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Robust hybrid mass damper

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ABSTRACT

In this paper, the design of a hybrid mass damper (HMD) is proposed for the reduction of the resonant vibration amplitude of a multiple degree-of-freedom structure. HMD includes both passive and active elements. Combining these elements the system is fail-safe and its performances are comparable to usual purely active systems. The control law is a revisited direct velocity feedback. Two zeros are added to the controller to interact with the poles of the plant. The developed control law presents the particularity to be simple and *hyperstable*. The proposed HMD is compared to other classical control approaches for similar purpose in term of vibration attenuation, power consumption and stroke.

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1. Introduction

Tuned mass dampers (TMD) or dynamic vibration absorber (DVA) are small oscillators, appended to a primary structure to dissipate the vibrational energy of a specific resonance. The main reduction of the primary structure oscillation amplitude is achieved by the spring force (or the inertia force) of the TMD and the viscous damper in the TMD makes the rest and also makes the TMD insensitive to changes in the disturbing frequency (see [1–5]). Since their first invention, these practical and robust devices have found numerous applications ranging from civil structures to precision engineering. The beauty is that the same design rules can be used to damp nanometer vibrations or meter vibrations. The most popular is the so-called Den Hartog's method [6]; other interesting rules can be found in [7,8]. Only the technology changes. The major drawback of TMD is that they are tuned to damp only one specific resonance. To some extent, broadband dampers (e.g. mass mounted on a layer of elastomer) may damp several resonance modes, with a limited efficiency. A more efficient method to treat several resonances is to introduce an actuator which controls the force that the reaction mass applies on the structure. The resulting device is often called an active mass damper (AMD). In order to add viscous damping to the structure, the natural way is to drive the actuator with a signal which is proportional to the velocity. However, for large values of the control gain, the AMD poles tend to be destabilized, even when the structure velocity is measured near the AMD. It is believed that this is the reason why many elaborated control strategies have been introduced to control AMDs (although it is rarely openly confessed): classical tuning [9], fuzzy controllers [10–12] pole placement [13] Lyapunov's method [14] optimal control [15–18], μ-synthesis [19], H-infinity [20] or sliding mode control [21].

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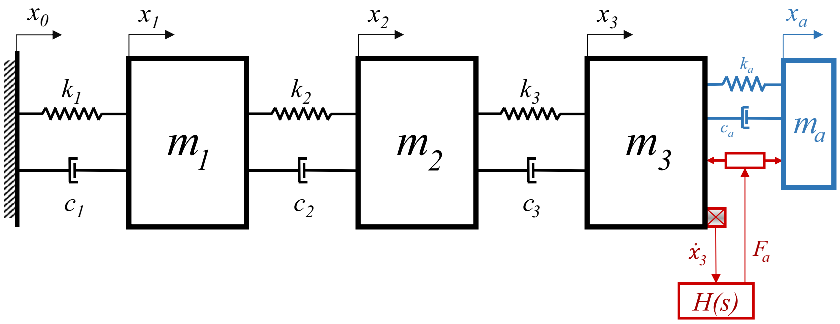

Fig. 1. Simplified model of a flexible structure with an AMD/HMD. (Blue: passive part, red: active part). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

On the other hand, a novel class of AMD has appeared which are trying to combine several objectives at the same time. These devices are gathered under the common name of hybrid mass damper (HMD), or hybrid vibration absorber (HVA) even though the objective pursued may significantly differ from one HMD to the other. For example, in [18], a HMD is presented, where an optimal control is used to combine structural damping with a restricted stroke of the actuator. In [22] a H-infinity optimal control is used to minimize both the response and the control effort.

Among the HMD, several of them have the property of being fail safe, which means that they still behave as TMD even when the feedback control is turned off. In [23] the control relies on the pole placement technique. In [24], a dual loop approach is preferred to increase the stability margins. In [25], a two degree of freedom system is studied, which can behave as an active mass damper to suppress the vibrations induced by small earthquakes, and as a tuned mass damper to suppress the vibrations of a targeted mode excited by a big earthquake. In the active configuration, a linear quadratic regulator is used with a large gain on the structural velocity. The control is switched off above a threshold value of the structure displacement, i.e. when the actuator cannot deliver the requested force anymore.

The controller proposed in this paper is fail safe and unconditionally stable. The strategy consists of placing a pair of zeros adequately in the controller in order to obtain interacting poles and zeros in the open loop transfer function. Besides these assets, the control approach proposed requires a low consumption and is extremely efficient under harmonic excitation, and is extremely simple compared to controller found in the literature.

The paper is organized as follows. Section 2 presents the simplified flexible structure which is used to illustrate the controller efficiency. Section 3 contains the analysis of the stability of active/hybrid mass damper. Sections 4 and 5 present the concept of the proposed controller and its design laws. Section 6 discusses the performances and limitations of the new controller and compares it to others. Section 7 draws the conclusions.

2. Description of the simplified flexible structure

Consider the system shown in Fig. 1. It represents a flexible structure, excited from the base x_0. The mass suspended has been divided in three parts in order to include three resonances. In the remaining of the paper, the following numerical values have been used: $m_1 = m_2 = m_3 = 100$ kg, $k_1 = k_2 = k_3 = 4 \cdot 10^6$ N/m; leading to the following eigen-frequencies $f_1 = 14.1$ Hz, $f_2 = 39.7$ Hz, $f_3 = 57.3$ Hz. Dashpot constants c_1, c_2, c_3 are tuned in order to provide a modal damping ratio of $\xi = 0.01$ to all modes.

A HMD is appended on m_3, as shown in Fig. 1. The passive part of the HMD (in blue) consists in a mass m_a, a spring with a coefficient k_a and a damper with a viscous damper coefficient c_a. The values of these parameters depend on the operating mode (see Section 3). The active part (in red) consists in an absolute velocity sensors mounted on m_3, a controller $H(s)$ and an actuator F_a between m_3 and m_a.

3. Stability analysis of active/hybrid mass damper

Consider the active mass damper (AMD) as shown in Fig. 1. Typically, the natural frequency of the device (m_a, c_a, k_a) is chosen significantly lower than the frequency of the first mode of the structure. The following numerical values have been used: $m_a = 9.2$ kg, $k_a = 2861$ N/m, $c_a = 197.8$ N s/m. Under these conditions, the AMD behaves as a nearly perfect force generator in the whole frequency range containing the dynamics of the structure. In order to damp the structural resonances, the actuator is driven by a force proportional to the absolute velocity of the structure measured on mass m_3

$$F_a(s) = H(s)\dot{x}_3(s) \tag{1}$$

where $H(s)$ is a simple gain.

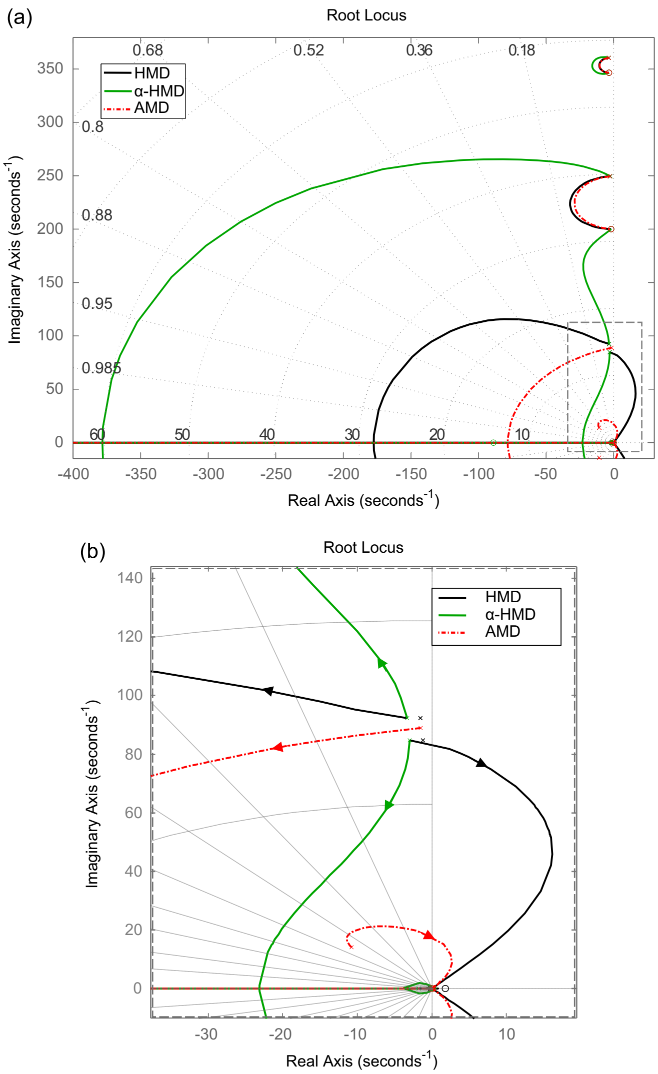

Fig. 2. (a) Root locus for AMD, HMD and α-HMD. (b) Detail near the origin of the root locus. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

The corresponding root locus is shown in Fig. 2(a). The root locus of the AMD (red dotted line) shows three loops associated to the flexible modes of the structure and a small loop at low frequency linked to the dynamics of the actuator. This kind of active device is very efficient to damp structural resonance. However, in case of failure (e.g. no current in the actuator), the first mode of the system is uncontrolled because the AMD parameters are not tuned to damp the first mode. Moreover, it can be seen that for high gains, the root locus passes through the imaginary axis and can create instabilities.

In order to obtain a fail-safe system, let us further consider a hybrid mass damper (HMD), which is the same as the AMD, except that the mechanical parameters are chosen according to the design rules of Den Hartog to damp the first resonance of the structure: $m_a = 1.84$ kg, $k_a = 14.3$ kN/m, $c_a = 19.8$ N s/m. The same control force as for the AMD is used for the HMD (1). This HMD is fail-safe because it will continue to behave as the TMD described above when the controller is turned off. The resulting root locus, also shown in Fig. 2(a and b) for comparison (continuous black line) shows clearly the limitation of this approach. On the zoom close to the origin shown in Fig. 2(b), one can see that the initial pole has been duplicated by the TMD. One sees also that the lower frequency pole goes rapidly in the right half-plane, leading to instability. The closed loop system will always be near this stability limit even with low gains.

This is due to the absence of zero between the pole of the inertial damper and the first pole of the structure. As explained in the introduction, several elaborations of the controller have been proposed to bypass this limitation.

In the next section, we will present a simple alternative controller which addresses the root cause of the problem by adequately placing a pair of zeros at the right location, which will allow recovering a guaranteed stability of the closed loop system.

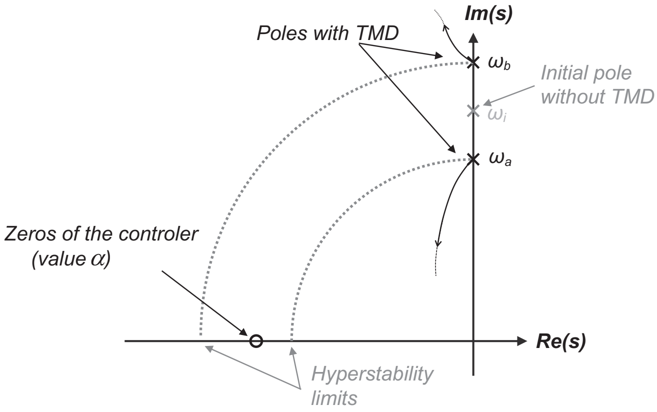

Fig. 3. Hyperstability limits on the pole map, without structural damping.

4. A robust hybrid mass damper (α-HMD)

As in the previous section, consider a HMD where the mechanical parameters have been tuned in such a way that the uncontrolled device behaves like a TMD tuned on the first mode. The controller is still using the absolute velocity of m_3, however, the filter is now defined as

$$H_\alpha(s) = g_\alpha \frac{(s+\alpha)^2}{s^2} \tag{2}$$

Compared to Eq. (1), one sees that a pair of poles and zeros have been introduced in Eq. (2). The parameter α is tuned to make the controller hyperstable. Its value will be discussed in Section 5. The tuning of gain g_α depends on the objectives of the controller and also on the capability of the transducer (see Section 6). In the remaining of the paper, we will call it α-HMD.

Taking $\alpha = 2\pi f_1 = \omega_1$ the resulting rootlocus of the α-HMD is also shown in Fig. 2 (continuous green line). Notice that, in order to avoid a constant component in the feedback loop, a high pass filter $H_{HP}(s)$ at very low frequency has been added. It is a first-order filter with a cut-off frequency equals to $\alpha/50$. The resulting control force is:

$$F_a(s) = H_\alpha(s) H_{HP}(s) \dot{x}_3(s) \tag{3}$$

This new controller has several advantages. Firstly, the whole root locus plot is in the left half-plane, meaning an unconditional stability of the feedback system (infinite gain margin). Secondly, the two branches move directly away from the imaginary axes (this will be further discussed in Section 5). Thirdly, we can observe that the damping authority on all modes is equal or larger than for the AMD. Fourthly, as it is based on a TMD device, in case of failure of the active part, the first mode is still passively controlled by the TMD.

The very small loop close to the origin is due to the high-pass filter introduced by Eq. (3). This filter does not change the controller behavior for the flexible modes.

5. Designing the α-HMD

Due to the presence of a TMD tuned on the mode to control, the pole of this mode ω_i is replaced by two new poles. One has a frequency slightly lower than the resonance frequency of the initial system, and one has a frequency slightly higher. Let us call them ω_a and ω_b respectively. The resulting root locus is shown in Fig. 3. As long as $\alpha \in [\omega_a; \omega_b]$, the branches of the root locus go immediately inside the left half-plane.

Actually, using the Routh criterion, it can be readily found that the stability condition is:

$$\omega_a \le \alpha \le \omega_b \tag{4}$$

Notice that this criterion applies regardless of the mode on which the α-HMD is tuned. Fig. 4 further illustrates the departure angles of the root locus branches for extreme cases $\alpha = \omega_1$ and $\alpha = \omega_2$. In these cases, one of the departure angles is equal to $\pm\pi/2$ and its branch is tangent to the imaginary axis.

In case of lightly damped systems, the limits defined by Eq. (4) can be extended due to the fact that the poles (ω_a and ω_b) are no more on the imaginary axis. The α parameter can be optimized depending on the application. In the following, it has been chosen that $\alpha = \omega_i$ ensuring the condition expressed in Eq. (4) is verified. The next section studies the performance of the α-HMD.

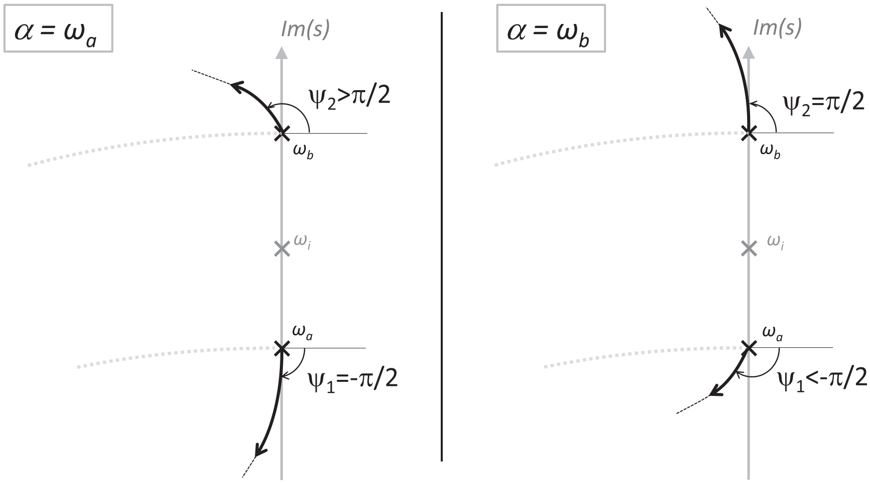

Fig. 4. Departure angles on the root locus, left: $\alpha = \omega_a$, right: $\alpha = \omega_b$, without structural damping.

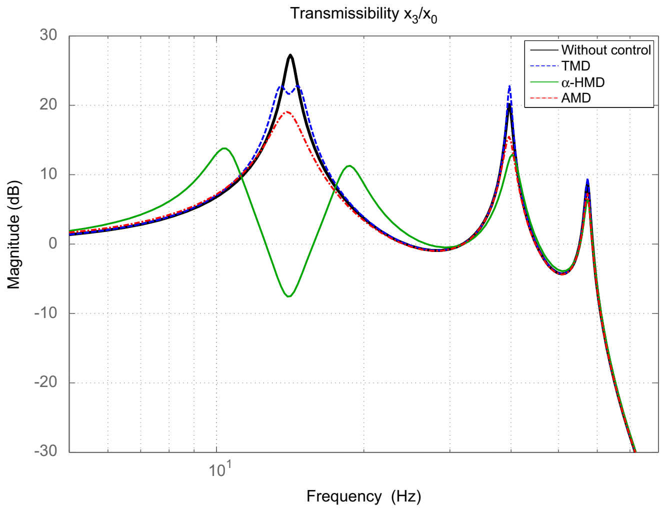

Fig. 5. Transmissibility function x_3/x_0 for various control devices.

6. Performances and limitations of the α-HMD

In the following the α-HMD is compared with the passive TMD and the purely active AMD. Each of these devices considered to be mounted alternatively on the system is shown in Fig. 1. As these devices are clearly different, the device parameters have been chosen arbitrarily. The gain g_α and the gain of the AMD are both tuned to double the damping of the first mode of the system with TMD. To sum up, the first mode of the system has a damping factor of 1 percent when it is uncontrolled, of 3.5 percent when it is controlled with the passive system, and of 7 percent when it is controlled with the active and hybrid systems.

TMD or hybrid TMD are usually designed for applications where the disturbance is harmonic, or narrow band (e.g. helicopters). However, in order to get some insight into the system dynamics, the analysis is carried out in two steps. The first one analyzes the system response in the frequency domain while it is subjected to a uniform broadband disturbance on x_0. Then we will analyze the system response in the time domain, under a harmonic excitation from x_0 at the resonance frequency of the first mode. For both steps, the following outputs have to be considered:

- The displacement of the mass x_3 where the control device is attached. It represents directly the performances of the control in terms of structural damping.
- The relative displacement $x_a - x_3$ that represents the stroke of the actuator. It is an important parameter for the practical design.
- The actuator force necessary to obtain the control performances. This is also an important parameter for the practical design.

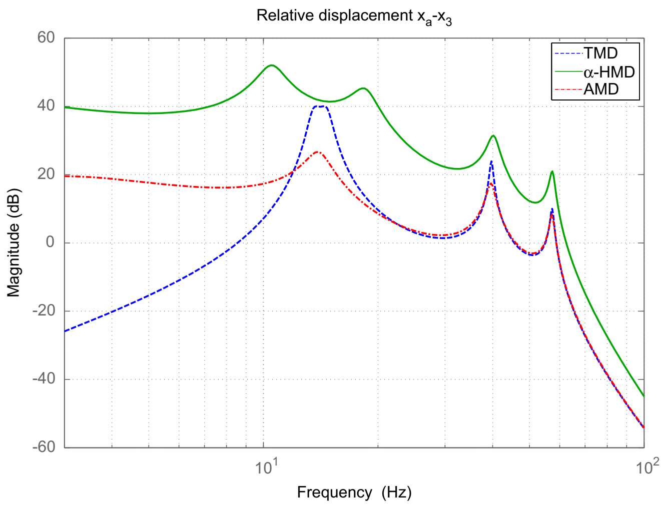

Fig. 6. Relative displacement spectrum of the actuator $(x_a - x_3)$ for a broadband ground disturbance (x_0).

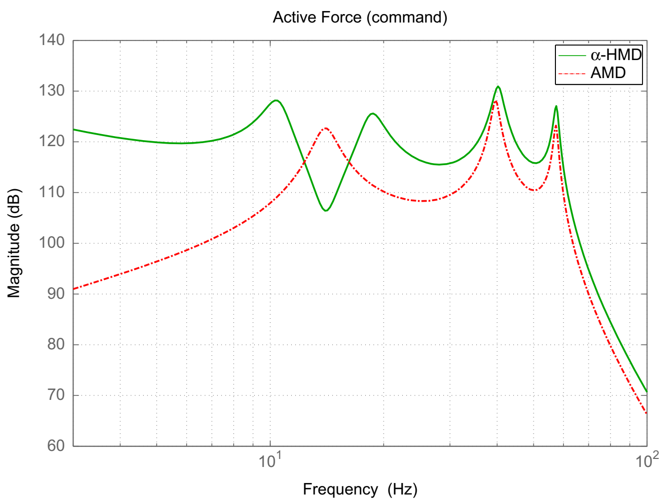

Fig. 7. Active force spectrum of the actuator (command) for a broadband ground disturbance (x_0).

6.1. *Broadband analysis*

Fig. 5 shows the transmissibility function x_3/x_0 between m_3 and a solicitation coming from the ground x_0. For the α-HMD controller one sees that the initial peak of the controlled mode is replaced by a huge antiresonance due to the introduction of the zeros at this pulsation, associated with the dynamical effect of the TMD. The resulting absorption capacity is greatly improved. The two main resulting peaks are well separated as expected when looking at the root locus (Fig. 2). Moreover, as for the AMD, the α-HMD damps the other flexible modes, whereas it is not the case for the purely passive TMD.

Fig. 6 compares the relative displacement spectrum of $(x_a - x_3)$ of the TMD, the AMD and the α-HMD, assuming that the system is excited from the base by a uniform random noise of unitary spectral density. One can see that the superior performance of the α-HMD controller is obtained at the cost of a much larger relative displacement of the moving mass (m_a) over a wide frequency range.

On the other hand, the active force spectrum of the actuator is plotted in Fig. 7. At the resonance frequency of the first mode, the α-HMD controller requires a smaller force than the classical AMD, because the natural frequency of the passive part of the α-HMD is close to the target resonance frequency of the structure. For other frequencies, the α-HMD controller requires a larger force, this is a consequence of the fact that, in this example, its mass is five times smaller than the mass of the AMD and that the gains have been tuned to provide an equivalent damping.

Nevertheless, these drawbacks (huge relative displacements and large active forces on a wide frequency range), do not appear as critical. Indeed, the envisaged applications for the α-HMD are mainly harmonics and not necessarily broadband. This is why, in the following, we study to the behavior of the α-HMD controller in the case of an harmonic disturbance.

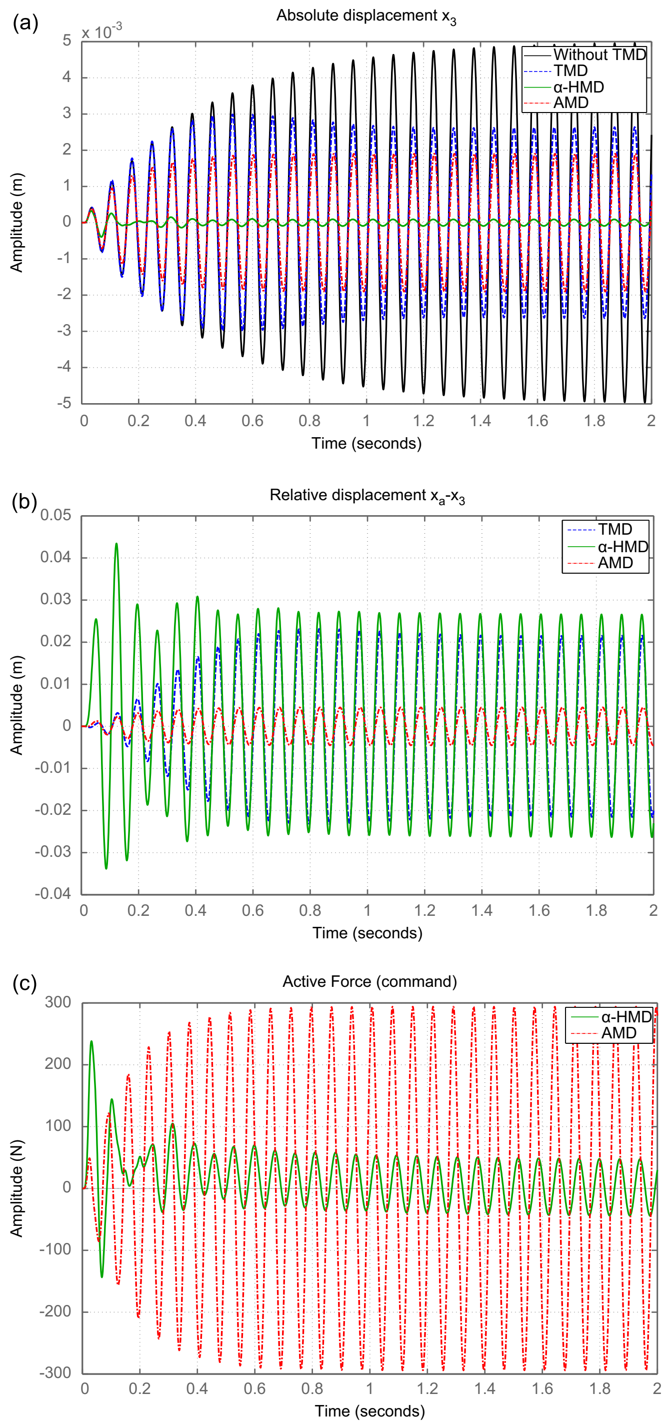

Fig. 8. Time response to a ground disturbance: $\ddot{x}_0 = A\ \sin(\omega_1 t)$: (a) absolute displacement x_3; (b)Relative displacement of the actuator $(x_a - x_3)$. (c) Active force of the actuator (command).

6.2. Performances under harmonic disturbance

Harmonic disturbance represents a more realistic situation in a field of application of a TMD (hybrid or not). The input is a harmonic ground motion of x_0 at the first resonant frequency of the uncontrolled structure (14.1 Hz). Its amplitude is chosen in order to obtain an acceleration equals to 1 g in the steady state on mass m_3 for the uncontrolled structure. The outputs are analyzed in the time domain during their transient to steady-state responses.

Fig. 8(a) compares the time history of the absolute displacement x_3 without TMD, with TMD, with AMD and with the proposed α-HMD. The residual motion of m_3 is about 10 times smaller with the α-HMD than with the AMD. The corresponding peak acceleration of m_3 is 0.53 g with TMD, 0.38 g with the AMD, and 0.185 g with the α-HMD.

As it was already emphasized in the previous section, the price to pay for this superior performance is a much larger stroke of the actuator ($x_a - x_3$), as shown in Fig. 8(b). However, interestingly, it also corresponds to a much smaller force delivered by the actuator, except during the first oscillations. The control effort is large at the beginning to cancel rapidly the oscillations. Afterwards, the actuator simply increases the motion of reaction mass to counteract the disturbance (stiffness force part of the reaction mass) and to increase the energy dissipation (viscous force part of the reaction mass). Notice finally that, whatever the value of the control gain, the closed loop system is unconditionally stable.

7. Conclusion

A new control law for hybrid vibration absorber has been presented in this paper, which has been referred to as α-HMD. Besides its simplicity, the proposed control law belongs to the restricted class of hyper stable control, which means that the stability of the closed loop system is guaranteed. Design guidelines have been presented to tune the control parameter adequately for an arbitrary plant.

Then, the performances of the α-HMD have been discussed on a three degree-of-freedom model, and compared to a TMD, and an AMD. It has been shown that, for better damping performance, the α-HMD requires smaller active forces and hence less energy for the active element than the AMD, which makes it ideal for damping vibrations of embedded equipments. The unique price to pay is to allow a larger deflection of the reaction mass. The experimental validation of this strategy on a scaled system is left for a future work.

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