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# Mechanical Based Circular Notch Hinge Design

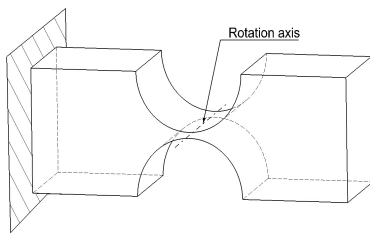
*At the microscale, classical bearings such as ball bearings, sliding bearings and other pivots may be difficult to manufacture. Among all possible solutions to replace them, the notch hinge seems to be very attractive through its many advantages. The major limitation is on the rotation angle. This paper presents a new simplified analytical model for the prediction of the maximum angular displacement and the angular stiffness of a notch hinge. To validate our model, an experimental set-up has been developed. Finally, the results are compared to other theories.*

## 1 Introduction

Miniaturization and integration of functions (mechanical, sensing, control, computing) within confined spaces is becoming increasingly important in modern systems. Examples are mobile phones, wearable sound and video systems, digital organizers, robot endoscopes, micro-pumps for medical applications, aerospace equipment, etc. In a micro-system, many functions need to be fundamentally reconsidered. Scale laws make some physical principles useless for microsystems, while other principles, although without interest in macrosystems, may be extremely useful for miniaturized systems.

This is the case of the hinge function. Classical bearings such as ball bearings, sliding bearings and other pivots may be difficult to manufacture at the micro scale. As it is very difficult to manufacture small parts with good tolerances, the guiding precision may be insufficient for a particular application. In micromachines, friction may become very important compared to other forces and torques. In some applications, e.g. in medical devices, cleanliness exigencies practically prohibit the use of greasy lubricants. Consequently, there is an important need to develop frictionless micro bearings. Finally, the assembly of small components may become very difficult, and a device composed of a minimal number of components becomes paramount.

Among all possible solutions, the notch hinge, see fig. 1,



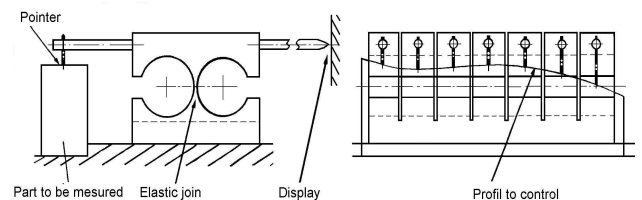
**Fig. 1 Geometry of the circular notch hinge**

seems to be very attractive : no friction, no lubricant required, minimum space required, only one part to manufacture, no assembly, etc.

This last advantage is decisive. Indeed, the

best way to simplify the assembly is to avoid, or better, to eliminate any assembly. Of course the major limitation of the presented solution concerns the rotation angle.

The circular notch hinge has been known for some years. It was already used in a profile control mounting, see fig. 2 by R. Pazot [3]. This mounting is called an indicating pointer mounting and Pazot adds that this joint avoids any axis or return spring and so removes any clearance and simplifies the mounting.



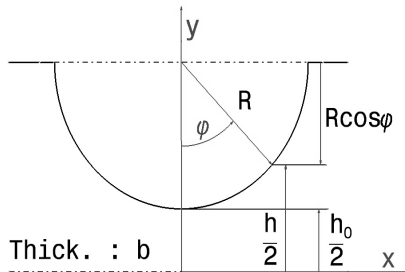
**Fig. 2 Profile control mounting**

Many other applications can be found in the literature as to the use of such hinges. Among other examples, Xu and King have used them in their piezoactuator displacement amplifiers [10], Ryu and al. have designed an XYθ wafer stage based on a flexure hinge [12]. The reader can find many other examples of applications in [6] and [11]. The description of the notch hinge and its modelling was described early in 1965 by Paros and Weisbord [5]. Since, it has been the object of several studies, for example, Smith and al. [18]-[19], Zhang and Fasse [15], Koster [14], Heinen [6], etc. The notch hinge plays a very important role in a part which includes many other functions. Thus, the modelling of the notch hinge is necessary in order to understand how it works. This paper first presents a new simplified analytical model for the prediction of the maximum angular displacement and the angular stiffness of a notch hinge. This model is then compared to the results of an experimental setup and finally confronted with expressions determined by other scientists.

## 2 Asymptotic forms

A first step will be to develop formulae for the case where the thickness  $h_0$  (figure 3) is very small compared to the radius  $R$ . This gives a valuable insight to the behaviour of notch hinges.

Figure 3 presents the main geometry of the joint. The notch hinge is defined by its radius,  $R$ , minimal thickness  $h_0$ , and the thickness in the direction perpendicular to the section  $b$ .



### Fig. 3 Geometric variables

The X-axis is the axis of symmetry and the Y-axis connects the centres of the two circular arcs,  $\varphi$  is the direction angle of a particular point.

The motion of interest is the rotation about the

Z-axis. We therefore basically wish to determine the total deflection angle  $\alpha_f$  of the beam under a given load,  $M_f$ . This is calculated by a classical expression of strength of materials [4]. Only the notch hinge zone (which extends from  $-R$  to  $R$  along the X-axis) is assumed to deflect under the action of the bending moment and so, the rest of the beam is assumed to have infinite stiffness. The integral expression is similar to known integrals of lubrication theory [2] and therefore, Sommerfeld's change of variables [1] is used to solve this expression. After some simple developments, the notch stiffness is found to be

$$\frac{M_f}{\alpha_f} = \frac{2}{9\pi} Eb \frac{h_0^{5/2}}{\sqrt{R}} \quad (1)$$

where E is the elastic modulus.

The maximum stress in the notch is given by

$$\sigma_{\max} = \frac{6M_f}{bh_0^2} = \frac{4}{3\pi} E\alpha_f \sqrt{\frac{h_0}{R}} \quad (2)$$

Thus, a maximum bending angle can be calculated, depending on the material and on the requested geometry

$$\alpha_f = \frac{3\pi}{4} \frac{\sigma}{E} \sqrt{\frac{R}{h_0}} \quad (3)$$

The other stiffness are obtained in the same manner, as fully described in [7].

For torsional stiffness (X axis), we obtain

$$\frac{M_t}{\alpha_i} = \frac{8}{9\pi} Gb \frac{h_0^{5/2}}{\sqrt{R}} \quad (4)$$

$G$  being the torsion modulus.

Calculation of the stretching stiffness (X axis) leads to

$$\frac{N}{u} = \frac{1}{\pi} E b \sqrt{\frac{h_0}{R}} \quad (5)$$

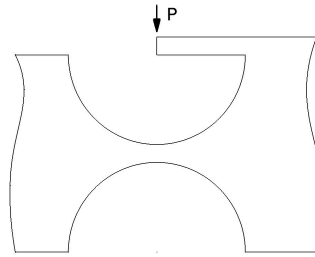
where  $N$  is the applied load (stretching load) and  $u$  the corresponding displacement.

The bending stiffness in the transverse direction (Y axis) is given by

$$\frac{M_{ft}}{\alpha_{ft}} = \frac{1}{12\pi} Eb^3 \sqrt{\frac{h_0}{R}} \quad (6)$$

where the hinge bends through an angle  $\alpha_{fi}$  under the load  $M_{fi}$  along the Y-axis.

A shearing effect appears when a load is applied in the Y-axis direction. Figure 4 presents a theoretical case where shear



**Fig. 4 Shear loading of the beam**

appears, causing the translation of one side of the beam along the Y-axis, without rotation of the extremities.

The load leads to two effects: bending and shear. Nevertheless, flexion is predominant, i.e., the shear effect may be neglected. After a few developments described in [7], the displacement at the

midpoint of the hinge (in the Y-direction, under a load  $O$ ) being denoted by  $v$ , the technological shear stiffness is found to be

$$\frac{O}{v} \approx \frac{2}{3\pi} Eb \left( \frac{h_0}{R} \right)^{3/2} \quad (7)$$

The same development is made along the Z direction to lead to,

$$\frac{P}{w} = \frac{1}{12\pi} \frac{Eb^3}{R^2} \sqrt{\frac{h_0}{R}} \quad (8)$$

Finally, we calculate the displacement for an in-plane bending moment. After a few calculations, the following result is obtained

$$v = \frac{M_f}{Eb} \frac{R^{3/2}}{h_0^{5/2}} \left[ \frac{9\pi}{2} - 6\sqrt{\frac{h_0}{R}} \right] \quad (9)$$

The largest term in equation (9) is  $\alpha(R).R$  (the first term).

This means that the rotation centre is, with  $\mathcal{O}(\sqrt{h_0/R})$  precision, the centre of the notch hinge.

### 3 Correction Coefficients

The preceding results were established under the hypothesis that the ratio  $h_0/R$  remains very small. But, due to machining considerations, there is a lower bound of obtainable hinge thickness, so that in actual applications, higher values of  $h_0/R$  may prevail, up to 0,5.

The preceding asymptotic expressions are then no longer valid. However, a system of correction factors, depending only on the ratio  $h_0/R$ , has been developed by two of the authors [7], in the following way. First, the exact values of the integrals involved in the preceding computations are developed in closed form. The correction factors are then obtained via a numerical approximation scheme.

The corrected formula for the bending stiffness is

$$\frac{M_f}{\alpha_c} = K_1 \frac{2}{9\pi} E b \frac{h_0^{5/2}}{\sqrt{R}}, K_1 = 1 + 0,137 \frac{h_0}{R} \quad (10)$$

Conversely, expression (3) must be replaced by

$$\alpha_f = \frac{1}{K_1} \frac{3\pi}{4} \frac{\sigma}{E} \sqrt{\frac{R}{h_0}} \quad (11)$$

The torsional stiffness correction factor is the same as for the bending stiffness because the same integral is involved. In the same way, a factor  $K_2$  was found for the stretching stiffness

$$\frac{N}{u} = K_2 \frac{1}{\pi} E b \sqrt{\frac{h_0}{R}}, \quad K_2 = 1 + 0,97 \sqrt{\frac{h_0}{R}} \quad (12)$$

The same factor  $K_2$  holds in the case of transverse bending stiffness.

Formula (7) for shearing stiffness along Y direction becomes

$$\frac{O}{v} = K_3 \frac{2}{3\pi} E b \left( \frac{h_0}{R} \right)^{3/2}, \quad K_3 = \frac{1 + 0,5 h_0/R}{1 + 2,2 h_0/R} \quad (13)$$

Factor  $K_3$  is also valid in shearing stiffness along Z direction.

Finally, relation (9) becomes

$$v = \frac{M_f}{E b} \frac{R^{3/2}}{h_0^{5/2}} \left[ \frac{1}{K_1} \frac{9\pi}{2} - K_4 6 \sqrt{\frac{h_0}{R}} \right], \quad K_4 = \frac{1}{1 + 0,5 h_0/R} \quad (14)$$

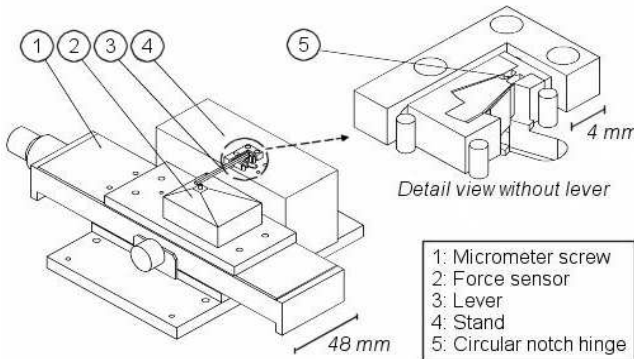
The maximal error, compared to the exact solution, remains below 1,6 % for all the correction coefficients [7].

The manufacturing method influences the roughness of the surface defining the hinge. Although small, this peak to valley height ( $R_p$ ) corresponds to a non-negligible decrease of the minimal thickness,  $h_0$  (for example, if  $h_0=0,1$  mm and  $R_p=2,75$   $\mu$ m, on each face, the difference is about 5,5%). The difference is taken into account as follows:

$$\frac{M_f}{\alpha_f} = K_1 K_r \frac{2}{9\pi} E b \frac{h_0^{5/2}}{\sqrt{R}}, \quad K_r = \left( 1 - 5 \frac{R_p}{h_0} \right) \quad (15)$$

## 4 Experimental Setup

Stainless steel hinges with thickness ranging from 66 to 175  $\mu$ m were manufactured using Wire Electrical Discharge Machining (WEDM). The part to be manufactured is placed in a dielectric solution and a voltage difference between a conductive part and the wire produces an electrical arc forming smelts and vaporizing the material locally.



As Ryu and al. [9] have explained, WEDM is the best process to machine notch hinges without any serious impact on motion induced by machining error. The sole error is a body

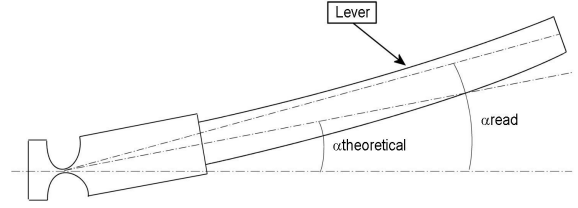


Fig. 6 Bending of the lever

shape error. Tosun and al. [10] have determined a relation between the CLA roughness obtained and the two important parameters in the WEDM : the pulse duration,  $t_i$ (ns), and the open circuit voltage,  $u_i$ (V). The relation is

$$R_a = 0,048 t_i^{0,3613} u_i^{0,3213} (\mu\text{m}) \quad (16)$$

From measurements, the ratio between  $R_a$  and  $R_p$  is found to be  $R_a/R_p = 0,43$ .

Finally, the parameters used in machining the notch hinge,  $t_i=0,7$   $\mu$ s and  $u_i=80$  V, lead to  $R_p=4,86$   $\mu$ m. After the WEDM machining, a shot-peening treatment is applied to the surface, which decreases  $R_p$  to about 2,75  $\mu$ m.

The experimental setup is depicted in figures 5 and 7. The hinge is fixed at one end. A beam (3) is fixed to the other end and the force is applied by a micrometer screw (1). A sensor (2) manufactured by CSEM is used to measure the applied force. The main characteristics of this sensor are described in table I.

Tab. I Sensor characteristics

Type	LC-01
Nominal Capacitance	$\pm 0,1$ N
Theoretical resolution	<10 $\mu$ N
Linearity error	0,5 %
Hysteresis	<0,2 %
Rigidity	0,2 mm / 0,1 N
Supply	5 V/ 5 kHz

Figure 8 presents the results of the experiment conducted for one particular hinge (0,066 mm thickness). For each angular position of the hinge, three measurements of the return torque were made. The graphs present only the mean values for each position, the standard deviance being below 0,01 Nmm. This is then compared to the torque evaluated using the corrected model (10) ("model 1").

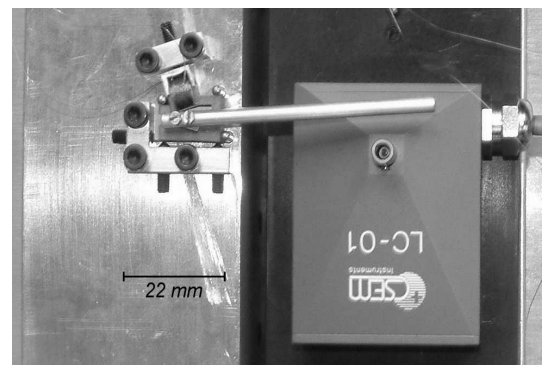


Fig. 7 Top view of the test bed

One can easily see that the agreement between measurements and model is good for small angle displacements, smaller than  $0,2^\circ$ . However for large displacements, the model overestimates the experimental data. Three reasons may explain the difference.

The beam used to apply the force to the hinge may bend. The deflection for a straight beam fixed at one end and free at the other is, with  $P$  the applied load,  $l$  the length of the beam and  $I$  the area moment of inertia,  $v = -Pl^3/3EI$ .

The force is not applied at the hinge axis, so this may introduce a torsional moment and thus a displacement which can be modelled as

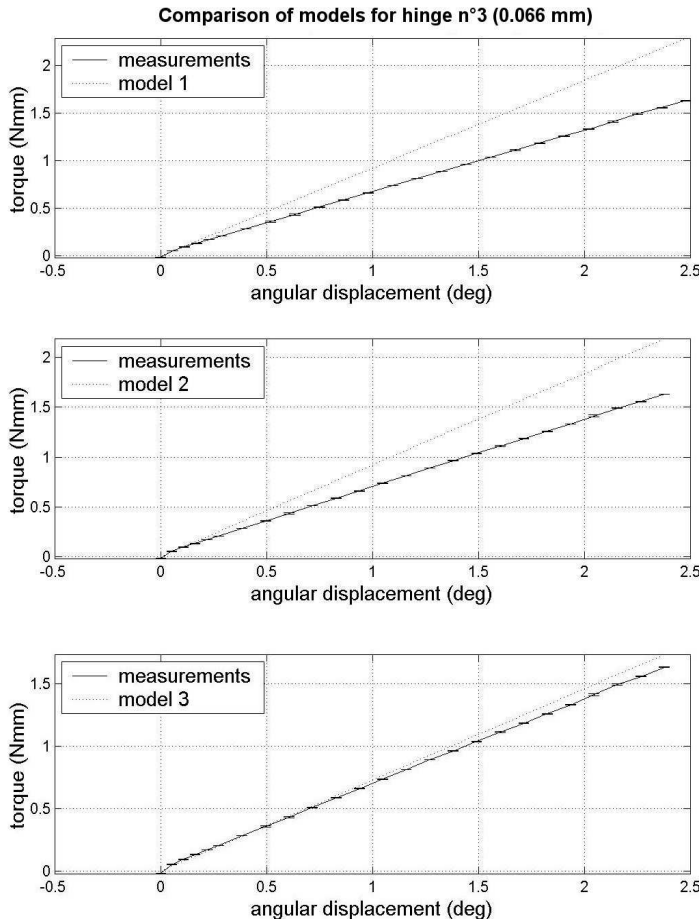
$$v_t = \frac{\theta_r (b+d)(l_1+l_2)}{2l_2} \quad (17)$$

with  $\theta_r = M_t/R_t$ ,  $M_t = P(b+d)/2$ ,  $d$  the rod diameter,  $R_t$  the torsional stiffness of the hinge,  $l_1$  the rod length and  $l_2$  the distance between the centre of the hinge and the attachment point of the rod.

As a consequence, the displacement has three components: one resulting from deformation of the hinge (this one is interesting), a second one from the bending of the lever and the third from the torsion of the hinge. The actual displacement is now  $v_{read} = v_t + v_f + v_{theo}$ .

The last step is now to transform this linear displacement into an angular displacement using the formula

$$\alpha_f = a \tan(v/(l_1+l_2)) \quad (18)$$



**Fig. 8 Model evolution**

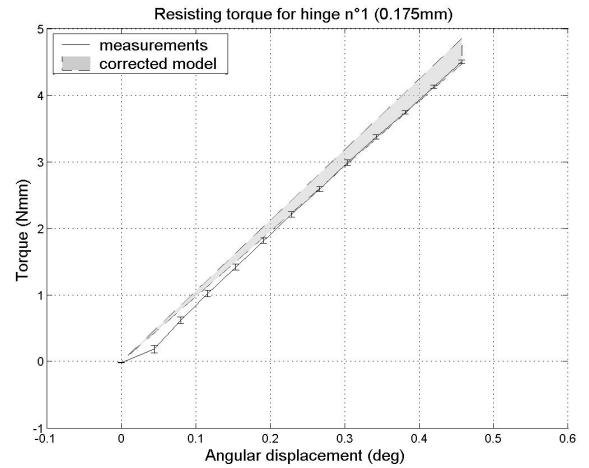
In order to compare the measurements with the model, the measured torque must be plotted against the hinge displacement which is  $v_{theo} = v_{read} - v_t - v_f$ . The same procedure can be used for angular displacements. Figure 6 shows the difference between the angular displacement measured at the end of the lever and the effective angular displacement of the hinge.

“model 2” (figure 8) considers these two modifications (bending of the lever and torsion of the hinge).

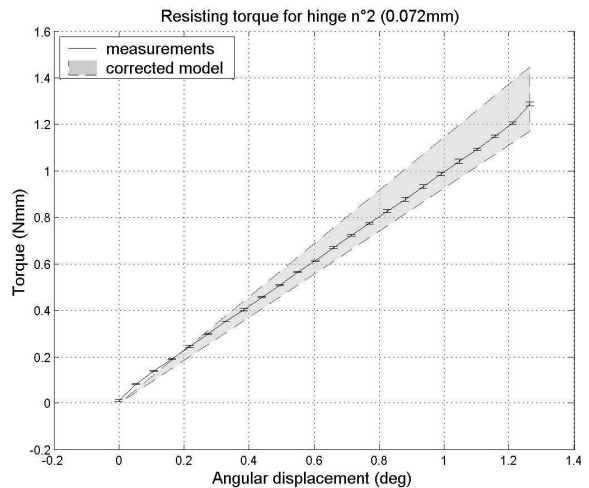
The hinge surface finish is not perfect. Roughness should be taken into account (it is not negligible compared to the thickness of the hinge). So the theoretical formula can be modified and yields a different boundary for the measurements. Returning to formula (15), another correction coefficient has been introduced ( $K_r$ , with  $R_p = 2,75 \mu m$ ).

In “model 3”, the complete expression of the torque (15) is considered. The results of this global method are illustrated on figure 8.

The influence of the roughness is particularly interesting. The smaller the thickness of the hinge, the more important the influence of the roughness. This influence can be seen as a reduction of the thickness of the hinge thus inducing a reduction of the return torque. But it is very difficult to determine the actual value of the hinge roughness. We therefore preferred to consider the following scheme : the actual return torque lies between the modelled torque without



**Fig. 9**



**Fig. 10**

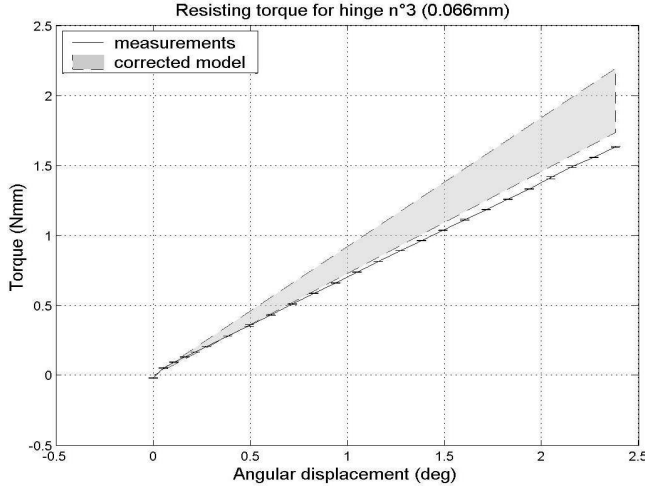


Fig. 11

taking into account the roughness and the torque calculated using the corrector coefficient  $K_r$ .

Figures 9 to 12 present the torque comparisons (model versus experiments) for the four different hinges that were manufactured. As explained before, to fulfil the requirements, the measurements should lie in the range defined by the model. This range is smaller for the hinge with a larger thickness. In this case,  $h_0$  has a larger value compared to  $R_p$  and so the influence of roughness is less important.

## 5 Comparison With Existing Solutions

We can find, in the literature, many references regarding notch hinges. Some are interesting as to the machining method of the notch hinges [17]-[8], others try to predict the stiffness of the hinges [5]-[7] and [18]-[19], principally for the design of such joints.

All the authors give the same relation for the asymptotic form of bending stiffness (eq.(1), hereafter named Heinen's solution, because it is the only one he published [6]). When we neglect this limit, the given solutions are almost all different. The various solutions can then be compared and discussed.

In 1965, Paros & Weisbord [5] gave a well-know solution. If  $\beta = h_0/2R$ , for a complete circular notch hinge,

$$\frac{\alpha_f}{M_f} = \frac{3}{2EbR^2} \left[ \frac{1}{2\beta + \beta^2} \right] \left\{ \frac{1}{1+\beta} \frac{3+4\beta+2\beta^2}{(2\beta+\beta^2)} + \frac{6(1+\beta)}{(2\beta+\beta^2)^{3/2}} \tan^{-1} \sqrt{\frac{2+\beta}{\beta}} \right\} \quad (19)$$

$$\frac{\alpha_{fi}}{M_{fi}} = 24 \left\{ \frac{1+\beta}{\sqrt{2\beta+\beta^2}} \tan^{-1} \sqrt{\frac{2+\beta}{\beta}} - \frac{\sqrt{2}}{2} \right\} \quad (20)$$

This solution appears to be an exact solution of the system of differential equations but is challenged as being the best solution for the hinge by Zangh & Fasse [15] because Paros & Weisbord used linear beam theory, which is accurate only

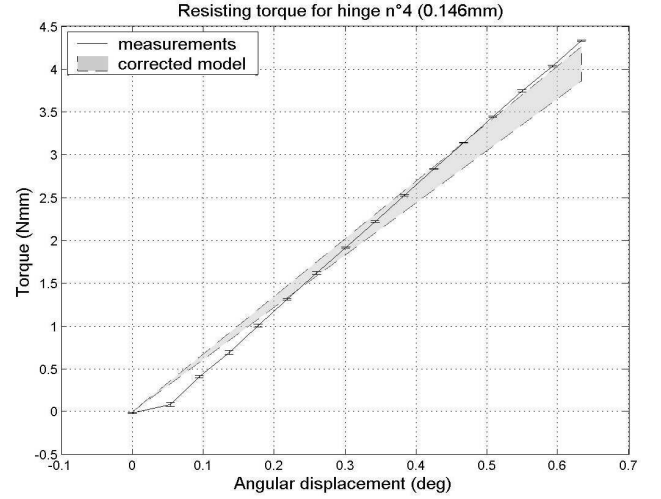


Fig. 12

for long beams with gradually varying cross section. Zangh & Fasse calculated the stiffness of the hinge by modelling it as an elastic system of three dimensional finite elements subjected to applied loads. The observed displacements allow to calculate the corresponding stiffness. By fitting the set of points obtained, they produced the following functions, with  $\gamma = b/2R$ .

$$\frac{N}{u} = Eb \left( 0,145\beta + 0,357\beta^{0.5} \right) \quad (21)$$

$$\frac{O}{v} = Eb^3 \left( 0,109\beta^2 - 0,102\beta^{1.5} + 0,388\beta - 0,005\beta^{0.5} \right) \left( 0,230\gamma^2 - 1,396\gamma + 2,583 \right) \quad (22)$$

$$\frac{P}{w} = Eb \left( -0,067\beta^{1.5} + 0,283\beta - 0,034\beta^{0.5} \right) \quad (23)$$

$$\frac{M_t}{\alpha_t} = Gb^3 \left( 0,052\beta^{1.5} + 0,047\beta - 0,0082\beta^{0.5} \right) \left( 0,274\gamma^2 - 1,397\beta + 2,496 \right) \quad (24)$$

$$\frac{M_f}{\alpha_f} = Ebh_0^2 \left( -0,035\beta + 0,111\beta^{0.5} \right) \quad (25)$$

$$\frac{M_{fi}}{\alpha_{fi}} = Eb^3 \left( 0,013\beta + 0,030\beta^{0.5} \right) \quad (26)$$

Koster [17] also gives functional approximations for the stiffness parameters within a range he calls the "realistic regime",  $0 < h_0/2R < 0,5$ .

$$\frac{N}{u} = 0,48Eb\beta^{0.5} \quad (27)$$

$$\frac{P}{w} = 0,56Eb \frac{\beta^{0.5}}{1,2+1/\beta} \quad (28)$$

$$\frac{M_t}{\alpha_t} = 0,0467 \frac{E}{G} b \frac{\beta^{0.5}}{1,2+1/\beta} \quad (29)$$

$$\frac{M_f}{\alpha_f} = 0,093Eb h_0^2 \beta^{0.5} \quad (30)$$

$$\frac{M_{fi}}{\alpha_{fi}} = 0,04Eb^3\beta^{0,5} \quad (31)$$

In our case, we limit the ratio  $h_0/R$  to 0,5. We think that it is not realistic to have a thickness  $h_0$  equal to the radius,  $R$ , because the stiffness of the notch hinge must be limited (eq.(10)). In the same way, eq. (11) shows that to reduce the stress for a given displacement, the ratio will be kept under a limit specified by the same equation. Finally, the last but not least important argument, the stress concentration remains below 9 % (Peterson, [13] p 56, fig. 37) when the ratio stays below 0,5.

Earlier, Smith & Chetwynd [18]-[19] obtained an empirical formula similar to that for a simple cantilever beam, also based on a finite element method. The relation is,

$$\frac{M_f}{\alpha_f} = \frac{EI}{2KR} \quad (32)$$

$I$  is the moment of inertia of the smaller section of the hinge,  $K$  is determined by the FE analysis and can be expressed as

$K = 0,565 \frac{h_0}{R} + 0,166$ . Finally, the expression is equivalent to

$$\frac{M_f}{\alpha_f} = Ebh_0^2 \frac{\beta}{1,992 + 13,56\beta} \quad (33)$$

All the expressions of bending stiffness are represented on fig. 13.

For  $h_0/R$  ranging between 0,025 and 0,07, our expression of bending stiffness seems agree well with the measurements. For three hinges, it is the best solution, with Koster for two of them. Further tests with other hinge thickness which covered the full range of the realistic regime may be carried out to entirely validate our expression.

One can see, from fig. 13, that the solutions of Koster, Zhang & Fasse and Smith & Chetwynd move away from ours, when the ratio  $h_0/R$  increases. The choice of the best solution should also be made in agreement with the higher values of this ratio. As long as we do not have any experimental results in this range, these remains a doubt as to the best solution.

The experiment consists of a test with a large set of measurement point for each hinge, as shown in appendix, in order to have an acceptable approximation of the stiffness. So it is difficult to realise the same analysis for a large number of hinges and thus the result may not be complete. This shows the way for further tests, with tests of torsional stiffness, transversal bending stiffness, ...

The advantage of our solution compared to these of Paros & Weisbord and Zhang & Fasse is the facility to find the value of the ratio  $h_0/R$  for a given stiffness. Smith & Chetwynd give a solution for only the bending stiffness. This make a relatively too simple model.

Finally, the result of Koster seems to be the best but we bring the possibility to determine the minimal thickness from all geometric and structural parameters, for an given angular displacement. Indeed, we are thinking that it's the most important for a designer.

## 6 Conclusions

An asymptotic model for the prediction of the maximum angular displacement and the angular stiffness of a notch hinge has been developed. This model is valid for very small  $h_0/R$  ratios. However, due to machining considerations, classical ratios could reach 0,5. For this value, asymptotic expressions are not valid and correction factors have been evaluated. Four hinges, manufactured using the Wire Electrical Discharge Machining (WEDM) technique, were used to validate our model. A comparison between

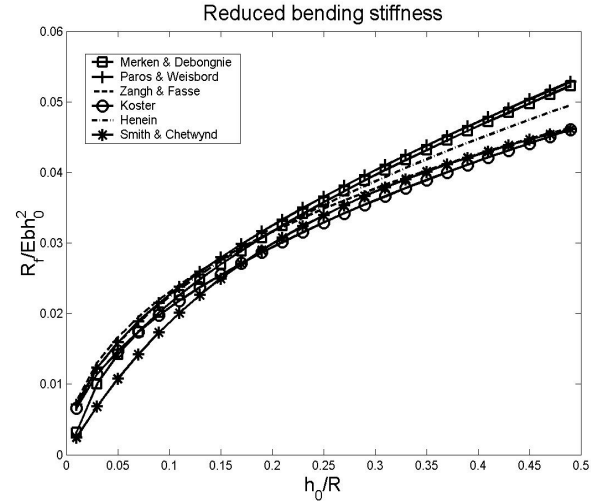


Fig. 13 comparison between five different models

experimental results and simulation shows good agreement for small angle deviations but not for angular deviation greater than 0,5 degrees. In addition, agreement for small thickness was worse than for higher thickness. Three reasons may explain this error: the bending of the rod used to apply the torque to the hinge, the fact that the force is not applied to the hinge axis which introduces torsion, and the hinge roughness. These three factors were modelled and taken into account to correct the model. The comparison with other theories show that there remains an uncertainty on the expression to be chosen as best approximation for the bending stiffness of the circular notch hinge.

## 7 Acknowledgements

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## 8 Nomenclature

$\alpha_f$	Rotation angle around <b>Z</b> -axis
$\alpha_{fi}$	Rotation angle around <b>Y</b> -axis
$\alpha_t$	Rotation angle around <b>X</b> -axis
$\beta$	Ratio $h_0/2R$
$\gamma$	Ratio $b/2R$
$\varphi$	Azimuth direction angle around <b>Y</b> -axis
$\theta_r$	Rotation angle around <b>X</b> -axis due to the off-centreness applied force

$\sigma$	Stress in the notch hinge	$O$	Shearing load along <b>Y</b> -axis
$\sigma_{max}$	Maximum stress in the notch hinge	$P$	Shearing load along <b>Z</b> -axis
$b$	Thickness of the hinge	$R$	Radius of the circular notch hinge
$E$	Elastic modulus (Young modulus)	$R_a$	CLA roughness
$G$	Torsion modulus	$R_p$	Peak to valley height
$h_0$	Minimal thickness of the notch hinge	$R_t$	Torsion stiffness of the hinge <b>X</b> -axis
$I$	Cross section inertia area moment	$t_i$	Pulse duration in WEDM
$K$	Factor in Smith and Chetwynd's theory	$u$	Displacement along <b>X</b> -axis
$K_1$	Bending factor in Merken and Debongnie's theory	$u_i$	Open circuit voltage in WEDM
$K_2$	Stretching factor in Merken and Debongnie's theory	$v$	Displacement along <b>Y</b> -axis
$K_3$	Shearing factor in Merken and Debongnie's theory	$v_{read}$	Displacement given by the sensor
$K_4$	Displacement factor in Merken and Debongnie's theory	$w$	Displacement along <b>Z</b> -axis
$K_r$	Roughness factor in Merken and Debongnie's theory	<b>X</b>	<b>X</b> -axis of the system of coordinates, aligned with the axis of symmetry of the hinge
$M_f$	Bending torque around <b>Z</b> -axis	<b>Y</b>	<b>Y</b> -axis of the system of coordinates, aligned with the line connecting the centres of the two arcs defining the hinge
$M_{fi}$	Bending torque around <b>Y</b> -axis	<b>Z</b>	<b>Z</b> -axis of the system of coordinates, perpendicular to the <b>X</b> - and <b>Y</b> -axes.
$M_t$	Torsion torque around <b>X</b> -axis		
$N$	Stretching load along <b>X</b> -axis		

## APPENDIX

### Results of the measurement

Thickness in mm

$\alpha_f$  angular displacement, in degrees

$M_f$  reaction torque, in Nmm

hinge 1 : 0,066				hinge 2 : 0,072				hinge 3 : 0,146				hinge 4 : 0,175	
$\alpha_f$	$M_f$	$\alpha_f$	$M_f$	$\alpha_f$	$M_f$	$\alpha_f$	$M_f$	$\alpha_f$	$M_f$	$\alpha_f$	$M_f$	$\alpha_f$	$M_f$
0.0011	-0.0188	1.1660	0.8146	-0.0003	0.0094	0.7172	0.7252	0.0002	-0.0188	0.5528	3.7435	0.0001	-0.0188
0.0006	-0.0141	1.1660	0.8146	-0.0011	0.0188	0.7176	0.7204	-0.0001	-0.0141	0.5522	3.7529	-0.0004	-0.0094
0.0006	-0.0141	1.1660	0.8146	-0.0003	0.0094	0.7176	0.7204	-0.0004	-0.0094	0.5539	3.7247	-0.0001	-0.0141
0.0533	0.0518	1.2773	0.8900	0.0532	0.0848	0.7726	0.7770	0.0540	0.0800	0.5960	4.0213	0.0444	0.2543
0.0528	0.0565	1.2773	0.8900	0.0532	0.0848	0.7730	0.7722	0.0523	0.1083	0.5948	4.0402	0.0489	0.1742
0.0533	0.0518	1.2773	0.8900	0.0536	0.0800	0.7730	0.7722	0.0548	0.0659	0.5954	4.0307	0.0505	0.1460
0.1085	0.0942	1.3887	0.9653	0.1083	0.1413	0.8277	0.8335	0.0950	0.3955	0.6369	4.3368	0.0801	0.6781
0.1080	0.0989	1.3892	0.9606	0.1083	0.1413	0.8285	0.8240	0.0941	0.4097	0.6377	4.3227	0.0852	0.5886
0.1085	0.0942	1.3892	0.9606	0.1087	0.1366	0.8288	0.8193	0.0933	0.4238	0.6377	4.3227	0.0849	0.5933
0.1642	0.1318	1.5000	1.0406	0.1638	0.1931	0.8831	0.8853	0.1373	0.6875			0.1169	1.0830
0.1637	0.1366	1.5005	1.0359	0.1638	0.1931	0.8843	0.8711	0.1356	0.7157			0.1214	1.0030
0.1642	0.1318	1.5005	1.0359	0.1642	0.1884	0.8843	0.8711	0.1384	0.6686			0.1222	0.9888
0.2198	0.1695	1.6123	1.1066	0.2188	0.2496	0.9386	0.9371	0.1783	1.0030			0.1547	1.4691
0.2198	0.1695	1.6113	1.1160	0.2192	0.2449	0.9397	0.9229	0.1772	1.0218			0.1585	0.4032
0.2198	0.1695	1.6118	1.1113	0.2196	0.2401	0.9390	0.9323	0.1791	0.9888			0.1595	1.3844
0.2755	0.2072	1.7236	1.1819	0.2747	0.2967	0.9936	0.9936	0.2195	1.3138			0.1923	0.8600
0.2755	0.2072	1.7231	1.1866	0.2743	0.3014	0.9948	0.9794	0.2190	1.3232			0.1963	1.7893
0.2755	0.2072	1.7236	1.1819	0.2747	0.2967	0.9944	0.9841	0.2203	1.2996			0.1961	1.7941
0.3869	0.2825	1.8349	1.2572	0.3298	0.3532	1.0487	1.0501	0.2613	1.6151			0.2297	2.2555
0.3869	0.2825	1.8344	1.2620	0.3298	0.3532	1.0502	1.0312	0.2599	1.6387			0.2334	2.1896
0.3869	0.2825	1.8349	1.2572	0.3298	0.3532	1.0499	1.0359	0.2621	1.6010			0.2337	2.1849
0.4988	0.3532	1.9466	1.3279	0.3852	0.4050	1.1045	1.0972	0.3028	1.9212			0.2681	2.6322
0.4978	0.3626	1.9461	1.3326	0.3860	0.3955	1.1053	1.0877	0.3028	1.9212			0.2710	2.5804
0.4983	0.3579	1.9461	1.3326	0.3852	0.4050	1.1053	1.0877	0.3037	1.9071			0.2723	2.5569
0.6102	0.4285	2.0564	1.4173	0.4407	0.4568	1.1596	1.1537	0.3443	2.2273			0.3054	3.0278
0.6092	0.4379	2.0579	1.4032	0.4407	0.4568	1.1604	1.1442	0.3449	2.2178			0.3080	2.9807
0.6097	0.4332	2.0564	1.4173	0.4411	0.4520	1.1604	1.1442	0.3457	2.2037			0.3099	2.9477
0.7211	0.5086	2.1671	1.4974	0.4961	0.5086	1.2146	1.2102	0.3859	2.5333			0.3440	3.3998
0.7206	0.5133	2.1681	1.4880	0.4961	0.5086	1.2154	1.2007	0.3864	2.5239			0.3459	3.3668
0.7206	0.5133	2.1681	1.4880	0.4965	0.5038	1.2154	1.2007	0.3870	2.5145			0.3478	3.3338
0.8324	0.5839	2.2799	1.5586	0.5512	0.5651	1.2674	1.2949	0.4277	2.8347			0.3832	3.7623
0.8319	0.5886	2.2804	1.5539	0.5512	0.5651	1.2685	1.2808	0.4282	2.8253			0.3848	3.7341
0.8319	0.5886	2.2799	1.5586	0.5516	0.5603	1.2685	1.2808	0.4277	2.8347			0.3861	3.7105
0.9438	0.6592	2.3911	1.6340	0.6067	0.6169			0.4697	3.1314			0.4216	4.1390
0.9433	0.6639	2.3916	1.6292	0.6070	0.6121			0.4695	3.1361			0.4224	4.1249
0.9438	0.6592	2.3916	1.6292	0.6070	0.6121			0.4692	3.148			0.4243	4.0919
1.0052	0.7346			0.6617	0.6734			0.5112	3.4374			0.4595	4.5252
1.0547	0.7393			0.6621	0.6686			0.5107	3.4468			0.4613	4.4922
1.0547	0.7393			0.6625	0.6639			0.5112	3.4374			0.4621	4.4781

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