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ABSTRACT

Model updating using a structural model is based on the analysis of the discrepancies between analytical and experimental results. In order to compare these quantities, a matching process is necessary.

The updating procedure can be based on mode shapes or on operating deflection shapes. The operating deflection shapes correspond to the deformations that the structure suffers when it is excited with an harmonic force. The direct use of operating deflection shapes for model updating is interesting since they contain information on all the excited mode shapes. Moreover, their reliability can be evaluated by the coherence function. Thus the errors associated to modal identification are avoided.

The local error indicator considered in this paper is an extension of a technique already used with expanded mode shapes. Physical insight is straightforward since the error measure corresponds to a local strain energy. Then, model correction can be performed by minimizing energy residuals.

1. INTRODUCTION

A finite element model (FEM) updating procedure goes through several logical steps. The first step consists in expanding the size of the measured response vectors or in reducing the size of the analytical model in order to match the results of both models. The second step is to apply correlation techniques to measure the quality of the simulation. In the third step, an error localization method allows to locate errors on the structure and to select the design variables that should be corrected. In the last step, the correction is performed in order to generate a new model. Then the updating procedure can be initiated again in an iterative way.

Basically, there are two kinds of dynamic responses in a structure : mode shapes (with their associated resonance frequencies) and the operating deflection shapes (or Frequency Response Function vectors). In a conservative system, modal parameters $\left(\omega_v, \{\phi\}\right)$ are solutions of the homogeneous equation :

$$[K]\{\phi\} = \omega_v^2 \, [M]\{\phi\} \tag{1}$$

where $[M]$ and $[K]$ are respectively the generalized mass and stiffness matrices. These matrices are explicit functions of the structural parameters like density, elasticity modulus, plate thickness, moment of inertia, etc.

The operating deflection shapes (ODS) correspond to the stationary dynamic responses $\{V\}$ that the structure suffers when it is excited by an harmonic force $\{F\}$ of unitary magnitude :

$$\left(-\omega^2 \, [M] + [K]\right)\{V\} = \{F\} \tag{2}$$

The operating deflection shapes can be directly measured on the actual structure.
This paper presents an ODS based model updating method by considering three steps : expansion, error localization and model correction. This method is in fact an extension of a technique that was already applied successfully to mode shapes in [1].

2. EXPANSION METHOD

2.1. Theoretical background

Hamilton's principle applied to a continuous system (using the harmonic movement assumption) can be expressed as a variational problem in the spatial variables :

$$\delta\left(\mathrm{T}_{\max} - V_{\max}\right) = 0 \qquad (3)$$

where

$$\mathrm{T}_{\max} = \frac{1}{2}\int_V \rho \;\; \omega^2 \{v\}^T \{v\}\, dV \qquad (4)$$

is the kinetic energy, obtained by using the displacement field $\{v\}$ associated to the cartesian coordinates $\{x, y, z\}$.

$$V_{\max} = U + P \qquad (5)$$

is the total potential energy; in **equation (5)**, U represents the strain energy of the structure and P is the potential energy of the prescribed external loads (body forces $\{\overline{X}\}$ and surface tractions $\{\bar{t}\}$).

The strain energy

$$U = \int_V W(\{\varepsilon\})\, dV \qquad (6)$$

can be expressed in terms of the strain field

$$\{\varepsilon\} = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \gamma_{12}, \gamma_{23}, \gamma_{31}\}^T$$

$$\text{with} \quad \gamma_{ij} = \gamma_{ji} = \varepsilon_{ij} + \varepsilon_{ji} \qquad (7)$$

which verifies the compatibility condition

$$\{\varepsilon\} = [D]\{v\} \qquad (8)$$

where the spatial differentiation operator $[D]$ is defined by

$$\begin{bmatrix} \frac{\partial}{\partial x_1} & 0 & 0 & \frac{\partial}{\partial x_2} & 0 & \frac{\partial}{\partial x_3} \\ 0 & \frac{\partial}{\partial x_2} & 0 & \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_3} & 0 \\ 0 & 0 & \frac{\partial}{\partial x_3} & 0 & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} \end{bmatrix}^T \qquad (9)$$

and the displacement field $\{v\}$ verifies the prescribed boundary conditions

$$\{v\} = \{\bar{v}\} \text{ on } S_v \qquad (10)$$

The potential energy P of the prescribed external loads is obtained from the displacement field $\{v\}$:

$$P = -\int_V \{\overline{X}\}^T \{v\}\, dV - \int_{S_\sigma} \{\bar{t}\}^T \{v\}\, dS \qquad (11)$$

where S_σ represents the part of the surface where traction forces are prescribed.

A less restrictive variational principle may be written in the form

$$\delta\left(\mathrm{T}_{\max} - V_{\max} + D\right) \qquad (12)$$

where

$$D = \int_V \{\lambda\}^T \left(\{\varepsilon\} - [D]\{v\}\right) dV + \int_{S_v} \{\mu\}^T \left(\{v\} - \{\bar{v}\}\right) dS_v \qquad (13)$$

D is a dislocation potential in which the Lagrange multipliers $\{\lambda\}$ and $\{\mu\}$ are used to relax the compatibility **equation (8)** and the prescribed displacement condition (10). In the general principle (12), the three fields $(\{v\}, \{\varepsilon\}, \{\lambda\})$ are subject to independent variations. A two-field principle was introduced by Fraeijs de Veubeke [2] assuming that the Lagrange multipliers $\{\lambda\}$ verify *a priori* the equilibrium conditions :

$$\begin{cases} [D]^T \{\lambda\} + \rho \;\; \omega^2 \{v\} + \{\overline{X}\} = \{0\} & \text{in V} \\ \{\mu\} = [n]^T \{\lambda\} = \{\bar{t}\} & \text{on } S_\sigma \end{cases} \qquad (14)$$

where

$$[n]^T = \begin{bmatrix} n_1 & 0 & 0 & n_2 & 0 & n_3 \\ 0 & n_2 & 0 & n_1 & n_3 & 0 \\ 0 & 0 & n_3 & 0 & n_2 & n_1 \end{bmatrix} \quad (15)$$

is the matrix of direction cosines of the outward normal to the boundary.

Then, integrating by parts, the general principle (12) can be reduced to

$$\delta F(\{\lambda\},\{\varepsilon\}) = \delta\left(\int_V \left(\{\lambda\}^T\{\varepsilon\} - W(\{\varepsilon\})\right) dV\right) +$$

$$\delta\left(+ \frac{1}{2}\int_V \left(\{\overline{X}\} + [D]^T\{\lambda\}\right)^T \{v\}\, dV - \int_{S_v} \{\lambda\}^T [n]\{\overline{v}\}\, dS_v \right) \quad (16)$$

In this principle, only fields $\{\lambda\}$ and $\{\varepsilon\}$ are subject to variations. The displacement field $\{v\}$ is no more independent since it is associated to the stress field $\{\lambda\}$ through **equation (14).** The application of principle (12) results in

$$\min \int_V \|\{\sigma\} - \{\lambda\}\|\, dV \quad (17)$$

where

$$\{\sigma\} = [H]\{\varepsilon\},\ \{\varepsilon\} = [D]\{v\} \quad (18)$$

and $[H]$ is the Hooke's matrix.

If the prescribed displacement field $\{\overline{v}\}$ is set to the measured displacement field, **the field $\{v\}$ can be interpreted as the expanded experimental field, on which compatibility and structural equilibrium equations of the analytical model are imposed.**

For convenience, a fictive displacement field $\{u\}$ may be associated to field $\{\lambda\}$, which complies the relations :

$$\{\lambda\} = [H]\{\varepsilon^*\}, \{\varepsilon^*\} = [D]\{u\} \quad (19)$$

The fields $\{\lambda\}$ and $\{v\}$ verify *a priori* the equilibrium conditions (14), but field $\{u\}$ is different from $\{v\}$ since the constitutive equations are not verified *a priori* $(\{\lambda\} \neq \{\sigma\})$.

Written in terms of displacements, the problem defined by **equation (17)** takes the form

$$\min \int_V \| [H][D](\{u\} - \{v\}) \|\, dV \quad (20)$$

The true expanded experimental vector will be found using problem (20) as long as the model equilibrium equations coincide with those of the actual structure, which is not the case in general. As long as the model represents acceptably the actual behavior, the expanded vectors found using (20) will be close to the true vectors.

2.2. Application to a discretized system

The discretization of the displacement fields gives

$$\{u(\{x\})\} = [N(\{x\})]\{U\}$$
$$\{v(\{x\})\} = [N(\{x\})]\{V\} \quad (21)$$

where $[N(\{x\})]$ is the shape function matrix; $\{V\}$ and $\{U\}$ are the vectors of generalized coordinates resulting from the finite element model of the structure.

If metric $[H]^{-1}$ is used, problem (17) is equivalent to search for the minimum of the residual energy

$$\min\ (\{U\} - \{V\})^T [K] (\{U\} - \{V\}) \quad (22)$$

with the following constraints

$$[K]\{U\} = \omega^2 [M]\{V\} + \{F\} \quad (23)$$

$$\left(\{V_1\} - \{\overline{V}\}\right)^T \left(\{V_1\} - \{\overline{V}\}\right) = 0 \quad (24)$$

where $\{V_1\}$ is the partition of $\{V\}$ corresponding to the measured degrees of freedom and $\{\overline{V}\}$ is the vector of measured coordinates.

The residue vector $\{U\} - \{V\}$ can also be interpreted as the result of « filtering » the experimental vector by the equilibrium equation of the model.

If noise is considered, condition (24) may be relaxed by adding a second term to (22), i.e. :

$$\min\ (\{U\} - \{V\})^T [K] (\{U\} - \{V\}) +$$

$$\alpha\left(\{V_1\} - \{\overline{V}\}\right)^T [K_{red}] \left(\{V_1\} - \{\overline{V}\}\right) \quad (25)$$

where α is a weighting coefficient that indicates confidence in the measurements and $[K_{red}]$ is the model stiffness matrix reduced to the measured set of degrees of freedom. This matrix is used as a weight to obtain a well scaled objective.

In order to reduce computational costs, **equation (27)** can be transformed by expressing the expanded vector as a linear combination of vectors from the truncated FE modal basis. This approximation reduces considerately the number of unknowns.
An important remark is that the preceding development was done with the assumption of a conservative system. In practice, energy dissipation is always present, but its effects are notorious close to resonances. Criteria are needed to choose the frequency(es) at which (25) is evaluated.

3. ERROR LOCALIZATION

Error localization methods seek for the locations on the structural model *where* discrepancies between experimental and analytical results may be present. The convenient introduction of the instrument vector $\{U\}$ in **equation (23)** allows the definition of an error indicator that quantifies a residual strain energy density (element-by-element, substructure-by-substructure). The residual energy density for a given substructure s is defined by :

$$e_s = \left(\{U\}-\{V\}\right)^T [K_s]\left(\{U\}-\{V\}\right) \Big/ Vol_s \qquad (26)$$

$[K_s]$ is the stiffness matrix of the substructure, and Vol_s is the associated volume. As can be seen, the error indicator is closely related to the proposed expansion method.
The overall reliability of this indicator strongly depends on the previous expansion technique and on the energy distribution on the structure.

Remark

It can be noticed that the expansion method considered in this paper differs from other methods proposed in the literature by the metric used to quantify the residuals.
For instance, it is easy to show that the expansion method proposed by Hemez [3] reduces to the solution of :

$$\min \left(\{U\}-\{V\}\right)^T [K]^2 \left(\{U\}-\{V\}\right) \qquad (27)$$

In a similar way, Alvin's method [4] is based on

$$\min \left(\{U\}-\{V\}\right)^T \left(\{U\}-\{V\}\right) \qquad (28)$$

4. MODEL CORRECTION

Let us assume the existence of experimental mass and stiffness matrices that satisfy equilibrium equation :

$$[K_x]\{V\} = \omega^2 [M_x]\{V\} + \{F\} \qquad (29)$$

Using **equation (23),** the following system can be found for parameter corrections :

$$[K]^{-1}[\Delta Z(\omega,\Delta p)]\{V\} = \{U\} - \{V\} \qquad (30)$$

where

$[\Delta Z(\omega,\Delta p)] = [\Delta K(\Delta p)] - \omega^2[\Delta M(\Delta p)]$ is the correction matrix for the dynamic stiffness.

Equation (30) is exact as long as $\{V\}$ is the true vector. In practice, however, only an approximation of this vector is available. This leads to an iterative correction procedure.

A good property of the method is that it does not need a pairing of the ODS (See also [5]).

5. FREQUENCY SELECTION FOR ERROR LOCALIZATION

The Frequency Domain Assurance Criterion (FDAC) defined in [6] can be used to measure the correlation between two ODS :

$$\frac{\{V_a(\omega_a)\}^T\{\overline{V}(\omega_x)\}}{\left|\{V_a(\omega_a)\}\right|\left|\{\overline{V}(\omega_x)\}\right|} \qquad (31)$$

where

- ω_a corresponds to the "analytical" frequency at which $\{V_a\}$ is computed;
- ω_x corresponds to the "experimental" frequency at which $\{\overline{V}\}$ is measured.

The evaluation of **equation (31)** for a given set of analytical and experimental frequencies results in the *FDAC matrix.* From the definition, it follows that FDAC values are bounded by the interval $[-1,1]$. Values close to 1 indicate that the compared ODS correlate well not only in shape *but also in phase*.

For this reason, frequency intervals showing good agreement between analytical and experimental data may be used for error localization and updating purposes.

6. EXPERIMENTAL VALIDATION

The objective of the test-case presented in this section is to validate the failure detection method based on ODS and to compare the results with the classical approach based on mode shapes. For this purpose, the 3-D beam frame shown in **Fig. 1** is used. The main characteristics of the test-case are listed in table I.

The initial FE model corresponds to the complete truss structure without any damage. In order to test the error localization method based on ODS, a damaged truss structure was experimentally performed by removing a beam (beam n° 18 was removed in this example). The correlation (MAC) between the initial FE model and the damaged structure is shown in **Fig. 2** [7]. It can be seen that the first experimental mode is not predicted by the initial FE model. The corresponding FDAC matrix is shown in **Fig. 4**. From this figure, it appears that the frequency range from 0 to 150 Hz is convenient for error localization. Over 150 Hz, the correlation levels decrease drastically due to the strong influence of local mode shapes. The results of the error localization procedure are illustrated in **Figs. 6** and **7** where the error levels are indicated for each beam and each frequency in the range 10-150 Hz. For comparative purposes, the residual global energy on the whole structure at each frequency was normalized to 1. It can be seen from **Fig. 6** that the removed beam is detected in almost all the frequency interval. However, the frequency range from 20 Hz to 45 Hz, in which resonance peaks are concentrated show poor localization results. This can be explained by the non negligible effects of damping. In this range, some residual energy dispersion to surrounding beams is observed (e.g. beams 7 and 29). This is caused by the smoothing of the expanded vector according to criterion (22).

Regarding to the results obtained in the error localization step, the flexural stiffness of the "missing" beam is chosen as updating parameter. Using an iterative updating procedure based on **equation (30),** the "missing" beam is completely "removed" from the initial model after two iterations. As shown in **Figs. 3** and **5** by the MAC and FDAC matrices, the correlation of the updated model with the experimental results has been improved.

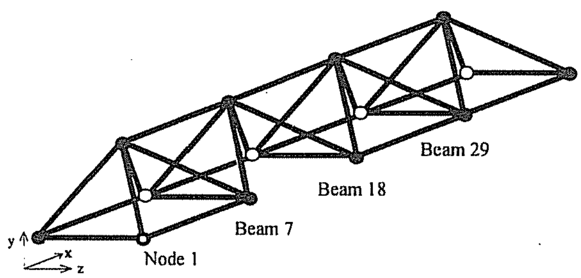

Fig. 1. Experimental structure

7. CONCLUSION

A method for the expansion of operating deflection shapes has been presented. It is based on the variational principles used in structural dynamics.

The objective of this method is the detection and localization of discrepancies (approximation, discretization or parametric errors) between the FE model and the actual structure.

The application of the proposed method to an experimental test-case shows its ability to localize the dominant errors in the model.

The model correction method associated to the expansion technique also shows good convergence properties.

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Structure
length = 2.82 m; width = 0.5 m; height = 0.25 m
FE model : SAMCEF V6.1
N° dofs : 1044 Beam and concentrated inertia elements No damping considered
Modal analysis : LMS-Cada-x 3.1
Reference point : node n° 1, direction y (a corner) N° measured coordinates : 45 nodes × 3 directions Frequency band : 0 - 200 Hz Resolution : 0.25 Hz

Table I. Characteristics of the test

3. Hemez, F.M., *Theoretical and experimental correlation between finite elements models and modal tests in the context of large flexible space structures*, Ph.D. dissertation, Univ. of Colorado, 1993.
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6. Pascual, R., Golinval, J.C., Razeto, M., *A frequency domain correlation technique for model correlation and updating*, 15st Intl. Modal Anal. Conf., Orlando, Florida, pp 587-592,1997.
7. Allemang, R., Brown, D., *A correlation coefficient for modal vector analysis*, 1st Intl. Modal Anal. Conf., Orlando, Florida, pp 110 - 116, 1982.

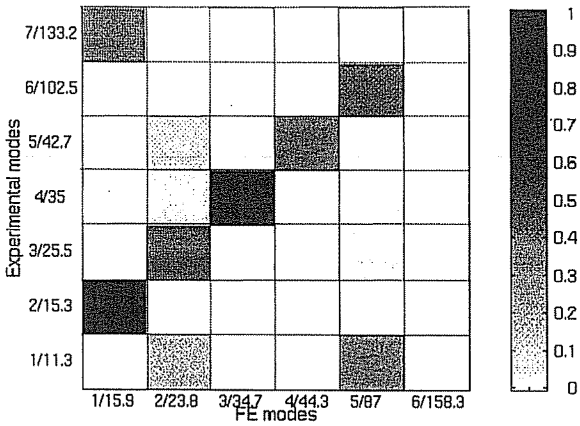

Fig.2 . Initial MAC

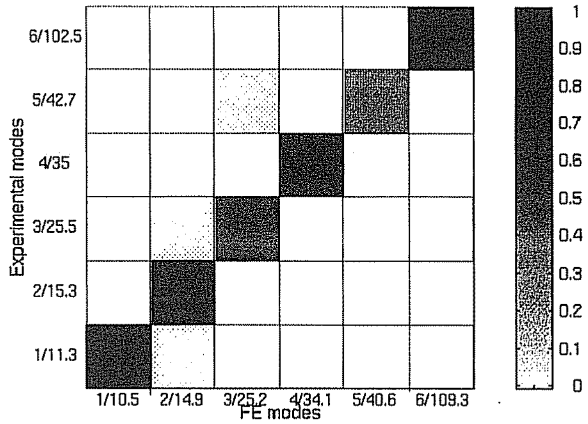

Fig. 3. Updated MAC

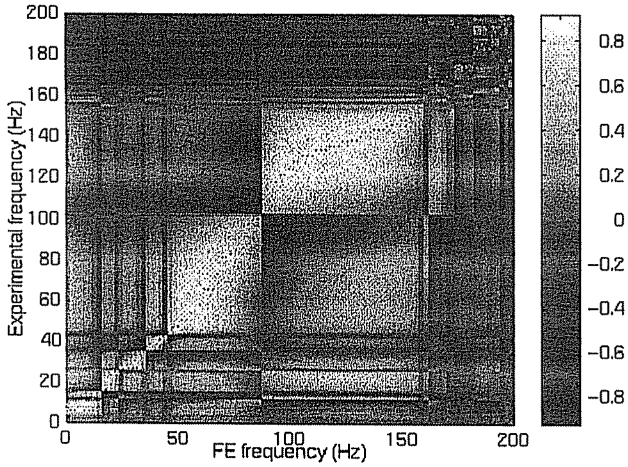

Fig. 4. Initial FDAC (all the frequency range)

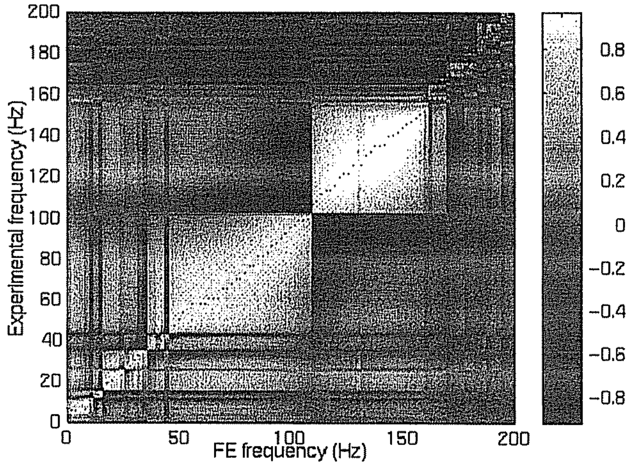

Fig. 5. FDAC after 'removing' beam 18

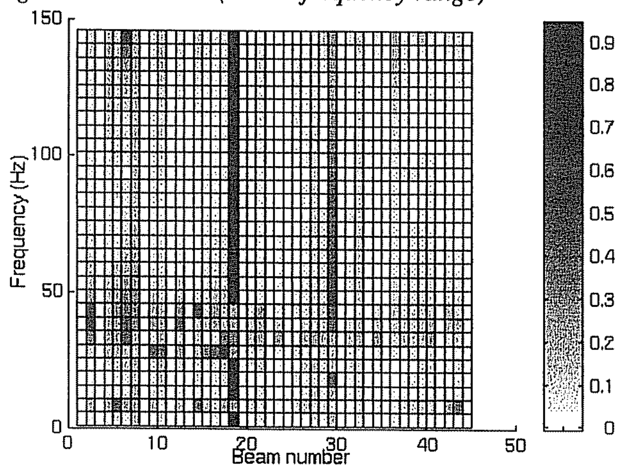

Fig. 6. Normalized residual strain energy

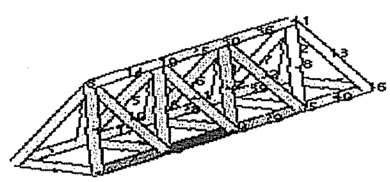

Fig. 7. Averaged residual strain energy distribution