Introduction to Environmental Modelling

Face-It Summer School 2019 : Marine Biogeochemical Cycling: from measurements to modelling

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What is a model ?

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A simplified representation of a complex phenomenon.

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What are models used for?

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A simplified representation of a complex phenomenon.

What are models used for?

- Understand Observations:
  - Confront Theory to Observations, ie. check hypotheses
  - Unified framework
- Complete Observations :
  - Upscale Observations
  - Quantify processes difficult to measure
- Assess Scenarios
  - Management
  - Predict the Future (or attempt to)
  - Reconstruct the Past

What is a model ?

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A simplified representation of a complex phenomenon.

What are models used for?

Understand-Quantify / Complete / Predict / Assess

What is a model ?

How simple should a model be ?

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How simple should a model be ?

"As simple as possible, but not simpler" [A. Einstein]

What is a model ?

How simple should a model be ?

"As simple as possible, but not simpler" [A. Einstein]

Arguments in favor of:

Simplicity

- Computation Time
- Facility of Analysis, description
- Occam's Razor
- Lack of knowledge / Observations

Complexity

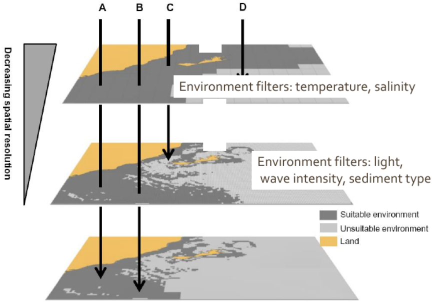
- Realism
- Accuracy
- Inner 'local' mechanisms support system 'global' properties

Type of models

Type of models

Statistical Models

- Basis : Observations & Statistics
- Example: Species Distribution modelling
- Hypothesis : environmental conditions act as the first filter to determine species distribution.
- Expressed as : Calibrated relationships.
- Use : Predict species distribution in unsampled sites
- Limitation : Extrapolation outside of obs. range ?



Species Distribution models (SDM), Hattab et al., 2014

Type of models

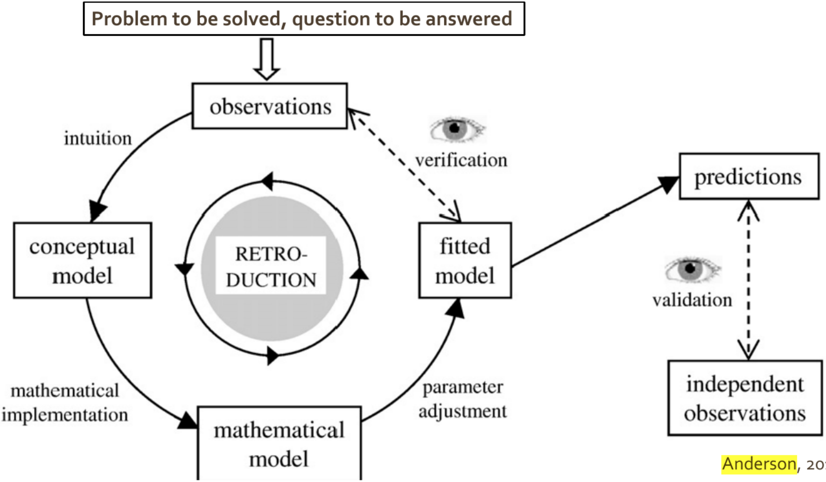
Mechanistic Models

- Basis : Knowledge on Processes
- Example: Meteo, Ocean circulation, growth, etc ..
- Hypothesis : Mechanisms and interactions does not changes, and rules the evolution of the system.
- Expressed as : (Often) Set of differential equations
- Use : Understand, Forecast, Scenario.
- Limitation : Demanding, needs loads of simplification, assumptions ..

Building a model

1. **Basics Concepts**
   - What is a model ?
   - Type of models
   - Building a model
2. **Conceptual model**
   - Research Questions
   - Scales
   - State Variables
   - Processes & Flows
3. **Mathematical model formulation**
   - State Variables
   - Processes & Rates
   - Processes & Rates
4. **Practical Works**
   - Thursday

Building a model



Anderson, 2010

Research Questions

Research Questions

Research Questions

- Clear formulation of the research question should lead decisions for all elements of the model

Research Questions

Research Context

Our study mainly focuses on

- increased production of organic matter (faeces and pseudofaeces)
- food depletion by the growth of biofouling
- impacts on biogeochemical processes via respiration and excretion.

Research Questions

Research Questions

- How deep can the blue mussels grow under mixed/stratified conditions,
- Will there be local depletion of food resources such as phytoplankton, zooplankton and detritus ?
- Will mussels on seabed have the same effect as mussel on the structure ?
- Does type of turbine and distance between them impacts on the accumulation of mussel biomass and on ecosystem and biogeochemical dynamics ?

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Scales

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Spatial Scales

- Relevant scales for system dynamics ?

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- Relevant scale for operating processes ?

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- Length scale of spatial resolution for available observations ?

Scales

Spatial Scales

- Relevant scales for system dynamics ?
- Relevant scale for operating processes ?
- Non-linearities ?
- Anisotropy? In forcings ? in processes ?
- Length scale of spatial resolution for available observations ?
- Memory !

Scales

Spatial Scales

- 1-dimension
- 25 m pylone
- 50 layers of 0.5 m each

Scales

Spatial Scales

- 1-dimension
- 25 m pylone
- 50 layers of 0.5 m each
- Horizontal length scales: characterized with parameters

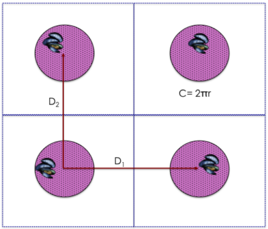

Figure 2.6 Influence of the mussels growing on monopiles

Scales

Temporal Scales

- Relevant scales for system dynamics

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- Periodicity in forcings ?

Scales

Temporal Scales

- Relevant scales for system dynamics
- Relevant scale for operating processes
- Non-linearities
- Periodicity in forcings ?
- CPU !

Scales

Temporal Scales

- Seasonal Temperature Cycle

Scales

Temporal Scales

- Seasonal Temperature Cycle
- Typical rates: Day $\rightarrow$ Weeks

Scales

Temporal Scales

- Seasonal Temperature Cycle
- Typical rates: Day $\rightarrow$ Weeks

$\rightarrow$ Simulations of a few years, timestep of 1 day.

State Variables

State Variables

State variables

- State variables define the state of our simplified system, at any time.
- Those are the descriptors for which we have to provide 'Rules of evolution', in the form of differential equation.
- Usually, those rules are derived from mass conservation principles

$\rightarrow$ State Variables needs to be expressed in a common conservative currency.

State Variables

State variables

3 Components:

- Physics
- Biogeochemistry
- Mussels

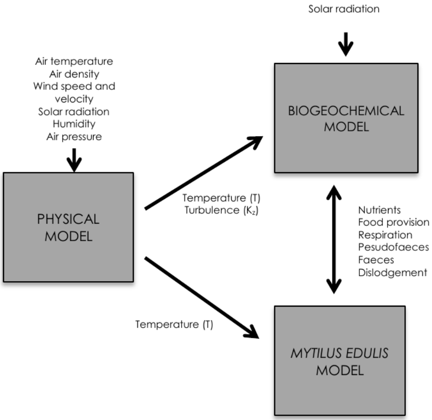

Figure 2.3 Description of the three submodels and their interactions
Source: Adaption from Meire et al. (2013)

State Variables

State variables

3 Components:

- **Physics**
  - No feed backs from others
  - → Can remains external
- Biogeochemistry
- Mussels

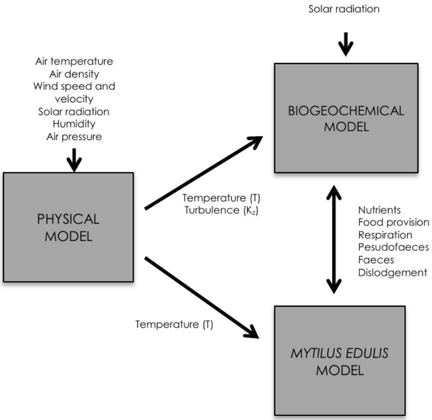

Figure 2.3 Description of the three submodels and their interactions

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State Variables

State variables

3 Components:

- Physics
- **Biogeochemistry**
  - NPZD Approach
  - Only N limits growth.

  → Currency: $[\mathrm{mmolN\,m^{-3}}]$
- Mussels

State Variables

State variables

3 Components:

- Physics
- **Biogeochemistry**
  - NPZD Approach
  - Only N limits growth.
  - → Currency: $[\mathrm{mmolN\,m^{-3}}]$
- Mussels

- ammonium: $\mathrm{NH4}$
- nitrate: $\mathrm{NO3}$
- phytoplankton: PHYTO
- zooplankton: ZOO
- detr.: $\mathrm{PELDETRITUS}$
- bot. detr.: $\mathrm{BOTDETRITUS}$

State Variables

State variables

3 Components:

- Physics
- **Biogeochemistry**
  - NPZD Approach
  - Only N limits growth.

  → Currency: $[\mathrm{mmolN\,m^{-3}}]$
  - ⋆ Inorganic
    - ⋆ Living Organic
    - ⋆ Dead Organic
- Mussels

- ammonium: $NH4$
- nitrate: $NO3$
- phytoplankton: $PHYTO$
- zooplankton: ZOO
- detr.: $PELDETRITUS$
- bot. detr.: $BOTDETRITUS$

State Variables

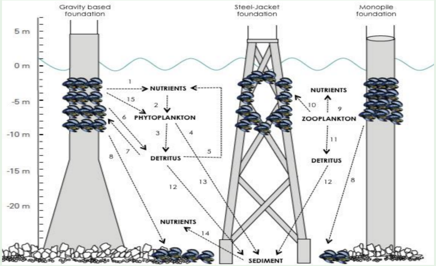
State variables

3 Components:

- Physics
- Biogeochemistry
- **Mussels**
  - ! Different domains !

→ Need to convert Biomass on pylons

→ $[\mathrm{ind\,m^{-2}}] \rightarrow [\mathrm{mmolN\,m^{-3}}]$



Processes & Flows

Processes & Flows

Mass Balance Equation

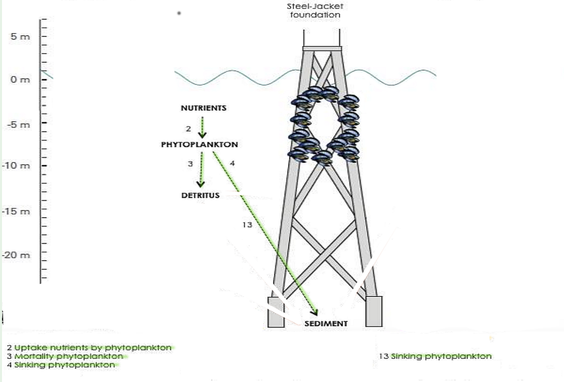
- Connect Flows among state variables
- Identify controls on those flows

Processes & Flows

Flows

Phytoplankton

- Uptake Nutrients for Growth
  - NH3 and NO3
  - NH3 first
  - Light limitation (depth)
- Sink
- Die

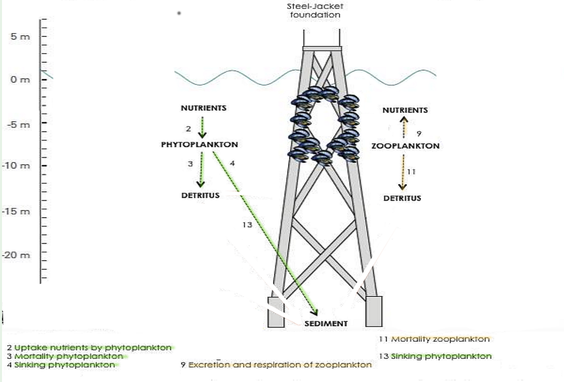


Processes & Flows

Flows

Zooplankton

- Graze on Phytoplankton
- Sink
- Egest nutrient
- Die

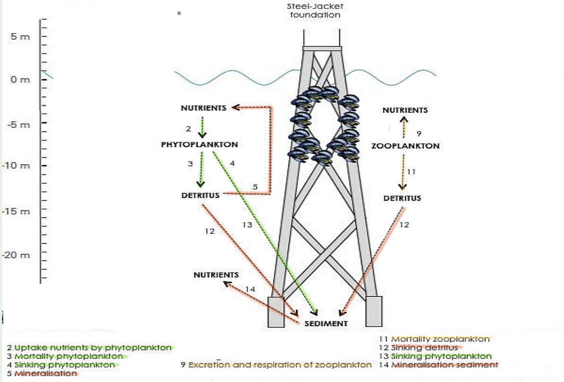


Processes & Flows

Flows

Detritus

- Decay
- Sink

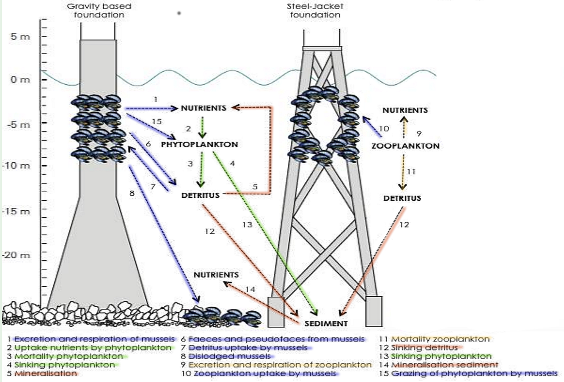


Processes & Flows

Flows

Mussels

- Excrete and respire
- Produce Faeces and pseudofaeces
- Graze on PHY, ZOO, DETRITUS
- Fall
- Die



Processes & Flows

Physical Transport

- All pelagic variable are transported by diffusion (vertically mixed).
- PHY and DETRITUS have additional vertical sinking

Processes & Flows

External Controls on Processes

- Temperature affect growth and decay rates.
- Turbulent diffusion coefficient controls vertical diffusion.
- Light availability limits planktonic growth.

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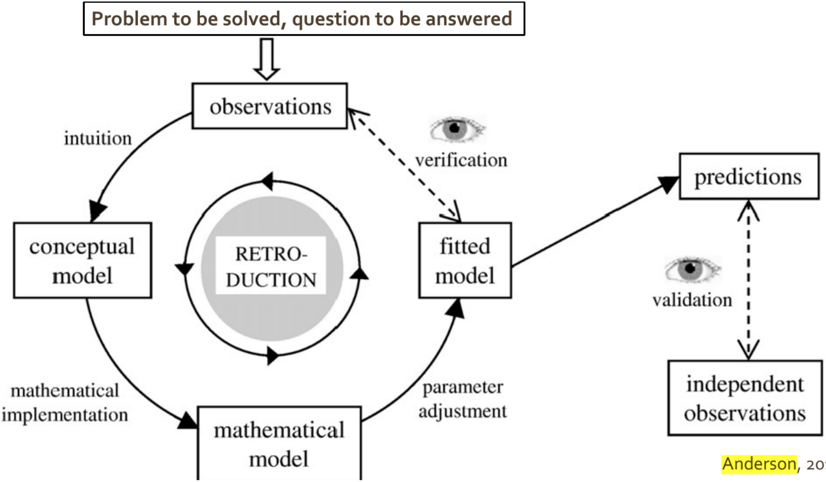

Anderson, 2010

State Variables

State Variables

State Vector

- Spatial domain is divide in N_{Cell} cells.
- Sytem defined by N_{Var} State Variables.
- The state of the system at time t, $C(t)$, can be stored numerically as a vector of size $N_{Cell}.N_{Var}$.

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State Vector

- NO3, NH4, PHY, ZOO, DET, and BIVALVE are defined along the vertical grid.
- PELDETRITUS is defined only at the bottom.
- Our state vector contains 6x50+1=301 element.

State Variables

Mussel Counts

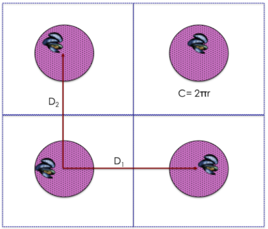

Figure 2.6 Influence of the mussels growing on monopiles

- Monopiles distant of D_1 and D_2, and of radius r.
- For a given layer (dz)
  - Surface of monopile section : $2.\pi.r.dz$
  - Volume of water : $D_1.D_2.dz$
- $100\ \mathrm{mmolN\,m^{-2}}$ mussels on monopile $\rightarrow 100x\left(\frac{2\pi r}{D_1.D_2}\right)\ \mathrm{mmolN\,m^{-3}}$ in water layer.

Processes & Rates

Processes & Rates

Equation of evolution for the State Vector

- $C(t)$ is the state vector at a given time t.

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- $C(t)$ is the state vector at a given time t.
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- $C(t + \Delta t) = C(t) + \frac{\partial C}{\partial t}\Delta t$

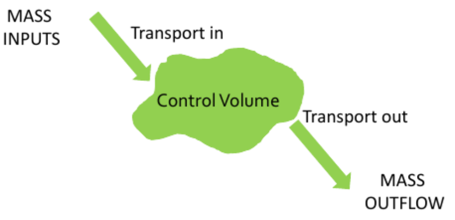
Processes & Rates

Equation of evolution for the State Vector

- $C(t)$ is the state vector at a given time t.
- $\frac{\partial C}{\partial t}$ is the temporal rate of change of the state vector.
- $C(t+\Delta t) = C(t) + \frac{\partial C}{\partial t}\Delta t$
- The equation of evolution for $C(t)$ has the form $\frac{\partial C}{dt} = f(C, t)$

Processes & Rates

Transport & Reaction

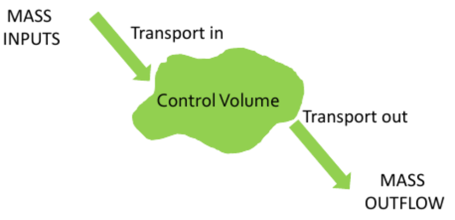


Transport

$\frac{dC}{dt}$ in the control volume $=$ *Mass inflow* $-$ *Mass outflow*

Processes & Rates

Transport & Reaction

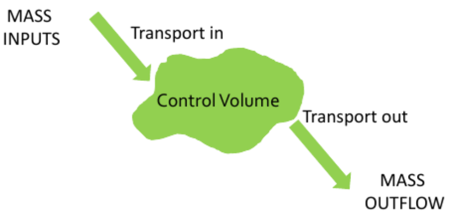


Transport & Reaction

$\frac{dC}{dt}$ in the control volume $= \textit{Mass inflow} - \textit{Mass outflow} \pm \textit{Reactions}$

Processes & Rates

Transport & Reaction



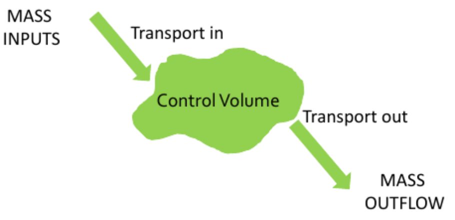
Transport

$\frac{dC}{dt}$ in the control volume $=$ *Mass inflow* $-$ *Mass outflow*

- Today, we won't deal with transport terms, only reaction rates

Processes & Rates

Transport & Reaction



Transport

$\frac{dC}{dt}$ in the control volume = *Mass inflow* − *Mass outflow*

- We express Reaction rates through Mass Balance Equations

Processes & Rates

Color Code

Type	Unit	Example
State Variable	$[\mathrm{mmolN\,m^{-3}}]$	*PHY*
Processe	$[\mathrm{mmol\,N\,m^{-3}\,d^{-1}}]$	$Mortality_{PHY}$
Parameter	diff.	*sinkingRatePhyt*,$[\mathrm{m\,d^{-1}}]$
Work Variable	diff., mostly unitless	$f(T)$, [-]
Forcing	diff.	T

Processes & Rates

Mass Balance Equation for PHY

$$\frac{\partial\, PHY}{\partial t} = Diffusion_{PHY} + Sinking_{PHY}$$
$$+NPP - Grazing_{by\ ZOO} - Grazing_{by\ BIVAL} - Mortality_{PHY}$$

Processes & Rates

Mass Balance Equation for PHY

$$\frac{\partial\, PHY}{\partial t} = \underbrace{Diffusion_{PHY} + Sinking_{PHY}}_{\text{Transport}} + NPP - Grazing_{by\ ZOO} - Grazing_{by\ BIVAL} - Mortality_{PHY}$$

Processes & Rates

Mass Balance Equation for PHY

$$\frac{\partial\, PHY}{\partial t} = Diffusion_{PHY} + Sinking_{PHY}$$
$$+\underline{\mathbf{NPP}} - Grazing_{by\ ZOO} - Grazing_{by\ BIVAL} - Mortality_{PHY}$$

NPP

$$NPP = maxUptake.PHY.min(f(I), f(DIN)).f(T)$$

$maxUptake$	Maximum Uptake of Dissolved Inorganic Nitrogen	d^{-1}
$f(I)$	Light limitation	[-]
$f(DIN)$	DIN limitation	[-]
$f(T)$	Temp. effect on growth	[-]

Processes & Rates

NPP

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Light limitation

$$f(I) = tanh\left(\frac{I(z,t)}{I_{opt}}\right)$$

$I(z,t)$	Light	$\mathrm{W\,m^{-2}}$
I_{opt}	Optimum light intensity	$\mathrm{W\,m^{-2}}$

Processes & Rates

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Nutrient Limitation

$$f(DIN) = \frac{NO3}{NO3 + ksNO3}.e^{-\psi.NH3} + \frac{NH3}{NH3 + ksNH3}$$

$ksNO3$	Half-saturation coefficient for NO3 uptake	$[\mathrm{mmolN\,m^{-3}}]$
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ψ	Inhibition coefficient for NH4	$[\mathrm{mmolN^{-1}\,m^{3}}]$

Processes & Rates

NPP

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$f(T)$	Temp. effect on growth	[-]

Temperature effect on Growth

$$f(T) = Q_{10}^{\left(\frac{T - T_{ref}}{10}\right)}$$

Q_{10}	Temperature coefficient	[-]
T_{ref}	Reference temperature	[C]

Processes & Rates

Mass Balance Equation for PHY

$$\frac{\partial PHY}{\partial t} = Diffusion_{PHY} + Sinking_{PHY}$$
$$+NPP - \underline{\mathbf{Grazing_{by\ ZOO}}} - Grazing_{by\ BIVAL} - Mortality_{PHY}$$

$Grazing_{by\ ZOO}$

$$Grazing_{by\ ZOO} = maxGrazing . \frac{PHY}{PHY + ksPHY} . ZOO . f(T)$$

maxGrazing	Maximum grazing rate by zooplankton	d^{-1}
ksPHY	Half-saturation for zoo grazing on phyto	$[\mathrm{mmolN\,m^{-3}}]$

Processes & Rates

Mass Balance Equation for PHY

$$\frac{\partial\, PHY}{\partial t} = Diffusion_{PHY} + Sinking_{PHY} + NPP - Grazing_{by\ ZOO} - \underline{\mathbf{Grazing_{by\ BIVAL}}} - Mortality_{PHY}$$

$Grazing_{by\ BIVAL}$

$$Grazing_{by\ BIVAL} = maxClear.BIVAL.PHY.\left(1 - \frac{BIVAL}{maxB}\right).f(T)$$

maxClear	Clearance rate of the mussels	$[\mathrm{mmolN\,m^{-3}\,d^{-1}}$
maxB	Carrying capacity	$[\mathrm{mmolN\,m^{-3}}]$

Processes & Rates

Mass Balance Equation for PHY

$$\frac{\partial PHY}{\partial t} = Diffusion_{PHY} + Sinking_{PHY} + NPP - Grazing_{by\ ZOO} - Grazing_{by\ BIVAL} - \underline{\mathbf{Mortality_{PHY}}}$$

$Mortality_{PHY}$

$$Mortality_{PHY} = mortalityRatePhyt.PHY.f(T)$$

mortalityRatePhyt Phyto mortality rate $[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Equation for NO3

$$\frac{\partial \, NO3}{\partial t} = Diffusion_{NO3} - (1 - \alpha).NPP + Nitrification$$

α Inhibition of NO3 uptake by the presence of NH3 [-]

Processes & Rates

Mass Balance Equation for NO3

$$\frac{\partial\, NO3}{\partial t} = Diffusion_{NO3} - (1 - \underline{\alpha}).NPP + Nitrification$$

α Inhibition of NO3 uptake by the presence of NH3 [-]

α

$$\alpha = \left(\frac{1}{f(DIN)}\right) \cdot \left(\frac{NH3}{NH3 + ksNH3}\right)$$

Processes & Rates

Mass Balance Equation for NO3

$$\frac{\partial NO3}{\partial t} = Diffusion_{NO3} - (1 - \alpha).NPP + \underline{\textbf{Nitrification}}$$

α Inhibition of NO3 uptake by the presence of NH3 [-]

Nitrification

$$Nitrification = NitR.NH3.f(T)$$

NitR Nitrification Rate $[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Equation for NH3

$$\frac{\partial\, NH3}{\partial t} = Diffusion_{NH3}$$
$$+Excretion_{ZOO} + Excretion_{BIVAL}$$
$$+Respiration_{ZOO} + Respiration_{BIVAL}$$
$$-Nitrification - (\alpha).NPP$$
$$+Mineral_{PELDETRITUS} + Mineral_{BOTDETRITUS}$$

Processes & Rates

Mass Balance Equation for NH3

$$\frac{\partial NH3}{\partial t} = Diffusion_{NH3} + \underline{\mathbf{Excretion_{ZOO}}} + Excretion_{BIVAL} + Respiration_{ZOO} + Respiration_{BIVAL} - Nitrification - (\alpha).NPP + Mineral_{PELDETRITUS} + Mineral_{BOTDETRITUS}$$

$Excretion_{ZOO}$

$$Excretion_{ZOO} = excretionRate_{ZOO}.ZOO.f(T)$$

$excretionRate_{ZOO}$	Excretion rate of zooplankton	$[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Equation for NH3

$$\frac{\partial\, NH3}{\partial t} = Diffusion_{NH3} + Excretion_{ZOO} + \underline{\mathbf{Excretion_{BIVAL}}} + Respiration_{ZOO} + Respiration_{BIVAL} - Nitrification - (\alpha).NPP + Mineral_{PELDETRITUS} + Mineral_{BOTDETRITUS}$$

$Excretion_{BIVAL}$

$$Excretion_{BIVAL} = excretionRate_{BIVAL}.BIVAL.f(T)$$

$excretionRate_{BIVAL}$ Excretion rate of zooplankton $[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Equation for NH3

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$Respiration_{ZOO}$

$$Respiration_{ZOO} = RespirationRate_{ZOO}.ZOO.f(T)$$

$RespirationRate_{ZOO}$ Respiration rate of zooplankton $[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Equation for NH3

$$\frac{\partial NH3}{\partial t} = Diffusion_{NH3}$$
$$+Excretion_{ZOO} + Excretion_{BIVAL}$$
$$+Respiration_{ZOO} + \underline{\mathbf{Respiration_{BIVAL}}}$$
$$-Nitrification - (\alpha).NPP$$
$$+Mineral_{PELDETRITUS} + Mineral_{BOTDETRITUS}$$

$Respiration_{BIVAL}$

$$Respiration_{BIVAL} = RespirationRate_{BIVAL}.BIVAL.f(T)$$

$RespirationRate_{BIVAL}$	Respiration rate of Mussels	$[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Equation for NH3

$$\frac{\partial NH3}{\partial t} = Diffusion_{NH3}$$
$$+Excretion_{ZOO} + Excretion_{BIVAL}$$
$$+Respiration_{ZOO} + Respiration_{BIVAL}$$
$$-Nitrification - (\alpha).NPP$$
$$+\underline{\mathbf{Mineral_{PELDETRITUS}}} + Mineral_{BOTDETRITUS}$$

$Mineral_{PELDETRITUS}$

$$Mineral_{PELDETRITUS} = mineralRatePel.PELDETRITUS.f(T)$$

mineralRatePel Mineralisation Rate for Pel. Detr. $[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Equation for ZOO

$$\frac{\partial\, ZOO}{\partial t} = Diffusion_{ZOO} + ZooGrowth - Excretion_{ZOO}$$
$$- Respiration_{ZOO} - Mortality_{ZOO} - Grazing_{ZOObyBIVAL}$$

Processes & Rates

Mass Balance Equation for ZOO

$$\frac{\partial\, ZOO}{\partial t} = Diffusion_{ZOO} + \underline{\textbf{ZooGrowth}} - Excretion_{ZOO}$$
$$- Respiration_{ZOO} - Mortality_{ZOO} - Grazing_{ZOObyBIVAL}$$

ZooGrowth

$$ZooGrowth = (1 - Faeces_{ZOO}).Grazing_{by\ ZOO}$$

$Faeces_{ZOO}$	Fraction of zooplankton faeces	[-]

Processes & Rates

Mass Balance Equation for ZOO

$$\frac{\partial ZOO}{\partial t} = Diffusion_{ZOO} + ZooGrowth - Excretion_{ZOO} - Respiration_{ZOO} - \underline{\mathbf{Mortality_{ZOO}}} - Grazing_{ZOObyBIVAL}$$

$Mortality_{ZOO}$

$$Mortality_{ZOO} = mortalityRateZoo.ZOO^2.f(T)$$

mortalityRateZoo	Mortality rate of zooplankton	$[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Equation for ZOO

$$\frac{\partial\, ZOO}{\partial t} = Diffusion_{ZOO} + ZooGrowth - Excretion_{ZOO}$$
$$- Respiration_{ZOO} - Mortality_{ZOO} - \underline{\mathbf{Grazing_{ZOObyBIVAL}}}$$

$Grazing_{ZOObyBIVAL}$

$$Grazing_{ZOObyBIVAL} = maxClear.BIVAL.ZOO.\left(1 - \frac{BIVAL}{maxB}\right).f(T)$$

Processes & Rates

Mass Balance Equation for BIVAL

$$\frac{\partial\, BIVAL}{\partial t} = GrazingBival + GrazingBivalZOO + GrazingBival_{DET}$$
$$- FaecesBP - FaecesBD - FaecesBZ$$
$$- ExcretionBIVAL - FallingBivalve - RespirationBIVAL$$
$$- PseudofaecesP - PseudofaecesZ - -PseudofaecesD$$

Processes & Rates

Mass Balance Equation for BIVAL

$$\frac{\partial\, BIVAL}{\partial t} = GrazingBival_{PHY} + GrazingBival_{ZOO} + \underline{\mathbf{GrazingBival_{DET}}}$$
$$- FaecesBP - FaecesBD - FaecesBZ$$
$$- ExcretionBIVAL - FallingBivalve - RespirationBIVAL$$
$$- PseudofaecesP - PseudofaecesZ - -PseudofaecesD$$

$GrazingBival_{DET}$

$$GrazingBival_{DET} = maxClear.BIVAL.PELDETRITUS.\left(1 - \frac{BIVAL}{maxB}\right).f(T)$$

Processes & Rates

Mass Balance Equation for BIVAL

$$\frac{\partial\, BIVAL}{\partial t} = GrazingBival_{PHY} + GrazingBival_{ZOO} + GrazingBival_{DET}$$
$$-\underline{\mathbf{FaecesBP}} - FaecesBD - FaecesBZ$$
$$-ExcretionBIVAL - FallingBivalve - RespirationBIVAL$$
$$-PseudofaecesP - PseudofaecesZ - -PseudofaecesD$$

FaecesBP

$$FaecesBP = pFaecesBP.GrazingBival_{PHY}$$

pFaecesBP Production faeces by consuming phytoplankton [-]

Processes & Rates

Mass Balance Equation for BIVAL

$$\frac{\partial\, BIVAL}{\partial t} = (1 - pFaecesBP - pPseudoBP).GrazingBival_{PHY} \\ +(1 - pFaecesBZ - pPseudoBZ).GrazingBival_{ZOO} \\ +(1 - pFaecesBD - pPseudoBD).GrazingBival_{DET} \\ -Excretion_{BIVAL} - FallingBivalve - Respiration_{BIVAL}$$

Processes & Rates

Mass Balance Equation for BIVAL

$$\frac{\partial\, BIVAL}{\partial t} = (1 - pFaecesBP - pPseudoBP).GrazingBival_{PHY}$$
$$+(1 - pFaecesBZ - pPseudoBZ).GrazingBival_{ZOO}$$
$$+(1 - pFaecesBD - pPseudoBD).GrazingBival_{DET}$$
$$-Excretion_{BIVAL} - Respiration_{BIVAL} - FallingBivalve$$

Processes & Rates

Mass Balance Equation for BIVAL

$$\frac{\partial\, BIVAL}{\partial t} = (1 - pFaecesBP - pPseudoBP).GrazingBival_{PHY} + (1 - pFaecesBZ - pPseudoBZ).GrazingBival_{ZOO} + (1 - pFaecesBD - pPseudoBD).GrazingBival_{DET} - Excretion_{BIVAL} - Respiration_{BIVAL} - FallingBivalve$$

FallingBivalve

$$FallingBivalve = pFall.BIVAL$$

pFall Falling Rate $[\mathrm{d}^{-1}]$

Processes & Rates

Mass Balance Eq. for PELDETRITUS

$$\frac{\partial\, PELDETRITUS}{\partial t} = Diffusion_{PELDETRITUS} + Sinking_{PELDETRITUS}$$
$$+ FaecesZ + FaecesBP + FaecesBD + FaecesBZ$$
$$+ PseudofaecesP + PseudofaecesZ + PseudofaecesD$$
$$+ Mortality_{PHY} + Mortality_{ZOO}$$
$$- Mineral_{PELDETRITUS} - GrazingBival_{DET}$$

Processes & Rates

Mass Balance Eq. for BOTDETRITUS

$$\frac{\partial BOTDETRITUS}{\partial t} = sinkingRate_{PHY}.PHY|_{Bottom}$$
$$+sinkingRate_{PELDETRITUS}.PELDETRITUS|_{z=Bottom}$$
$$+\sum_{i=1}^{N}\left[FallingBivalve|_{z=Bottom}\right]$$
$$-MineralisationBot$$

Processes & Rates

Processes & Rates

Equation of evolution for the State Vector

- $C(t)$ is the state vector at a given time t.
- $\frac{\partial C}{\partial t}$ is the temporal rate of change of the state vector.
- $C(t + \Delta t) = C(t) + \frac{\partial C}{\partial t} \Delta t$
- The equation of evolution for $C(t)$ has the form $\frac{\partial C}{dt} = f(C, t)$
- It remains to
  - Assign initial conditions to variables : $C(t = O)$
  - Use the formulations of $\frac{\partial C}{\partial t}$ to compute the next time steps ...

Thursday

Thursday

- Run the model
- Plot model outputs
- Play with parameters
- Extract and store model outputs for further use

Thursday

That's all Folks !