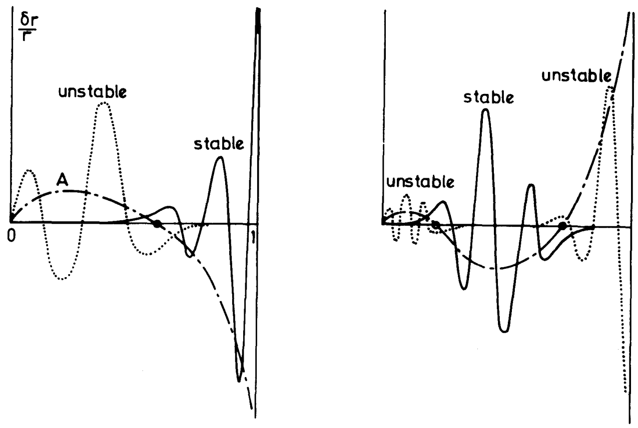

Fig. 3. Eigenfunctions for *g*-modes when *A* changes sign.

these regions and remaining appreciable in between (cf. Fig. 4). If *A* is positive in the two regions considered, this may be significant for an increased mixing across the intermediate stable zone when the latter becomes sufficiently narrow (tunnel effect).

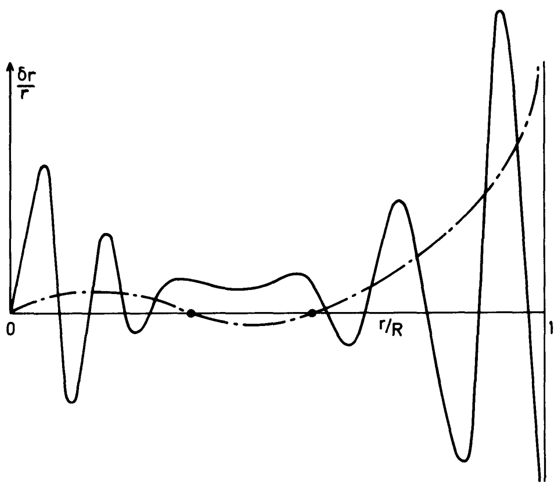

Fig. 4 - Interresonnance between two regions with $A>0$.

When σ^2 is positive, the adiabatic oscillations described above, once excited, would go on indefinitely. But the non-adiabatic terms neglected up to now will either damp or amplify these oscillations according to $\exp(i\sigma t).\exp(-\sigma' t)$. In stars, thermonuclear energy generation and the energy transfer, most often by radiation, play the major role. In that case,

$$\sigma' = (2\sigma_a^2\ J)^{-1} \int_0^M (\Gamma_3 - 1)(\delta\rho/\rho)_a\ \delta\left(\varepsilon - (1/\rho)\mathrm{div}\ \vec{F}\right)_a dm$$ and

$\delta\varepsilon$ has always a distabilizing influence while very often the flux term is stabilizing. The run of the adiabatic amplitudes is important since it determines the "weight" of the various stabilizing or distabilizing factors. If the star is vibrationally (pulsationally) unstable (overstable) ($\sigma' < 0$), either this instability is strong enough to lead finally to mass ejection or non-linear damping effects enter limiting the amplitude to some finite value which is the case of interest for regular variables.

The simplest and best known case is that of radial oscillations. On the main sequence, there is a limiting mass above which stars become vibrationally unstable due to the ε term which then has a large weight. A few special g-modes can also be energized by thermonuclear reactions and their amplification to finite amplitudes must excite some kind of forced turbulence. White dwarfs are easily made vibrationally unstable in presence of thermonuclear reactions especially in the external layers (accretion of H, for instance) either radially or, more efficiently, towards g-modes which, as $A \simeq 0$ in the interior, are restricted to the very exterior (rapid blue variable perhaps). The same is probably true of neutron stars.

However, up to now, none of the classical regular variables can be accounted for on the basis of vibrational instability due to the $\delta\varepsilon$ term. In most cases, the source of the excitation and maintenance is the "valve" mechanism which consist in blocking the energy in at compression and letting it out at expansion. In stars, this can result from the so-called Γ (reduced so that $|\delta T/T|$ decreases) or χ (opacity increases at compression) - mechanisms which are at work in the ionization zones of H and He in stars occupying the cepheids strip and its extensions and, perhaps, also in red supergiants. But many of the following papers will deal with this.

Non radial p-modes coupled to rotation have been advocated with some good reasons for the β Cephei stars in which however a reliable source of vibrational instability is still lacking. High p-modes of large ℓ are also involved in the solar five minute oscillation.

Secular (sometimes called thermal) instability corresponds to the direct effects of the non-conservative terms, the inertial terms being neglected. It is important for stellar evolution and it can be approached by methods such as that of linear series with success. A paper will deal with this later.

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