A posteriori verification methodology for astrochronology:
a step further to improve the falsifiability of cyclostratigraphy

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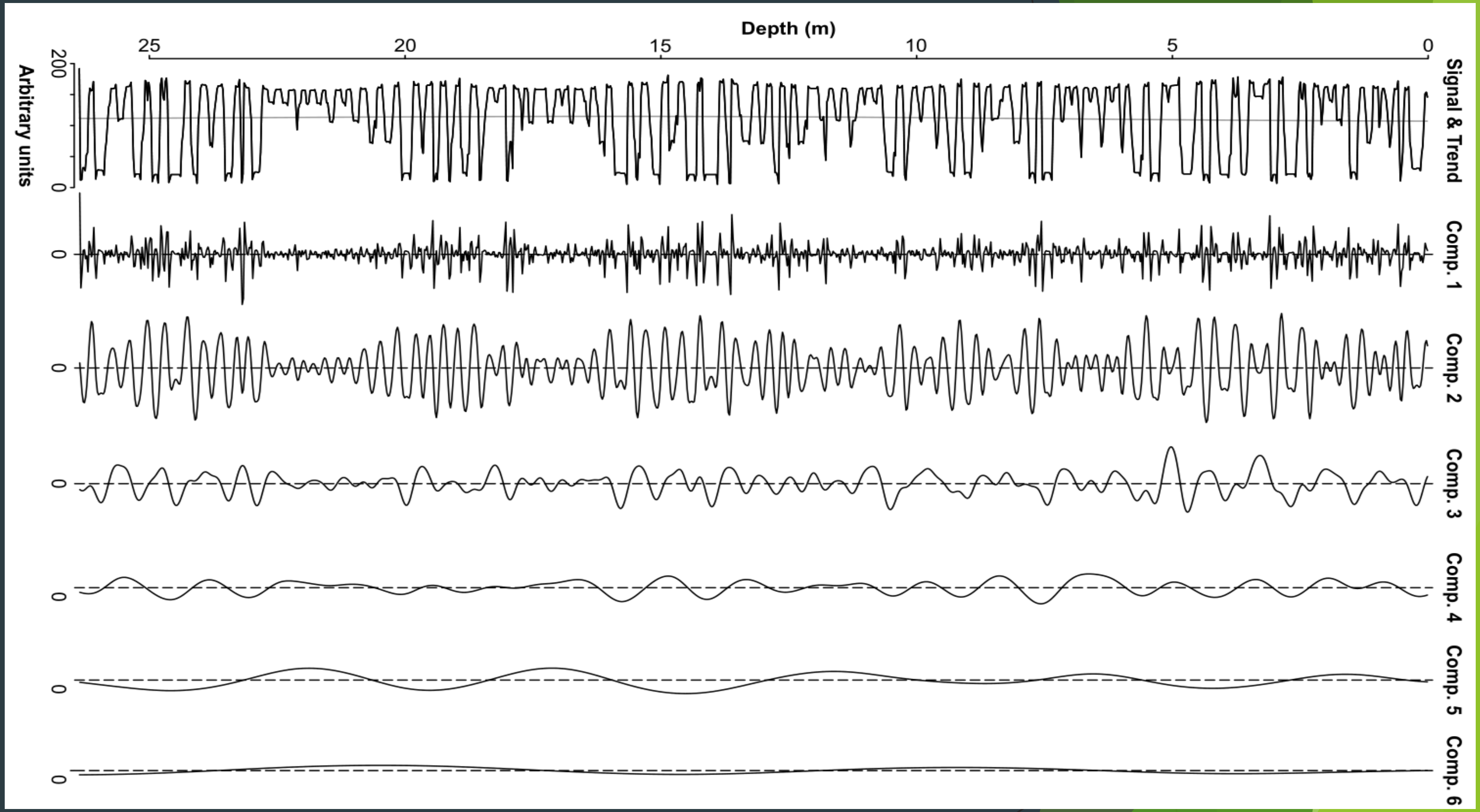
Decomposition for verification

- Astrochronology heavily relies on time-series analysis to detect Milankovitch cycles.

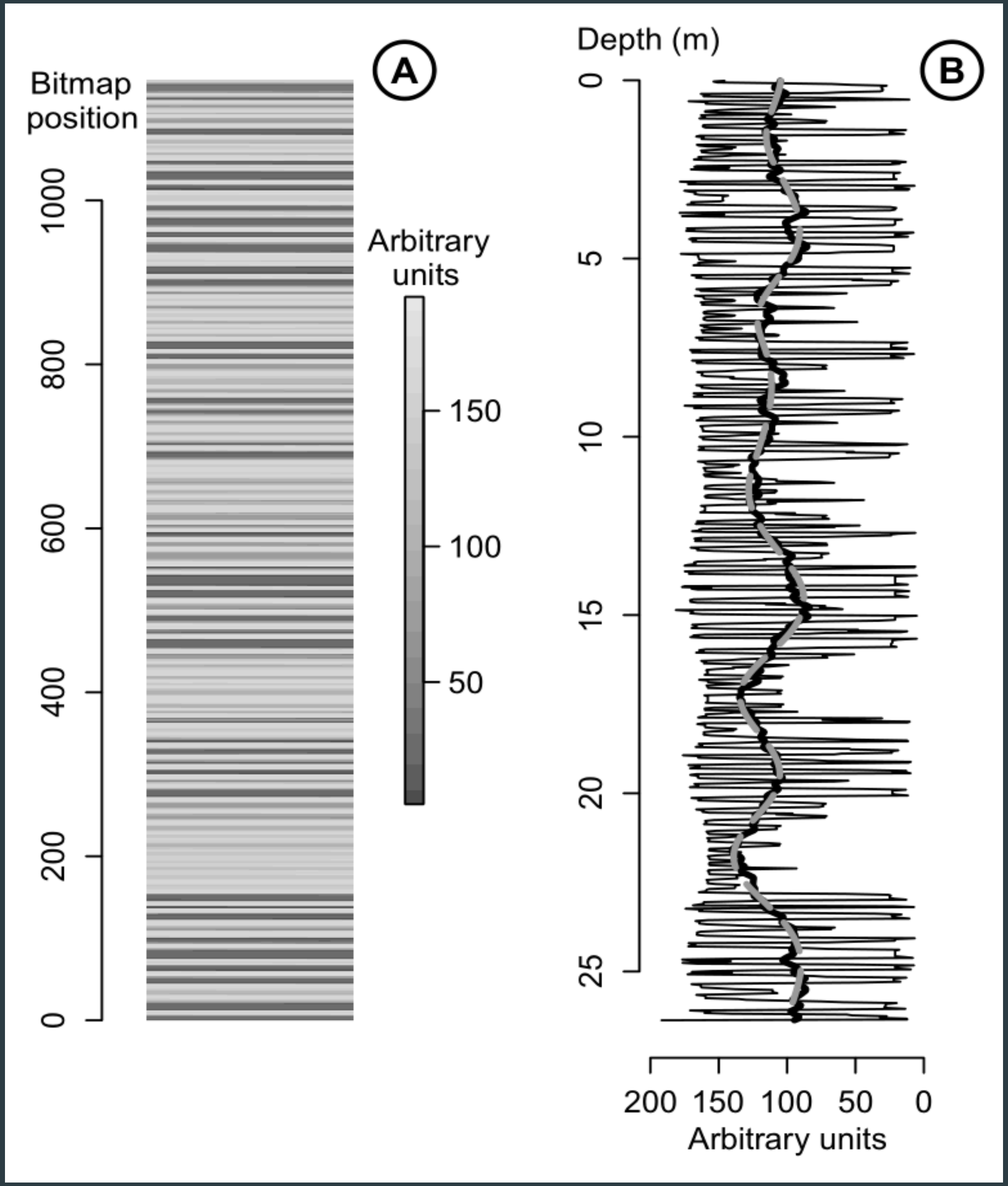
- Different criteria are used during time-series analysis to check whether the cycles can be identified as the Milankovitch cycles. However such verification should also allowed to be made *a posteriori*, i.e. on the final result of the cyclostratigraphic analysis, independently of how it was performed. This would allow scientists to be able to check and reproduce each other results, improving the falsifiability of the discipline.

- Such an *a posteriori* verification can be performed via decomposition.

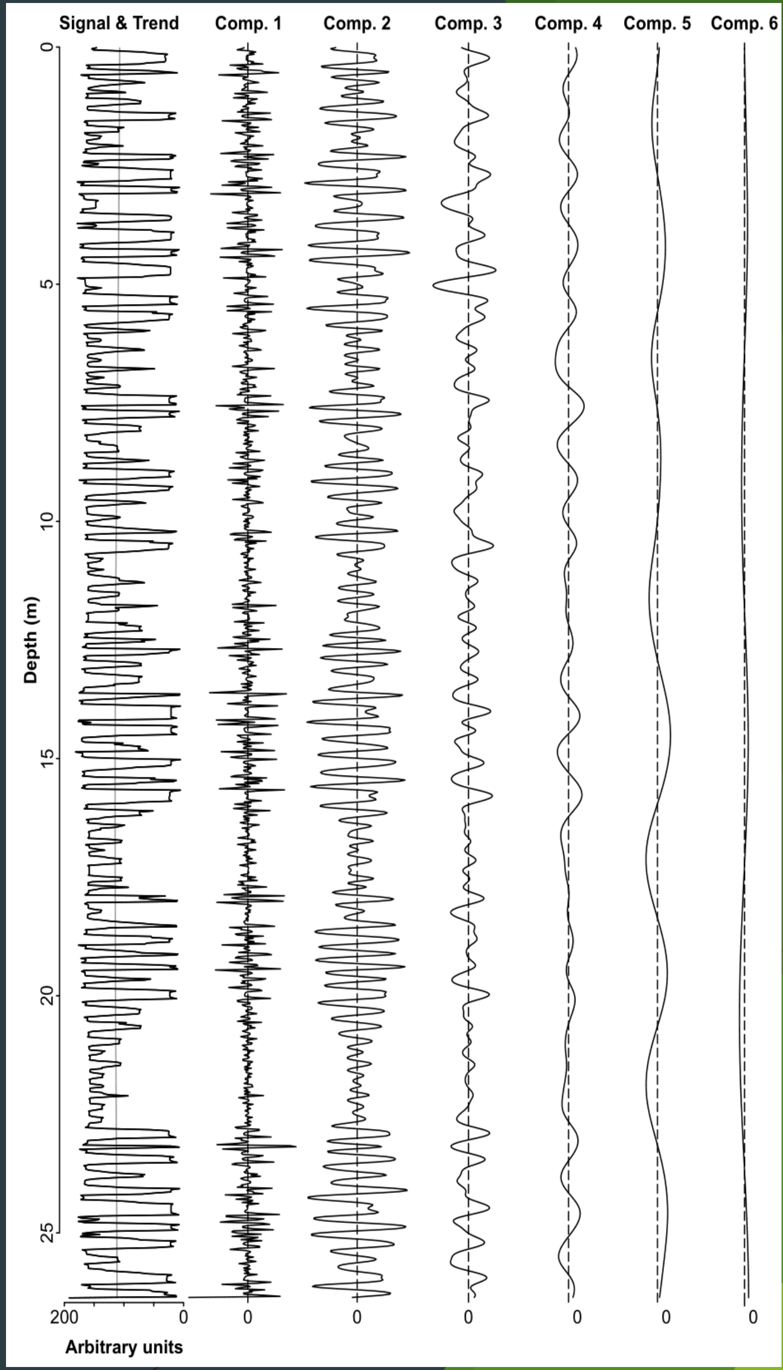
- **Decomposition**: a set of functions that can be added back to reconstitute a given signal, or a procedure to obtain such a set.



Decomposition of the Case 1 of the Cyclostratigraphy Intercomparison Project (Sinnesael et al., 2019)



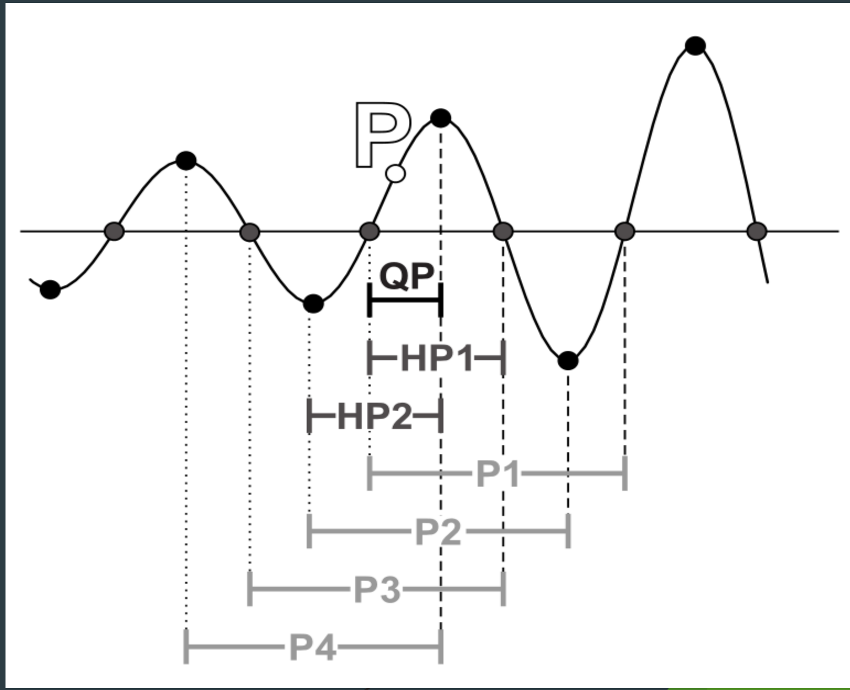
Processing of Case 1 of the Cyclostratigraphy Intercomparison Project (Sinnesael et al., 2019)



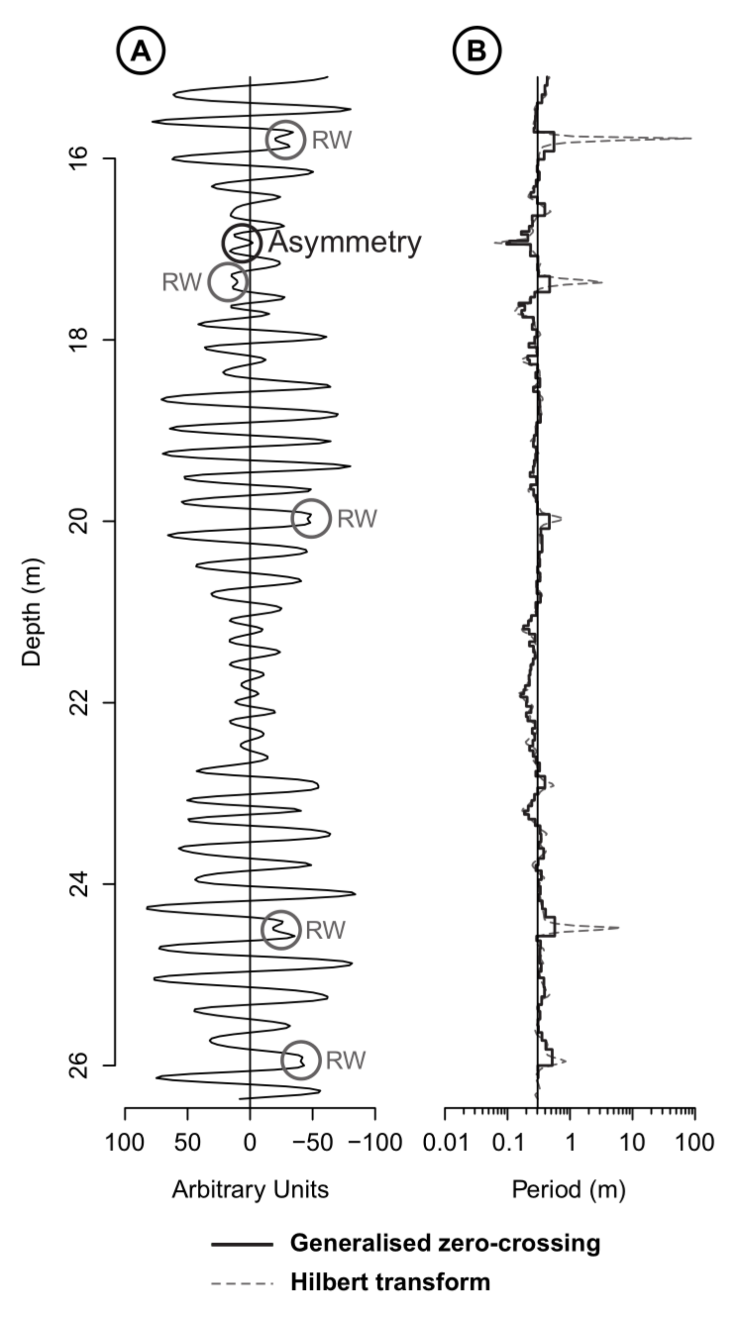
- **If the components of a decomposition are close enough to Intrinsic Mode Functions (IMFs), Instantaneous Frequency (IF) can be computed.**

- **IMF (Intrinsic Mode Function):** function for which (I) zero-crossing and local extrema follow each other in a minima / zero-crossing / maxima / zero-crossing pattern and (II) the upper and lower envelopes, respectively defined by the local maxima and the local minima, cancel each other at any point, meaning that they are symmetrical.

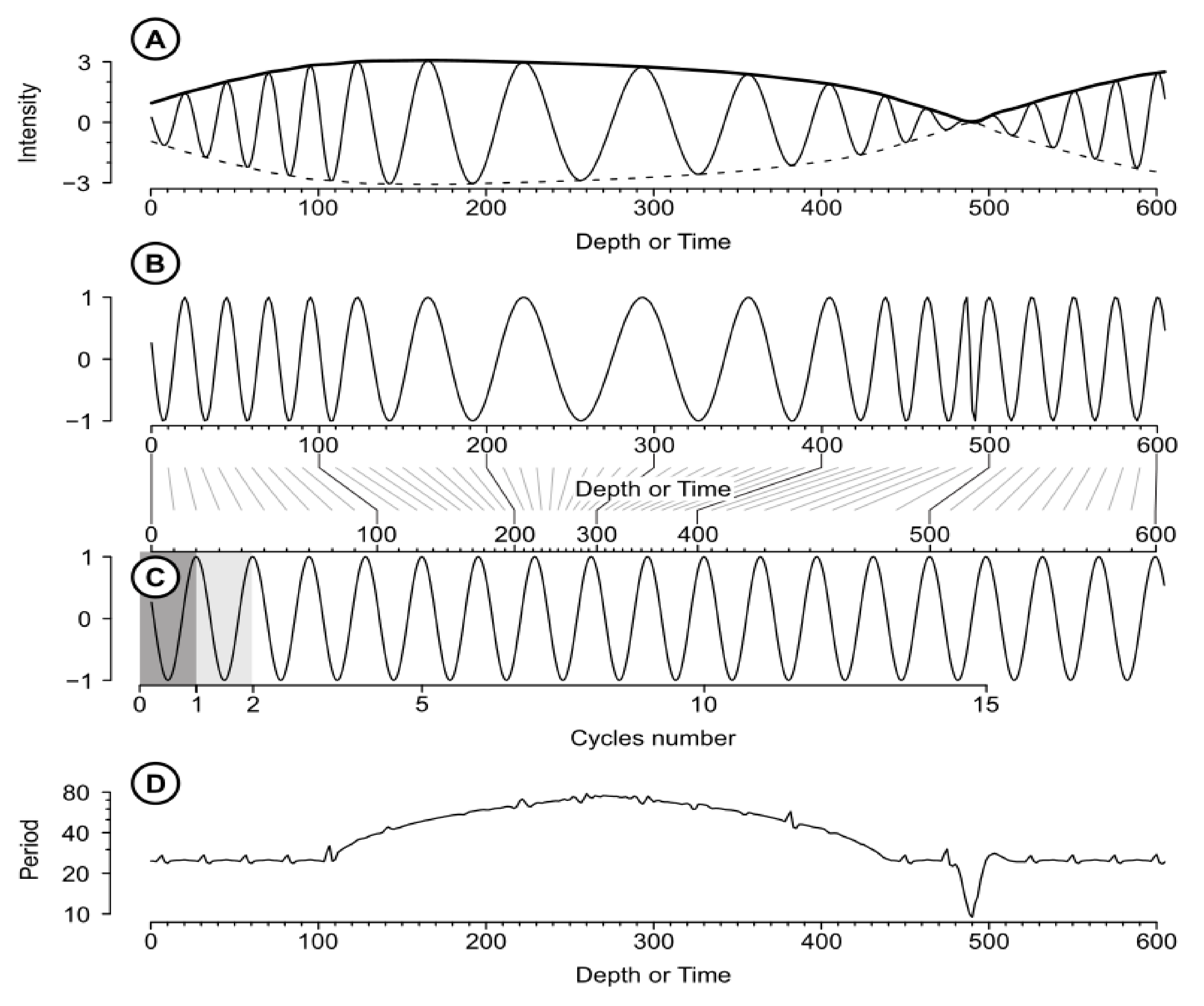
Illustration of the frequency computation by the generalized zero-crossing (GZC) method. For each point P, the frequency can be computed by making a weighted average of the quarter of the cycle period (QP), two half periods (HP1 & HP2), and four full periods (P1 to P4).



- **There are different methods for computing Instantaneous Frequency** (see Huang et al., 2009 for more precision);
  - Generalized zero-crossing (GZC) is the most robust, and can be used as a reference to test other methods.
  - Other methods exist, such as the Hilbert transform. Hilbert transform is more local than GZC.



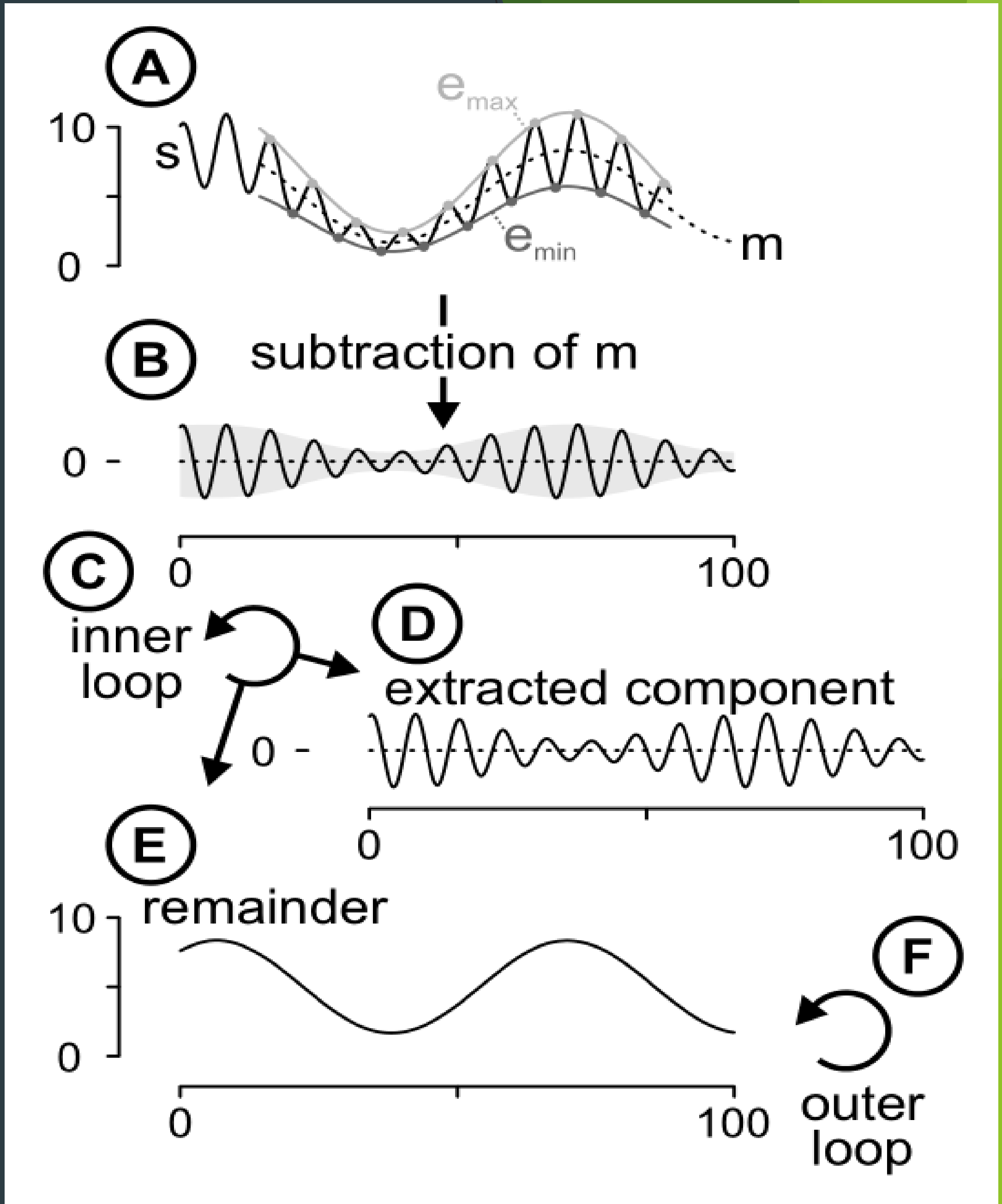
- Instantaneous frequency and amplitude can be understood as frequency and amplitude modulation of a pure sine wave.



Decomposition can be perfomed via Empirical Mode Decomposition:

(A) From the signal [s], the local extrema are used to define upper [emax] and lower [emin] envelopes, and the mean of these envelopes [m]. (B) The mean [m] is subtracted from the signal [s] to obtain a first prototype of component. (C) The process is iterated on the prototype component in the inner loop. (D) From the inner loop is extracted a component. (E) A remainder is defined as the signal minus the extracted component. (F) The remainder is used to perform a new round of component extraction.

But a large set of other methods can be used, such as filtering.



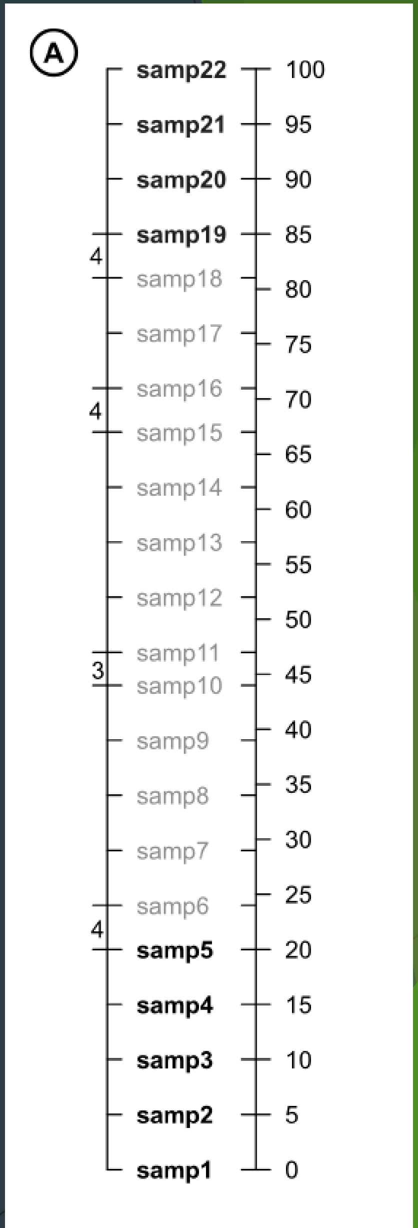
Decomposition for verification

- Decomposition is representative of an entire signal.
- Decomposition can be designed to be meaningful: i.e. by making each component representative of a single process, such as precession, obliquity, white noise,…

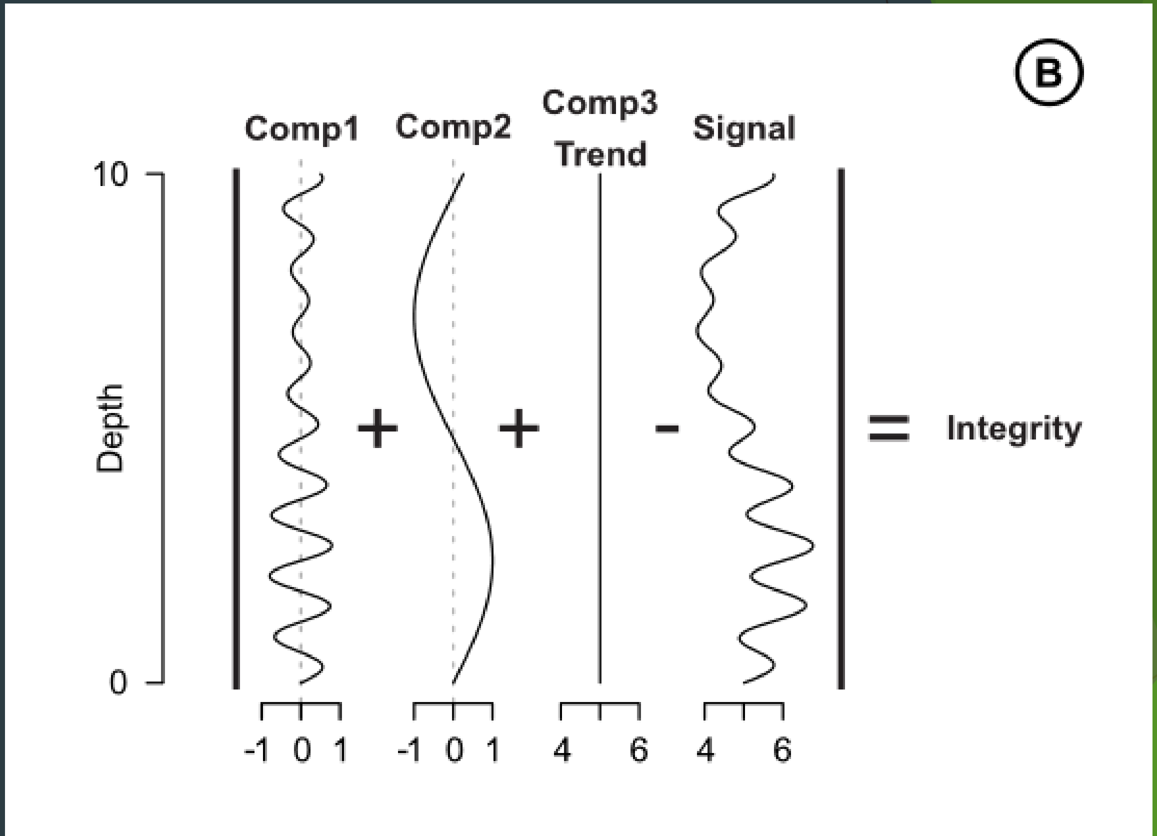
- A meaningful decomposition can serve for comprehensive testing of any interpretation.

- We propose a set of tools to assess the quality of a decomposition.

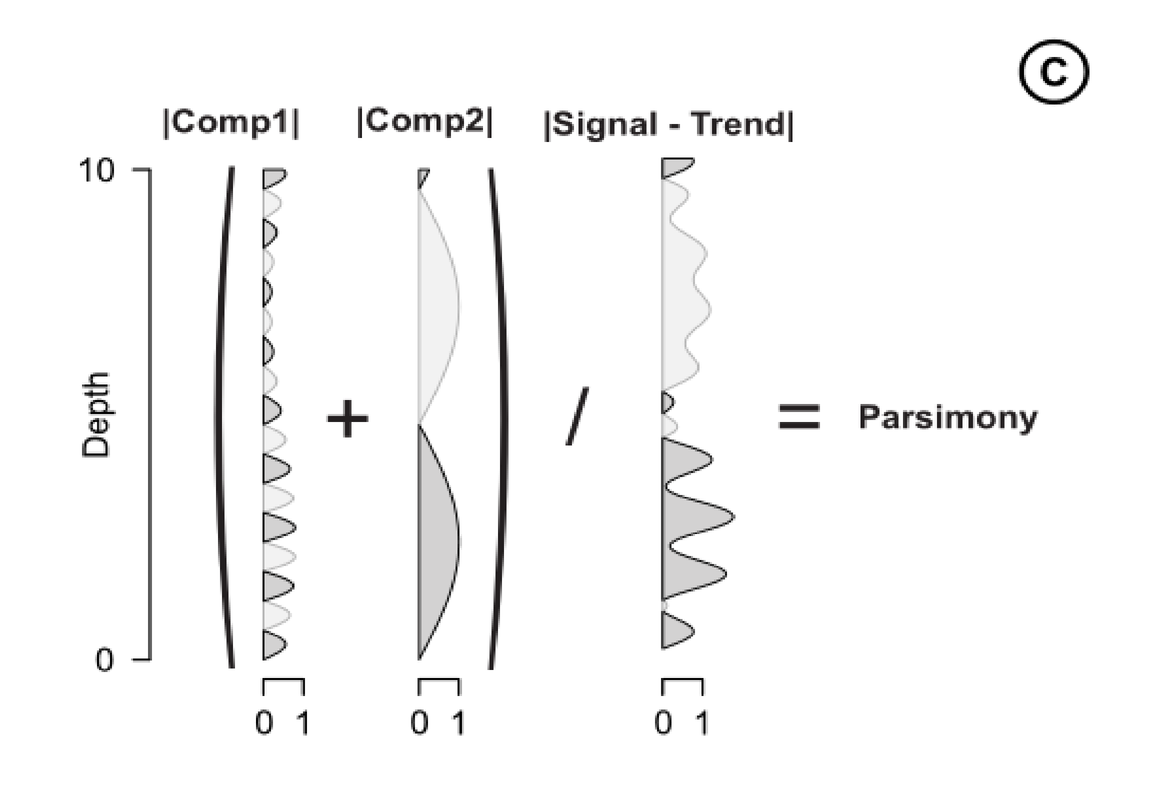
Reversibility is the concept that all initial data points are preserved, even after linear interpolation and tuning. This allows to revert back to the original signal and discuss the significance of each data point. To facilitate reversibility we introduce the concept of quanta (smallest depth or time interval having significance for a given sampling) and an algorithm computing the highest rational common divisor of given values in R: “divisor” (available in the R StratigrapheR package).



Integrity quantifies to what extent the sum of the components is equal to the signal. It is defined as the cumulated difference between (1) the signal, and (2) the summed components of the decomposition. EMD fulfils integrity by design, except for errors caused by floating-decimal arithmetic.

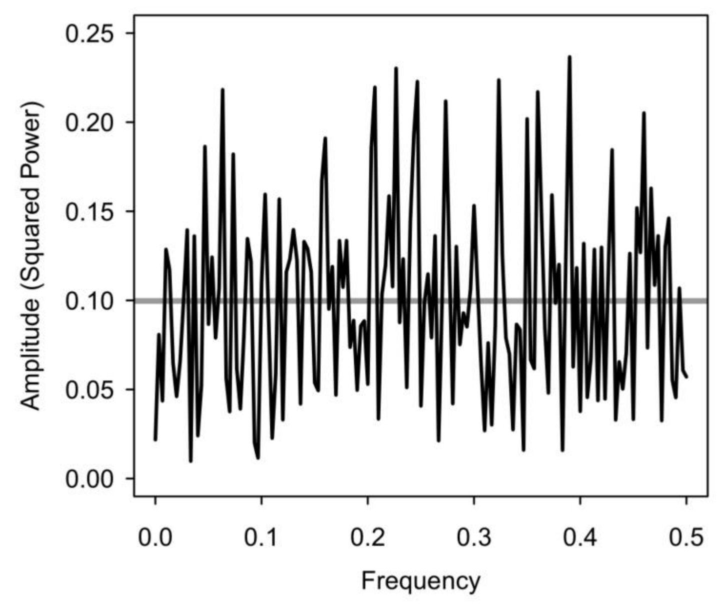


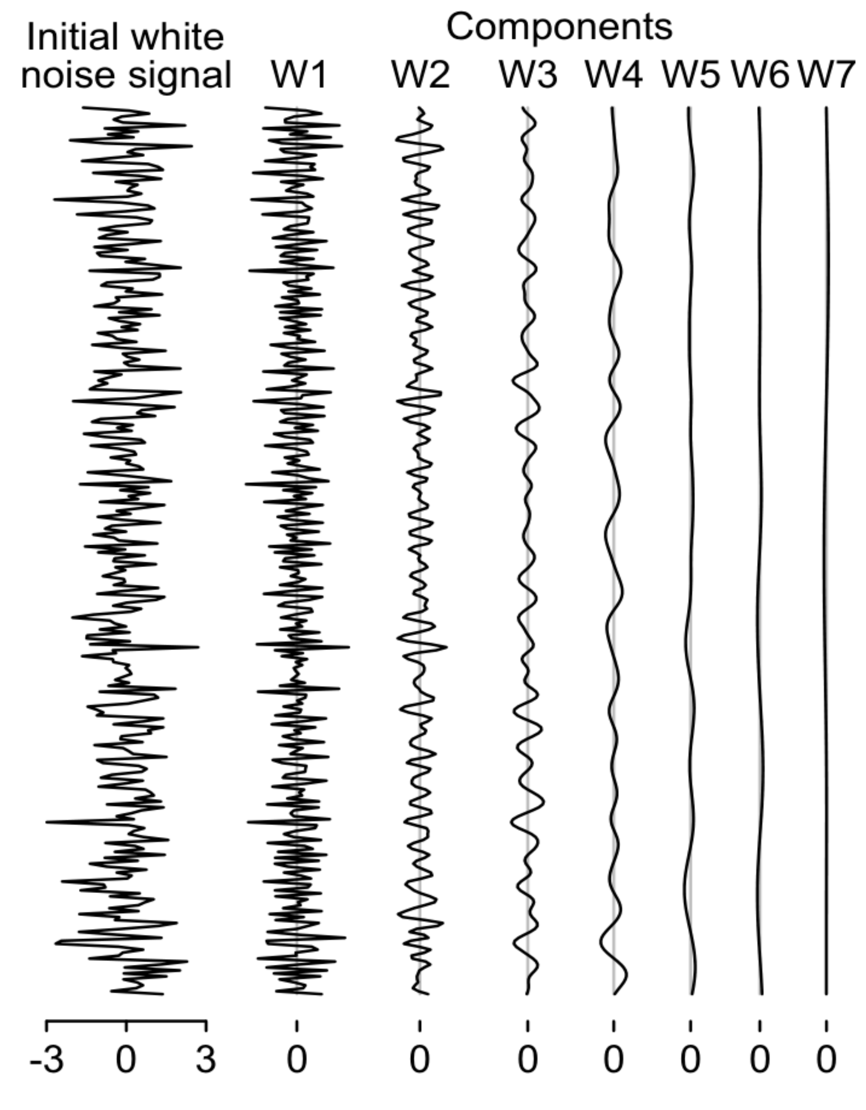
Parsimony checks that the decomposition does not generate components that heavily cancel out. We propose to quantify it as the ratio between (1) the cumulated absolute values of each component (except the trend), and (2) the cumulated absolute values of the signal (minus the trend). The trend should be ignored in the calculation, because an added trend decreases the parsimony estimation of a similar decomposition.



The parsimony of decomposition reveals an interesting distinction between Fourier transform and Empirical Mode Decomposition (EMD). In noisy signals, here a white noise of 300 points, the parsimony of decomposition made via EMD is relatively low, of 1.74 (74% absolute intensity in the decomposition in excess compared to the original signal), compared to a decomposition made by Fourier transform (i.e. made of sines and cosines only) which has a parsimony of 12.26 (1126 % excess absolute intensity).

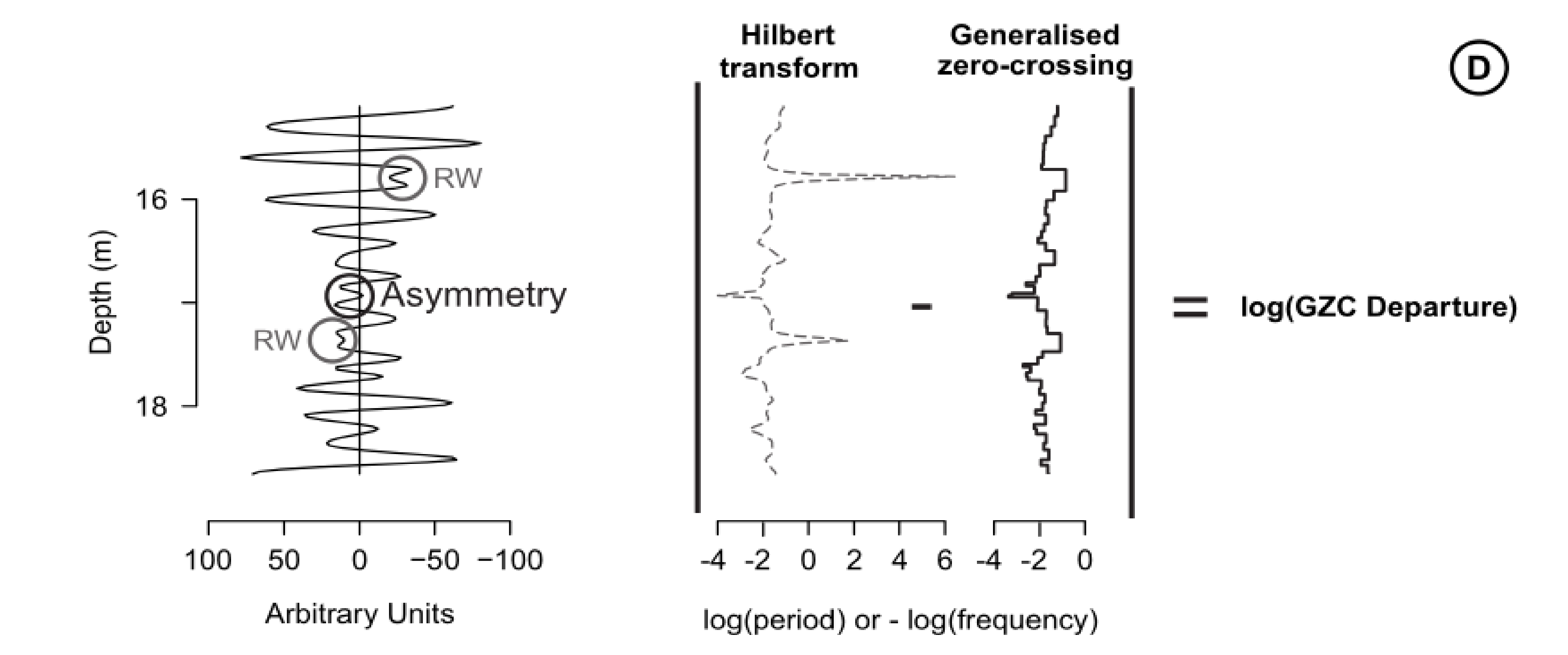
This implies that a lot of cross-cancellation of the sines and cosines occur in Fourier transform. Compared to EMD, this makes Fourier transform theoretically more prone to create artefacts in noisy signals.





Power spectra (obtained via direct Fourier transform) and decomposition (obtained via EMD) of an identical 300 points white noise signal.

GZC departure checks that the Instantaneous Frequency computation is valid, by comparison to the GZC method. It is the exponential of the mean of the absolute differences of the logarithms of frequencies obtained using (I) a robust generalized zero-crossing method (GZC, with an algorithm which simplifies the components into extrema separated by zero-crossings) and (II) a more local method such as the Hilbert Transform (or any method used to compute IF).

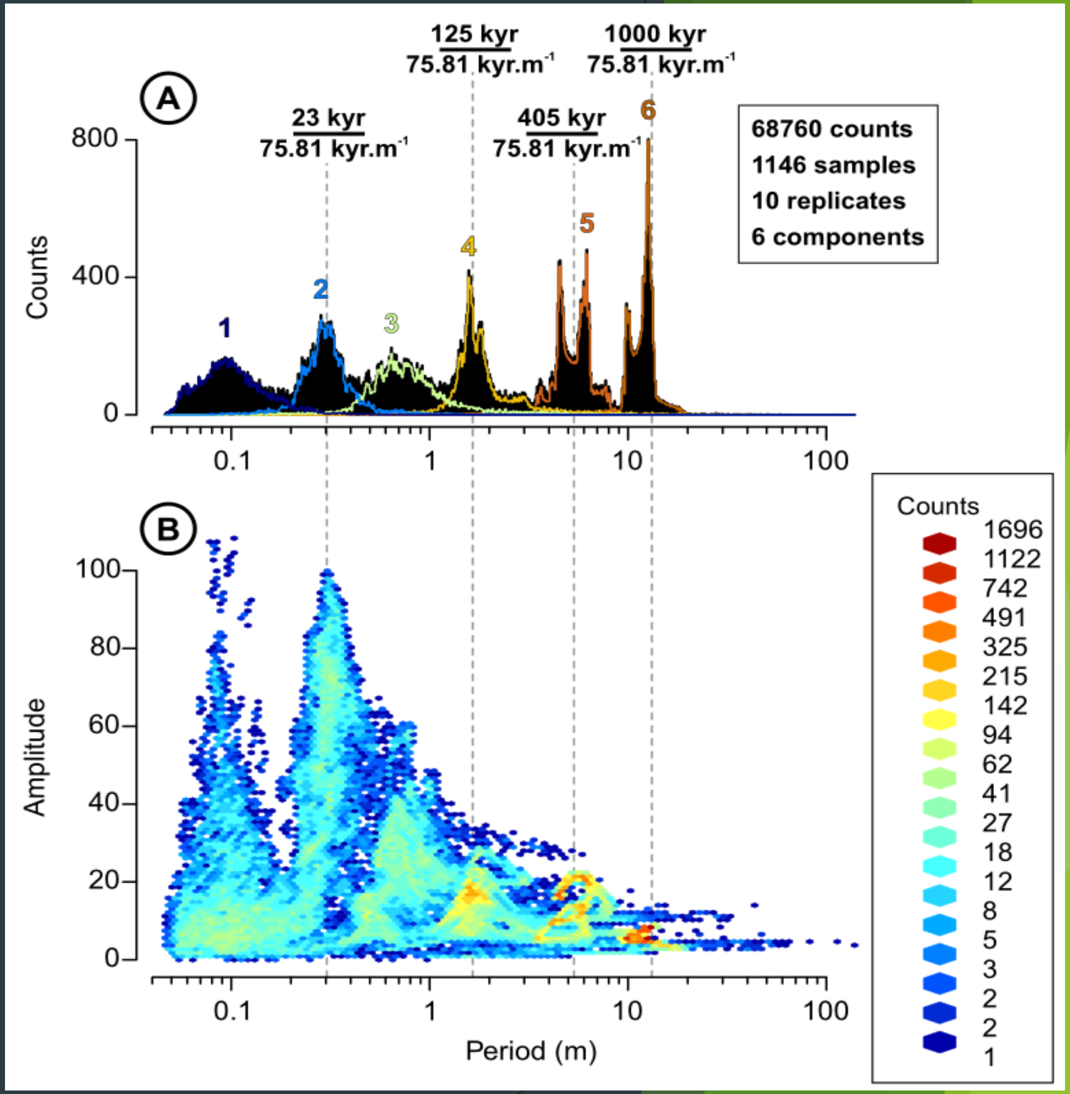


Decomposition for verification

- Reversibility, integrity, parsimony and GZC departure are meant to be general checks for a decomposition.

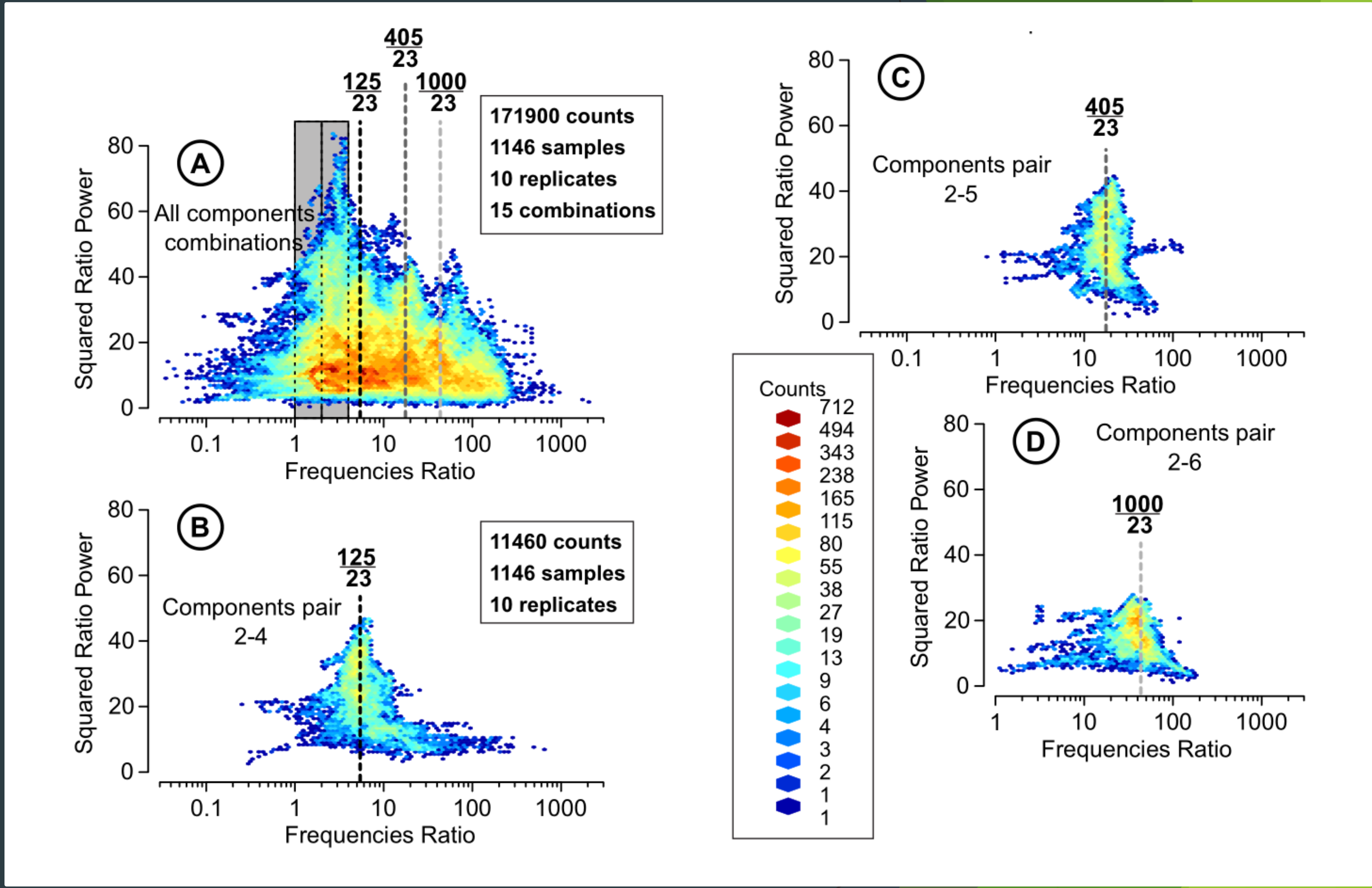
- Further verification can be performed using:
  - The instantaneous frequency of components.
  - The instantaneous amplitude of components.
  - The instantaneous ratio of frequencies of components, taken two by two.
  - Direct comparison of components with reference curves such as the astronomical solutions.

The spectral population plot is a bivariate distribution plot of amplitude against frequency, taken for each data point. It can be used to estimate the main frequencies in the case of relatively stable sedimentation rates.



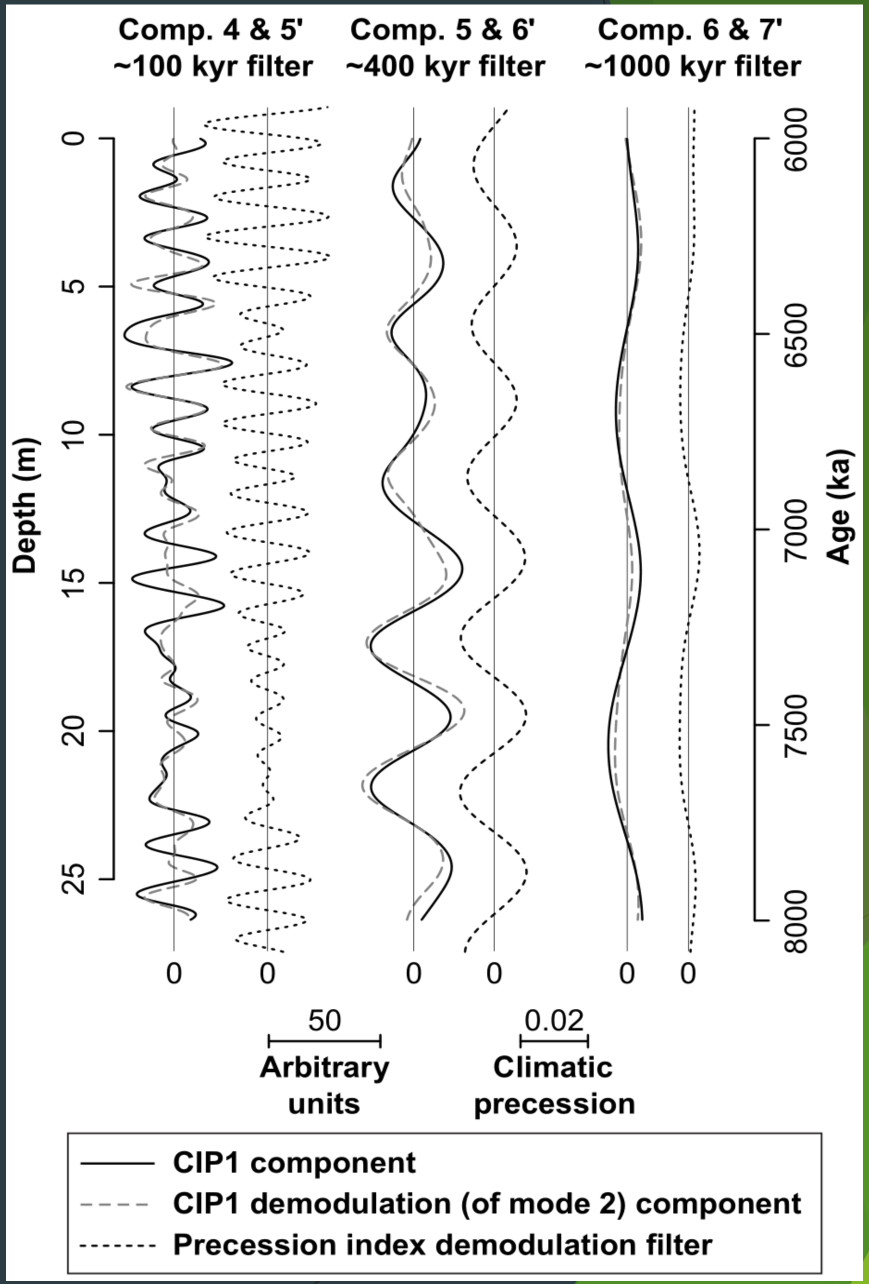
Spectral population of the decomposition of the Case 1 of the Cyclostratigraphy Intercomparison Project (Sinnesael et al., 2019)

The ratio population plot is a bivariate distribution of the squared ratio power (squared root of the amplitudes of the two components multiplied together) against the frequency ratio. Squared ratio power allows to discriminate significant ratios based on amplitude. Separate component pairs can further be isolated in all distribution plots for better visualization (see B, C & D in the figure).



The components can be directly compared to reference curves such as the astronomical solutions.

Instantaneous amplitude can easily be extracted, and decomposition can be applied on it to check the amplitude modulation that is expected from certain astronomical parameters.



Conclusion

- Decomposition is a versatile verification tool for astrochronology: it has the potential of combining several verification tools (spectral population, ratios of frequency, comparison to astronomical solution, extraction of amplitude) in one.

- A decomposition can be obtained via EMD, but also more classical filtering methods deriving for instance from Fourier transform. However Fourier transform decompositions seem to be less parsimonious, which would lead to more artefacts.

References

- Huang, Norden E., Zhaohua Wu, Steven R. Long, Kenneth C. Arnold, Xianyao Chen, and Karin Blank. 2009. 'On Instantaneous Frequency'. *Advances in Adaptive Data Analysis* 01 (02): 177–229. https://doi.org/10.1142/S1793536909000096.
- Sinnesael, Matthias, David De Vleeschouwer, Christian Zeeden, et al. 2019. 'The Cyclostratigraphy Intercomparison Project (CIP): Consistency, Merits and Pitfalls'. *Earth-Science Reviews* 199 (December): 102965. https://doi.org/10.1016/j.earscirev.2019.102965.