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GENERALIZED SHAPE OPTIMIZATION USING XFEM AND LEVEL SET METHODS

P. Duysinx, L. Van Miegroet, T. Jacobs and C. Fleury
Automotive Engineering / Multidisciplinary Optimization
Aerospace and Mechanics Department
University of Liège

OUTLINE

- Introduction
- eXtended Finite Element Method (XFEM)
- Level Set Method
- Problem Formulation
- Sensitivity Analysis
- Applications
  - Implementation
  - Plate with a hole
- Conclusion

INTRODUCTION

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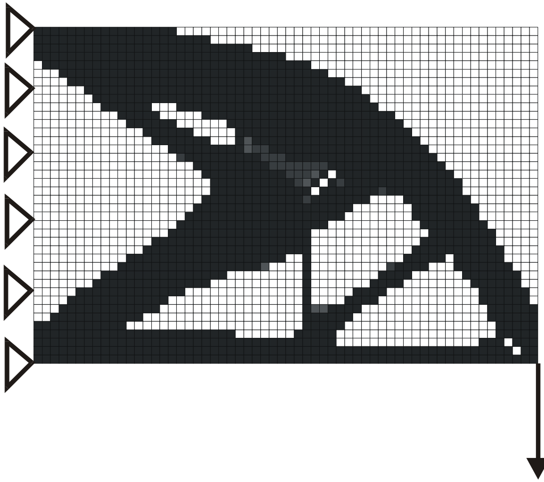
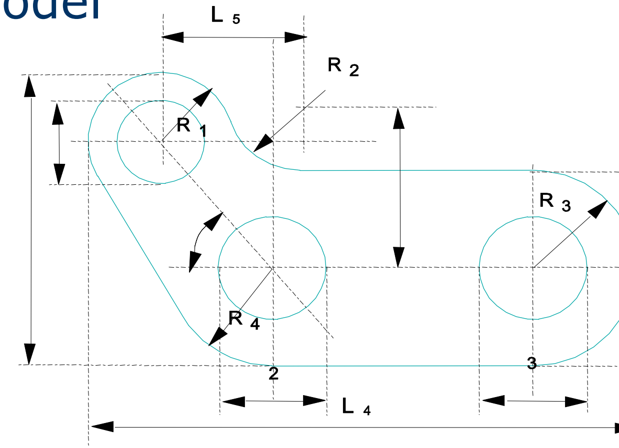
- TOPOLOGY OPTIMIZATION (Bendsoe & Kikuchi, 1988)
  - Optimal material distribution
  - Optimal topology without any a priori
  - Fixed mesh
  - Design variables
    - = Local density parameters
    - Many thousand design variables
  - Simple design problem:
    - Minimum compliance s.t. volume constraint
    - Local constraints are difficult to handle
    - Geometrical constraints (often manufacturing constraints) are difficult to define and to control
  - Preliminary design: interpretation phase necessary to come to a CAD model
  - Great industrial applications



INTRODUCTION

- SHAPE OPTIMIZATION (Braibant & Fleury, 1984)
  - Modification of boundaries of CAD model
  - Fixed topology a priori
  - Design variables
    - = CAD model parameters
    - Small number of design variables
  - Quite complicated design problems:
    - Large number of global and local constraints
    - Geometrical constraints easily included
  - Detailed design
  - Mesh management problems
    - Mesh modification / mesh distortion
    - Velocity field
  - Industrial applications are stepping



INTRODUCTION

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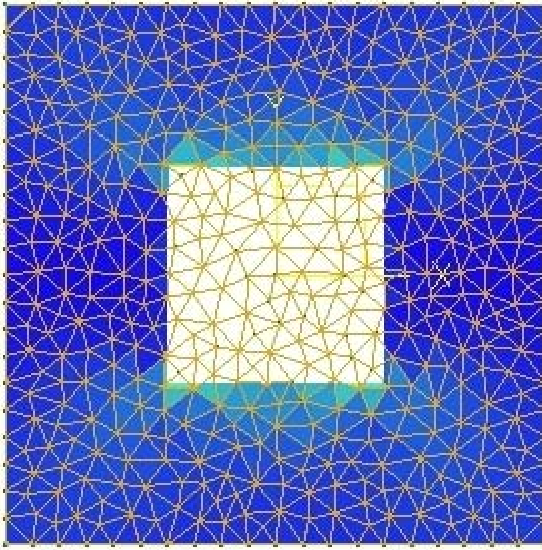
- EXTENDED FINITE ELEMENT METHOD (XFEM)
  - alternative to remeshing methods
- LEVEL SET METHOD
  - alternative description to parametric description of curves
- XFEM + LEVEL SET METHODS
  - Efficient treatment of problem involving discontinuities and propagations
  - Early applications to crack problems. Moes et al. (1999)
  - Applications to topology optimisation Belytschko et al. (2003), Wang et al. (2003), Allaire et al. (2004)

INTRODUCTION

- THIS WORK
  - XFEM + Level Set methods = alternative method to shape optimisation
  - Intermediate approach between shape and topology optimisation
  - XFEM
    - work on fixed mesh
    - no mesh problems
  - Level Set
    - smooth curve description
    - modification of topology is possible
  - Problem formulation:
    - global and local constraints
    - small number of design variables



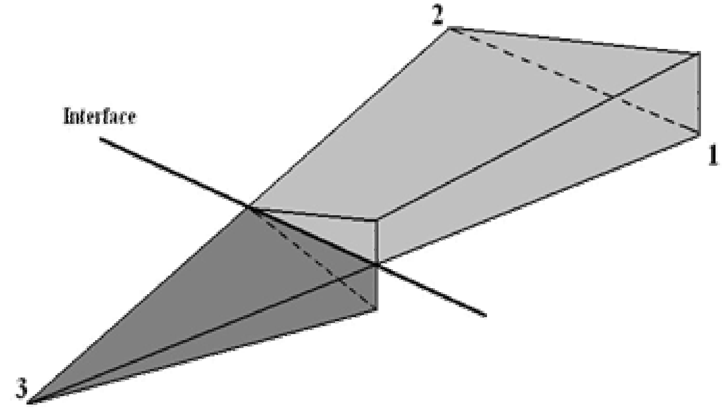
EXTENDED FINITE ELEMENT METHOD

- Early motivation : study of propagating crack in mechanical structures → avoid the remeshing procedure

- Principle :
  - Allow the model to handle discontinuities that are non conforming with the mesh
  - Add internal degree of freedom a_i
  - Add special shape functions $H(x)N_i(x)$ (discontinuous)

$$u = \sum_{i \in I} u_i N_i(x) + \sum_{i \in L} a_i H(x) N_i(x)$$

$$K \cdot q = g \Leftrightarrow \begin{bmatrix} K_{uu} & K_{ua} \\ K_{au} & K_{aa} \end{bmatrix} \begin{bmatrix} u \\ a \end{bmatrix} = \begin{bmatrix} f_u^{ext} \\ f_a^{ext} \end{bmatrix}$$



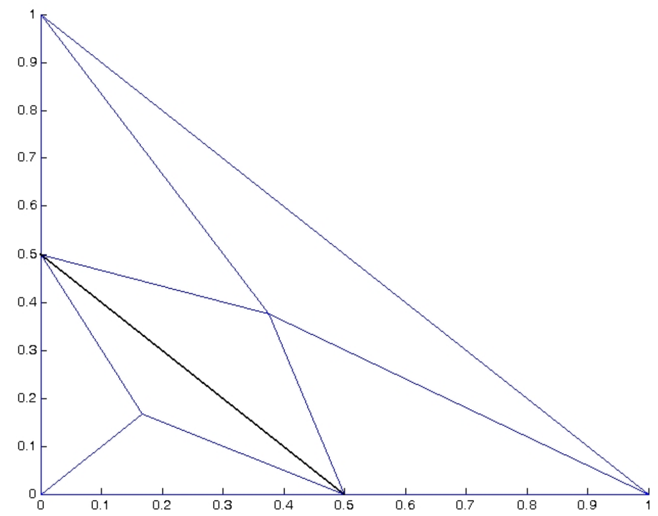
EXTENDED FINITE ELEMENT METHOD

- Representing holes or material – void interfaces
  - Remove empty elements
  - Keep partially filled elements
  - Use XFEM numerical integration

$$u = \sum_{i \in I} N_i(x)\, V(x)\, u_i \qquad V(x) = \begin{cases} 1 & \text{if node inside material} \\ 0 & \text{if node not in material} \end{cases}$$

- Numerical integration
  - Division into sub-triangles
  - Integration over sub-triangles

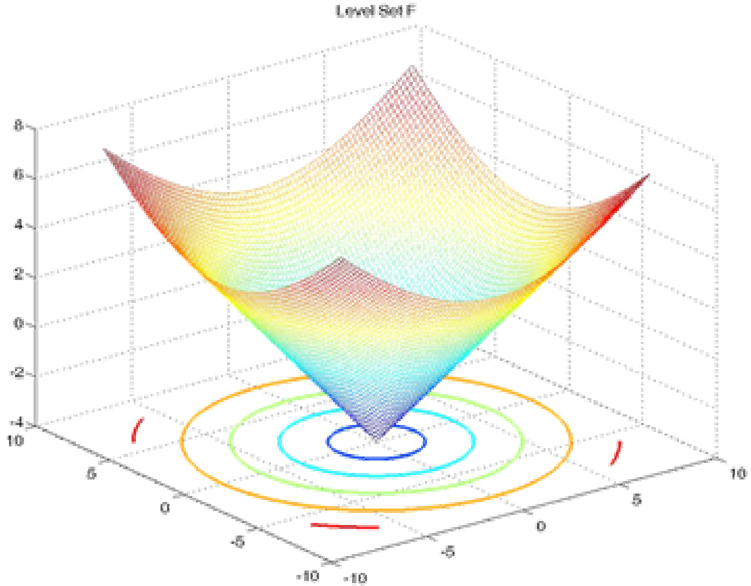
Gauss points



THE LEVEL SET METHOD

- Principle (Sethian, 1999)
  - Introduce a higher dimension
  - Represent the interface as the zero level a function $\psi(x,t)=0$

- Possible practical implementation:
  - Approximated on a fixed mesh by the signed distance function to curve Γ:



$$\psi(x,t)=\pm \min_{x_\Gamma \in \Gamma(t)} \left\| x - x_\Gamma \right\|$$

- Advantages:
  - 2D / 3D
  - Combination of entities: e.g. min / max

THE LEVEL SET METHOD

- Evolution of interface

$$\frac{\partial \psi}{\partial t} + F\left\|\nabla \psi\right\| = 0$$

$$\psi(x,t) = 0 \qquad \text{given}$$

  - ***F***: speed function of Γ in the outward normal direction to interface

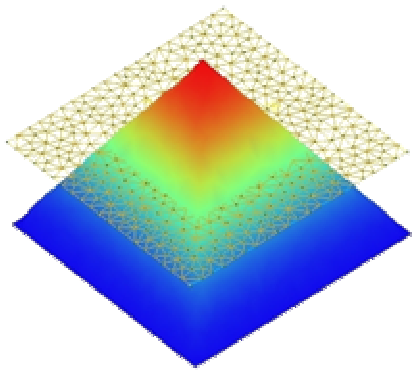
- In XFEM framework,
  - Each node has a Level Set dof
  - Interpolation using classical shape functions

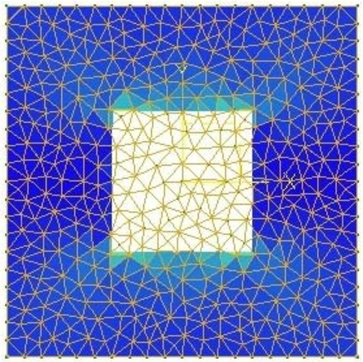
$$\psi(x,t) = \sum_i \psi_i \, N_i(x)$$

  - Material assigned to a part of the Level Set (positive or negative)

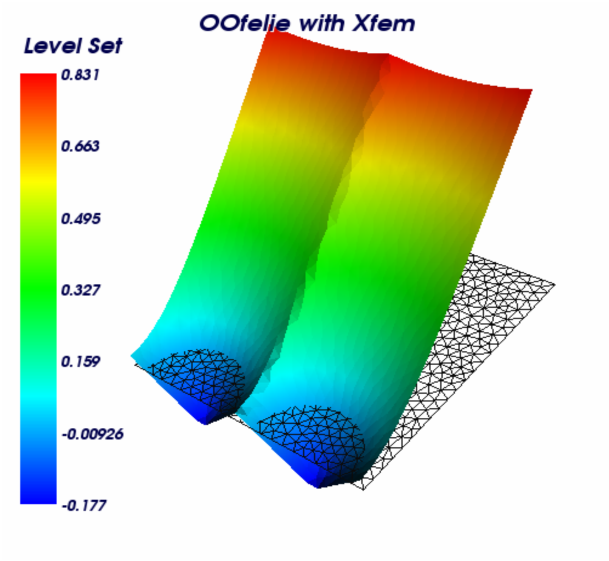
THE LEVEL SET METHOD

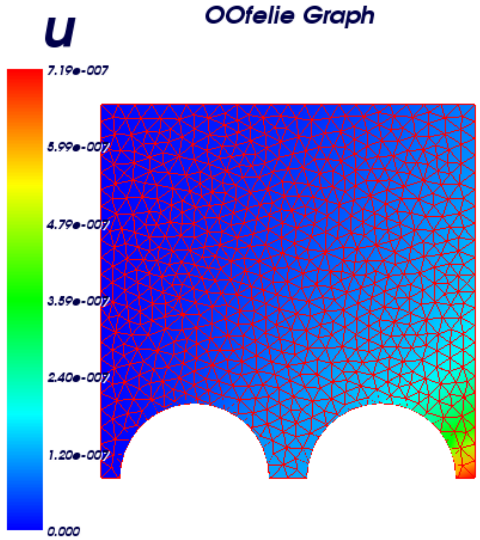
- Level Set of a square hole





- Combination of two holes





PROBLEM FORMULATION

- Geometry description and material layout :
  - Using Level Sets
  - Basic Level Set features: circles, ellipses, rectangles, etc.
- Design Problem
  - Find the best shape to minimize a given objective functions while satisfying design constraints
- Design variables:
  - Parameters of Level Sets
- Objective and constraints
  - Mechanical responses: global (compliance) or local (displacement, stress)
  - Geometrical characteristics: volume, distance
- Problem formulation similar to shape optimization but simplified thanks to XFEM and Level Set!

PROBLEM FORMULATION

BECAUSE OF XFEM AND LEVEL SET

- The mesh has not to coincide with the geometry
- Working on a fixed mesh

- Sensitivity analysis: no velocity field and no mesh perturbation required

- Topology can be altered as entities can be merged or separated → generalized shape
- Introduction of new holes requires a topological derivatives

- Topology optimization can be simulated using a design universe of holes and an optimal selection problem (Missoum et al. 2000)

SENSITIVITY ANALYSIS

- Classical approach for sensitivity analysis in industrial codes: semi analytical approach

- Discretized equilibrium

$$\mathbf{K}\,\mathbf{u} = \mathbf{f}$$

- Derivatives of displacement

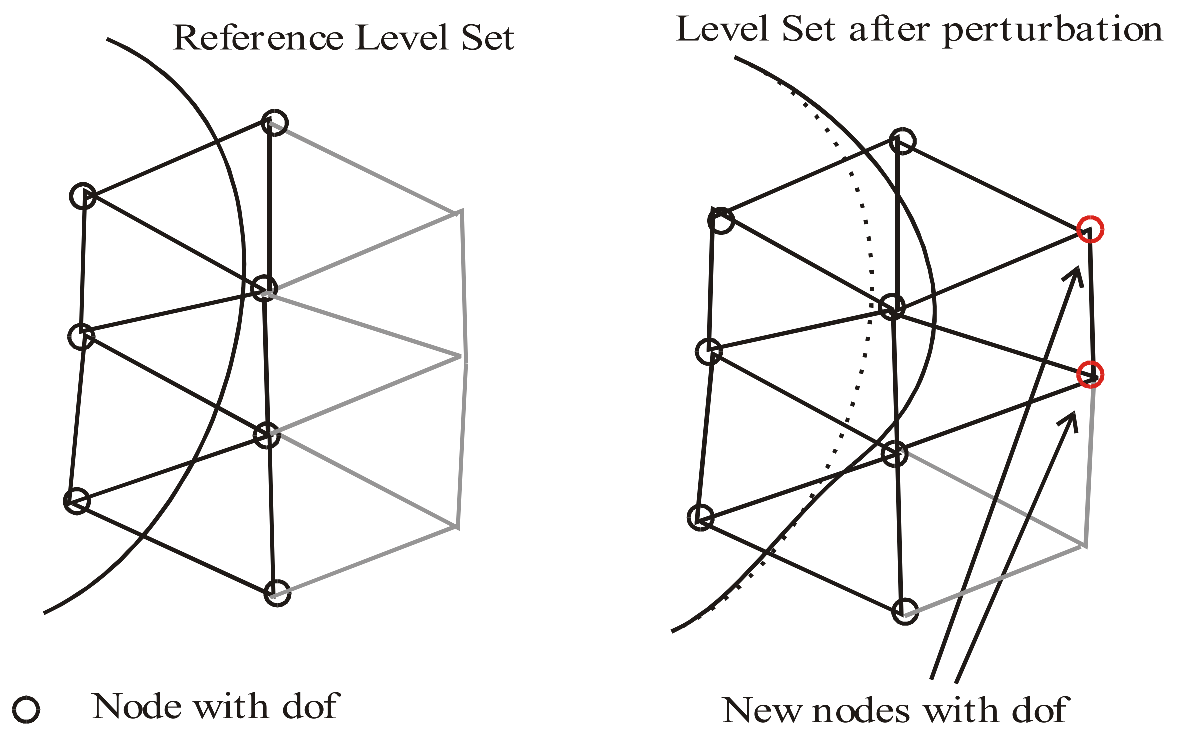
$$\mathbf{K}\,\frac{\partial \mathbf{u}}{\partial x} = \left(\frac{\partial \mathbf{f}}{\partial x} - \frac{\partial \mathbf{K}}{\partial x}\,\mathbf{u} \right)$$

- Semi analytical approach

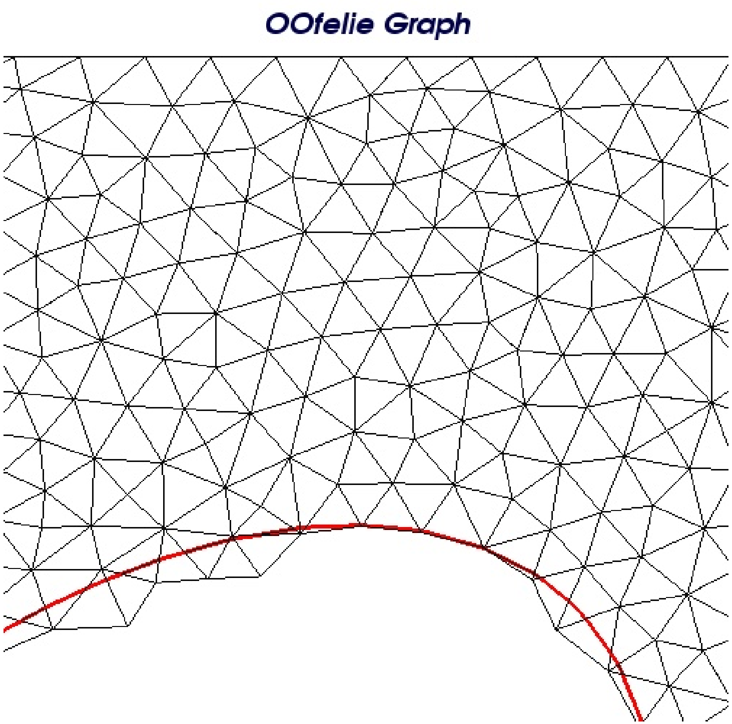
$$\frac{\partial \mathbf{K}}{\partial x} \approx \frac{\mathbf{K}(x+\delta x) - \mathbf{K}(x)}{\delta x} \qquad \frac{\partial \mathbf{f}}{\partial x} \approx \frac{\mathbf{f}(x+\delta x) - \mathbf{f}(x)}{\delta x}$$

SENSITIVITY ANALYSIS

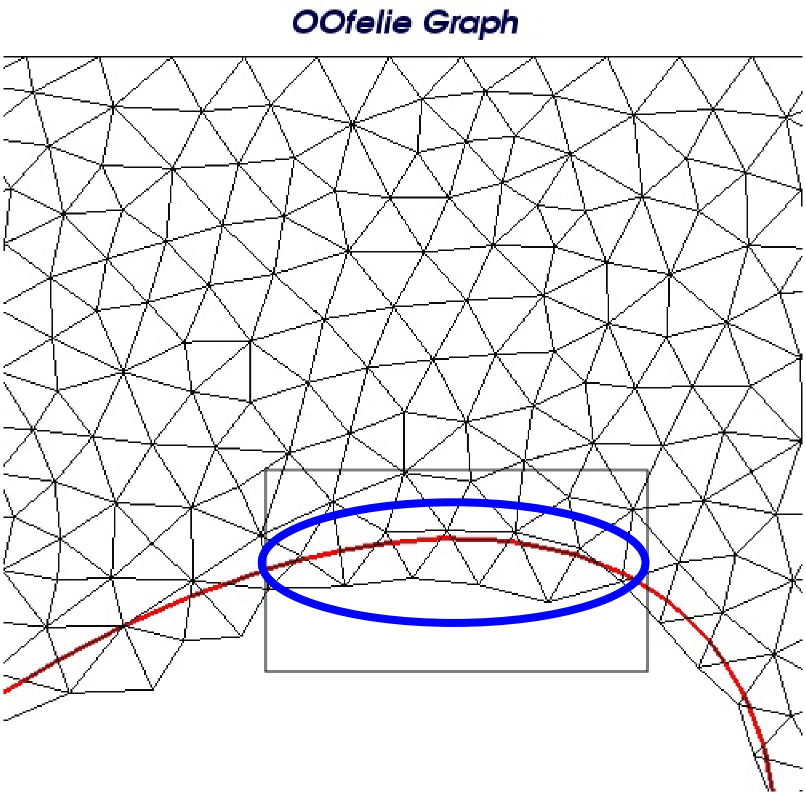
- Fixed mesh → no mesh perturbation
- However finite differences of stiffness matrix have to be made with a frozen number of dof (internal dof)
- Critical situations happen when new empty elements become partly filled with solid after perturbating of the level set :



SENSITIVITY ANALYSIS



Reference configuration



After level set perturbation

SENSITIVITY ANALYSIS

- Strategies to freeze the number of dof
  - analytical derivatives of stiffness matrix:
    - not general!
  - boundary layer in which all elements are retained
    - rigid modes, larger size of the problem
  - boundary layer with softening material (SIMP law)
    - lost of void / solid approximation
  - ignore the new elements that become solid or partly solid
    - small errors, but minor contributions
    - practically, no problem observed
    - efficiency and simplicity
    - validated on benchmarks

SENSITIVITY ANALYSIS

- Summary of the semi-analytical approach strategy

$$\frac{\partial \mathbf{K}}{\partial x} \approx \frac{\mathbf{K}(x+\delta x)-\mathbf{K}(x)}{\delta x}$$

Element initially	→ Solid	→ Cut	→ Void
Solid		OK	OK
Cut	OK	OK	OK
Void	Ignored	Ignored	

APPLICATIONS

- Implementation
  - Preliminary investigations by coupling a standard XFEM code by Moës with a general open optimisation code (Boss Quattro)
  - New implementation in a multiphysic finite element code in C++ (OOFELIE from Open Engineering)
  - Available: 2D problems with a library of quadrangles and triangles.

- Solution of optimisation problem:
  - Sequential convex linearization
  - CONLIN optimiser

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & g_0(\mathbf{x}) \\
s.t. \quad & g_j(\mathbf{x}) \leq \overline{g}_j \\
& \underline{x}_i \leq x_i \leq \overline{x}_i
\end{aligned}
$$

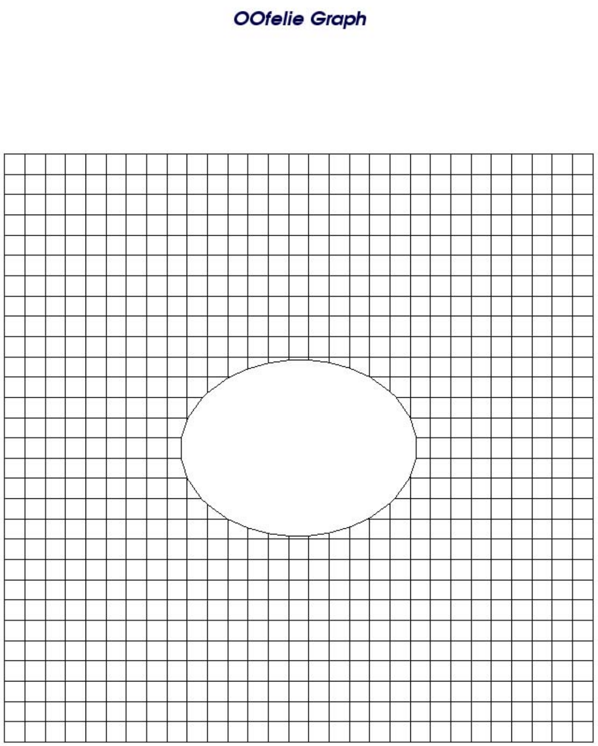
APPLICATIONS

CLASSICAL PROBLEM OF PLATE WITH A HOLE REVISITED

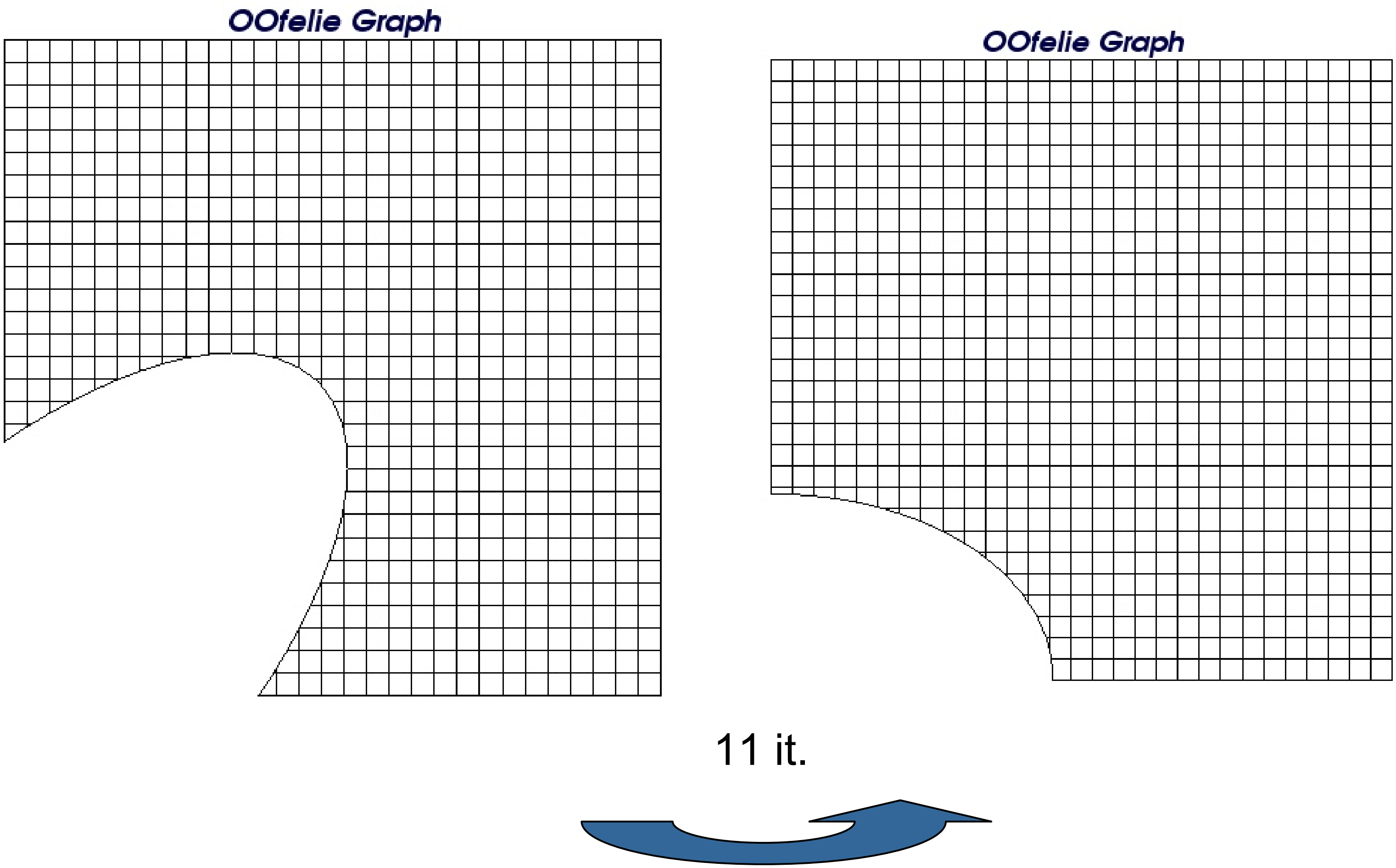
- Square plate with a hole
- Bidirectional stress field
- $\sigma_x = 2\ \sigma_0$ $\sigma_y = \sigma_0$
- E= 1 N/m², ν=0.3

- Minimize compliance
  - st volume constraint
- Design variables: major axis a and orientation θ

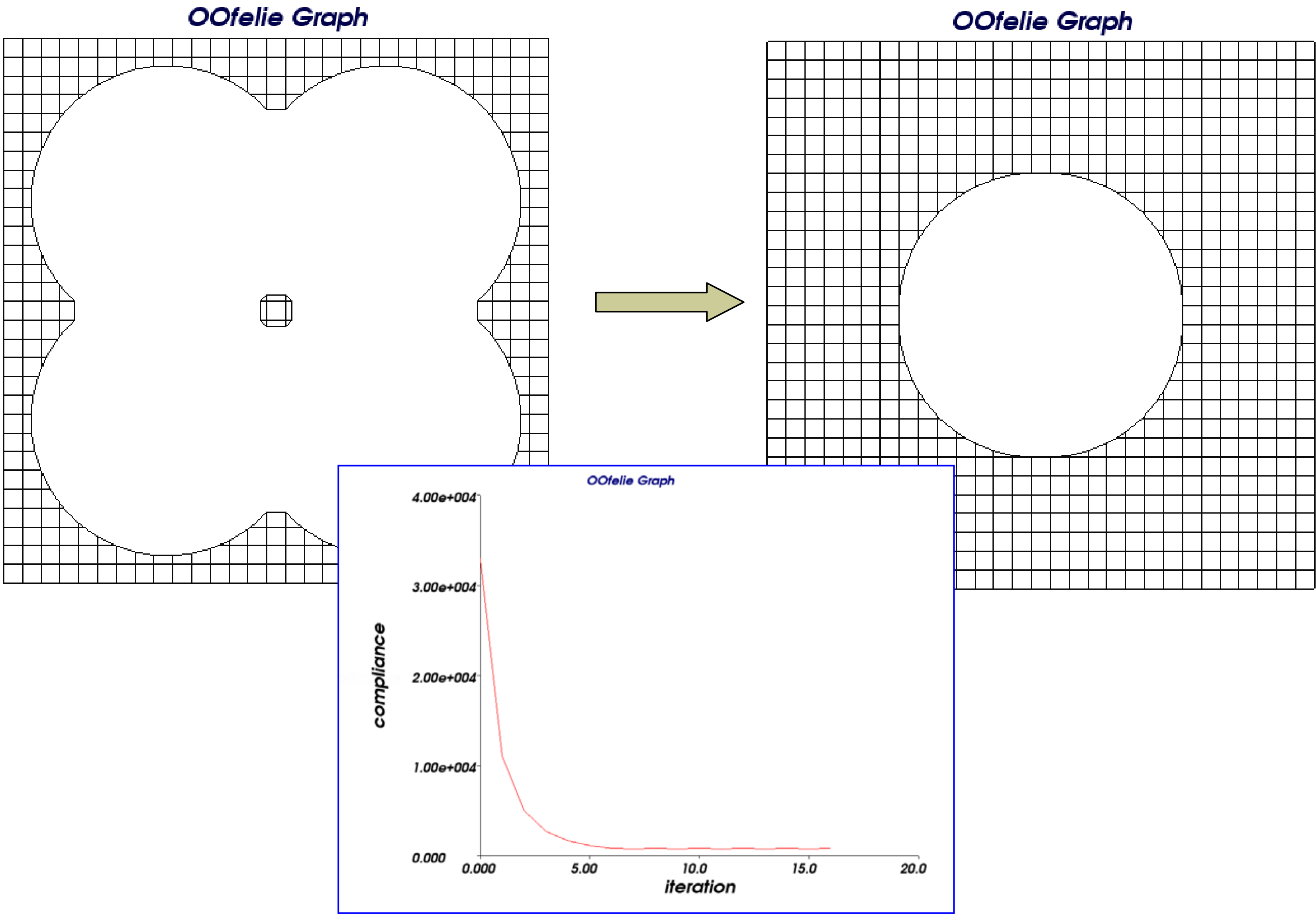
- Mesh 30 x 30 nodes



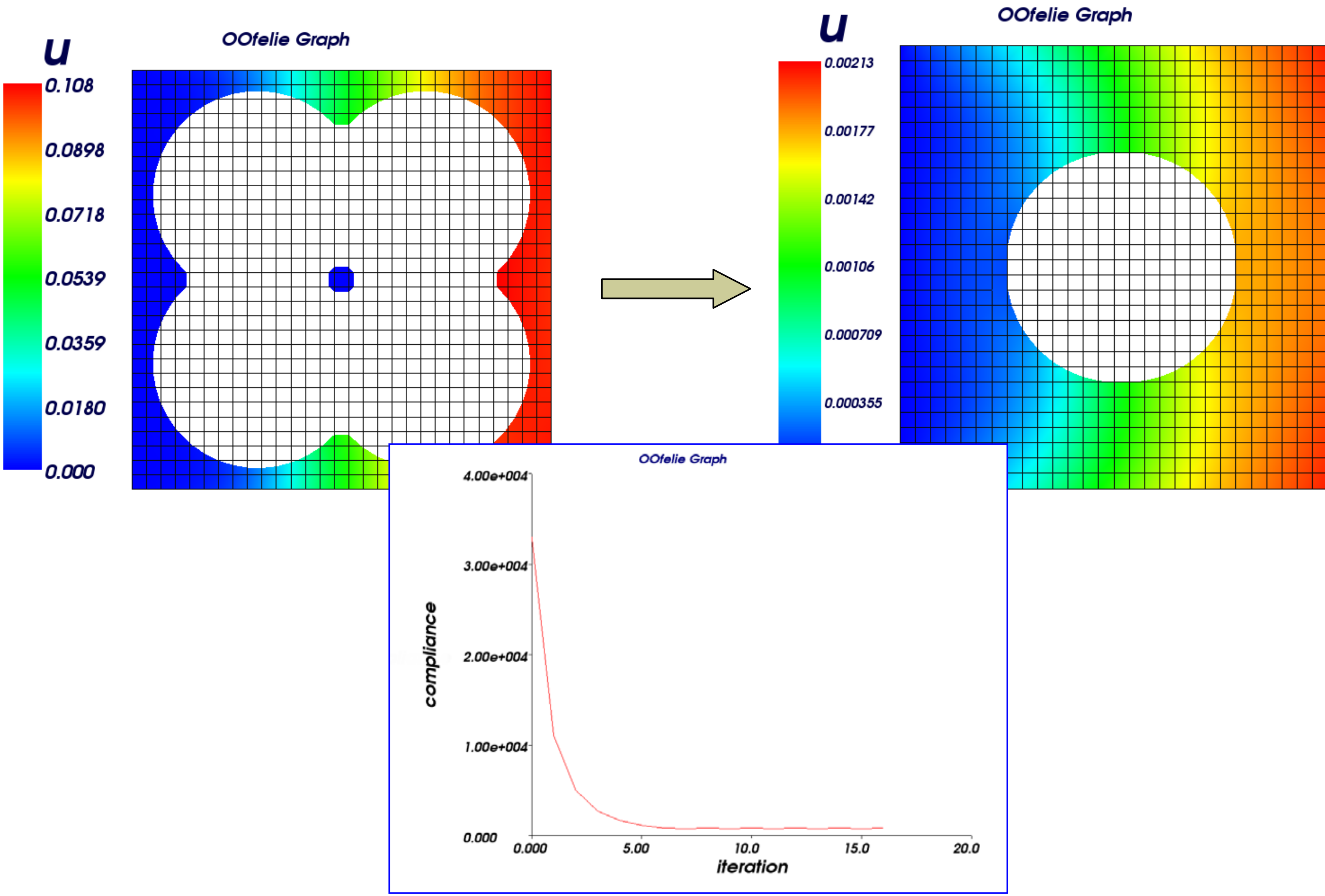
APPLICATIONS



APPLICATIONS



APPLICATIONS



CONCLUSION

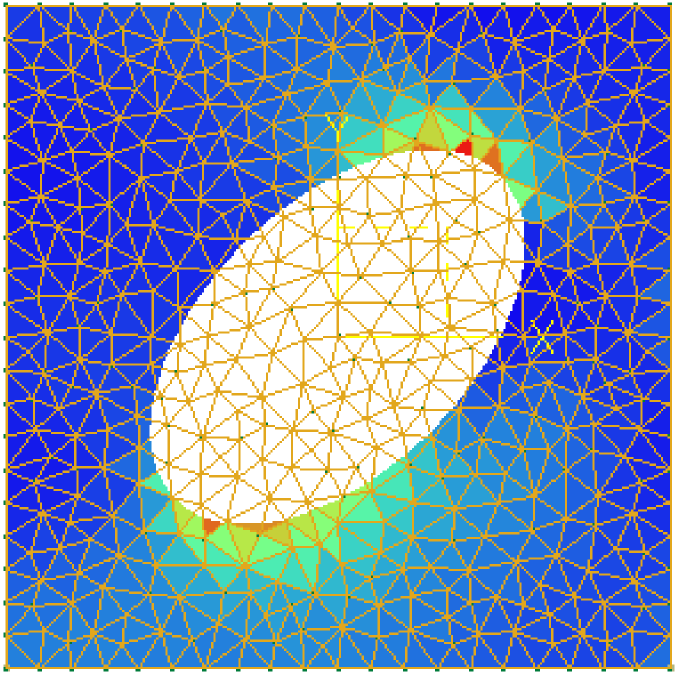
- XFEM and Level Set gives ride to a generalized shape optimisation technique

- Intermediate to shape and topology optimisation
  - Work on a fixed mesh
  - Topology can be modified:
    - Holes can merge and disappear
    - New holes cannot be introduced without topological derivatives
  - Smooth curves description
  - Void-solid description
  - Small number of design variables
  - Global or local response constraints
  - No velocity field and mesh perturbation problems

CONCLUSION

- Contribution of this work
  - New perspectives of XFEM and Level Set
  - Investigation of semi-analytical approach for sensitivity analysis
  - Implementation in a general C++ multiphysics FE code
- Concept just validated
- Perspectives:
  - Sensitivity analysis (to be continued)
  - 3D problems
  - Stress constrained problems
  - Dynamic problems
  - Multiphysic simulation problems with free interfaces

APPLICATIONS



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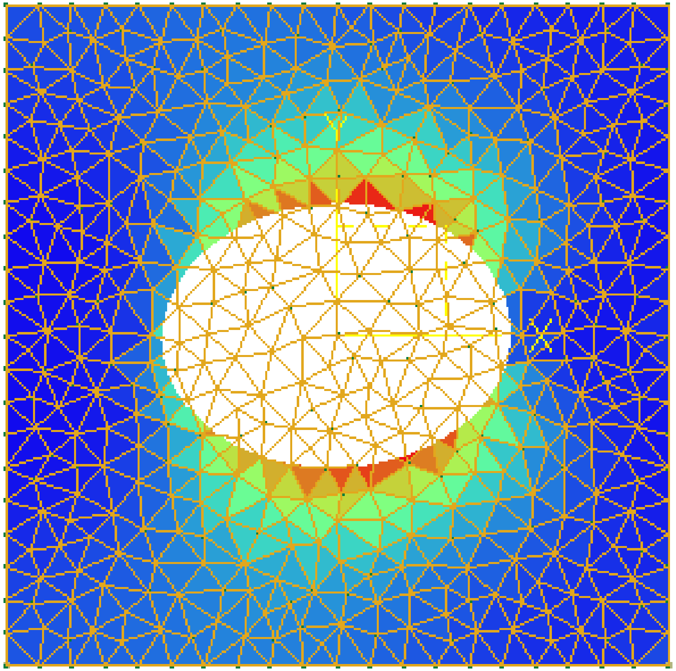
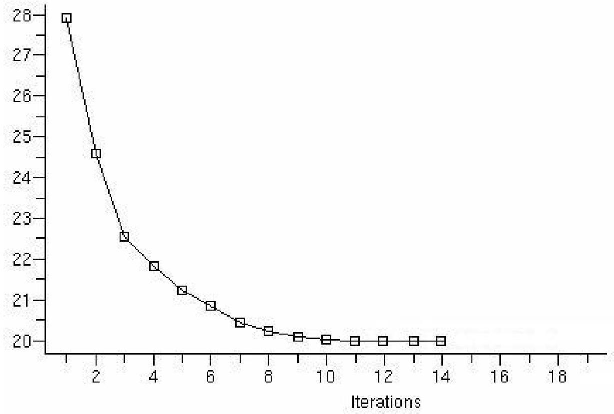


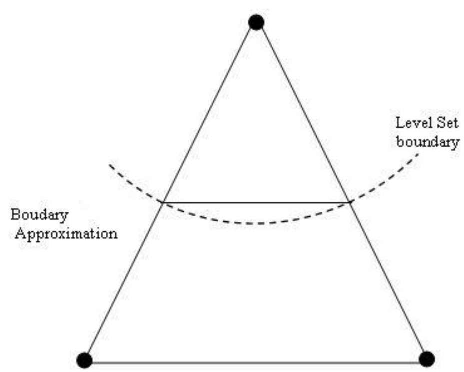


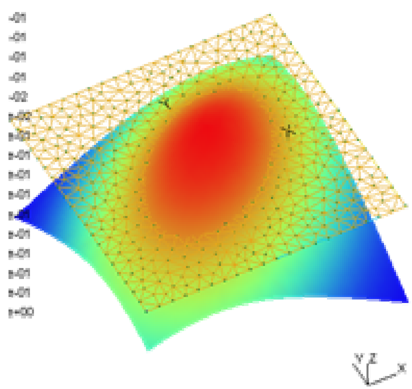
APPLICATIONS

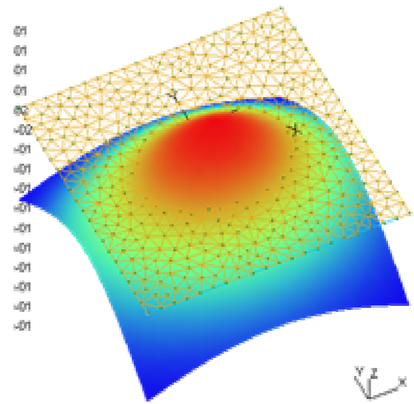
- Discretization error of geometry
  Over-estimating geometric values :

		Xfem		*Fem*	
		Iteration 1	Iteration 11	Iteration 1	Iteration 9
Objective function	Minimise U	27.9	20.2	26	18.3
Constraint	Surface < 3.45	3.59	3.45	3.50	3.45
Variable	1e-4$< \theta <$ 90	45	1e-4	45	0
Variable	1e-4<a< 1	0.5	1.06	0.5	0.88

- Representating interfaces inside an element :

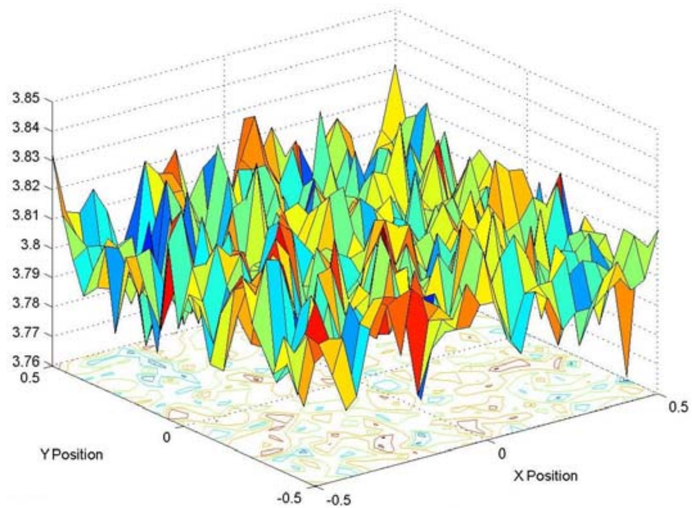


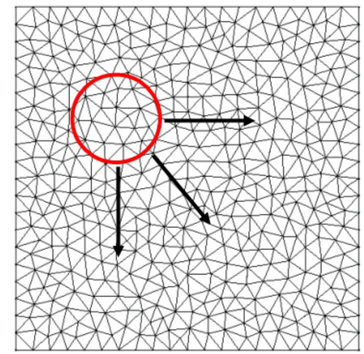


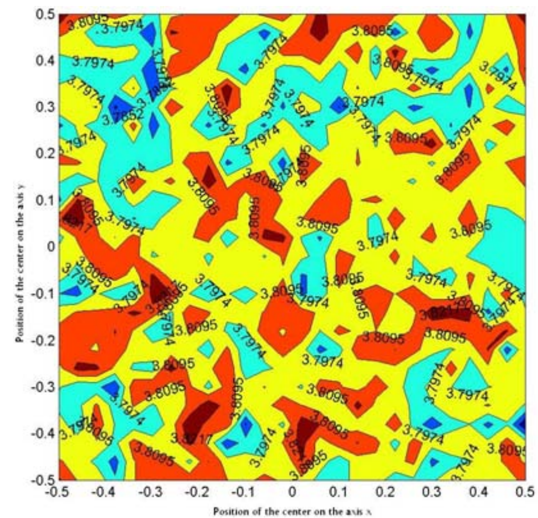


APPLICATIONS

- Linear interpolation of the Level Set may introduce discontinuity :
  - Parametric study of the surface of the plate
  - Variation of 1%



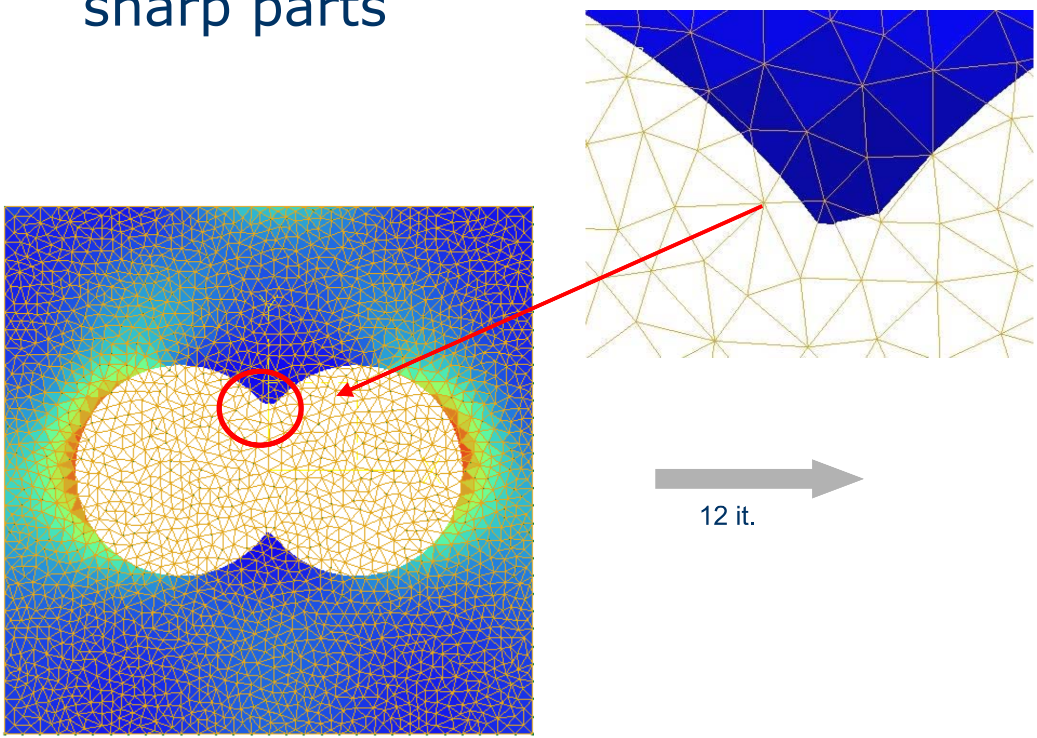




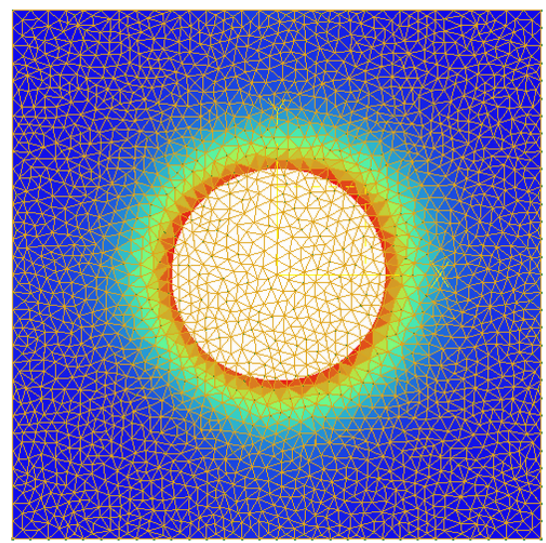
- Take care of numerical noise

APPLICATIONS

- Toplogy optimization
  - Two variables : *center* x_1, *center*x_2
  - Min. potential energy under a surface constraint
  - Uniform Biaxial loading : $\sigma_x = \sigma_0$, $\sigma_y = \sigma_0$
- Mesh refinement for the Level Set representation of sharp parts

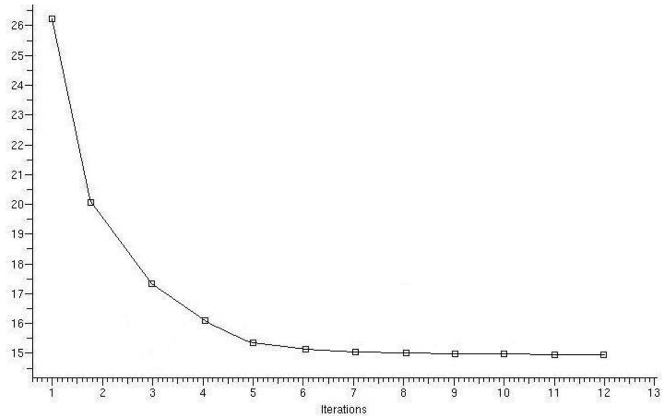


Meaning of this is may be questionnable!



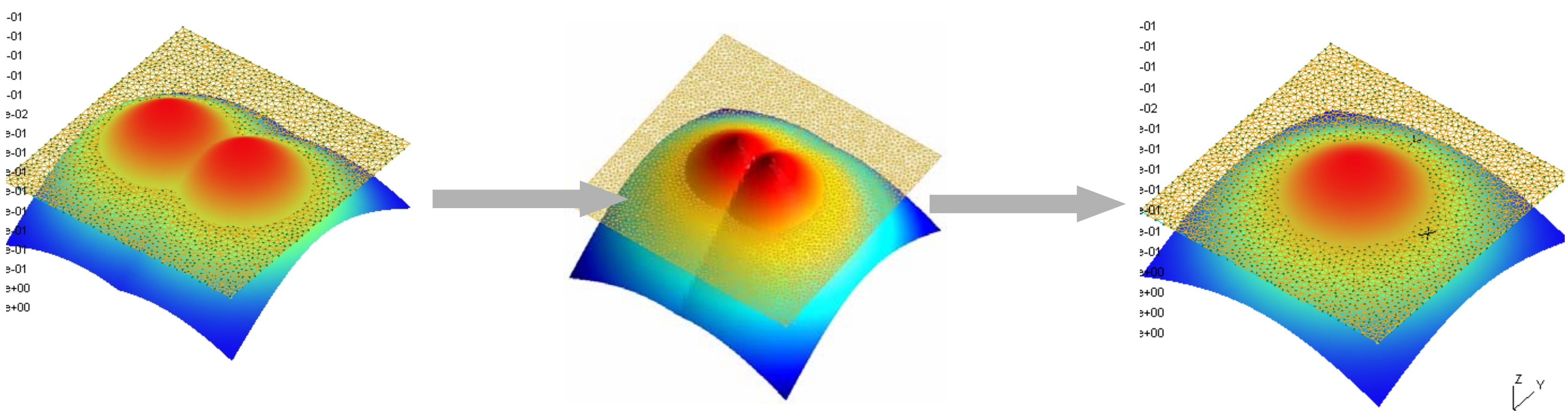
APPLICATIONS

Evolution of the objective function



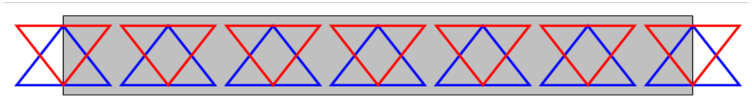
		Iteration 1	Iteration
Objective function	Minimise U	26,6	14,9
Constraint	Surface> 7.8	6.9	7.95
Variable	$-0.5<x_1$ position<0.5	0.5	-0.066207
Variable	$-0.5<x_2$ position<0.5	-0.5	0.045791

Evolution of the Level Set



APPLICATIONS

- Design universe of holes (Missoum et al., 2000)
  - Selection and sizing of basic Level Set entities with a GA in classical topology
- Find a result as close as possible to MBB topology solution
  - 14 triangles are « well » placed.
  - Variables : presence of a triangle



- The optimum is reached after 36