Improving BEL1D accuracy for geophysical imaging of the subsurface

Hadrien MICHEL\(^{(1,2,3)}\), Thomas HERMANS\(^{(2)}\), Thomas KREMER\(^{(4)}\), Frédéric NGUYEN\(^{(1)}\)

(1) ULiège – Liège, Belgium
(2) UGent – Gent, Belgium
(3) F.R.S.-FNRS – Brussels, Belgium
(4) UNantes – Nantes, France
Table of content

• Basics:
  – sNMR – detecting water from the surface
  – BEL1D (Michel et al., 2020, Computers & Geoscience)

• Numerical benchmark:
  – Applying BEL1D
  – Improving with Iterative Prior Resampling (IPR)
  – Comparison with McMC

• Case study: Mont Rigi

• Conclusion and perspectives
sNMR

- sNMR = surface Nuclear Magnetic Resonance
- Detecting groundwater from the surface
- After inversion:
  - Water content distribution
  - Relaxation time distribution

Improving BEL1D accuracy
• Adaptation from BEL (Schiedt et al., Quantifying Uncertainty in Subsurface Systems, 2018)

• Building a relationship between:
  – Synthetic models
  – The associated datasets

• Extracting the posterior from this relationship

Improving BEL1D accuracy
BEL1D
Sampling models and forward modelling

\[ d = G(m) \]

Improving BEL1D accuracy
BEL1D
Reducing dimensionality (PCA)

- From 10,000 dimensions in the dataset to around 10
  - Keeping 90% variability

- Not applied to the models
  - Uncorrelated prior
  - Poor performances
BEL1D
Canonical correlation analysis

- Linking the models parameters to the reduced datasets $\rightarrow$ CCA
BEL1D

Extracting the posterior in reduced space

- Transform the field dataset (PCA and CCA)
- Report in the CCA space
- Extract the obtained distribution (Kernel Density Estimation)

Improving BEL1D accuracy
BEL1D
Back-transform into original space

- Apply the inverse transform (CCA) to sampled models in reduced space
BEL1D
Improving with IPR, the concept

- IPR = Iterative Prior Resampling
- Inspired by Iterative Spatial Resampling (Mariethoz et al., 2010, Water Resour. Res.) and Sampling Importance Resampling (Dosne et al., 2016, J PHARMACOKINET PHAR)
- Process:
  - Adding the sampled models to the prior
  - Re-running the BEL1D operations
  - Repeat until convergence:
    - Threshold on the difference between the obtained distributions (Wasserstein distance in normalized space)
Numerical benchmark

• Simple 3-layer model

• Experimental design:
  – Same transmitter/receiver loop
  – 50 m diameter → penetration depth about 50 meters
  – Noise = 10nV (Gaussian)

• Prior defined accordingly but still large

<table>
<thead>
<tr>
<th>Layer #</th>
<th>Thickness e [m]</th>
<th>Water content W [%]</th>
<th>Decay time T₂* [ms]</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Min</td>
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<td>Max</td>
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<td>0</td>
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<td>2</td>
<td>0</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>Half-space</td>
<td>/</td>
<td>Inf</td>
<td>/</td>
</tr>
</tbody>
</table>

Improving BEL1D accuracy 11
Numerical benchmark
Applying BEL1D
Building the CCA space relationship
Numerical benchmark
Applying BEL1D

- Reduced uncertainty
- Still very large!
- RMSE above noise level

→ Room for improvement

Improving BEL1D accuracy
Numerical benchmark
Improving with IPR

1st iteration

Last iteration

Improving BEL1D accuracy
Numerical benchmark
Improving with IPR

- Narrow uncertainty
- Sensitivity lower in depth
  - From experimental design

Improving BEL1D accuracy

Root Mean Square Error [V]
Numerical benchmark
Comparing with McMC

- Using DREAM\(_{(zs)}\) (e.g., Vrugt, 2016, ENVIRON MODELL SOFTW and Laloy et al., 2018, Water Resour. Res.)

- Tuned to convergence:
  - Number sequences: 20
  - Samples per chain: 10,000
  - Jump rate: 0.1

Improving BEL1D accuracy
Numerical benchmark
Comparing with McMC

- The last iteration coincide with results from DREAM
- However:
  - CPU time is lower (250 seconds for BEL1D vs 500 for DREAM\(_{zs}\))
  - Difficulty to tune to convergence in DREAM
Case study: Mont Rigi

• Natural reserve in the Eastern part of Belgium
• Metric peat above Cambrian bedrock
• Experiment:
  – Single transmitter/receiver
  – 20 meters in diameter
  – Noise ~ 18 nV

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Case study: Mont Rigi

First iteration

- Significant reduction of uncertainty
- Still an observable \((W_1, e_1)\) link
- Trend for the relaxation time
- CPU time = 30 sec

Last (6th) iteration

Improving BEL1D accuracy
Conclusion and perspectives

- BEL1D combined with IPR is:
  - Accurate (comparison with McMC)
  - Efficient (CPU time)
  - Easy to tune to convergence

- Significant improvement over BEL1D without iterations

- Currently developing other use case (MASW, EM)
- Near future: Smooth models, 2D, etc.
Conclusion and perspectives

2D case – Time-Lapse ERT

- Heat tracer experiment
- Captures the behavior of the heat tracer.
Conclusion and perspectives
Work in progress – Surface waves preview

• Mirandola (Italy) case study from INTERPacifc (Garofalo et al., 2016, SOIL DYN EARTHQ ENG)
• Comparison of the results with IPR with the different experts curves

Improving BEL1D accuracy
Conclusion and perspectives
Work in progress – Surface waves preview

• Efficient reduction of the dataspace from the prior (gray) to the posterior at the last (15th) iteration
• The error model fits nicely the posterior dataspace
  – The noise is not of the same kind as the one in sNMR
Conclusion and perspectives
Work in progress – Surface waves preview

- Field benchmark available: Depth to the bedrock = 118m (Garofalo et al., 2016, SOIL DYN EARTHQ ENG)
- Accurately reproduced by BEL1D
1D geological imaging of the subsurface from geophysical data with Bayesian Evidential Learning

Hadrien Michel\textsuperscript{a,b,c,\textdagger}, Frédéric Nguyen\textsuperscript{a,d}, Thomas Kremer\textsuperscript{a}, Ann Elen\textsuperscript{d}, Thomas Hermans\textsuperscript{b}

\textsuperscript{a} University of Liège, Urban and Environmental Engineering Department, Faculty of Applied Sciences, Liège, Belgium
\textsuperscript{b} Ghent University, Department of Geology, Ghent, Belgium
\textsuperscript{c} F.R.S.-FNRS (Fonds de la Recherche Scientifique), Brussels, Belgium
\textsuperscript{d} KU Leuven, Department of Earth and Environmental Sciences, Leuven, Belgium

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Improving BEL1D accuracy
Thanks for your attention!

Improving BEL1D accuracy
Conclusion and perspectives

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