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Optimization of mechatronic systems: application to a modern car equipped with a semi-active suspension

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1. Abstract

The research aims at developing a global mechatronic approach to model, simulate and optimize complex industrial applications. The approach is illustrated with the simulation and the optimization of a modern car (an Audi A6) equipped with a controlled semi-active suspension. An optimization procedure is used to find the best sub-system parameters in order to improve the comfort of the passengers while preserving the car ride and handling performances. Two different modeling and optimization approaches are used and compared. The first one is realized in the MATLAB-SIMULINK environment and is based on a symbolic multibody model of the chassis while the hydraulic actuators, and the controller are integrated using S-functions. Optimization is also carried out in MATLAB using algorithms available in MATLAB libraries, especially a genetic algorithm (GA). On the other hand, the second approach relies on a multibody model based on the Finite Element method whereas the optimization can be realized with an industrial open optimization tool.

2. Keywords: Multidisciplinary optimisation, Mechatronics, Multibody systems, Control, Vehicle.

3. Introduction

In the last decade, machines have turned from purely mechanical systems to complex mechatronic systems, which integrate mechanical and electrical components, electronic devices, control systems and software tools. This enhancement of the mechanical functions allows achieving better motion and vibration control. Moreover, the mechatronic approach is very modular by nature and fits new business challenges. Plenty of models are based on a small number of platforms and can be developed with excellent quality and reliability. This explains that mechatronics concept is so successful and finds numerous applications in robotics, machine tools, and transportation systems.

As mock-ups are expensive and time-consuming, *virtual prototyping and simulation tools* are very attractive to design mechatronic products [1]. However mechatronic machines are quite complex systems because of the strong couplings between its components. Therefore, to predict accurately the system performances, it is necessary to take into the mutual interactions of the components in a *multiphysics* and *multidisciplinary* approach. One has to address the *integrated simulation* of the full mechatronic system instead of considering each single part solely.

The design of the control system is generally a major issue of the mechatronic machine since its design has to be carried out at the system level. This problem has been addressed in numerous works, but still continues to be an active research topic, mainly because of the intrinsic non-linear character of some components (e.g. multibody systems) and because of the difficulty to find robust and efficient controllers for the problem.

Then, another important challenge is to find quickly and efficiently the right component parameters that achieve the best (or at least improved) system performances. The major difficulty of mechatronic problems stems from the intricate interactions and couplings of the components, which make difficult to understand intuitively the influence of parameter modifications. In order to achieve this task efficiently, mathematical optimization techniques are natural tools to provide a rationale methodology to solve these complex design problems. As showed by Van Brussel et al. [2], a novel research direction tries is to use an *optimization upper-layer* over integrated simulation of the controlled mechatronic systems to determine the best design parameters. Its major difficulties come from the nonlinear character of the components,

Figure 1: Audi A6 (left) and the same vehicle on 4 poster rig for testing and its instrumentation with corner accelerometers (right).

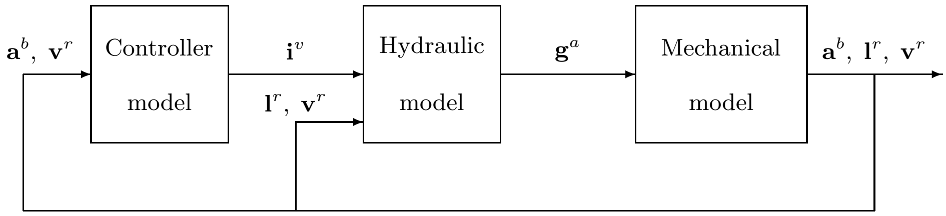

Figure 2: Mechatronic model of the car equipped with a semi-active suspension. $\mathbf{a}^b$ is the vector of car-body accelerations, $\mathbf{l}^r$ is the vector of rattle extensions, $\mathbf{v}^r = \dot{\mathbf{l}}^r$ is the vector of rattle velocities, $\mathbf{i}^v$ is the vector of electrical currents, and $\mathbf{g}^a$ is the vector of damper forces.

from the multiphysics / coupled character of the problem that is used in mechatronic problems. The main goal of the research is to emphasize a *global mechatronic approach* to simultaneously model, simulate and optimize complex industrial applications. The problem remains still quite unexplored. The problem being very general, our research is driven by applications. Therefore this paper presents one application that is used as a guideline for the developments: The modeling, the integrated simulation and the optimization of a semi-active suspension system of a car.

Comfort and road handling performances of a passenger car are mainly determined by the damping characteristic of the shock absorbers. Moving from passive shock absorbers to active or semi-active suspension systems can much improve the comfort and the handling and make possible to adapt it to road conditions. The selection of appropriate control parameters against mechanical design variables to meet often-conflicting restrictions is a complex problem, which can be solved efficiently using optimization.

In this study two different approaches are used and compared to realize the modeling and the optimization. At first a multibody system (MBS) modeling approach based on a symbolic tool is used. The behavior models of the MBS, the hydraulic actuators, the sensors and the controller are integrated in the MATLAB-SIMULINK environment using S-functions. Optimization is also carried out in the same environment. Because the evaluation procedure is fast, a genetic algorithm (GA) can be used. On another hand, the second approach relies on a multibody simulation tool based on the Finite Element method (FEM) approach (here SAMCEF-MECANO) whereas the optimization is realized with SAMCEF-BOSS QUATTRO, an open optimization tool.

The paper is organized as follows. In section 4, the modeling of the mechatronics system, a car equipped with semi-active suspensions is presented. The different component models are described: The vehicle chassis, the hydraulic systems of the semi active shock absorbers. The integrated simulation of the vehicle is also described. Section 4 gives some insights into the nonlinear controller. Section 6 stands the optimization problem formulation while the numerical results are presented in section 7. Finally the conclusions are given in section 8.

4. Mechatronic system modeling

The system under consideration is an Audi A6 car (see Fig. 1) equipped with four semi-active hydraulic dampers, whose force-velocity characteristics can be modified and controlled via an electro-valve. As semi-active hydraulic actuators with current controlled valves are considered, the suspension

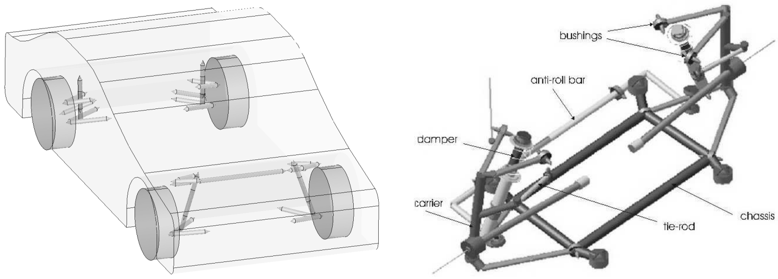

Figure 3: Multibody model of the car chassis (left) and the multi-mink rear suspension

system is moved from a purely mechanical system to a mechatronic design. Fig. 1 shows also a test-vehicle on a test-rig with 4 hydraulic shakers which are capable of independently exciting the 4 car-wheels with a desired road profile.

As mentioned in the introduction, a particular attention is paid to the formulation of the 'mechatronic' model of the car, which contains (see Fig. 2):

- a mechanical sub-model: the vehicle (chassis-suspensions-wheels) for which a multibody approach is used.
- an electro-hydraulic sub-model: to describe the behavior of the semi-active shock absorbers, oil pressures and flows in the damper chambers are computed taking into account oil compressibility, active and passive valves characteristics and gas accumulator behavior.
- a control sub-model: the latter have been tailored to satisfy some comfort and/or ride criteria (input: car body acceleration and damper velocity - output: electro-valve current).

This model, whatever the formalisms and tools which produced it, will be in the heart of the optimization cost function and thus must be efficient, while taking the main dynamical properties of the car suspension into account.

4.1. Vehicle model

The chassis and the suspension system of the Audi A6 are modeled as a multibody system (see Fig. 3, left). It includes the car-body and chassis, rear and front multi-link suspension mechanisms (including the passive springs) (see Fig. 3, right), the slider-crank direction mechanism for the front wheels and the wheel model.

From the multibody point of view, this 3D model is rather complex since it involves: around 50 revolute or prismatic joints; 18 closed-loops; 4 wheel-ground contact models, involving a vertical compliant force and a lateral slip model. The current model involves about 600 mechanical dof, and it could be extended to include the stiffness of the suspension bushings, the flexibility of the chassis, and coupled longitudinal-lateral model for the wheels. However, in the framework of this simplified study, the current model is sufficient.

The multibody system modeling has been realized following two approaches. The first one is a *symbolic method* that allows building equations of motion in symbolic format, whereas the second one is a numerical approach based on the *Finite Element method* (FEM). The symbolic approach has been carried out with the ROBOTRAN software [3] and allows building equations of motion in alphanumeric format (e.g. C-code). The symbolic format has the advantages of portability and efficiency. The finite element approach [4] produces the equation of motion as complex numerical procedures. It is realized here in the commercial SAMCEF-MECANO package. However numerical procedures are able to deal with more general class of problems, and they are especially suitable to model the dynamics of a flexible mechanism with complex topology in a systematic way.

In both approaches (symbolic or FEM based), the generation of the equations of motion lead to a

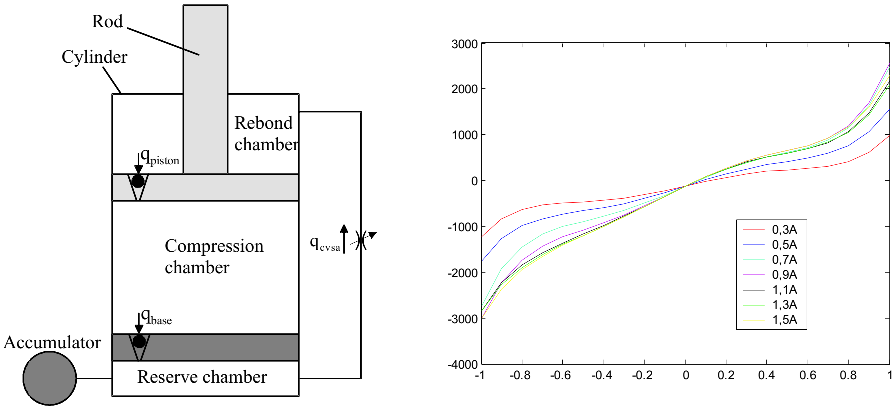

Figure 4: Semi-active damper (right) and (left) fitted curves of the shock absorber damping force as a function of rattle velocity for different CVSA valve current. A low/high current to the CVSA valve corresponds to a small/large restriction yielding a low/high damping ratio.

differential-algebraic system of the form:

$$M(q,t)\ddot{q} + g(\dot{q}, q, t, F_{ext}) \quad = \quad J^T(q,t)\,\lambda \tag{1}$$

$$h(q,t) \quad = \quad 0 \tag{2}$$

in which M denotes the system mass matrix, q are the generalized – absolute or relative – coordinates, g is the 'dynamical' vector containing centrifugal, Coriolis, gyroscopic terms as well as the contributive forces and torques (in particular those arising from the shock absorbers), h is the set of algebraic constraints, J is the associated Jacobian matrix and λ are the Lagrange multipliers.

4.2. The semi-active shock absorbers

The semi-active shock absorber hardware (see figure 4 left) corresponds to that of a passive shock absorber in which the piston and base valve are each replaced by a check valve. A current controlled CVSA valve (continually variable semi-active valve) connects the two damper chambers. The current to this valve is limited between $i^- = 0.3A$ and $i^+ = 1.6A$, which corresponds to the least and most restrictive positions of the valve (i.e. open and closed), respectively. When the rod moves up (positive rattle velocity), the piston check-valve closes and oil flows through the CVSA valve. Because the volume of the rod inside the cylinder (rebond valves) reduces, oil is forced from the accumulator into the cylinder through the base check-valve. When the rod moves down (negative rattle velocity), the piston check-valve opens. Because the volume of the rod inside the cylinder increases, the base check-valve closes and oil flows from the cylinder into the accumulator through the CVSA valve.

The dynamic behavior is quite complex, and a detailed nonlinear model [5], available as C-functions, has been calibrated by the manufacturer of the shock absorber, Tenneco Automotive Company (Saint-Trond, Belgium). This model can be presented in nonlinear state-space format, defining the inputs $\mathbf{u}^{(damp)} = [l^r \quad \dot{l}^r \quad i^v]^T$ (l^r, the rattle extension, i^v, the electrical current in the CVSA valve), the state variables $\mathbf{x}^{(damp)} = [p^{reb} \quad p^{comp}]^T$ (p^{reb} and p^{comp} the pressures in the rebond and compression chambers) and the output $\mathbf{y}^{(damp)} = [g^a]^T$ (g^a the force exerted by the damper):

$$\begin{aligned} \dot{\mathbf{x}}^{(damp)} &= \mathbf{f}^{s(damp)}(\mathbf{u}^{(damp)}, \mathbf{x}^{(damp)}) \\ \mathbf{y}^{(damp)} &= \mathbf{f}^{o(damp)}(\mathbf{u}^{(damp)}, \mathbf{x}^{(damp)}) \end{aligned} \tag{3}$$

However, when coupled with the multibody dynamics model (1-2), the model (3) assuming oil compressibility results in a set of a stiff differential-algebraic system. Therefore, a fitted set of curves (see Fig. 4, right) has been realized on the basis of experimental data. The fitted relationships provides quasi-static damper force with respect to the CVSA electrical current, and the rattle velocity:

$$g^a = g^a(i^v, v^r) \tag{4}$$

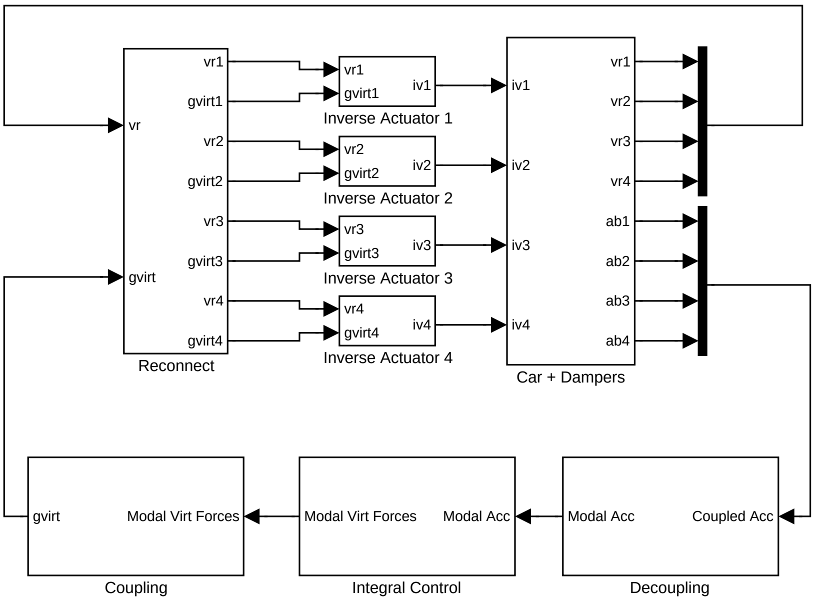

Figure 5: Controller of the semi-active suspension. "vr" stands for the rattle velocities, "ab" for the car-body accelerations, and "iv" for the valve currents.

They can provide a satisfactory model of the shock absorber in the design perspective.

4.3. Integrated simulation

As explained in [6], several approaches are available to carry out the integrated simulation of the mechatronic system. Here depending on the choice that is performed for the multibody system dynamics, two approaches are used. As regards the symbolic approach, since the latter enables to generate a stand-alone dynamical model (in C language in the present case), the full model is exported towards the SIMULINK environment via S-functions, ready-to-use for fast-running simulations, implementing a controller. Conversely to the symbolic approach, when using the FEM approach for the MBS modeling, the integrated simulation of the mechatronic consists in importing the hydraulic and the controller models (created as C-functions) into the MBS simulation package. This procedure is presently used in MECANO. However, a novel procedure developed by Bruls [7] sounds more interesting: it is a extended formalism of the FEM to integrate block diagram description, so that the time-integration of the system equations is realized in a fully coupled and rigorous manner within a unified environment.

5. Controller

A flexible model-free control structure has been tailored by Lauwerys et al. [8] based on physical insights in the car and semi-active suspension dynamics. Its parameters are physically interpretable and will be chosen according to guidelines given by test pilots or here by simulation and optimization methods. As illustrated in Fig. 5, the controller structure consists of three stages: a feedback linearization (inverse actuator models), a transformation into modal space (coupling and decoupling operations), and a linear integral control.

The car and the dampers are represented as a black box, whose inputs are the four CVSA electrical currents i^v, and whose outputs are the rattle velocities $v^r = \dot{l}^r$, and the accelerations measured at the four corners of the car a^b (see Fig. 1).

The feedback linearization technique seeks for virtual inputs which have the property to influence the outputs in a linear way. If one accepts that the nonlinearity of the mechanism is weak, the main source of nonlinearity lies in the actuator. According to Lauwerys *et al.* [8], an efficient feedback linearization law is obtained by inversion of a simplified quasi-static model of the actuators (4). For $v^r \neq 0$ it comes:

$$i^v = (g^a)^{-1}\,(g^{virt}, v^r) \tag{5}$$

where g^{virt} is the new virtual input, which can be interpreted as a virtual damper force. For $v^r = 0$, the controllability is defective, and the singularity of $(g^a)^{-1}$ is avoided thanks to a regularization strategy. If this inverse model cancels the non-linearity of the actuator, the virtual input g^{virt} is actually proportional to the damper force. Therefore, besides the advantage of a good linearity between g^{virt} and the output, a force control strategy can be established on this basis.

Since the motion of the car simultaneously involves the forces applied on the four wheels, the definition of the virtual control forces g^{virt} for the four shock absorbers is a linear but coupled multi-input/multi-output problem. This problem can be simplified by a transformation into a modal space defined by the heave (pumping), roll and pitch of the car-body. In this modal space, the system is made of 3 uncoupled single-input/single-output subsystems, for which 3 independent integral controllers are designed.

The several stages of the controller are easily described using the block diagram language in MATLAB-SIMULINK. The inverse actuator model is implemented as a specific element, which directly invokes the C-function implemented in the actual controller. The SIMULINK code of the controller can be used directly in the MATLAB simulation with the symbolic model of the car. It can also be compiled in C-language and imported in MECANO as controller elements or alternatively it can be programmed as a user-routine.

6. Optimization

In order to provide the vehicle with improved ride quality and handling performance under various operating conditions, one would like to minimize or bound the following suspension characteristics [9]:

- the root-mean-square (RMS) value of the sprung mass acceleration, which is a direct measure of the discomfort and which can be used in comfort criteria;
- the RMS value of the suspension travel, which is bounded because of the finite space available under the car-body and the chassis;
- the RMS value of the tire-ground reaction or alternatively the dynamic tire deflection, which is an indirect measure of the contact force between the tire and the ground.

Generally (see for instance [10],[11]) the comfort is chosen as the most important criterion while the two other ones are controlled via design restrictions. However, for sport cars or off-road vehicles, the opposite choice can be to maximize the road handling with an acceptable level of comfort. Both design problems will be investigated in the following.

In order to achieve the optimization of suspension systems, two different kinds of optimization algorithms are often cited in the literature: 1/ Mathematical programming algorithms with or without gradient as in Ref. [11] and 2/ heuristic algorithms as Genetic Algorithms (GA) which do not require derivatives as in Ref. [10]. Here, the optimization procedure developed in [12] for mechanisms has been extended to mechatronic systems. It consists in running first a GA algorithm and then in performing a number of fine optimization runs with a Nelder-Mead algorithm starting from a set of initial designs which have been identified from the best GA results. The Nelder-Mead algorithm can also be replaced by a gradient-based optimization algorithm, and usually CONLIN or MMA, which are efficient and reliable in multidisciplinary optimization.

If derivatives are necessary, they are evaluated using finite difference techniques. Indeed, up to now, semi-analytical procedures are not available for mechatronic systems involving multibody systems, hydraulic components and control systems. The finite difference procedure is rather stable and simple to implement even though quite expensive in computing time.

The most efficient optimization procedure is quite dependent of the integrated simulation strategy. On the one hand, when using the symbolic approach for the MBS system modeling, one comes to a full MATLAB-SIMULINK model and it is quite natural to resort to an optimization procedure in the

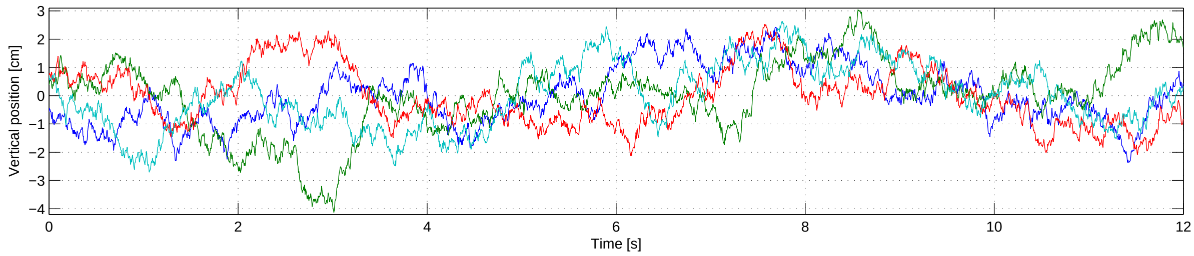

Figure 6: Stochastic road profile

MATLAB environment, taking benefit of the good MATLAB library. This advantage has been widely used here. On the other hand when a FEM based approach has been selected for the MBS modeling, the general character of the tool makes easier to couple the simulation tool to a general optimization package. Here the Boss Quattro package has been used to conduct the optimization because of its open character and its large library of optimisation algorithms. In our study, the two procedures have produced similar results, so that only the MATLAB approach will be presented in the following.

7. Numerical applications

7.1. Comfort optimization

The goal of this first optimization run is to improve by simulation the comfort of passengers of an Audi A6 car equipped with semi-active suspensions. Practically, the behavior of this car is simulated during 12 seconds on a given-roughness road profile, which is different for each wheel (see Fig. 6). This profile is made of a filtered stochastic white noise whose amplitude does not exceed 4.2 cm.

As explained in Sections 4 and 5, the complete model consists of the multibody dynamics of the car coupled with the hydraulic dynamics of the suspensions, which are controlled by feedback. The controller inputs are the vertical acceleration of the body corners and the velocity of the dampers and the outputs are the currents of the four electro-valves of the suspensions. Six parameters of the controller are tunable: the heave, pitch and roll integral gains, the cut frequency, a proportional pitch gain and the current bias. The optimization problem consists in finding their best values, with respect to comfort and handling criteria respectively.

The comfort of passengers is supposed to be in close relation with the vertical acceleration of the car body. The comfort indicator is thus the mean value of the RMS vertical acceleration measured at the 4 body corners. The objective function f_c to minimize is:

$$f_c\left(\mathbf{x}\right) = \frac{1}{4}\sum_{i=1}^{4}\frac{\sqrt{\int\limits_{2}^{12} a_i^2\left(\mathbf{x},t\right)\, dt}}{10} \tag{6}$$

where $\mathbf{x}$ is the vector of the six controller parameters and a_i is the vertical acceleration at the i^{th} body corner. Remark that this indicator is not observed from the beginning of the simulation to let the car behavior stabilize during two seconds.

To take care of the closed-loop mechanical systems, a penalty technique is used as suggested in [12]. The objective function is thus penalized every time the mechanical model cannot be assembled or when a damper length exceeds its limit. This penalty is computed proportionally to the discrepancy of the parameters from a chosen standard value. This method enables the optimizer to treat any design.

Because of the non-linearity and discontinuities of the objective function, a classical genetic algorithm is used. Thanks to this stochastic method, the entire design space is explored and the global optimum is approached. Then, a second optimization run is worked out with the deterministic method of Nelder-Mead to refine the solution. All of this is performed in the MATLAB-SIMULINK environment using the genetic MATLAB Toolbox. The Fig. 7 compares the simulations of the car behavior with the

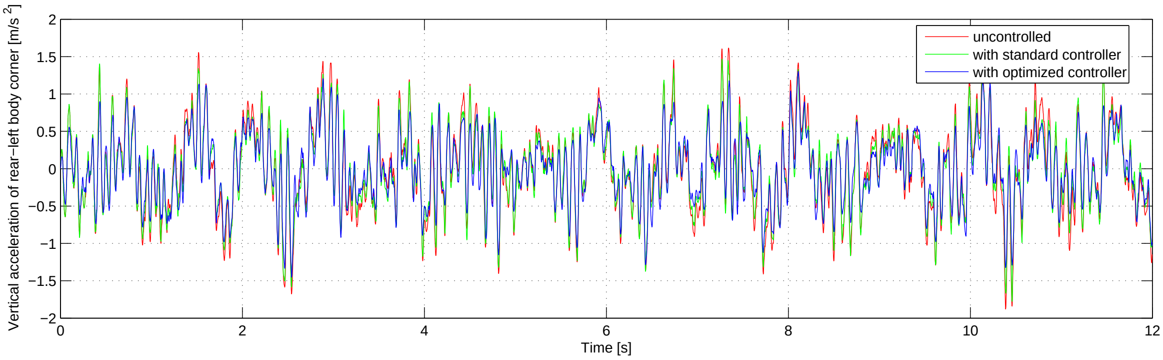

Figure 7: Comparison between car comfort performances with optimized controller, standard controller and without controller

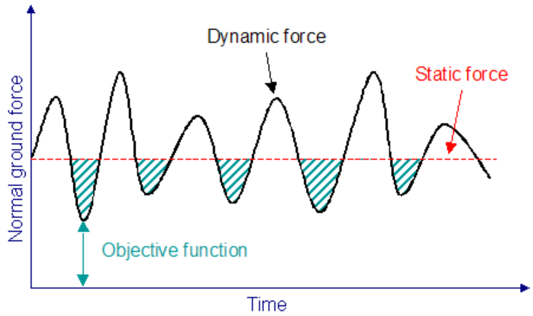

Figure 8: Example of static and dynamic contact forces

optimum controller, with a standard controller and without controller.

7.2. Handling optimization

The optimization procedure is the same as for the comfort optimization, but the objective function is now the handling performance of the car that can be evaluated on basis of the vertical ground reaction force on each wheel. This dynamic force is compared with the static force, which is the vertical reaction force on the wheel at equilibrium on a flat ground as illustrated on Fig. 8. When the wheel leaves the ground, the force vanishes to zero. Therefore, if the dynamic force is lower than the static force, the wheel tends to lose its adherence, which is bad. The objective function f_h to maximize is the minimum value of the ratio between static and dynamic ground forces at each wheel (see Figure 8):

$$f_h\left(\mathbf{x}\right) = \frac{1}{4}\sum_{i=1}^{4} \min_{2\leq t\leq 12} \frac{F_i^{dynamic}\left(\mathbf{x},t\right)}{F_i^{static}} \tag{7}$$

The results of the handling optimization are shown on Fig. 9. In this case, the optimizer produces a very small improvement. Further improvement should be possible if considering additional parameters as design variables, in particular those related to the mechanical and hydraulic sub-systems. In principle, the global modeling and simulating approach that is used allows considering easily any parameter in the optimization process, whatever its physical nature be: a body length, a spring stiffness, a piston area or a controller gain, etc. This is a very important feature of the 'mechatronic' approach in global system modeling that is promoted here.

Optimization results for both comfort and handling maximization are summarized in Table 1.

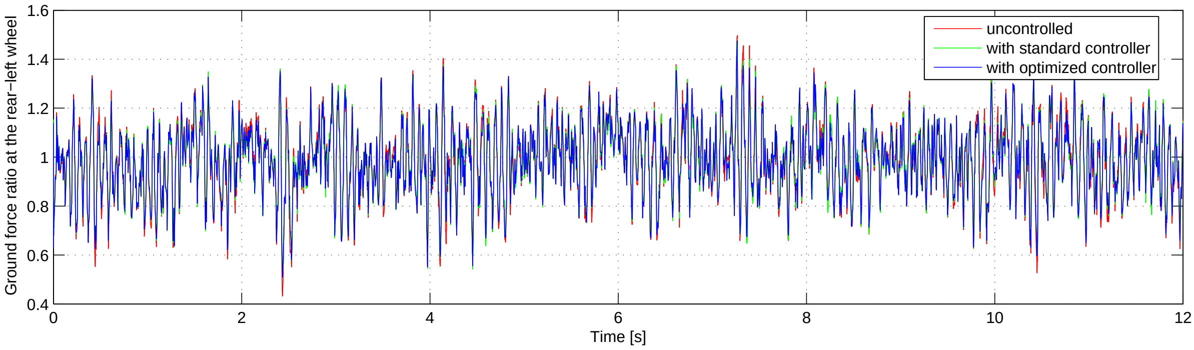

Figure 9: Comparison between car handling performances with optimized controller, standard controller and without controller

	Comfort RMS accelerations $[m/s^2]$	Handling Min Ground force ratio [%]
without controller	0.52	53.7
with standard controller	0.46	58.2
with optimized controller	0.41	59.0

Table 1: Numerical results of comfort and handling optimizations

8. Conclusions

The main goal of this research is to emphasize a *global mechatronic approach* to model, simulate and optimize complex mechatronic systems. The paper presents an application that has been selected to monitor the developments: an Audi A6 equipped with semi-active shock absorbers.

The modeling of the mechanical components has a major impact on the approach that is adopted later for the integrated simulation and the optimization. Among the computer modeling methods of complex multibody systems, *symbolic methods* allow building equations of motion in symbolic format, whereas *numerical methods* produce the equations of motion as complex numerical procedures.

The symbolic format has the advantages of portability and efficiency, and it provides mechanical models as very compact C-code routines, which are very fast and compact. Therefore it is rather efficient to choose a general environment as MATLAB-SIMULINK for the integrated simulation and the optimization. The sub-system models (MBS, hydraulic actuators, and controller) are integrated in the MATLAB-SIMULINK environment directly or using S-functions. Optimization is also carried out in the same environment taking advantage of the available optimization library. Especially the fast analysis procedure allows using Genetic Algorithms to explore the full design space to detect the global optimum. The genetic algorithm (GA) can also be used in combination of other algorithms like the Nelder-Mead algorithm to refine the optimal parameter knowledge.

Numerical procedures like the FEM-based description of MBS are able to deal with more general class of problems, and they are especially suitable to model the dynamics of a flexible mechanism with complex topology in a systematic way. The tool is more complex and the simulated integration is generally easier when importing the models of the other sub-systems into the MBS simulation tool. Here the MBS modeling and simulation have been realized in SAMCEF-MECANO. Because of the general character of the simulation tool, it is better to use an open multidisciplinary optimization environment like BOSS QUATTRO, to handle the optimization task. This last approach is rather general and one can take benefit of the large library of optimization strategies and algorithms (e.g. GCMMA, GA) available in BOSS QUATTRO. However the procedure is heavier and leads to more complicated and time consuming models.

Optimization of mechatronic systems is a quite complicated problem. Future work will be devoted to improve the actual optimization procedures. At first, including design variables of different natures and coming from different sub-systems will come to more important design improvements. Then computing time has to be reduced. For instance, sensitivity analysis, which is realized presently using finite differences, could be improved by taking advantage of semi-analytical approaches when available for some components (as MBS system). Finally multilevel approach, which is usual issue in the design of mechatronic system, could be investigated and introduced in the context of the global mechatronic optimization that is promoted here.

9. Acknowledgements

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