

Identification of tension in slender members with  
unknown boundary conditions using mode shapes

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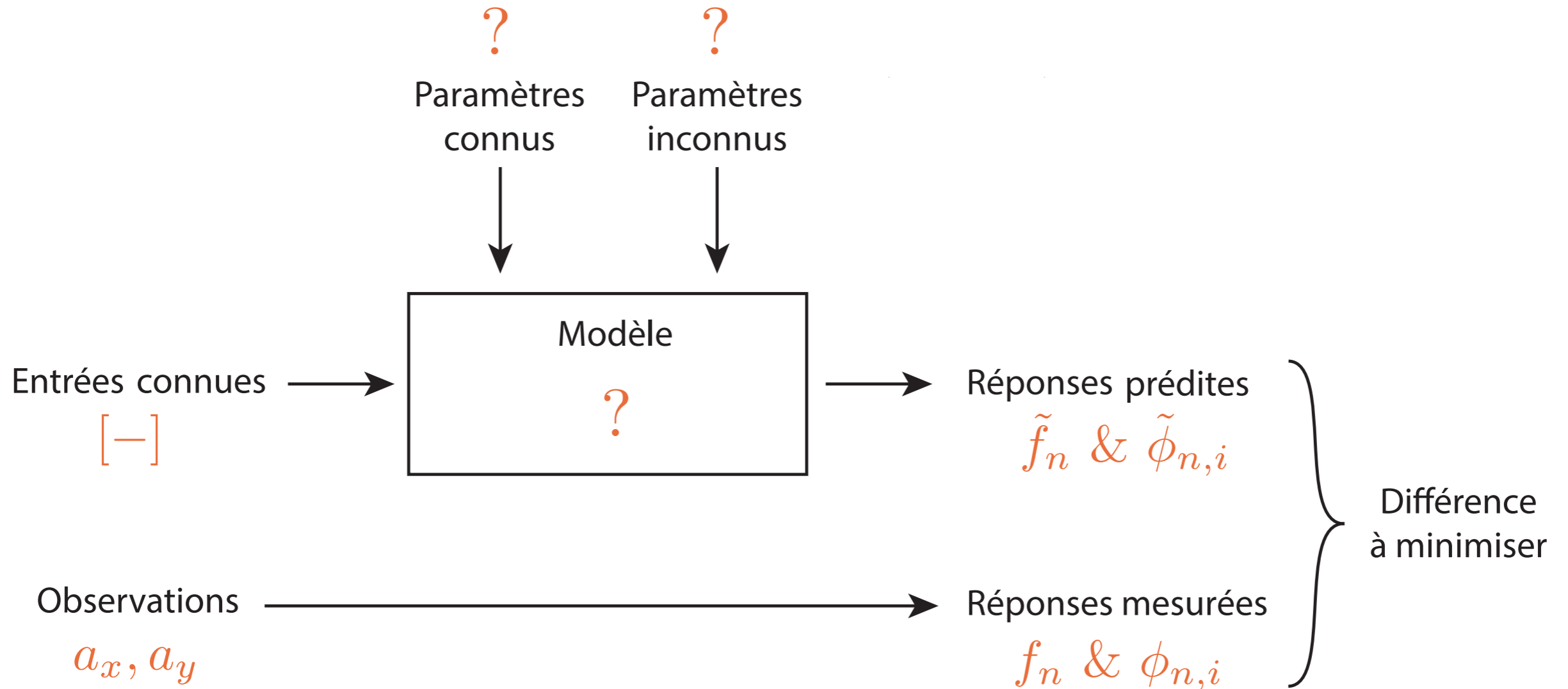
Vincent Denoël

# Mise en contexte

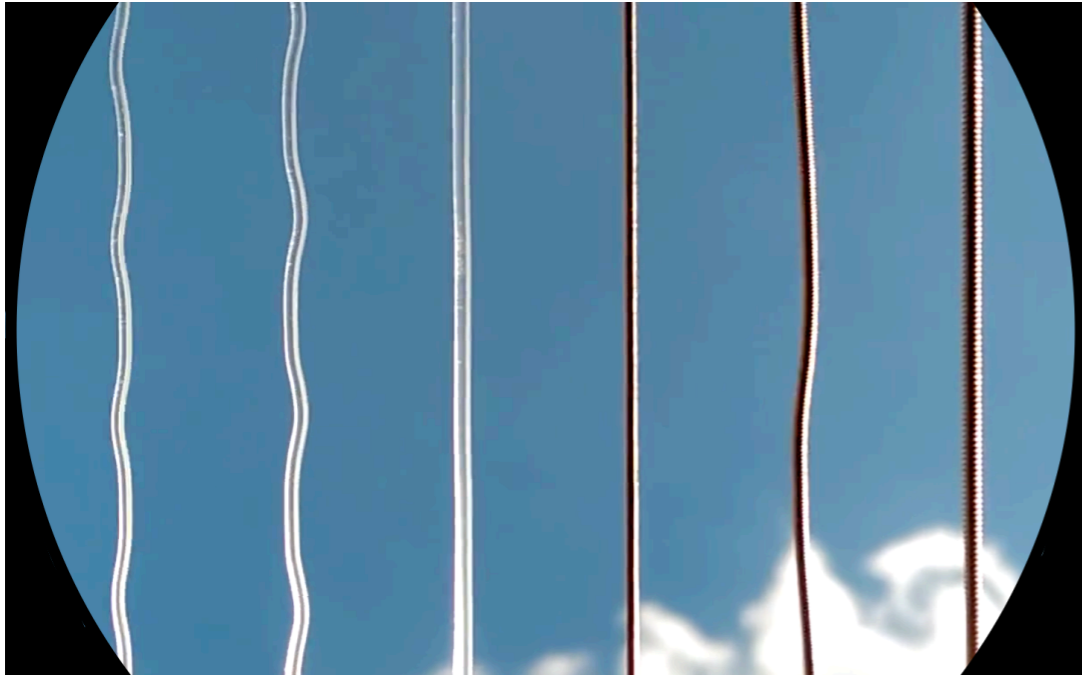


Monitoring à distance de la tension dans des haubans ou des suspentes au moyen de mesures vibratoires.

# Identification à pd mesures vibratoires



# Dynamique de la corde tendue



- (i) Câble extensible, tension  $>$  poids
- (ii) Raideur flexionnelle négligeable

$$\mu \frac{\partial^2 v}{\partial t^2} = H \frac{\partial^2 v}{\partial x^2}$$



$$f_n = \frac{n}{2L} \sqrt{\frac{H}{\mu}}$$

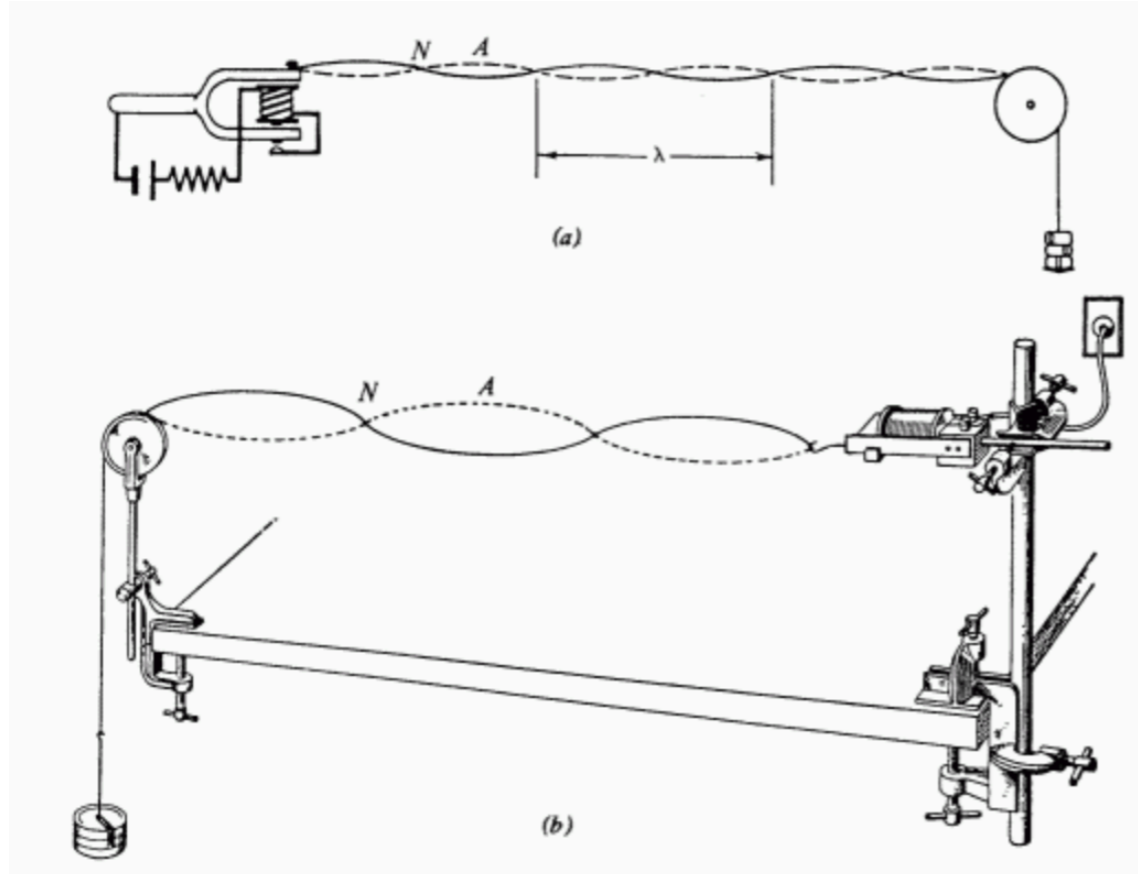
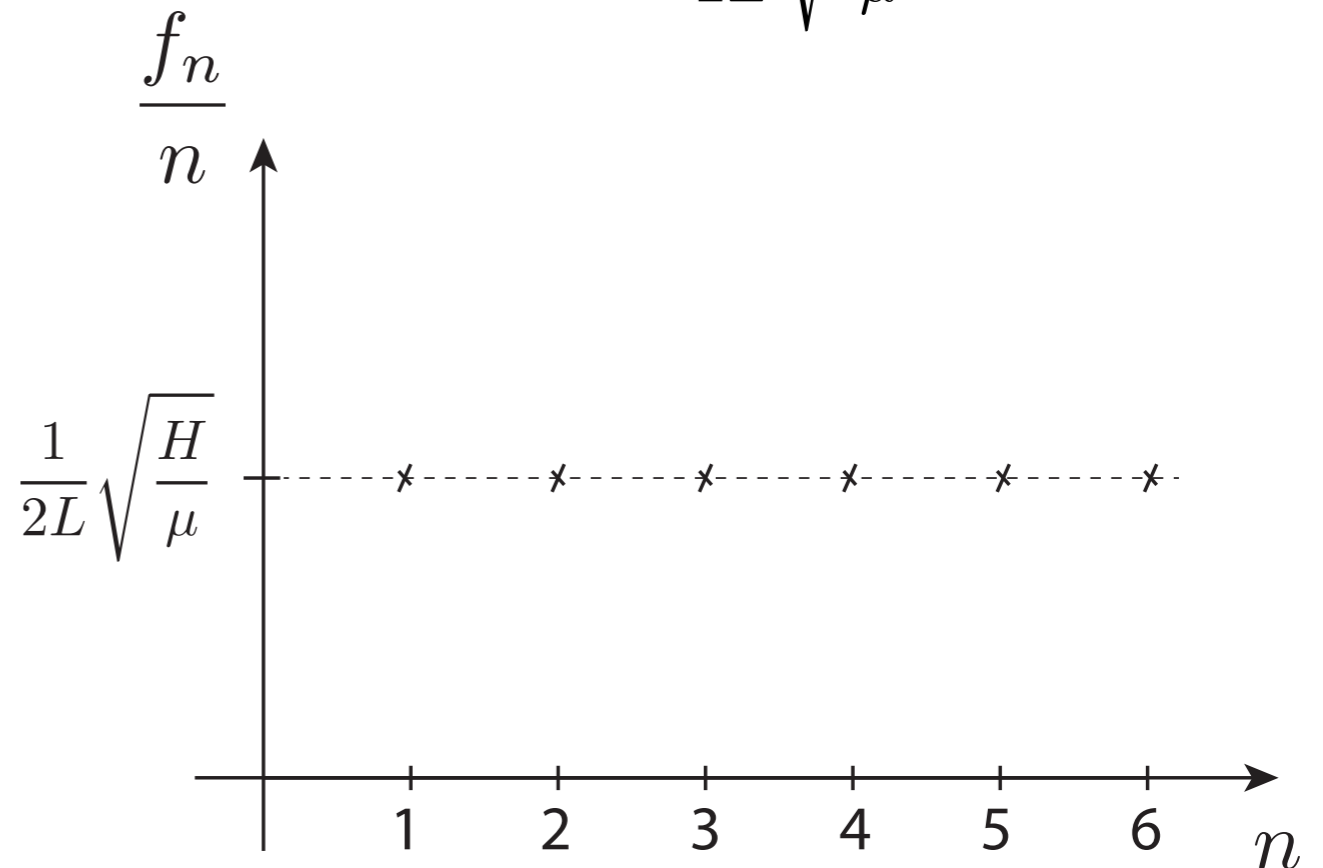


Fig. 1. Vibrator driving taut string. [Bernard and Epp]



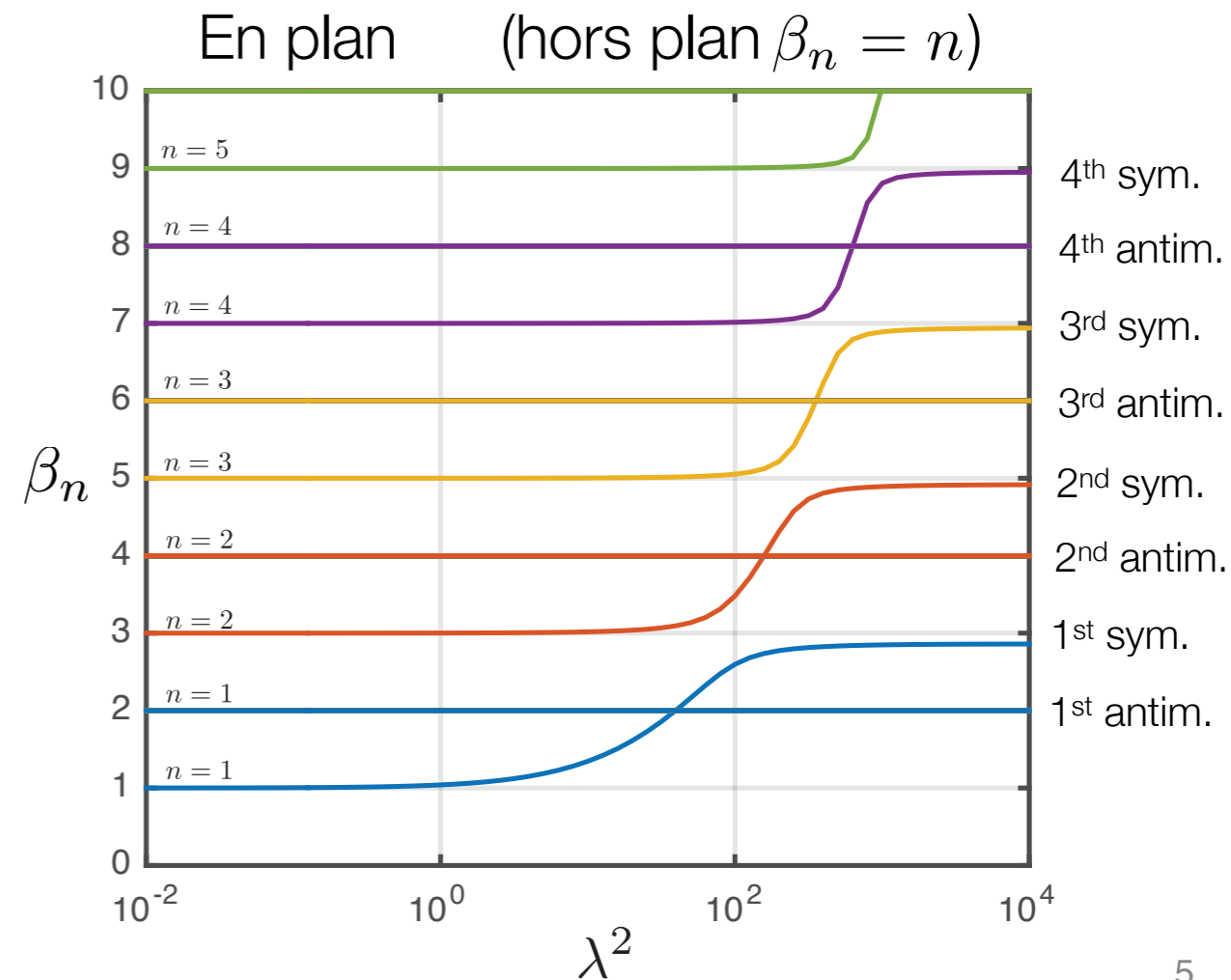
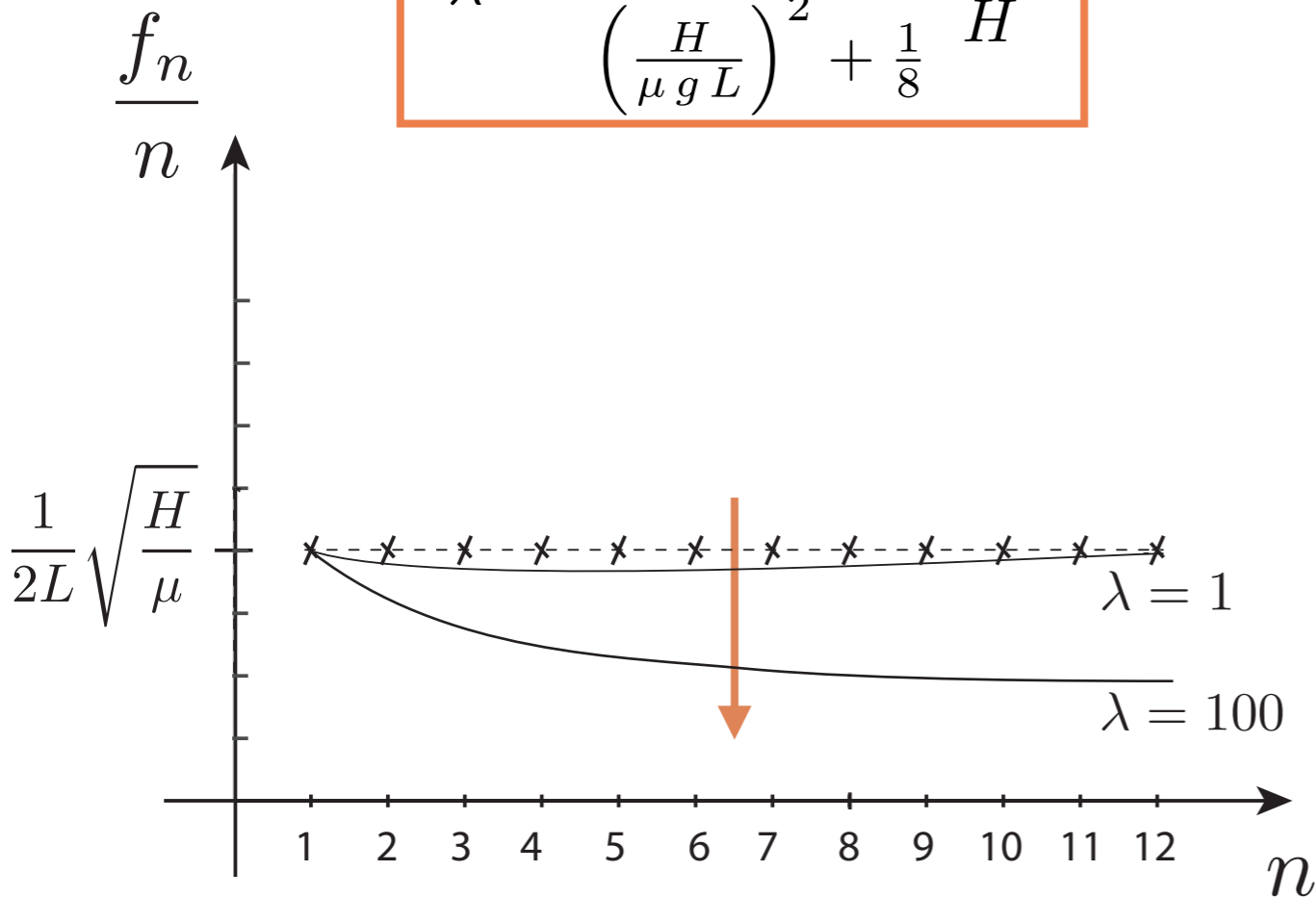
# (i) Effets de sagging

$$\mu \frac{\partial^2 v}{\partial t^2} = H \frac{\partial^2 v}{\partial x^2} + \boxed{h(t) \frac{d^2 y}{dx^2}}$$

$$f_n = \frac{\beta_n(\lambda^2)}{2L} \sqrt{\frac{H}{\mu}}$$

## Paramètre d'Irvine

$$\lambda^2 = \frac{1}{\left(\frac{H}{\mu g L}\right)^2 + \frac{1}{8}} \frac{EA}{H}$$



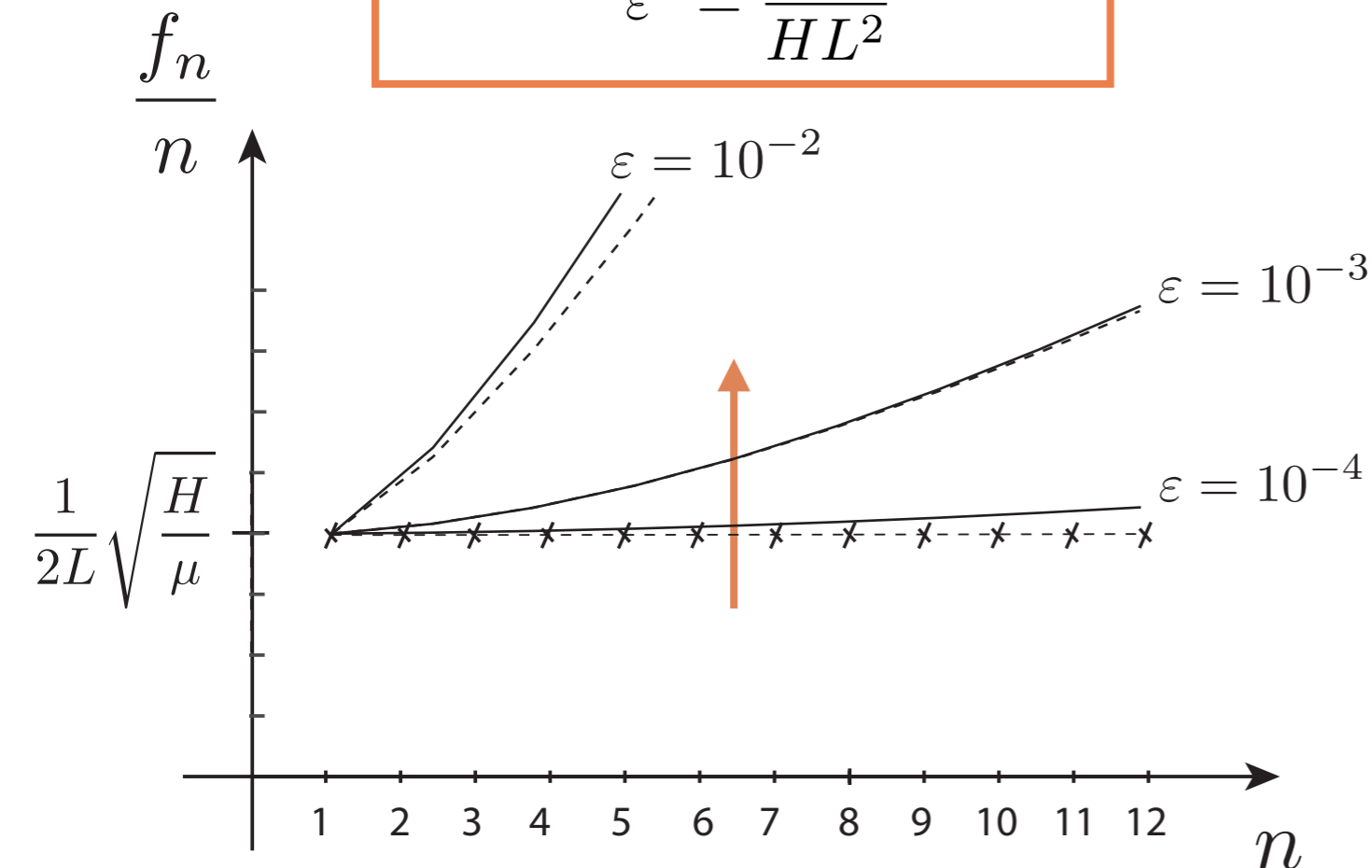
## (ii) Effets de flexion

$$\mu \frac{\partial^2 v}{\partial t^2} + EI \frac{\partial^4 v}{\partial x^4} = H \frac{\partial^2 v}{\partial x^2}$$

$$f_n = \frac{\beta_n(\varepsilon^2)}{2L} \sqrt{\frac{H}{\mu}}$$

**Paramètre de flexion**

$$\varepsilon^2 = \frac{EI}{HL^2}$$

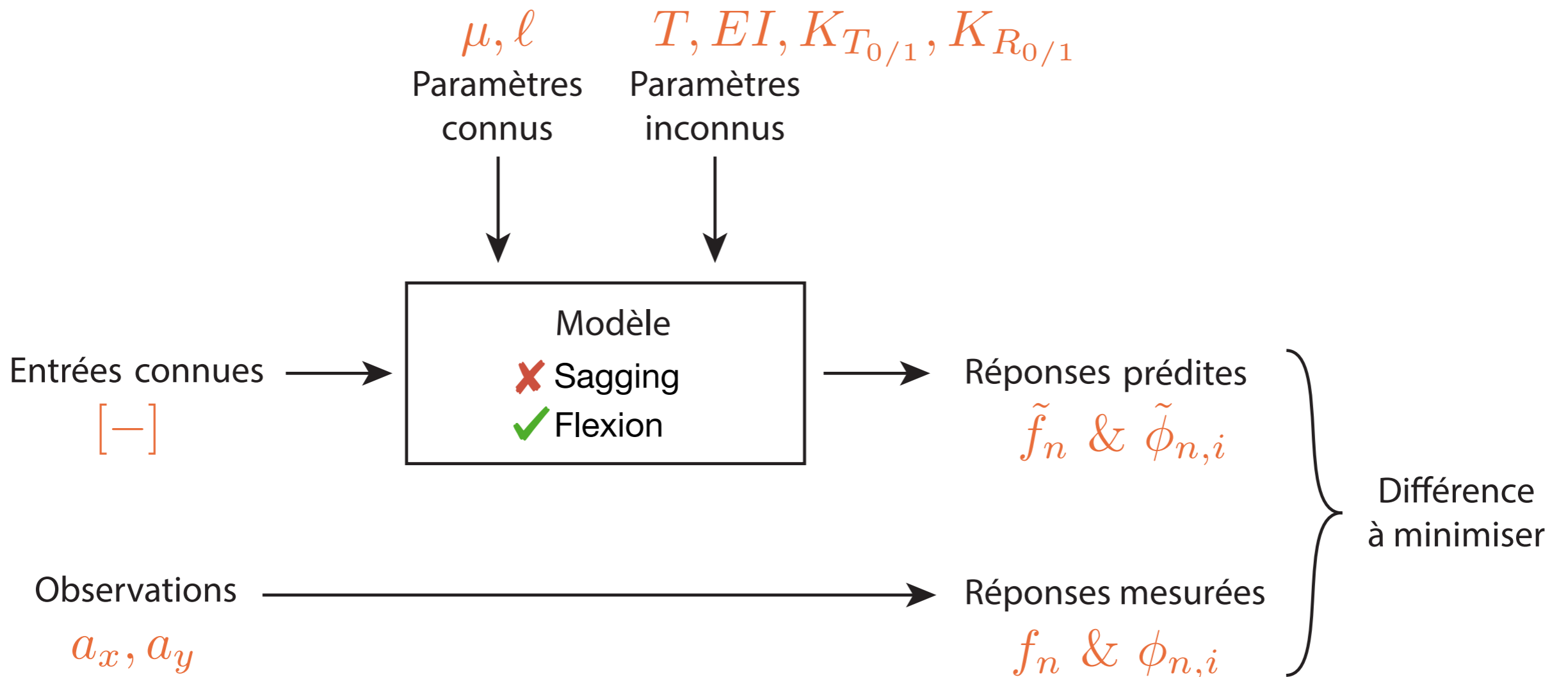
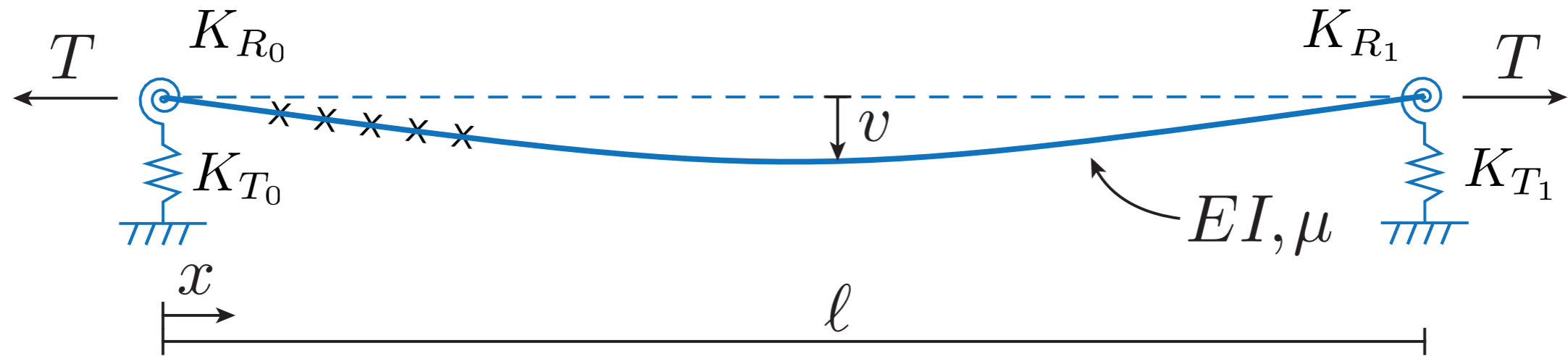


La rigidité des appuis modifie l'expression de  $\beta_n$ !

— bi-encasté

- - bi-rotulé  $\beta_n = n\sqrt{1 + n^2\pi^2\varepsilon^2}$

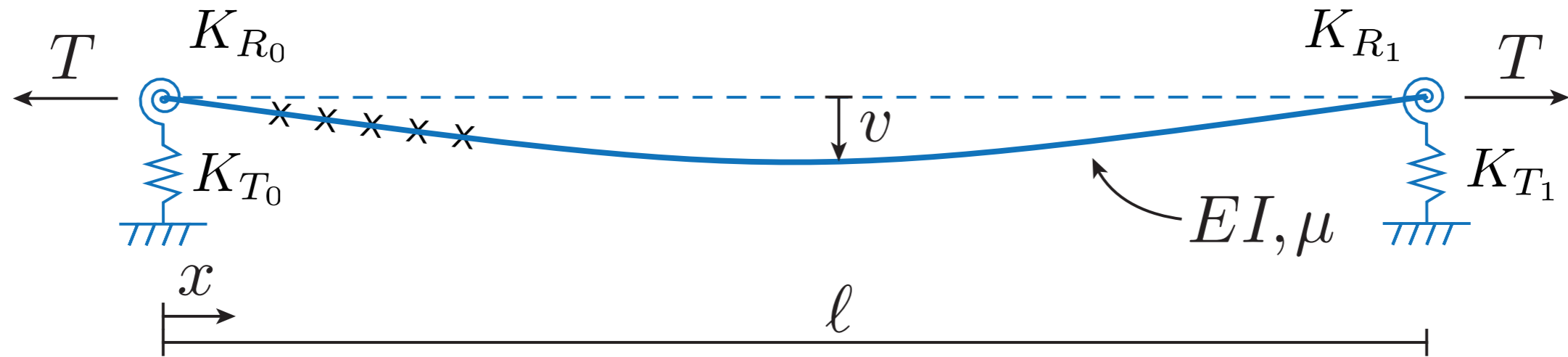
# Paramètres à prendre en compte



Analyse directe



# Équation du mouvement



$$\mu \partial_t^2 v + EI \partial_x^4 v - H \partial_x^2 v = 0$$

Adim.

$$\xi = \frac{x}{l} ; \omega_0 = \frac{\tau}{t} = \sqrt{\frac{H}{\mu l^2}} ; \varepsilon^2 = \frac{EI}{N l^2}$$

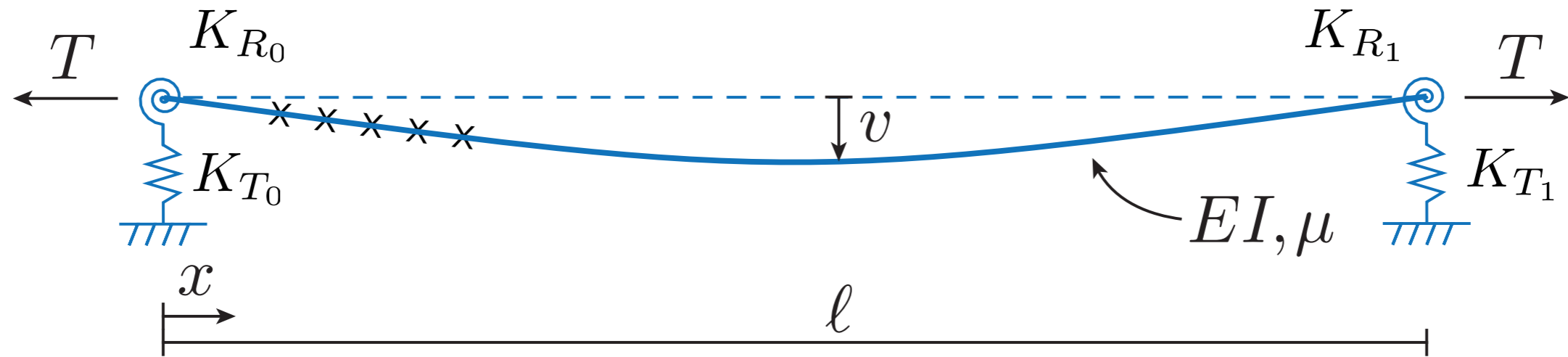
$$\partial_\tau^2 \bar{v} + \varepsilon^2 \partial_\xi^4 \bar{v} - \partial_\xi^2 \bar{v} = 0$$

Sep. var.

$$\bar{v}(\xi, \tau) = \phi(\xi) q(\tau)$$

$$\begin{cases} \ddot{q}(\tau) + \kappa^2 q(\tau) = 0 \Leftrightarrow q(\tau) = q_0 e^{i\kappa\tau} \\ \varepsilon^2 \phi^{(iv)}(\xi) - \phi''(\xi) - \kappa^2 \phi(\xi) = 0 \end{cases}$$

# Existence de couches limites spatiales



$$\varepsilon^2 \phi^{(iv)}(\xi) - \phi''(\xi) - \kappa^2 \phi(\xi) = 0$$

Si  $\varepsilon^2 \ll 1 \rightarrow$  couches limites à gauche et à droite

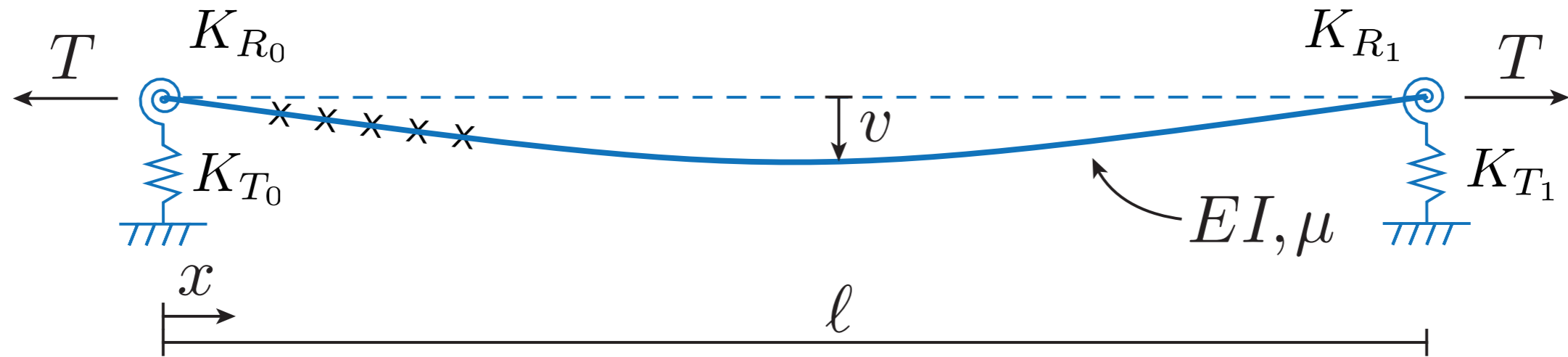
$$\phi(\xi) = A \sin(z_1 \xi) + B \cos(z_1 \xi) + C \exp(-z_2 \xi) + D \exp(-z_2(1 - \xi))$$

$$z_1 = \frac{\sqrt{2}}{2} \frac{1}{\varepsilon} \sqrt{\sqrt{1 + 4\kappa^2 \varepsilon^2} - 1}$$

$$z_2 = \frac{\sqrt{2}}{2} \frac{1}{\varepsilon} \sqrt{\sqrt{1 + 4\kappa^2 \varepsilon^2} + 1}$$

plutôt que  $\cosh(z_2 \xi)$  et  $\sinh(z_2 \xi) \rightarrow \infty, \xi > \varepsilon$

# Taille des couches limites



$$\varepsilon^2 \phi^{(iv)}(\xi) - \phi''(\xi) - \kappa^2 \phi(\xi) = 0$$

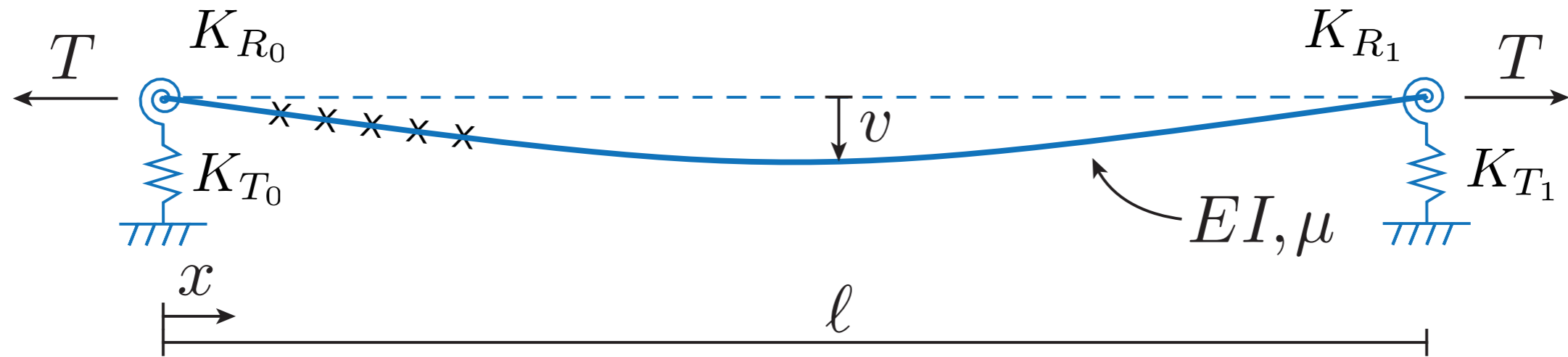
Si  $\varepsilon \kappa \ll 1$  en plus,

$$z_1 \approx \kappa$$

$$z_2 \approx \frac{1}{\varepsilon}, \text{ couches limites de taille } \varepsilon$$

$$\phi(\xi) = A \sin(z_1 \xi) + B \cos(z_1 \xi) + C \exp(-z_2 \xi) + D \exp(-z_2(1 - \xi))$$

# Taille des couches limites



$$\varepsilon^2 \phi^{(iv)}(\xi) - \phi''(\xi) - \kappa^2 \phi(\xi) = 0$$

$$\phi(\xi) = A \sin(z_1 \xi) + B \cos(z_1 \xi) + C \exp(-z_2 \xi) + D \exp(-z_2(1 - \xi))$$

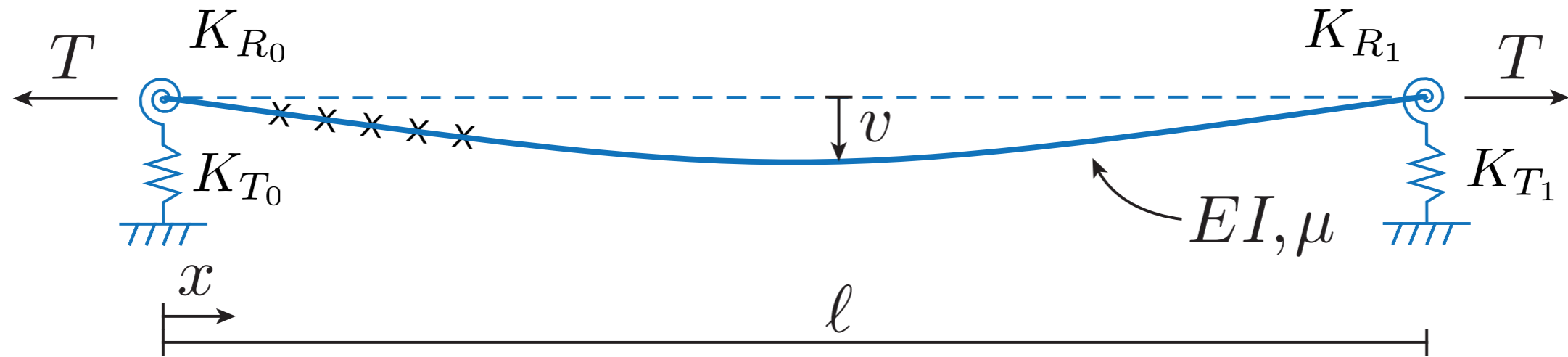
Si  $\varepsilon \kappa \approx 1$ ,

$$z_1 \approx \kappa \left( 1 - \frac{\kappa^2 \varepsilon^2}{2} \right)$$

$$z_2 \approx \frac{1}{\varepsilon} \left( 1 + \frac{\kappa^2 \varepsilon^2}{2} \right)$$

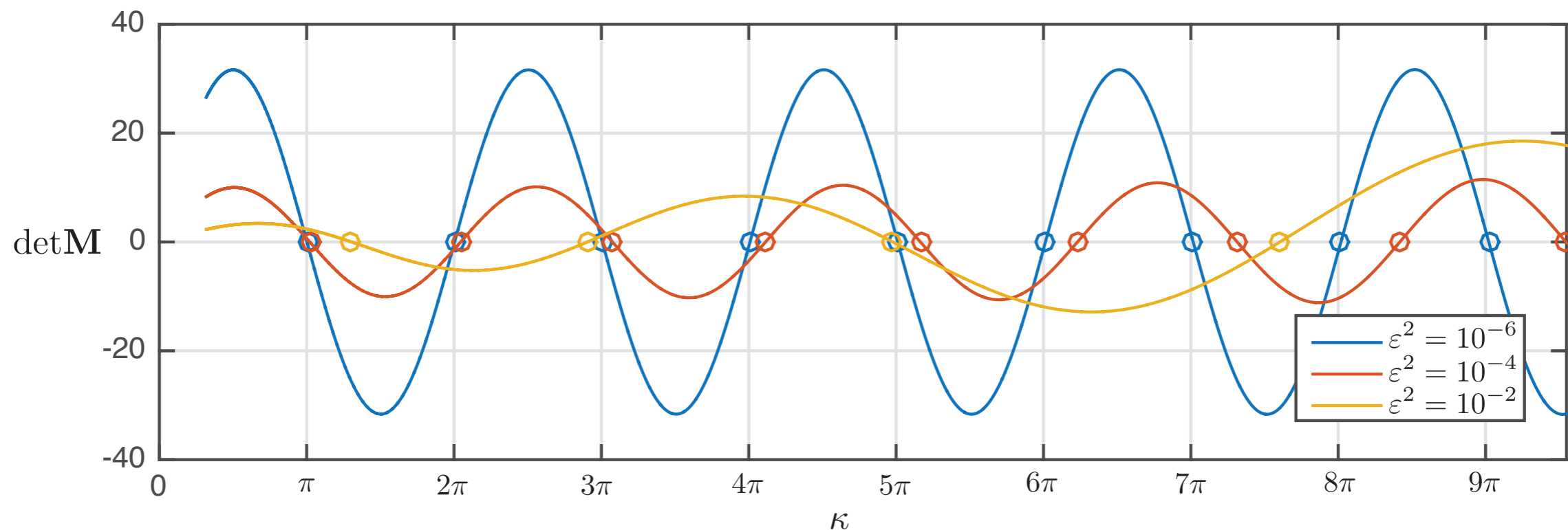
Les couches limites rétrécissent !

# Conditions d'équilibre aux limites

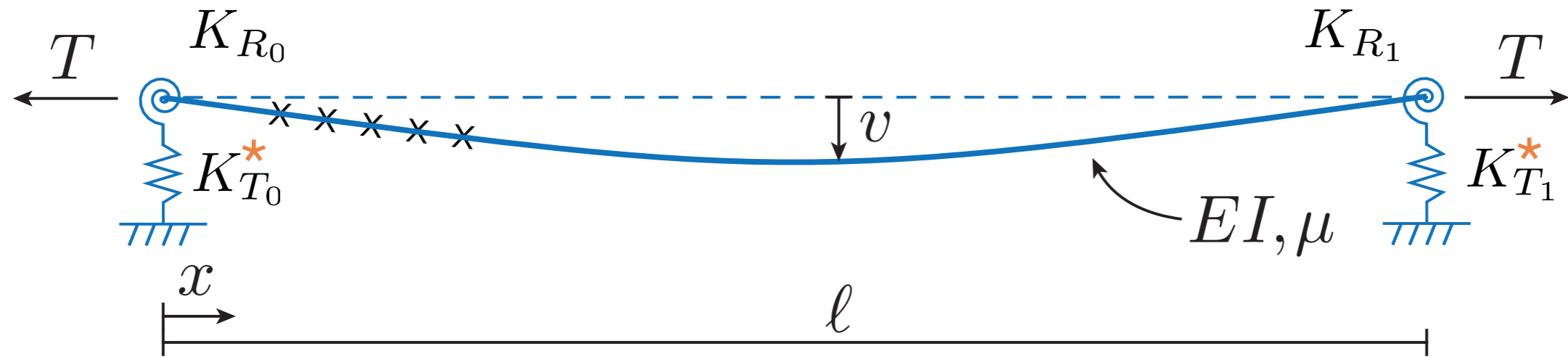


$$\phi(\xi) = A \sin(z_1 \xi) + B \cos(z_1 \xi) + C \exp(-z_2 \xi) + D \exp(-z_2(1 - \xi)) \quad \text{avec} \quad \begin{cases} \varepsilon^2 \phi'''(0) - \phi'(0) + \bar{K}_{R_0} \phi(0) = 0 \\ \varepsilon^2 \phi'''(1) - \phi'(1) - \bar{K}_{R_1} \phi(1) = 0 \\ \varepsilon^2 \phi''(0) - \bar{K}_{T_0} \phi'(0) = 0 \\ \varepsilon^2 \phi''(1) + \bar{K}_{T_1} \phi'(1) = 0 \end{cases}$$

Problème aux valeurs propres !  $\mathbf{M} \mathbf{c} = \mathbf{0}$



# Approche par perturbations



Effet global : R + T & O + 1

$$\kappa_k = k\pi \left( 1 + \textcircled{p}\varepsilon + \left( \frac{(k\pi)^2}{2} + p^2 \right) \varepsilon^2 \right)$$

T0

$$\xi \sim \varepsilon^2 \quad \phi_k(\xi) = k\pi \left( \frac{1 - \rho_0^*}{\rho_0^*} \right) \varepsilon$$

$$\xi \sim \varepsilon \quad \phi_k(\xi) = k\pi\xi + k\pi \left( \frac{1}{\rho_0^*} - 1 + \left( e^{\frac{-\xi}{\varepsilon}} - 1 \right) \rho_0 \right) \varepsilon$$

T0

R0

Analyse inverse

# Approche de Urbano/Li/Rebecchi

Mesure des fréquences propres et des amplitudes modales

$$\lambda_{ij} = \frac{\phi_i}{\phi_j} = \frac{C_1 \cos q_1 x_i + C_2 \sin q_1 x_i + C_3 \cosh q_2 x_i + C_4 \sinh q_2 x_i}{C_1 \cos q_1 x_j + C_2 \sin q_1 x_j + C_3 \cosh q_2 x_j + C_4 \sinh q_2 x_j}$$

5 points de mesure  $\rightarrow$  4 amplitudes modales normalisées  $\rightarrow$  4 conditions aux limites

Arrange les équations pour obtenir un problème aux valeurs propres  $\mathbf{S} \mathbf{c} = \mathbf{0}$

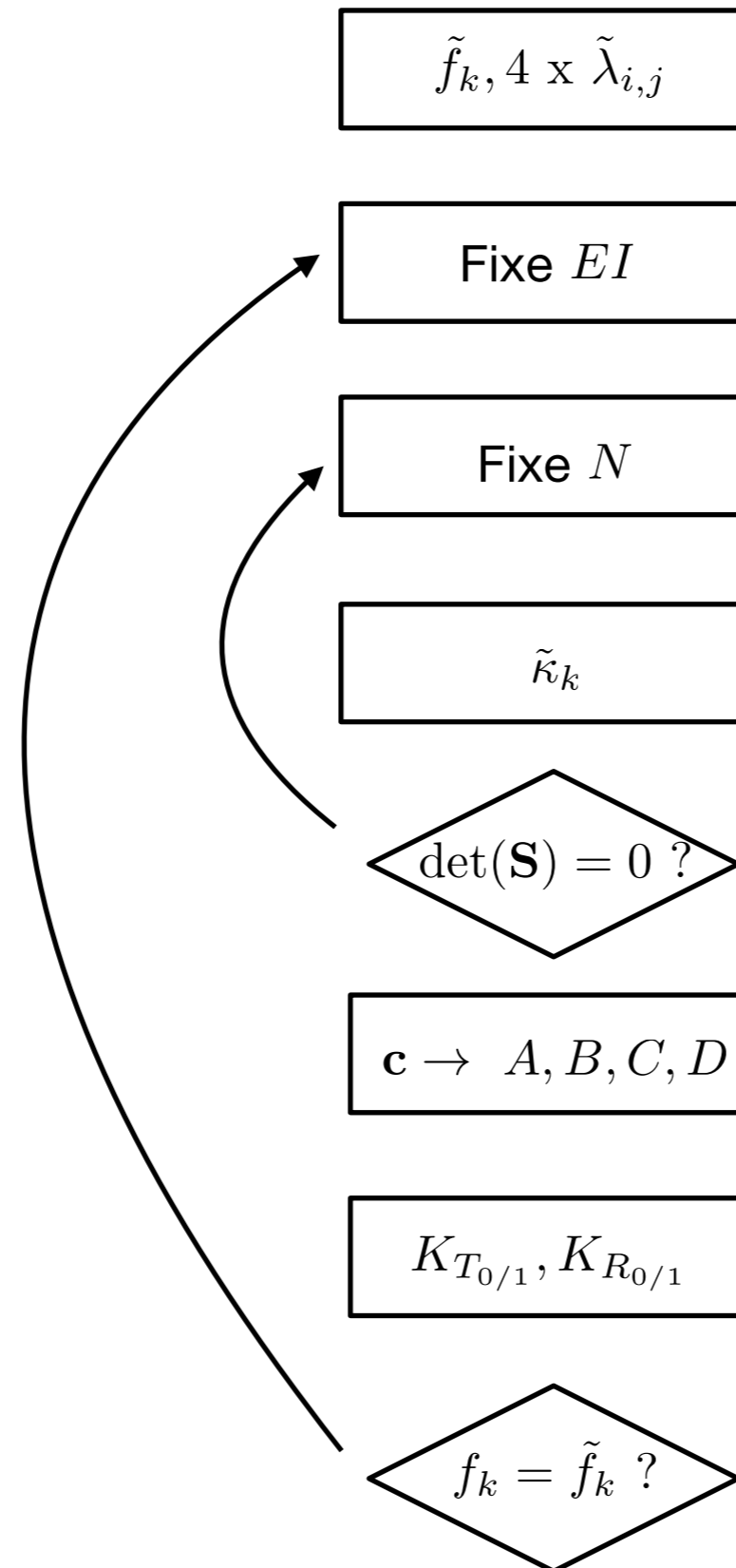
Considèrent qu'ils connaissent EI alors  $\det(\mathbf{S}) = 0 \Leftrightarrow T$

Ils ont des problèmes de convergence lorsque  $\varepsilon \ll 1$

$$\lambda_{ij} = \frac{\phi_i}{\phi_j} = \frac{A \sin(z_1 \xi_i) + B \cos(z_1 \xi_i) + C \exp(-z_2 \xi_i) + D \exp(-z_2(1 - \xi_i))}{A \sin(z_1 \xi_j) + B \cos(z_1 \xi_j) + C \exp(-z_2 \xi_j) + D \exp(-z_2(1 - \xi_j))}$$



# Organigramme



Si les capteurs sont assez proches des appuis !

Thank you !  
Questions ? Comments ?

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