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UPDATING INDUSTRIAL MODELS UNDER A GENERAL OPTIMIZATION ENVIRONMENT

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ABSTRACT

Model updating techniques are widely used by analysts in order to verify and eventually improve the correlation between analytical and experimental results.
The application of such techniques in large scale models is made difficult by the inverse nature of the problem which allows multiple solutions.
Also the increasing computational power of computers and the improvement of analytic algorithms to manage large problems generates an accelerated tendency to over-discretize the FE models, which reduces even more the ratio between measured and analytical degrees of freedom, and increase substantially the numbers of potential adjusting parameters.
Thus, the situation points towards the use of reduction techniques to solve the matching incompatibility without loosing accuracy of the analytical results.
This paper deals with the tuning of FE models using experimental measures. It considers the use of appropriate cost functions that express the discrepancies between the analytical and experimental models in the modal space. The technique is successfully implemented in a general purpose optimization package used for industrial applications. This environment allows an open choice of the design parameters, and to perform easily parametric studies, statistical analyses, multi-objective optimizations.

1. INTRODUCTION

Development of tools like FE and EMA has made correlation between analytical models and experimentally derived models an important task to validate the simulations. In order to reduce the discrepancies, various methods have been proposed. Reviews can be found [1], [2], [3], [4]. In general, all methods can be categorized into two approaches : global and local methods. Global methods, which are based on the correction of the global stiffness and mass matrices of the FE model, are recognized as physically meaningless. Local updating techniques use physical parameters of the input model. The use of such design parameters allows :

- Direct interpretability of the solution, so the feasibility of solution is easier to analyze.
- Judicious selection of the parameters; the analyst knowledge can be exploited.
- Direct use of the same optimization approach to improve the design, once updating is achieved. The updated model can be readily use for other design considerations : stress calculations, fatigue life prediction, etc.

Of course, the need for sensitivity calculation makes the task more complicated and time consuming. In general a finite differences process is used, and the solution is as good (or as wrong) as the model implicit capability, which depends on two types of errors :

- *Model structure errors*, which refers to the correctness and completeness of the model equations to represent the physical equations
- *Model parameter errors*, which include the uncertainty on boundary conditions, and inaccuracy on model parameters.

Among the model structure errors can be found the *discretization errors*, which are related to the number of elements, to their type, and their spatial distribution. One way to deal with this is to minimize the errors of the model equations by (i.e., discontinuities of the stress fields) by adding or deleting elements. This process is termed as *«adaptive mesh refinement»*. Other structure errors can be the assumption of linearity, the simplification of dissipative effects, etc.
A common practice of the model updating community has been to consider the initial model as enough powerful to represent the measured behavior. Then only parameter errors are handled.

2. THEORY

2.1. Equivalency of two models in structural dynamics

In terms of his modal base, the dynamic behavior of a linear mechanical structure can be expressed by his FRF matrix :

$$H(\omega) = \sum_i \frac{\phi_i \phi_i^T}{\mu_i\left(\omega_i^2 - \omega^2\right)} \tag{1}$$

We can conclude, then, in order to have the same behavior, two models should verify the three following conditions : equal eigenfrequencies, equal mode shapes, equal modal masses.

In order to derive a consistent model updating, it is advisable to include constraints requiring that the FE modal masses converge to the measured modal masses. In practice, however experimentally derived modal masses are the least accurate information from a modal analysis test [4], so only the first and/or second conditions are considered . Sas [5] concludes that obtaining equal natural frequencies and mode shapes is not enough to reproduce the same dynamic behavior.

A non modal approach considers the direct use of the measures, say the operating deflection shapes or FRFs. In this case modal analysis may be avoided.

The article considers the following schema :

1. Quantify the correlation between the current model and the measurements, in terms of modes and/or of operating deflection shapes [6], [7].

2. Selection of parameters to be updated, thanks to a sensitivity analysis.

3. Updating of the model by minimizing a residue expressing the discrepancies of the model. The process produces a new model whose level of correlation is measured in a new step 2. As can be seen the process is iterative.

2.2. Correlation

Correlation between the experimental dynamic measurements and the analytical results can be done in terms of mode shapes or in terms of operating deflection shapes (FRFs). The first are found using the second, after a procedure of experimental modal analysis.

2.2.1. Modal Assurance Criterion

Up to now, the most common technique used to assess the results of the updating procedure is the MAC [6]. It gives quantitatively a good idea of the global closeness between experimental and FE mode shapes :

$$MAC(i,j) = \frac{\left(\phi_i^T \phi_j\right)\left(\phi_i^T \phi_j\right)}{\left(\phi_i^T \phi_i\right)\left(\phi_j^T \phi_j\right)} \tag{2}$$

where j and i corresponds to the indices of two mode shapes that can be from the same origin (experimental or analytical) in order to check linear dependency, or mixed in order to check correlation between the two model modal bases.

MAC values oscillate between 0 and 1. An unitary value means perfect correlation. In general this situation does not appear, and a value greater than 0.8 is judged acceptable. Two corresponding modes will have a high degree of correlation. This property is exploited by MAC to allow a correct mode tracking which is a common need for the construction of residues to minimize.

2.2.2. Modal Scale Factor

If the experimentally identified modal mass is considered to be accurate (i.e., by comparing several identifications), the use of the Modal Scale Factor [8] can also be considered as a way to compare two corresponding modes. MSF gives a least square estimate of the ratio between two corresponding modes

$$MSF(i) = \frac{\left(\phi_i^T \phi_i\right)}{\left(\phi_j^T \phi_j\right)} \tag{3}$$

2.2.3. Frequency Domain Assurance Criterion

An alternative to use identified mode shapes is the direct use of the measured operating deflection shapes (the FRF vectors). To measure the correlation, it has been proposed in [7] to use the Frequency Domain Assurance Criterion:

$$FDAC(\omega_a, \omega_x, j) = \frac{\left\{H_a(\omega_a)\right\}_j^T \left\{H_x(\omega_x)\right\}_j}{\left|\left\{H_a(\omega_a)\right\}_j\right|\left|\left\{H_x(\omega_x)\right\}_j\right|} \tag{4}$$

which correlates two FRF vectors taking into account *the frequency shift* that is produced by the displacement of the modal base on the frequency axis when model parameters are perturbed.

When no damping is modeled, the experimental FRFs are converted to real, using the following formula :

$$\left\{H_{REAL}\right\}_j = abs\left(\left\{H_{COMPLEX}\right\}_j\right) sign\left(\mathrm{Re}\left(\left\{H_{COMPLEX}\right\}_j\right)\right) \tag{5}$$

2.2.4. Frequency Response Scale Factor

Unlikely the mode shapes whose norm can be freely set, a deflection shape has fixed amplitudes. In general the FE model does not consider damping, so the norm of the vectors will go to infinity as the frequency approaches a resonance. If the frequency is « enough » far, damping will not affect the amplitude of the vector, and in that case it is reasonable to compare the amplitudes of the operating deflection shapes in a least square sense :

$$FRSF(\omega_a) = \frac{\left\{H_a(\omega_a)\right\}_j^T \left\{H_a(\omega_a)\right\}_j}{\left\{H_x(\omega_x^*)\right\}_j^T \left\{H_x(\omega_x^*)\right\}_j} \tag{6}$$

where

ω_x^* is the experimental frequency corresponding to ω_a (maximum FDAC);

j refers to the measured column of [H] ;

A well correlated model will have no frequency shift between the operating deflection shapes, unitary FDAC and FRSF at all frequencies.

2.3. PARAMETER SELECTION

An crucial step to correct a model is the selection of the parameters that will be updated. Selecting too many parameters increase unacceptably the computation time and decrease the conditioning of the problem, which may cause « numerical solutions ». Selecting too few parameters limits the search space, and will probably not find an optimal solution. In order to have criteria to chose the parameters the use of *error localization techniques* is imposed. Some of them are reviewed in [3], [4], [9], [14].

In our case, a simple sensitivity analysis will be used also. It assures a faster convergence but it must be mentioned that the parameters with the highest sensitivities do not have to include those that present inaccuracies at all. This method is widely used because it is easy to implement and gives a close idea of the design corrections that must be made after an updating procedure.

2.4. UPDATING

Two types of correction methods are available : global and local.

Global methods compute updated systems matrices in a single step. A mathematical model with improved behavior is obtained but no physical insight is possible.

The local methods minimize the norm of a residue that expresses the discrepancy between experimental and analytical results in terms of physical parameters of the input model at a local level.

It is important to have cost functions showing a smooth topological search space with no local minima around the global minimum. It is also relevant to have non negligible sensitivity of the objective with respect to the parameters. Well scaled design parameters are needed in order to avoid numerical problems.

2.4.1. Correlation numbers as objectives

A residue that has been used extensively is presented in [10]. It is based on a weighted sum of deviations from measured eigenvalues and eigenvectors :

$$J = \sum_i \left(\omega_{a_i} - \omega_{x_i}\right)^T W_{\omega_i} \left(\omega_{a_i} - \omega_{x_i}\right) + \sum_i J_i \qquad (7)$$

where

$$J_i = \left(\phi_{a_i} - \phi_{x_i}\right)^T W_{\phi_i} \left(\phi_{a_i} - \phi_{x_i}\right) \qquad (8)$$

and W_{ω_i}, W_{ϕ_i} are diagonal weight matrices to include confidence and scaling considerations for the natural frequencies and eigenmodes, respectively.

Let us just consider the second term, and use the following weight matrix, in order to have a relative error for each mode :

$$W_{\phi_i} = \frac{1}{\phi_{x_i}^T \phi_{x_i}} [I] \qquad (9)$$

Then each term of the cost function is given by

$$J_i = \frac{\left(\phi_{a_i} - \phi_{x_i}\right)^T \left(\phi_{a_i} - \phi_{x_i}\right)}{\phi_{x_i}{}^T \phi_{x_i}} \qquad (10)$$

which is the relative error of the modes. It can also be written as

$$J_i = 1 + MSF_i \pm 2\sqrt{MSF_i \;\, MAC_{ii}} \qquad (11)$$

Since in general the experimental modal mass is not a reliable quantity (experimentally estimated modal masses have an error in the order of 20% [4]. Then it is convenient to use a modal scale factor set to 1. If the phase between the modes is correctly set also (a condition for the residue), then the relative error can be expressed as

$$J_i/2 = 1 - \sqrt{MAC_{ii}} \qquad (12)$$

From which it can be concluded, that maximizing MAC comes to minimize the relative error J_i.

Equation (12) shows that MAC can also be used to update. In fact, [11] has presented **equation (7)** using the following weight:

$$W_{\omega_i} = \left[\max\left(\omega_{x_i}^2, \omega_{a_i}^2\right)\right]^{-1} \qquad (13)$$

in order to have a bounded cost function.

In [12] the measured modal displacements are used only to identify mode shapes :

$$W_{\phi_i} = [0] \qquad (14)$$

This is done so because each modal displacement in general have an error in the range of 10%.

2.4.2. Penalty on parameter changes

Usually in FE analysis, some parameters are estimated more accurately than others. This may be reflected in the optimization by introducing a third term on the cost function which produces the following optimization problem :

$$\bar{J} = J + \left(p_j - p_o\right)^T W_p \left(p_j - p_o\right) \qquad (15)$$

The weighting matrix W_p is usually chosen to be diagonal with the reciprocals of the estimated variances as elements. The reciprocal is used because accurate data has low variance but requires large weighting in the algorithm. The same idea can be applied to the residue which is known to have noise. A difficulty is the estimation of the variances, so the engineering ability to set confidence factors to both parameters and residues is very powerful to guide the minimization process [3].

3. IMPLEMENTATION

An important aspect to any model updating method is its practical implementation. Aspects like correspondence between dofs, model reduction, mode and frequency tracking are crucial for a successful updating. The optimization strategy was implemented using the BOSS/QUATTRO environment which allows a very flexible programming.

The minimization of the objective(s) function(s) considers two nested loops:

The outer loop consists in
-performing the analytical eigenanalysis
- normalizing and matching analytical and measured modes
- calculating design parameter sensitivities
- detecting solution convergence

The inner loop consists in the calculation of the parameter corrections by a constrained optimization method. Once the corrections are made control goes back to the outer loop [10].

BOSS updating

BOSS is a tool oriented towards multi-model optimization (updating is inscribed into this case). The main idea is to import from one or more models the interesting *parameters* that control the *results*, without dealing with the nature of any of these variables, nor with the complexities of the analysis software needed to produce the *results*.
In order to import *parameters* and *results*, BOSS communicates with each model data bank and result files trough « drivers ». Launching applications to have *results* available are done using « scripts ».
BOSS also has the power to set new *parameter* values, which will produce new *result* values that will be inscribed into a loop of a parametric study, a montecarlo simulation, or an optimization.
Once the needed inputs are into memory, a « parser » allows the building of complex *functions* that may be used into an optimization procedure.
As it may be seen the architecture is ideal for updating purposes. An schema oriented towards updating can be seen in **Fig. 1.** Updating considers two models : the experimental, which is fixed and is not parameterized, and the analytical which is a function of the design parameters. The correlation between them is done by a LINKing program which solves the dof correspondence and the mode pairing, and calculates appropriate residues like MAC, for example.
Then this residues are imported into the BOSS data base to be used by the BOSS OPTIMIZATION task. Processes are launched via scripts, and data communication trough drivers.
Since the experimental (reference) model is « fixed », a work file is generated before the OPTIMIZATION loop. This is done by a script that reads data from a universal file and writes a boss neutral result file that let eigendata be imported into the BOSS data base, and also write an internal work file which is going to be used by the LINKing program.

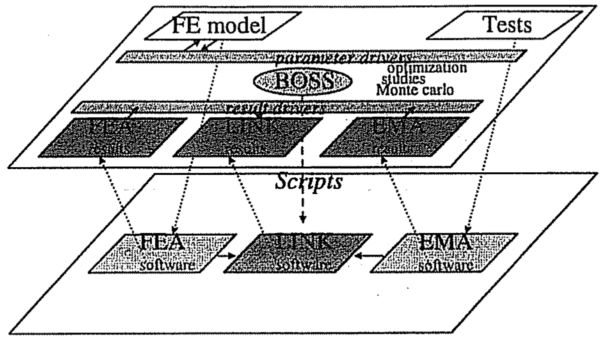

Figure 1. Boss updating diagram

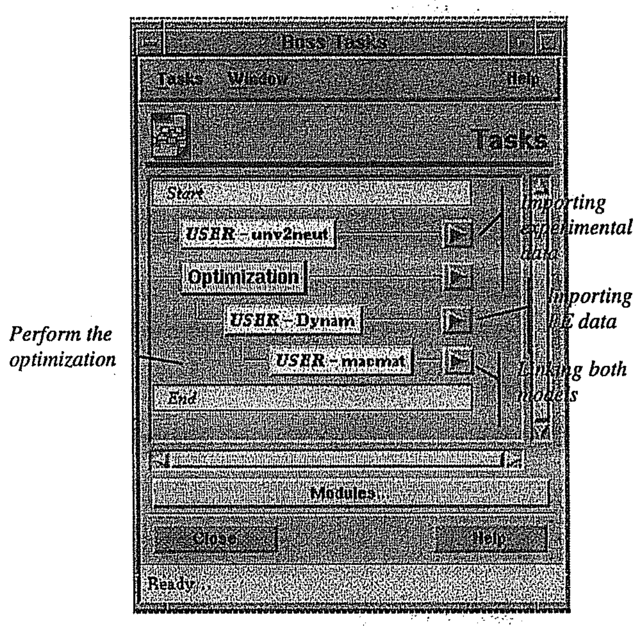

Figure 2. GUI example for modal correlation and updating

Apart from the FE file with the modes and the experimental work file the LINKing program needs also as input the correspondence between nodes, which include also the euler angles between the measured local coordinates systems and the FE global coordinate system. In this way, FE modal displacements are projected into the experimental local coordinates. If an experimental dof is zero then it is considered as not measured, and the FE dof will also be set to zero. (This is useful when measures on each node are mono or bi-axial like in the SNECMA test case presented later).
The program also writes the measured partition of the FE mode in a universal file format. This allows the analyst to visualize it and have a good idea of the quality of the measured dofs to represent the FE modal base.
The BOSS graphical user interface is shown in **Fig. 2.**

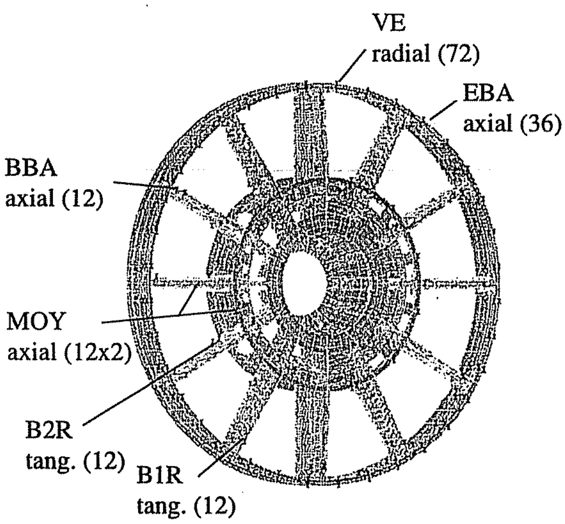

Figure 3. The fan frame and the actual sensor configuration

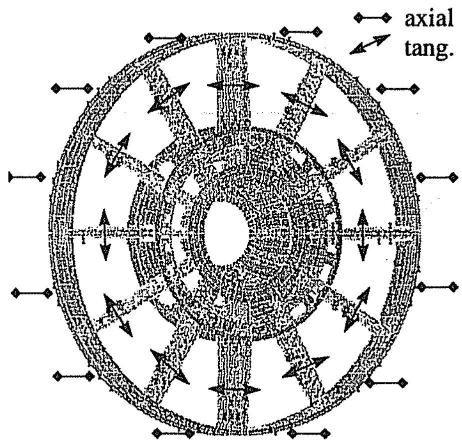

Figure 4. Efi setup for 169 dofs and 20 modes

4. APPLICATIONS

4.1. SNECMA TEST CASE

4.1.1. Presentation

The test case corresponds to a fan frame of a civil aircraft engine. Some of the characteristics of the SAMCEF FE model are : N° of elements : 19090, N° of dofs: 54714.
The structure represents a common situation on real life industrial test cases. Measurements were done on 169 nodes. Each node is instrumented with one monoaxial sensor that is oriented according to a cylindrical coordinate system. Measured dofs are organized into *components* as seen in **Fig.3**. Due to the cylindrical symmetry of the structure, double peaks are frequent.

4.1.2. Sensor placement

In order to compare the selected sensor distributions and its measured directions to the results from an automatic sensor location technique (which was not available by the time of the tests), the well known *Efi* method [13] was tested using 20 modes of the original FE model. Results are shown in **Fig. 4**.
The following observations are made:

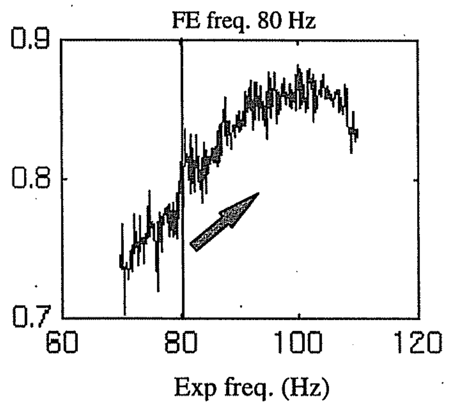

Fig. 5. FDAC correlation at FE 80 Hz

- On the external hoop, the most representative displacements are those on the axial axis.

- On the arms, local flexion displacements (tangential direction) are more important. On the big arms, measurements are most representative near the ends, while for the small, the measurements should be made at the middle.

- Comparing to measured dofs, all measurements made in the radial direction are not relevant for the representation of the chosen modal base. This set of measurements represent more or less 40% of the experimental data.

4.1.3. Correlation

The experimental mode shapes identified at SNECMA show a good degree of correlation with the FE eigenmodes.
In terms of operating deflection shapes, the correlation is evaluated using FDAC. Given a nominal operation speed of 4800 rpm (80 Hz), the FDAC vector shown in **Fig. 5** is obtained. A frequency shift of about 25% exists. The maximal correlation reaches a value of .88 which is judged acceptably good.

4.1.4. Parameter selection

In order to reduce the computation time during the optimization of the model, reducing the number of parameters is considered.
Since the FE model equilibrium equations are expressed in cartesian coordinates and the measurements where made in a cylindrical reference system (with just one direction measured on each node), the use of the MECE error localization technique [14] were not possible, at least not with all measures.
Then, for this case the use of the sensibility of the MAC and the eigenfrequency error with respect to the parameters is considered. In this way the list of parameters will be reduced.
BOSS handles sensitivities by using the finite differences approach :

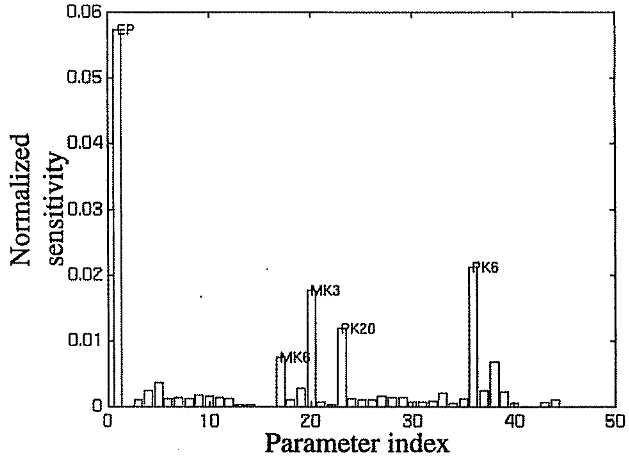

Fig. 6. Sensitivity analysis for the correlation (MAC) of the first experimental mode

$$\frac{\partial y}{\partial x} \approx \frac{y_p - y_o}{x_p - x_o} = \frac{\Delta y}{\Delta x} \qquad (16)$$

where

y goes for a result,

x represent a model parameter.

Since parameters may have an important difference of magnitude order, scaling of the sensitivities seems to be more easy to understand. We are interested in relative changes of the result, to relative changes in the parameters, using the non nule initial values as reference

$$\frac{\partial y_r}{\partial x_r} = \frac{\partial\left(y/y_o\right)}{\partial\left(x/x_o\right)} \approx \frac{\left(y_p - y_o\right)/y_o}{\left(x_p - x_o\right)/x_o} = \left(\frac{\Delta y}{\Delta x}\right)\left(\frac{x_o}{y_o}\right) \qquad (17)$$

The first factor in the right side of **equation (17)** is given in the log file of the BOSS optimization task. Reference values (the second factor) can also be found there.

Equation (17) was used for the frequency error for the 6 experimental modes that were considered to be well paired. An example of the sensibility results is shown in **Fig. 6.** The most perturbing parameters for each result are indicated on the bar chart. For confidentiality reasons, model updating results are not presented in this article.

5. COMMENTS

This paper demonstrates how an industrial partner like SNECMA is able to update the modal behavior of a model of considerable size (in this case an individual component of an aircraft engine), in an optimization environment like BOSS.
Implemented techniques are based on well known correlation techniques, that can also be used for model correction purposes.

Due to the flexibility of the BOSS environment, the user is allowed to select active results and weight them according to engineering criteria.

Thanks to the use of reduced model results, working with big scale models does not induce any problem *a priori,* as shown for the SNECMA test case. Anyway, due to the complex inverse nature of the problem, confidence in the results is dependent on the skill of the analyst to control the different sources of errors. Then, the use of model updating techniques as a black box is avoided.

In the future, the available updating tools will be used to assess the quality of a whole engine model after assembling the updated components which are tested in a free-free condition. The analysis of consistence between different updating procedures, performed on the same model in different conditions and /or different levels of reduction will also be considered.

6. ACKNOWLEDGEMENT

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7. BIBLIOGRAPHY

[1] Imregun, M., Visser, W.J., *A review of model updating techniques*, Shock and Vibration digest, 23(1), 1991.

[2] Mottershead, J.E., Friswell, M.I., *Model updating in structural dynamics : a survey*, Journal of sound and vibration, 167(2), pp 347-375, 1993.

[3] Friswell, M.I., Mottershead, J.E., *Finite element model updating in structural dynamics*, Kluwer academic publishers, 1995.

[4] Heylen, W., Lammens, S., Sas, P., *Modal analysis theory and testing*, K.U.L. , Leuven, Belgium, 1995.

[5] Janter, T., Sas, P., *Uniqueness aspects of model updating procedures*, AIAA Journal, vol. 28, no. 3, pp 538-543.

[6] Allemang, R. and Brown, D., *A correlation coefficient for modal vector analysis*, 1st Intl. Modal Anal. Conf., Orlando, Florida, pp 110 - 116, 1982.

[7] Pascual, R., Golinval, J.C., Razeto, M., *A frequency domain correlation technique for model correlation and updating*, 15th Intl. Modal Anal. Conf., Orlando, Florida, pp 587-592.

[8] Allemang R., *Investigation of some multiple input/output frequency response function experimental modal analysis techniques*, Ph.D. Dissertation, University of Cincinnati, 1980.

[9] Lallement,G ., Link, M., Caesar, B., et al., *Procedures for updating dynamic mathematical models* ; ESA contract report, 1991.

[10] Cobb, R. G., Canfield, R. A., Liebst, B. S., *Finite element model tuning using automated structural optimization system software* , AIAA Journal, Vol. 1, No. 1, 1996.

[11] Grossman, D.T., *An automated technique for improving modal test/analysis correlation* , AIAA paper 82-0640, 1982.

[12] Dascotte, E., Vanhonacker, P., *Development of an automatic model updating program* , 7th IMAC, Las Vegas, Nevada, pp 596-602, 1989.

[13] Kammer, D.C., *Sensor placement for on-orbit modal identification and correlation of large space structures*, AIAA Journal of Guidance, control, and dynamics, vol. 14, no. 2, pp. 251-259, march-april 1991.
[14] Collignon, P., Golinval, J.C., *Comparison of model updating methods adapted to local error detection*, 21th Intl. Sem. Modal Analysis, Leuven, Belgium, 1996.