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Sparks as radioemitters

Quand une étincelle servait de radioémetteur

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Abstract. In the early experiments on radiocommunication, Hertz, Branly, Lodge, Marconi used as emitter a spark produced by a high electric tension. Why did this spark emit an electromagnetic radiation? The phenomenon is well understood in the case of the device used by Hertz in 1887. It is not so clear in the device used by Branly in 1890. Hertz used a set of two big metallic spheres (or other big objects) as a capacitor whose discharge produced a spark in a burster. It is argued that in Branly's device the spheres of the burster are also a rather efficient capacitor. The main differences with Hertz's device are: (i) the frequency spectrum is much broader and higher; (ii) the capacity is about ten time smaller; (iii) a quantitative analysis, if feasible, would be very difficult.

Résumé. Les pionniers des Radiocommunications (alors appelées Télégraphie sans fil) utilisaient comme émetteur une étincelle produite par une haute tension électrique. Pourquoi cette étincelle émettait-elle une onde électromagnétique de haute fréquence? Le phénomène est bien compris dans le cas du dispositif utilisé par Hertz en 1887. Il l'est moins dans le cas du dispositif utilisé par Branly en 1890. Hertz utilisait une paire de grosses sphères métalliques (ou d'autres objets volumineux) comme condensateur, dont la décharge oscillante produisait une étincelle dans un éclateur. Nous suggérons que dans le dispositif de Branly les sphères de l'éclateur constituent aussi un condensateur assez efficace. Les principales différences avec le dispositif de Hertz sont que (i) le spectre de fréquence est bien plus large et plus élevé. (ii) La capacité est environ dix fois plus faible. (iii) Une analyse quantitative, si elle est possible serait fort difficile.

Keywords. Electromagnetism, Radiocommunication, Discharge, Self induction, Hertzian waves.

Mots-clés. Electromagnétisme, Radiocommunication, Décharge, Self induction, Ondes hertziennes.

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1. The birth of radiocommunications

In the year 1887, Heinrich Hertz demonstrated the possibility of emission and reception of electromagnetic waves [1]. The emitter was a linear conductor joining two big metallic objects (for instance two identical spheres of diameter 30 cm) and interrupted by a gap of width a few

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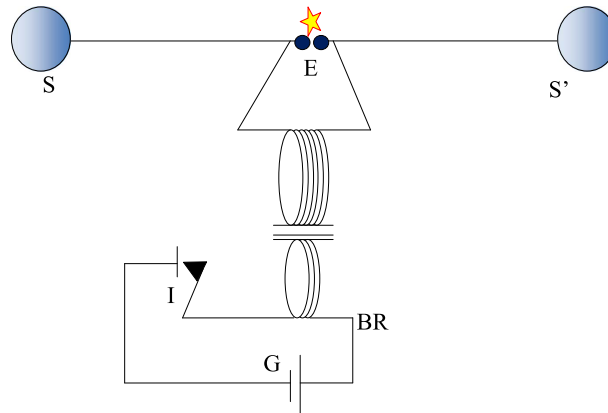


Figure 1. Hertz's emitter of electromagnetic radiation. When the switch I closes the primary circuit, the induction coil BR generates a high electric tension which charges the metallic spheres S and S' until a spark bursts in the gap E.

millimeters (Figure 1), in which a burster was inserted. In the “most appropriate device”, the burster consisted of two metallic spheres of diameter 3 cm at a distance of 7.5 mm. A high tension was applied through an induction coil (Ruhmkorff coil) and a spark short circuited the gap. The metallic spheres discharged in an oscillatory way and the frequency was sufficiently high to produce the emission of electromagnetic waves of wavelength a few metres.

Three years later, in 1890, Edouard Branly made a somewhat similar experiment [2], which was important because the detector was very different from that used by Hertz, and more efficient. However what is to be discussed in the present article is not the detector, but the emitter. Branly's emitter, as Hertz's one, contained a burster in which sparks were produced either by a small Wimshurst machine, or by a Ruhmkorff coil, or by another device which will be described later. The experiment using a Ruhmkorff coil has been reproduced recently by Dorbolo *et al.* [3] and is shown by Figure 2.

Why is a high frequency electromagnetic radiation emitted? In Hertz's experiment, it is clear: the electric discharge is oscillatory because the electric line which joins the spheres has a self induction coefficient L and a low electrical resistance, so that the tension between the two ends of the gap is $V = LdI/dt$ while the electric charge of each sphere of capacity C is $Q = CV$. Since the intensity is $I = -dQ/dt$, the equation $Ld^2Q/dt^2 = -Q/C$ follows. The solution is an oscillatory discharge with frequency

$$\omega = 1/\sqrt{LC}. \quad (1)$$

In the case of Branly's experiment the situation is not so clear. Branly himself did not care about electromagnetic wave and just observed that, when the spark burst, the electrical resistance of a metallic powder in the room near by dropped. Apparently the question has not been clarified. Jean Cazenobe, a historian of science, specialist of electromagnetism, even doubted that Branly produced electromagnetic waves. “We cannot absolutely deny the presence of electromagnetic waves in the experiments of 1890; and it is temptful to affirm it” (*nous ne pouvons nier absolument la présence d'ondes électriques... dans les expériences de novembre 1890; et nous sommes tentés de l'affirmer* [4]). Olivier Darrigol, another historian of science, author of a book on the history of electromagnetism [5], has a similar opinion and does not believe that the device of Figure 2 can emit electromagnetic radiations [6]. On the other hand, Branly's observations [2, 7] can hardly be explained without assuming the emission of electromagnetic waves.

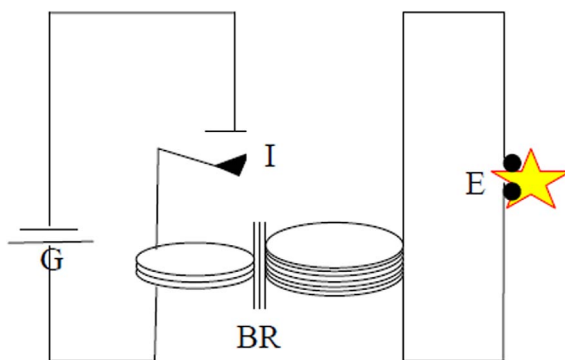


Figure 2. The emitter of electromagnetic radiation used by Dorbolo *et al.* [3] and presumably by Branly. The same as Hertz's device except that the metallic spheres S and S' are lacking.

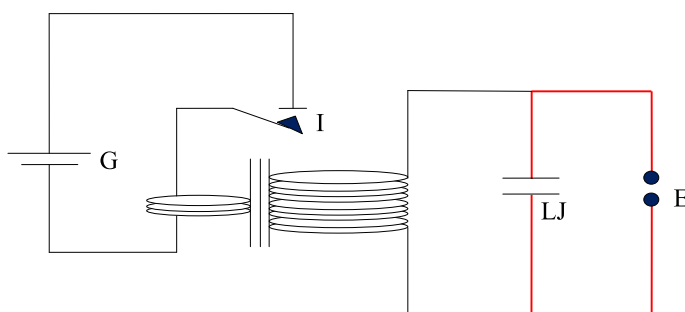


Figure 3. An emitter of electromagnetic radiation, presumably used by Branly. The same as Figure 2 except that the Leyden jar LJ has been inserted. The high frequency current is restricted to the red part on the right.

In the next sections it will be argued that the mechanism which has been described in the case of Hertz's device may also work in the case of Figure 2. Indeed the burster, as described above, has a non-negligible electric capacity C , even though smaller than Hertz's big spheres, and it also has a small self induction coefficient L , as well as the spark. It will be argued that the above argument may be valid, taking into account the capacity of the burster and the self induction coefficient of both the spark and the burster.

However, Figure 2, which is taken from the paper by Dorbolo *et al.* [3], is not the only device used by Branly. In his note [2,7] he wrote "I use either a small Wimshurst machine, or a Ruhmkorff coil, or the device which I already used earlier". He referred to two of his previous paper, in which no figure can be found. However in Branly's notebook a Leyden jar (i.e. a capacitor) is often mentioned, and his favourite device seems to have been that of Figure 3.¹

In Section 2 the relevant equations are written. In the articles that we could find [1, 5, 8], the discharge line (SS' in Figure 1) is treated as disconnected from the induction coil. An analogous approximation for the device of Figure 2 is proposed in Section 3, where the burster is shown

¹Cazenobe [4] gives the following description of Branly's spark generator: "*Le schéma théorique du montage peut être décrit comme une association de deux circuits couplés électriquement par l'intermédiaire d'un élément commun : le circuit du secondaire de la bobine et celui de l'éclateur à pointes étant tout deux fermés sur un unique condensateur en forme de jarre.*" This description corresponds to Figure 3.

to behave as an independent system with a good approximation during the discharge. Thus, the discharge of the spheres is oscillating if the resistance R of the spark is small enough. The resistance of a spark has been determined a long time ago [9, 10] and is between 1 and 5 Ω . In Section 4 the values of C and L are discussed. They can be evaluated with a good accuracy in the case of Hertz's device, while in the case of Figure 2 one can only guess plausible orders of magnitude. In Section 5 the power respectively emitted as Hertzian radiation and Joule effect (including light emission) is discussed, and our picture is found to be plausible. The calculation of the current in the spark ignores the energy loss resulting from Hertzian emission, which is then deduced from the calculated intensity. This procedure follows the usual presentation [1, 5, 8] but clearly lacks self-consistency. A self-consistent method is proposed in Section 6 and qualitatively gives the same results.

In the following, the orders of magnitude of L , C is discussed. Because R does not vanish, the oscillations are damped, but they are also damped for another reason, i.e. because the oscillatory discharge generates electromagnetic waves. Thus, the above equation $Ld^2Q/dt^2 = -Q/C$ is only approximate. However it is a good approximation.

2. Equations of the electrical network

The induction coil includes a primary circuit with a generator G which maintains a constant tension V_1 , and a switch I which closes the circuit at time $t = 0$, and a secondary circuit at the ends of which the tension is $V_2(t)$. The intensities $i_1(t)$ and $i_2(t)$ in the primary and secondary circuits are related to V_2 by the equations

$$L_1 \frac{di_1}{dt} + R_1 i_1 + M \frac{di_2}{dt} = V_1 \quad (2)$$

and

$$L_2 \frac{di_2}{dt} + R_2 i_2 + M \frac{di_1}{dt} + V_2 = 0 \quad (3)$$

where R_1 and R_2 are the resistances of the two circuits, L_1 and L_2 are their self induction coefficients and M is the mutual induction coefficient. The coefficients R_1 , R_2 , L_1 and L_2 are positive, and M will also be taken positive, and can be made such by choosing an appropriate orientation of the circuits.

The potential V_2 in (3) is given by different formulae in the cases of Figures 1, 2 and 3. The case of Hertz's experiment (Figure 1) has been discussed in Section 1, ignoring the capacity of the burster. The case of Figure 3 is analogous. In this section and in the following one the device of Figure 2 will be considered. The charges Q and $-Q$ of the spheres of the burster (which can no longer be neglected) are given by

$$Q(t) = CV_2(t) \quad (4)$$

where C is the capacity of the pair of spheres.

The current through the gap satisfies the equation

$$V_2 = LdI_2/dt + RI_2 \quad (5)$$

where $R = \infty$ when there is no spark, so that $I_2 = 0$. When there is a spark, R is its resistance and L its self induction coefficient. R and L are assumed to be constant, which is reasonable if the discharge current has such a high frequency that R and L do not change much on a period. We shall come back to this point in Section 4.

Although the spark closes the circuit, the intensity is not uniform in that circuit. The intensity I_2 in the spark is higher than the intensity i_2 in the secondary circuit of the induction coil. This allows for the discharge according to the equation

$$dQ/dt = i_2 - I_2. \quad (6)$$

Thus there are 5 equations (2)–(4) for 5 quantities i_1 , i_2 , I_2 , V_2 and Q . These equations correspond to the description made in the literature [1, 4, 5] but they are not quite satisfactory. Indeed, as will be seen, they do not take into account the damping which results from the emission of hertzian radiation. This defect will be corrected in Section 5.

The features of the solution (e.g. oscillating discharge or not) depend on the parameters L_1 , L_2 , L , R_1 , R_2 , R , C . The resistances R_1 , R_2 , R will be assumed to be small (in the case of R , this assumption will be discussed later).

If the gap were too wide, the spark would never burst and at long time one would reach a stationary state with $i_2 = I_2 = 0$ and $i_1 = V_1/R_1$. However the case of interest is when the term $R_1 i_1$ is small in (2). An appreciable algebraic simplification is obtained if this term is neglected. Then i_1 can be eliminated between (2) and (3), yielding

$$L'_2 di_2/dt + R_2 i_2 + M V_1/L_1 + V_2 = 0 \tag{7}$$

where

$$L'_2 = \frac{L_1 L_2 - M^2}{L_1}. \tag{8}$$

3. Oscillating discharge

Elimination of V_2 and Q between (4)–(6) yields

$$LC \frac{d^2 I_2}{dt^2} + RC \frac{dI_2}{dt} + I_2 = i_2. \tag{9}$$

At $t = t_0$ the resistance R of the spark is assumed to drop suddenly from infinity to a small, constant value R .

It will be argued that, for realistic values of the parameters the right hand side i_2 can be replaced by a constant $i_2(t_0)$, where t_0 is the time when the spark bursts. Before giving the argument, the solution of (9) in this approximation will be described. It is oscillatory if, for $t > t_0$,

$$R < R_c = 2\sqrt{L/C} \tag{10}$$

and the solution of (9) is

$$I_2 = \lambda \sin \omega t \exp(-t/\tau) + i_2(t_0) \tag{11}$$

where

$$\tau = 2L/R, \tag{12}$$

$$\omega^2 + 1/\tau^2 = 1/LC \tag{13}$$

and λ is determined by the initial conditions at $t = t_0$. Since V_2 and I_2 must be continuous, it follows from (5) that $\lambda = V_2(t_0)/(L\omega)$.

To justify the approximation $i_2(t) = i_2(t_0)$, Equation (7) will be used. Combined with (5) it yields

$$L'_2 di_2/dt + R_2 i_2 + L dI_2/dt + R I_2 + M V_1/L_1 = 0. \tag{14}$$

The set of equations (14) and (9) determines i_2 and I_2 . The solution has the form

$$I_2 = \lambda \sin(\omega t - \varphi) \exp(-t/\tau) + \eta \exp(-t/\tau_2) + A \tag{15}$$

$$i_2 = \lambda' \sin(\omega t - \varphi') \exp(-t/\tau) + \eta' \exp(-t/\tau_2) + A \tag{16}$$

where $A = M_1 V_1/[L_1(R + R_2)]$ and the other parameters can be determined by insertion of (15) and (16) into (14) and (9), and by the initial conditions. In particular, η'/η and τ_2 are given by the two equations

$$\frac{1}{\tau_2} = \frac{R_2 + R\eta/\eta'}{L'_2 + L\eta/\eta'} \tag{17}$$

which results from (14), and

$$\frac{\eta'}{\eta} = 1 + \frac{LC}{L\tau_2^2} - \frac{RC}{L\tau_2} \quad (18)$$

which results from (9). The case of interest is when R and R_2 are small ($R \ll L\omega_0$, in agreement with (10), and $R_2 \ll L_2\omega_0$, with $\omega_0 = 1/\sqrt{LC}$). For the sake of simplicity the equations for ω and λ'/λ will be given in the limit $R = R_2 = 0$. They are

$$\frac{\lambda'}{\lambda} = -\frac{L}{L_2'} = 1 - LC\omega^2. \quad (19)$$

Since L/L_2' is small, it follows from (17), (18) and (19) that λ'/λ is small, η'/η is close to 1, and τ_2 is much larger than ω .

It can be concluded that, at the right hand side of (16), the first term is small and the second one is nearly constant as I_2 oscillates. Formula (11) is therefore a good approximation.

4. Orders of magnitude

However, the previous results only hold if the resistance R of the spark and its self induction coefficient L satisfy the inequality (10). In this section it will be argued that it is so.

The potential of an isolated, metallic sphere of radius ℓ and charge Q is $Q/(4\pi\epsilon_0\ell)$. For a pair of remote spheres the difference in electric potential is therefore $Q/(2\pi\epsilon_0\ell)$ and the capacity is

$$C = 2\pi\epsilon_0\ell. \quad (20)$$

This formula should be a correct order of magnitude even for the spheres of the burster which are close together. The value (20) is quite low and the energy $CV^2/2$ seems hardly compatible with an visible spark and with the emission of observable Hertzian radiation. Indeed the spark bursts for an electric field of about 3000 V/mm [11], which corresponds to a maximum tension between 3000 and 20000 V. What is perceived as a single spark is presumably a succession of sparks corresponding to successive discharges separated by time intervals during which the burster does not conduct and the spheres are charged.

The evaluation of the self induction coefficient L (when the burster is conducting) is easy in the case of Hertz's device (Figure 1). It is essentially the self induction coefficient of the wire joining the two big spheres, which has a length D and a diameter $d \ll D$. Assuming this wire to be straight, L can be found in a paper from 1908 [12]. The formula is²

$$L = \frac{\mu_0}{\pi} kD \quad (21)$$

where

$$k = \ln \frac{4D}{d} - \alpha \quad (22)$$

and α depends on the distribution of the current inside the wire. At low frequency the distribution is uniform and $\alpha = 0.75$. At high frequency, the skin effect is strong and $\alpha = 1$. From now on, α will be neglected, as is correct for a long wire.

From formulae (1), (20) and (21) one obtains the frequency

$$\omega = \frac{c}{\sqrt{2k\ell D}}. \quad (23)$$

Formula (21) is in agreement with the observation that L/μ_0 has the dimension of a length. A reasonable guess is therefore

$$L = \mu_0 D f(D/d) \quad (24)$$

where f is an unknown function.

²A factor μ_0/π has been introduced instead of a factor 2 in consistency with the international system of units.

In the case of Figure 2 one can try to make a guess using dimensional arguments. The lengths in this device are the length of the spark, the width of the spark, and the diameter of the spheres of the burster. In his note of 1890 [2] Branly says nothing about the burster. As seen in Section 1, Hertz was more explicit. In his burster all relevant lengths have the same order of magnitude of 1 cm. In a note published in 1891 [7], Branly gave a more precise description of his burster, which consisted of two spheres (whose diameter was not given) at a very small distance, typically 1 mm. A short distance has the advantage of minimizing the resistance.

It will be assumed that there is a single typical length ℓ and that, similarly to (24) the order of magnitude of L is

$$L \approx \frac{\mu_0}{2\pi} \ell \tag{25}$$

where the factor $1/2\pi$ has been introduced in analogy with (21).

Using formulae (20), (21) and (24) one obtains the orders of magnitude of C and L in Hertz's device (Figure 1) and in Branly's device (Figure 2). The frequency can then be deduced from (1).

Hertz's device (big spheres present)

Hertz has used various devices. Here the dimensions will be chosen as those indicated by Joubert [8], namely

$$D = 1 \text{ m}; \quad 2\ell = 0.3 \text{ m}; \quad d = 5 \text{ mm}.$$

Formula (20) yields

$$C \approx 0.83 \times 10^{-11} \text{ s}/\Omega$$

and formula (21) (with $\alpha = 0$) yields

$$L \approx 2.27 \times 10^{-6} \Omega \cdot \text{s}$$

so that formula (1) yields

$$\omega \approx 2.3 \times 10^8 \text{ s}^{-1}. \tag{26}$$

This value corresponds to a wavelength $\lambda = 2\pi c/\omega = 8.2 \text{ m}$, which could be measured by interferences in Hertz's lab.

Finally (22) yields $k = \ln 800 = 6.38$.

Case of Figure 2 (no big spheres)

The burster will be assumed to consist of spheres of diameter ℓ at a distance of the order of ℓ too. It is probably not quite consistent with Branly's experiment in which the distance seems to have been very small. Since Branly gives no information about the diameter of the spheres, two values will be considered, $\ell = 3 \text{ cm}$ (the value recommended by Hertz) and $\ell = 1 \text{ cm}$. Then formulae (20), (25) and (1) yield

(i)

$$\ell = 3 \text{ cm}; \quad C \approx 1.7 \times 10^{-12} \text{ s}/\Omega; \quad L \approx 6 \times 10^{-9} \Omega \cdot \text{s}; \quad \omega \approx 10^{10} \text{ s}^{-1}; \quad \lambda \approx 20 \text{ cm}. \tag{27}$$

(ii)

$$\ell = 1 \text{ cm}; \quad C \approx 5.6 \times 10^{-13} \text{ s}/\Omega; \quad L \approx 2 \times 10^{-9} \Omega \cdot \text{s}; \quad \omega \approx 3 \times 10^{11} \text{ s}^{-1}; \quad \lambda \approx 6 \text{ cm}.$$

In both cases the critical resistance $R_c = 2\sqrt{L/C}$ which appears in condition (10) is of the order of 100 Ω . This is much larger than the resistance R of the spark, which, as already mentioned, is a few Ohms [9, 10].

Case of Figure 3

In the case of Figure 3, the Leyden jar discharges through the red circuit, which has length D and a self induction coefficient L roughly given by formulae (21) and (22), where α may be neglected. The frequency of the discharge current is given by (1) where the capacity C can be evaluated as $\epsilon_0 \epsilon S/d$, where d is the glass thickness, S the area and ϵ the dielectric constant. Assuming $S \approx 0.1 \text{ m}^2$, $d \approx 0.005 \text{ m}$ and $\epsilon \approx 5$, C is of the order of 1 nF, about 100 times larger than in the case of Figure 1. Assuming the same value of L , the frequency is expected to be about 10 times lower than the value (26), i.e. $2 \times 10^7 \text{ s}^{-1}$.

However, this argument neglects the capacity of the burster. It is indeed much smaller than that of the Leyden jar, but as will be seen, the power radiated by the discharge of the Leyden jar is very low. It is therefore of interest to take into account the electric charge of the burster. The equations will not be written, but they are linear and easy to solve. The result is essentially that the discharge current of the Leyden jar is, as explained above, localized in the red part of Figure 3 and has the moderate frequency evaluated above, but it is superposed with a high frequency current localized in the spark, which has the properties studied in Sections 2 and 3. More details will be given in the following sections.

5. Damping

As seen above, the spark bursts for an electric field of about 3000 V/mm [11]. Hertz used a burster with spheres distant of typically 7 mm, which corresponds to a maximum tension of about 20 000 V. Branly used a burster with spheres distant of typically 1 mm, which corresponds to a maximum tension of about 3000 V. As an example, the maximum value V_0 of the tension will be chosen as $V_0 = 10000 \text{ V}$, and the values (27) will be chosen for C and L . Then the energy stored in the spheres is $CV^2/2 \approx 10^{-4} \text{ J}$ and the maximum resistance which allows an oscillating discharge is $\sqrt{4L/C} \approx 130 \Omega$. If the resistance is small, the maximum intensity of the current is $I_0 \approx V_0/(L\omega) = 167 \text{ A}$. For various plausible values of the resistance R of the spark, the power produced by Joule effect is listed below

Resistance R	1 Ω	5 Ω	10 Ω	20 Ω	30 Ω
Power	116 W	580 W	1160 W	3320 W	5000 W.

These values are compatible with sparks of various intensities, fed by the Joule effect.

Let now the power emitted by radiation be evaluated. In Jackson's book [13] the power emitted by an electric current of frequency ω is calculated as a function of the dipole moment

$$\int \mathbf{r} \rho(\mathbf{r}, t) d^3 r = \mathbf{p} \exp(i\omega t) \quad (28)$$

where $\rho(\mathbf{r}, t)$ is the charge density at point \mathbf{r} and at time t .

The power is given by the following formula which has number (9.24) in the third edition of Jackson's book:

$$d\bar{W}/dt = \frac{\omega^4}{12\pi\epsilon_0 c^3} |\mathbf{p}|^2. \quad (29)$$

Since the charge of a conductor is concentrated at its surface, $\rho(\mathbf{r}, t)$ is mainly strong on objects with a large surface, i.e. (i) on the big spheres in the case of Figure 1; (ii) on the small spheres of the burster in the case of Figure 2. In the case of Figure 3 the charges are mainly in the Leyden jar LJ.

Thus, in the case of Hertz's device (Figure 1) the dipole moment can be written with a good approximation as $|\mathbf{p}| = DQ$, where Q is the charge of the big spheres and D is their distance. Insertion of this value into (29) yields

$$d\bar{W}/dt = \frac{\omega^4}{12\pi\epsilon_0 c^3} D^2 Q^2. \tag{30}$$

Since the energy stored in the spark is $\bar{W} = CQ^2/2$, the damping time τ is given, if the Joule effect is neglected, by

$$\frac{1}{\tau} = \frac{1}{\bar{W}} \frac{d\bar{W}}{dt} = \frac{1}{6\pi L\epsilon_0 c^3} D^2 \omega^2 = \frac{D\omega^2}{6kc} = \frac{c}{6k^2\ell}. \tag{31}$$

Formulae (23) and (31) yield

$$\omega\tau = 6k^{3/2}\sqrt{2\ell/D}. \tag{32}$$

The actual damping time is shorter because the Joule effect should be taken into account.

The calculation makes sense only if $\omega\tau \gg 2\pi$. In Hertz's case, inserting the typical values $D = 1$ m, $\ell = 0.15$ m, $k = 6.4$ into (32) one finds $\omega\tau = 52 \approx 16\pi$. Thus, after 8 periods of oscillation, the energy has been divided by $e \approx 2.7$ and the amplitude of the oscillations of the field has been divided by $\sqrt{e} = 1.65$.

In the case of Figure 2, assuming all lengths to have the same order of magnitude, a plausible assumption is that k and ℓ/D can be replaced by 1 in (32). It follows that $\omega\tau \approx 2\pi$, so that the spheres of the burster are almost completely discharged after few oscillations. So the emission spectrum is rather broad, in contrast with Hertz's case where the emitted radiation had a fairly well defined frequency and wavelength.

A rough evaluation of the radiated power (30) will now be given. The charge Q can be replaced by CV , where V has rather similar values in all cases, and ω can be replaced by (1) with an acceptable approximation. Thus the power is

$$d\bar{W}/dt \approx \frac{D^2 V^2}{12\pi\epsilon_0 c^3 L^2}.$$

In the cases of Figures 1 and 2, L is the self induction coefficient of a conductor of length D , so that, as discussed above, the order of magnitude of D/L is $1/\mu_0$. In the case of Figure 3 L is the self induction coefficient of a conductor of length of the order of 1 m, while D is the thickness of the glass of the Leyden jar, which is much smaller. Thus the energy radiated by the discharge of the Leyden jar is expected to be quite small. Most of the energy is dissipated as heat by the Joule effect. This observation is in contradiction with the fact that Branly's experiment was successful and did generate hertzian radiation. It is therefore of interest to consider the discharge of the burster, which is oscillating with a very high frequency as seen in Section 3. *A priori* this phenomenon does not seem to solve the problem because the burster loses its weak charge very quickly so that the very high frequency emission stops while the Leyden jar goes on discharging with a moderate frequency ω_1 . The only remaining possibility seems to be that the hypothesis of a constant resistance of the spark is not correct at this frequency ω_1 and that the spark stops after the discharge of the burster. This possibility will not be elaborated here.

Our procedure to neglect damping and then calculate it and find a strong damping is no satisfactory procedure. In Section 6 we try to devise a more consistent method. It confirms relation (32) in the limit $\omega\tau \gg 2\pi$ which corresponds to Hertz's case. More surprisingly, the consistent procedure also confirms the results obtained above in the case of Figure 2.

6. A consistent treatment of radiation damping

In Sections 2–4 the intensity has been calculated without taking into account the energy loss resulting from radiation, and in Section 5 the radiated power has been deduced from the intensity.

This is clearly not a self-consistent procedure. In this section, a consistent equation will be proposed. For the sake of simplicity R_2 will be assumed to vanish. The intensity i_2 will also be assumed to vanish, since it is not sufficient to prevent the discharge of the spheres. Thus the burster is treated as an isolated system. Then, elimination of V_2 and Q between (4), (5), (6) yields

$$LC \frac{d^2 Q}{dt^2} + Q = 0. \quad (33)$$

This equation implies that $LC\dot{Q}^2 + Q^2$ is a constant. Actually the energy of the system is³

$$W_0 = (L\dot{Q}^2 + Q^2/C)/2. \quad (34)$$

However, Equation (34) is an approximation (correct at low frequency) which neglects the loss of energy by radiation. This energy loss is given by the following, generalised version of (30)⁴

$$dW/dt = \frac{-D^2}{12\pi\epsilon_0 c^3} (dI/dt)^2 = \frac{-D^2}{12\pi\epsilon_0 c^3} \ddot{Q}^2. \quad (35)$$

This expression must be identified with the derivative of the energy W of the system (i.e. the burster). Therefore formula (34) must be replaced by

$$W = (L\dot{Q}^2 + Q^2/C)/2 + f(Q, \dot{Q}, \ddot{Q})$$

where df/dt is equal to expression (34) + something else. This apparently vague requirement has only one solution, namely

$$W = (L\dot{Q}^2 + Q^2/C)/2 - \frac{D^2}{12\pi\epsilon_0 c^3} \dot{Q}\ddot{Q}. \quad (36)$$

The variation δW in time δt is equal to $\dot{W}\delta t$, where \dot{W} designates the derivative of (36). But $\delta W/\delta t$ is also equal to (35). Therefore

$$\frac{-C\mu_0}{6\pi c} D^2 \frac{d^3 Q}{dt^3} + LC \frac{d^2 Q}{dt^2} + Q = 0. \quad (37)$$

Of course if the resistance of the spark is taken into account, an additional term proportional to dQ/dt should be added. We do not know if (37) has already been derived in this precise case, but the corresponding equation can be found in textbooks for a very similar problem, namely that of a point charge which oscillates around its equilibrium position. This is Equation (5) of Cohen-Tannoudji *et al.* [14].⁵

³Indeed if a capacitor is charged by a tension V , the energy is $\int V I dt = \int V dQ = \int Q dQ/C = Q^2/(2C)$ and if an intensity I is injected in a conductor the energy is $\int V I dt = \int L dI/dt I dt = \int L I dI = LI^2/2$.

⁴Derivation of formula (35): if a wire of length D centred at the origin is subject to an AC current of frequency ω it may be considered as made of a uniform density ρ of oscillating, electric charges with a displacement $x = a \cos \omega t$ and therefore a velocity $v = -\omega \sin \omega t$. In the time dt the charge which travels through a given point is $\rho v dt = -a\rho\omega \sin \omega t dt$. The intensity in the wire is therefore $I(t) = -I_0 \sin \omega t$ with $I_0 = a\rho\omega$. The electric field radiated at a very distant point \mathbf{r} is [14] perpendicular to \mathbf{r} and its value is

$$E(\mathbf{r}) = \int \frac{\rho dx}{4\pi\epsilon_0 c^2} a\omega^2 \frac{\cos \omega(t-r/c)}{r} = \frac{\omega I_0 D}{4\pi\epsilon_0 c^2} \frac{\cos \omega(t-r/c)}{r}$$

where D is assumed to be small with respect to the wavelength c/ω . This can be written as

$$E(\mathbf{r}) = \frac{D}{4\pi\epsilon_0 c^2 r} \frac{\partial I(t-r/c)}{\partial t}.$$

This last equation does not contain the frequency. It is valid for any electric current even with many Fourier components, provided that all wavelengths are larger than the length D of the wire. From Maxwell's equations one deduces the magnetic field $H = \epsilon_0 c E$, the Poynting vector EH which is the energy flux per unit area, and integration over the sphere of radius r yields (35).

⁵The equation of Cohen-Tannoudji *et al.* contains an inertial term which has the same form as the second term of (37). In fact the calculation presented above neglects the mass of the electrons (or rather the effective mass derived from the band structure). If it were taken into account, the second term of (37) would be slightly modified.

Equation (37) has an oscillating, damped solution. The existence of such a solution has therefore been demonstrated in a consistent way. However, the general solution of (37) is

$$Q(t) = \lambda \cos(\omega t - \varphi) \exp(-t/\tau) + \mu \exp(At) \tag{38}$$

where A is the real solution of

$$\frac{-C\mu_0}{6\pi c} D^2 A^3 + LC A^2 + 1 = 0 \tag{39}$$

which is positive. Formula (38) is physically acceptable only if $\mu = 0$. In principle, the constants λ , φ and μ should be determined by the initial conditions, and it is not clear that they imply $\mu = 0$. The problem is related to the fact that expression (36) has no lower bound, and is therefore not acceptable for an energy.

Imposing the condition $\mu = 0$, relation (37) can be written as

$$Q + \frac{d^2 Q}{d(\omega_0 t)^2} - \epsilon \frac{d^3 Q}{d(\omega_0 t)^3} = 0 \tag{40}$$

where $\omega_0 = 1/\sqrt{LC}$ and

$$\epsilon = \frac{\mu_0}{6\pi c L} \frac{D^2}{\sqrt{LC}}$$

or inserting (20) and (21)

$$\epsilon = \frac{1}{6\pi k \sqrt{k\ell/D}}. \tag{41}$$

For small ϵ , an expansion of ω and $1/\tau$ in powers of ϵ can be obtained from (39), where $A = i\omega - 1/\tau$. By this method one recovers formula (5) at lowest order. In the case of Figure 2, if one assumes $k = 1$ and $\ell = D$ as a rough evaluation, it follows that $\epsilon = 1/3$. The numerical solution of (39) yields the frequency $\omega = 0.94\omega_0 = 0.94/\sqrt{LC}$ and $\omega\tau = 6.9$, which is not very different from 2π as found in Section 5.

In the above calculation the current i_2 in the secondary circuit of the induction coil has been neglected. In this approximation the intensity I_2 in the spark is equal to dQ/dt and satisfies the same equation (37) as Q . This equation can be modified to account for the non vanishing values of the resistance R and the current i_2 and one obtains

$$\frac{-C}{3G} \frac{d^3 I_2}{dt^3} + LC \frac{d^2 I_2}{dt^2} + RC \frac{dI_2}{dt} + I_2 = i_2 \tag{42}$$

with

$$G = \frac{2\pi c}{\mu_0 D^2}. \tag{43}$$

Since i_2 is constant with a good approximation the variable $y = I_2 - i_2$ satisfies the equation

$$\frac{-C}{3G} \frac{d^3 y}{dt^3} + LC \frac{d^2 y}{dt^2} + RC \frac{dy}{dt} + y = 0. \tag{44}$$

The condition for an oscillating solution can be obtained as follows. The solutions of (44) have the form $y = \exp(At)$ where A is a root of

$$f(A) = \frac{-C}{3G} A^3 + LC A^2 + RC \frac{dy}{dt} + 1 = 0. \tag{45}$$

There is an oscillating solution if the minimum $f(A_1)$ of $f(A)$ is positive, where A_1 is given by $f'(A_1) = 0$ or

$$A_1 = LG - \sqrt{L^2 G^2 + RG}. \tag{46}$$

The condition $f(A_1)$ yields after some algebra the following, more explicit condition for an oscillating discharge:

$$R^2 < \frac{L}{C} \frac{3(1 + \sqrt{u})^2}{1 + 2\sqrt{u}} \tag{47}$$

where

$$u = 1 + \frac{\mu_0 R D^2}{2\pi c}. \quad (48)$$

Surprisingly, perhaps, condition (47) is somewhat less stringent than (10) (which can be retrieved by replacing u by 1 in (47)). Since (10) is much simpler, it can be retained as a sufficient condition for an oscillating discharge.

7. Conclusion

In the case of Figure 2 the burster discharging through a spark can be considered as an isolated system with a good approximation. The self induction coefficient of the spark cannot be evaluated as precisely as for the discharge line in Hertz's device (Figure 1) but plausible orders of magnitude can be given, which are compatible with a oscillating discharge, however with a much higher frequency in the case of Figure 2 than in Hertz's experiment. The damping by radio-emission also occurs after a smaller number of oscillations.

The case of Figure 3 is mysterious. The relatively simple calculation presented here, in which the spark has a constant resistance while the current oscillates, predicts a very weak radiation power. At the end of Section 4 we have proposed a speculative explanation of the success of Branly's experiment. An alternative possibility is that Branly has used the device of Figure 2 rather than Figure 3. The choice between the two possibilities would imply new experiments.

Most of the calculation has been carried out by the traditional method which ignores the energy loss resulting from hertzian emission, and eventually deduces this energy loss from the calculated intensity and frequency. In the last section a more consistent method is also presented, which gives rather similar results.

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