SUMMARY AND CONCLUSIONS

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These proceedings comprise the detailed contributions to a very successful conference, each with its own abstract and a general summary may look superfluous here even if it had some interest in the closing session or as a means of conveying the main results of the conference beforehand to a larger class of readers (Gough, 1983).

I shall endeavour at least to present it against a brief historical background so that the present trends might stand out better as well as the tremendous progresses that have taken place during the last few years thanks to the extraordinary refinements of the observational techniques and the development of powerful methods of numerical statistical analysis. No doubts the reader will also expect a few conclusions although, as in any fast developing subject, the future will probably bring surprises beyond our imagining.

Just twenty three years ago, Leighton (1961, cf. also Leighton et al, 1962) at the "Fourth Symposium on Cosmical Gas Dynamics" in Varenna announced the discovery, on the solar surface, of local fields of periodic motions of relatively small horizontal extent with velocities nearly up to 1 km s $^{-1}$ and with periods around five minutes. At the time, it must have appeared already that, if global modes of non-radial oscillations were involved, they could only be acoustic p-modes of very high degrees (ℓ and m very large in $P_0(\cos \theta)$ exp im ϕ , $-\ell < m < \ell$) and consequently of very short horizontal wave-lengths.

But for quite a time, the tendency was to think of these localized velocity fields rather in terms of wave-trains excited hapharzardly by underlying granules. The first attempts at a somewhat more detailed interpretation (cf. Schatzman and Soufrin (1967); Stein and Leibacher (1974)) rested and justifiably so, in this case, on the plane approximation. They contributed to bring the knowledge and insights gained in geophysical contexts to bear on the problem and they certainly shed a new light on the propagation of p- and g-waves, on the associated energy transfer and on the possibilities of their trapping in various zones. Ulrich (1970) was probably the first to point out that, in the sun, trapping can occur for p-waves towards the top of the hydrogen convection zone below the photosphere.

The search for excitation and maintenance mechanisms of the types so successful for globally pulsating stars like Cepheids or RR Lyrae stars, perhaps also the report by Hill and Stebbins (1974) of periodic variations in the shape of the Sun with much longer periods encouraged the tendency to look at the problem in terms of global oscillations, a point of view adopted explicitly by Wolff (1972) roughly ten years after Leighton's discovery.

Fig. I illustrates the typical behaviour of these modes with appreciable amplitudes only in a very shallow external layer $\Delta r/R \simeq 0.01$. Nevertheless these modes, at the top of the convec-

tion zone, are sensitive to the whole structure of the latter and favour its extent down to great depths as well as fairly high helium abundances leading to high central temperatures increasing the excess neutrinos. These aspects of the high p-modes and the remaining difficulties were not extensively discussed during the meeting. They are touched upon in the introductory talk by Gough and in the review presented by Rhodes (Rhodes E. J. et al.) and incidentally referred to in the paper by Shibahashi, Noels and Gabriel while they are put to use by Hill, Gough and Toomre to infer variations of horizontal velocities with depth and position suggesting a giant cell convective pattern superimposed on a differential rotation. A discussion of the vibrational stability of these modes, extended also to intermediate & values, by Antia, Chitre and Narasimha reveals overstability preferentially in a range of frequency centred on 3mHz.

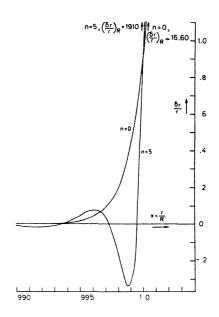


Fig. 1. Two p-modes for $\ell=800$ and n=0 (P=364 s) and n=5 (P=208.6 s) (courtesy of Dr. Gabriel)

Moreover, at the time of Leighton's work, based on a series of simultaneous observations at a great many point of the solar surface, these same p-modes were the object of another approach by Evans and Michard (1962) who concentrated instead on high dispersion spectra of a small region of the solar disk putting the emphasis on the vertical behaviour of the perturbation at a given point by studying different lines formed at different depths. An early interesting analysis of their data by Mein (1966) using a power spectrum technique yielded a diagnostic diagram (ν , k) very similar to those obtained in 1968 by Frazier and which were to culminate a few years later in Deubner's work (1975).

The point of view adopted and the methods used in a number of investigations reported here by Deubner and Endler, by Staiger, by Gouttebroze, Dame and Malherbe, by Poletto and Bruner are basically closer to those of Evans and Michard although their results at times relate more especially to the chromosphere and the transition region above and encompass some g-modes as well as the high degree p-modes. This approach allows, sometimes, to evaluate the net energy transferred by these waves through the transition region. In particular, the quantitative analysis of Poletto and Bruner shows that this is much too small to heat the corona. Campos thinks that this must be insured by the electric dissipation of Alfvén waves and he presents a corresponding model.

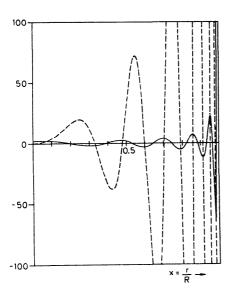
The role of these oscillations in the response of various atmospheric structures to inhomogeneities, activities or large scale motions at the solar surface are discussed by Leibacher, Gurman and Gouttebroze and by Schmieder, Mein, Malherbe and Ribes.

Apart from these high degree p-modes which can only be exhibited by high spatial resolution on the solar disk, global or quasi-global observations of the sun have also revealed (Brooks et al., 1976; Severny et al., 1976; Claverie et al., 1979; Grec et al., 1980) various types of oscillations over quite a range of periods from the still mysterious one with the 160 minute period to another important and dense group of oscillations of short periods around five minutes. Of course, the latter with their short periods can only be p-modes but they must correspond to very low ℓ values (0 to at most 3 to 4, very long $\lambda_h \approx R_0$) to avoid the cancelling of their effects when observed over the whole disk. On the other hand these low degree p-modes must have a great many nodes along the radius (order n between ≈ 10 and 30) to have these short periods.

The observational evidence for these modes, the very delicate techniques of resonance scattering for some selected spectral line used to detect the corresponding very small velocity fields (a few cm s $^{-1}$ to 1 m s $^{-1}$) and the extraordinary precise results (despite the numerous corrections to be applied) deduced from a power spectrum analysis are reviewed in the invited papers by Isaak and by Fossat. They both put some emphasis too on the interest of extending this type of observations to stars. The advantages of observations from two nearly antipodal stations are underlined by van der Raay et al. while Kuhn discusses the merit of using spectroscopic CCD observations and the possible variations in the velocity power density from the North-South to the East-West directions or with L. Woodard and Hudson on the one hand and Froehlich on the other bring beautiful confirmation of the velocity results as well as complementary information on the basis of brightness data collected either by the ACRIM on the SMM satellite or by balloons. Alurkar et al. also report periodic changes in radiobrightness at three radio frequencies, with five periods in the interval 3 to 13 minutes and Schmidt-Kaler and Winkler using a simple diffusion telescope at La Silla also find periodicities in the same range at various optical wave-lengths.

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As can be seen on Fig. 2, the behaviour of these low degree, high order p-modes is very different from that of the high degree ones (Fig. 1), the relative amplitudes $\delta r/r$ and $\delta p/p$ remaining ones (Fig. 1), the relative amplitudes $\delta r/r$ and $\delta p/p$ remaining fairly large down to great depths.



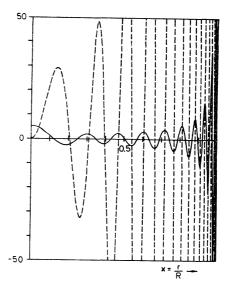


Fig. 2. The full lines represent $\delta r/r$ and the dashed lines, $\delta p/p$; left : $\ell = 2$, p_{10} , $\nu = 1666$, 86 μz , P = 600 s right : $\ell = 2$, P_{20} , $\nu = 3015$, 51 μz , P = 332 s (courtesy of Dr. R. Scuflaire).

This explains the interest of theorists who may expect to draw more information on the solar interior from these modes (cf. Rhodes et al., Shibahashi et al., Gabriel et al., these proceedings). Furthermore, these low ℓ modes according to the first order asymptotic approximation

$$v_{n,\ell} = (n + \frac{1}{2}\ell + \frac{n_e}{2} + \frac{1}{4})v_o, v_o = \{2\int_0^R \frac{dr}{\sqrt{\gamma\rho/\rho}}\}^{-1}$$
 (1)

have frequencies for a given ℓ and adjacent order very closely equidistant ($\Delta v = v_0 \approx 136 \, \mu Hz$), the odd ℓ modes falling in between the even ones at distance $v_0/2 \approx 68 \, \mu Hz$. On the other hand, an increase $\Delta \ell$ in the degree of the mode can be compensated by a decrease $\Delta n = \Delta \ell/2$ in the order n. This means that if modes can only be perceived up to a limited low &, say 4, the power spectrum will tend to exhibit only a finite number of peaks as indeed observed. This property lends itself to very accurate and critical comparison tests between theory and observation.

Let us note that Rhodes et al. have also taken into account the modes of intermediate degrees recently discovered by Duvall and Harvey (1983) and which exhibit a behaviour intermediate be-Let us note that Rhodes et al. have also taken into account and Harvey (1983) and which exhibit a behaviour intermediate between those illustrated on Figs. 1 and 2. Thus in the future, the whole range of p-modes of all degrees from $\ell = 0$ to $\ell = 1000$ should impose fairly strict constraints on solar models.

But the highlights of the conference, I believe, was the new information on g+modes. Apart from the acoustic p-modes driven essentially by the pressure, there exists another class of oscillating motions, called g^+ -modes, driven essentially by the buoyancy characterized by the Brunt-Väisälä frequency N^2 -Ag where A is the argument of Schwarzschild criterion for convection (A>0) and g the local gravity. A surface mode referred to as the f-mode separates the p and g⁺ spectra and, observationally, is more likely to be associated with the p spectrum.

Asymptotically these frequencies are given for p-modes by (1) and for g-mode, provided N^2 is positive everywhere, by

$$(v_{n,\ell})_{g^{+}} = \frac{\sqrt{\ell(\ell+1)} \cdot {}^{f}R(N/r)dr}{\pi^{2}(2n + \ell + n_{e} + \frac{1}{2})}$$
 (2)

where n is the effective polytropic index at the surface. These asymptotic formulae valid for n>l, show that for a given l the v's tend to infinity with n by discrete values separated by parated by

$$\Delta v_{p} = \frac{1}{2 \int_{0}^{R} \frac{d\mathbf{r}}{\sqrt{\gamma p/\rho}}}$$
 (3)

while the g^{+} s tend to zero with, in the limit,

$$\Delta v_{g} = -\frac{\sqrt{\ell(\ell+1)} \cdot \int^{R} (N/r) dr}{2 \cdot n \cdot (n+1) \cdot \pi^{2}}$$
(4)

For a given order n, both ν and ν , increase with ℓ but the ν_+ 's less rapidly than the ν_+ 's. The g^+g_m odes oscillate only in convectively stable regions $(A<0, N^2>0)$ and tend to become evanescent in a convectively unstable region $(A>0, N^2<0)$ even if $|N^2|$ is extremely small.

When a convection zone separates two stable radiative regions, the g spectrum appears, in the first order approximation and excluding resonances (M. Tassoul 1980; Ledoux and Perdang, 1980), to divide itself into two, one corresponding to the stable interior 0 \ r \ \ r \ , with

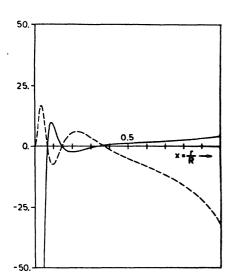
$$(v_{n,\ell})_{g^{+}} = \frac{\sqrt{\ell(\ell+1)} \cdot \int_{0}^{r_{c}} (N/r) dr}{\pi^{2}(2 \cdot n + \ell - 1/2)}$$
 (5)

84 MmSAI..55..357L and the other, to the outer zone with

$$(v_{n,\ell})_{g^+} = \frac{\sqrt{\ell(\ell+1)} r_c^*(N/r) dr}{\pi^2 (2 n + n_c)}$$
 (6)

There might be some ambiguity on the order n of the mode since in this approximation nodes are only counted in the re→ gion on which the integrals are taken while in the actual solution one node might fall outside. However the total number of nodes in the approximate and the exact solution may still remain the same.

Thus, in the sun, we expect to find two classes of g -modes, one associated with the radiative interior below the hydrogen convection zone and the other with the upper photosphere. The behaviour of the internal g-modes is illustrated on Fig.3.



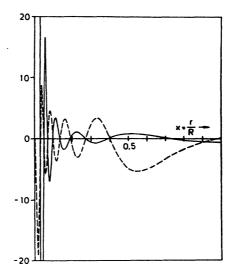


Fig. 3. The full lines represent $\delta r/r$ and the dashed line, $\delta p/p$; left : $\ell = 2$, g_3 , $\nu = 221.88 \,\mu Hz$, $P = 75.12 \, \text{min}$; right : $\ell = 2$, g_9 , $\nu = 110.58 \,\mu Hz$, P = 150.72 min. (courtesy of Dr. R. Scuflaire).

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We notice immediately that $\delta r/r$ always increases inwards but for the low order mode $(n=3)(\delta p/p)$ is still large in the external layers while for the higher order mode (n=9) both $(\delta r/r)$ and $(\delta p/p)$ are small close to the surface with respect to their values in the interior. This difference depends really on the surface boundary condition in which cancellation of terms may be more or less efficient according to the order n. This may have some incidence on the vibrational stability of these different modes.

On the basis of the decreasing amplitude $\delta r/r$ from the interior to the surface, the prospect of finding these modes by photospheric observations seemed rather poor. Nevertheless, g^+ -modes have been advocated on occasion especially for the long 160 min. period and for a few others, a little shorter, announced by Hill. The great difficulty here is the extreme selectivity in excitation which seems very difficult to explain. But more recently, Grec et al. discussing the low-frequency part of their power spectrum (periods 30 to 300 minutes) alluded to the possibility of explaining its spiked structure in terms of other g^+ -modes.

By cleaning this part of the spectrum as much as possible using the best available data (Birmingham, Stanford, ACRIM) and without underestimating the subsisting difficulties related to the crowding of lines, their splitting and the effects of side bands, Delache makes a good case in favour of the presence of g-modes using, as a theoretical guide line, expression (5) which in terms of periods

$$(P_{n,\ell})_{g^+} = (v_{n,\ell})^{-1}$$
, becomes
 $(P_{n,\ell})_{g^+} / \ell(\ell+1) = 2\pi^2 (n + \ell - \frac{1}{4}) P_0$

with

$$P_{\bullet} = \{ \{ r_{c}(N/r)dr \}^{-1} \}$$

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Data of the ACRIM radiometer analyzed by Froelich and Delache, despite the difficulties in identification confirm the previous results and the first value of P. and may perhaps constitute the strongest evidence for the existence of these g+-modes.

A theoretical discussion by Berthomieu et al. of low degree g-modes shows that, as expected from the general behaviour of these modes (cf. Fig. 3), a turbulent diffusive mixing in the central part of the sun can have rather large effects on the frequencies. As far as the mixing is concerned, Delache wondered if its origin could not be the g-modes themselves, the corresponding important differential velocity fields in the interior creating the necessary turbulence, an alternative to the differential rotation often invoked for instance, in these proceedings, by Roxburgh.

Severny et al. also report the discovery in the 110-210 minutes range of periodical motions many of which can be made to coincide with theoretical gomodes of low degrees but rather large order. They also draw attention to the possible appearance of combinatory frequencies due to non-linear coupling between gomodes with close frequencies.

A similar problem is addressed by Däppen and Perdang mainly, up to now, for radial modes whose frequencies become rapidly equidistant by a $\Delta \nu$ given by (3) thus also valid for all p-modes. The period corresponding to that $\Delta \nu_{\underline{\ }}$ is fairly long and the non-linear carried to fairly high order modes often reveal, calculations for simple models, a peak in the theoretical power spectrum in the range of the 160 min. oscillation. However many other suggestions as to the origin of this mode have been made : close passage of another star (Kosovichev and Severny), gravitational radiation from an appropriate binary star (Isaak). Recalling the commensurability of this period with the rotation periods of the planets, Kotov stated that this can be extended to the asteroids and that even earthquakes seems to occur at more or less constant phase with respect to that period. Among the effects of external factors, f Van der Raay et al. describe the intricate effects of an actual collision with a comet on the 5 min. low degree p-modes.

Unfortunately, Dr. Henry Hill could not be present at the meeting but the type of observations at the solar limb that he started are still going on as reported by Stebbins and Yerle. On the other hand, Gough and Latour discuss theoretically the possibility of desentangling the various modes in such observations and identify the modes most likely to be found.

Van der Raay et al. claim coherence on several years of a velocity signal of 13 day period and about 5 m s amplitude. This raised considerable discussions especially since Anderson, in his paper, argues that this could all be due to the influence of active regions on line-of-sight velocity measurements. But the study by Herrero et al. implies that such effects would change too rapidly during the solar cycle to be compatible with the stability of the data collected by the Birmingham group.

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Rotation can lift completely the azimuthal degeneracy (v_n) independent of m), splitting v_n into (2 ℓ +1) very close components of which only (ℓ +1) are detectable in integrated light on the whole solar disc. On the other hand, generally, a magnetic field will introduce only (ℓ +1) distinct components (cf. for instance, Perdang, 1968). It is perhaps worth recalling here that this effect was first applied to a class of variable stars, the ℓ Cephei stars.

In a general review of the problem including many personal contributions, Dziembowski arrives at the somewhat surprising conclusion that second order effects of rotation (centrifugal force and distortion) can be, in some cases, of the same order or larger than the linear effects which are related simply to the Coriolis force. The simultaneous presence of a magnetic field make things even more complex and can lead, if it is inclined on the rotation axis to a splitting into (l+1)(2l+1) distinct lines, (l+1) of which might be recognizable in integrated light. Let us note that Gough and Taylor studying the same problem were finding results departing less strongly from the linear case.

The application to the sun remains somewhat critical. When Claverie et al.(1981) discovered rotational splitting, they reported 3 components for l=1 and 5 for l=2 while, according to the above, only 2 and 3 respectively should have been present. Some kind of general assymetry with respect to the axis of rotation seemed necessary to explain the observation and Isaac (1982) proposed an intense internal non-aligned magnetic field. But then why didn't they detect more components as indicated above?

On the basis of their results for the second order rotational effects, Goode and Dziembowski point out that previous results for p-modes suggesting strong increase of the angular velocity with depth leading to difficulties as far as the quadrupole moment is concerned would have to be reconsidered.

On the other hand the splitting of g^+ -modes reported by Delache, Scherrer and Froelich shows only the expected (l+1) components for pure rotation with a mean rotation period for the radiative interior around 9 days. A somewhat slower rotation than that arrived at by Roxburgh assuming the Sun to be marginally stable to local perturbations all through.

Krause et al. have analysed the proper motion of sunspots groups and their possible correlations with torsional oscillations (otherwise ignored in this conference) and the solar cycle via the dynamo action in the convective zone.

There remain difficult problems concerning the interaction of the oscillation with the non-grey radiation in the very external layers and with convection below the photosphere. Christensen-Dalsgaard and Frandsen discussing the first aspect conclude that, while eigenvalues would be little affected, eigensolutions could be altered to the point of influencing the diagnostics of the oscillations. A possible way of tackling the second aspect, developed by Poyet et al., leading unavoidably to intricacies and the need for large computers, was presented by Dolez.

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Finally, one should not forget the possible analogy with stars as examplified by Fossat's results for α -Centaury where he has detected a periodic velocity field of amplitude 100 ms and period 14 min. or by the discussion by Shibahashi of the rapid oscillations of Ap stars discovered by Kurtz.

With respect to the future, extrapolating the recent past, it seems to me that there are no doubts whatsoever that further progresses in observational techniques and in the analysis of results, the establishment of an appropriate worldwide network of stations devoted to our problem and including possibly a space satellite will yield an increasing amount of more and more precise informations in this fascinating field of solar or perhaps more generally of stellar oscillations. Perhaps there is a danger that theory might lack behind especially in these difficult inversion problems. Let us hope that the imagination and the intuition of the theorists, constrained by better and better observations, will find short cuts to a comprehensive solution among the wild and exciting ideas and suggestions that they will no doubts continue to propose, sometimes to throw a bridge to some other unsolved problem. For instance, the excess neutrino problem has been with us for quite a long time and perhaps there are solutions through rather conventional approaches. Nevertheless ever since I became interested in asymptotic modes, I have played with the idea that, perhaps, acoustic waves might transport a non negligible amount of energy from the central part to the surface reducing thus the necessary temperature gradient and yielding a cooler core, perhaps enriched in H or He but emitting less neutrinos. Other people and H. Hill in particular have entertained similar ideas. Now that we know that the sun is the seat of so many modes of oscillations does it still look so impractical or far-fetched?

For instance, if the internal g⁺-modes are definitely there is it not likely that, with their large amplitudes in the central part, they are excited and maintained through nuclear processes? Could they not couple then either directly or through induced turbulence with acoustic modes, of the 5 minute class perhaps, feeding some of their excitation energy to the latter which could propagate without much dissipation up to the very external region where they would get damped, dissipating, their mechanical energy into heat? The total energy transfer could be appreciable even if each p-wave carries very little since the transit time for acoustic waves across the sun is much smaller than that of a photon.

What about conclusions? What can be said but that we have been witnessing another example in which the curiosity, the ingeniosity, the obstinacy of a few men have opened a new field of fruitful investigations on a subject which notso long ago might have appeared as somewhat trite to some of us. And this has already yielded information beyond the wildest dreams of a decade ago and, as it has become particularly apparent during this conference, we are in presence of an accelerating process which,

With the help of new or refined observational techniques, large computers and hard thinking and in combination with the solving computers and hard thinking and in combination with the solving of the neutrinos problem is going to lead to tremendous advances in the knowledge of the Sun and of its interior including probably the rotation law and the distribution of the magnetic field. This will provide in turn a safer basis for all our stellar work.

What a fascinating thought that perhaps, if appropriate means are provided, some of us might really come to know how the Sun, our own star, is made up! Isn't this really worth a special space mission?

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