A drop does not fall in a straight line: a rationale for the width of stalagmites

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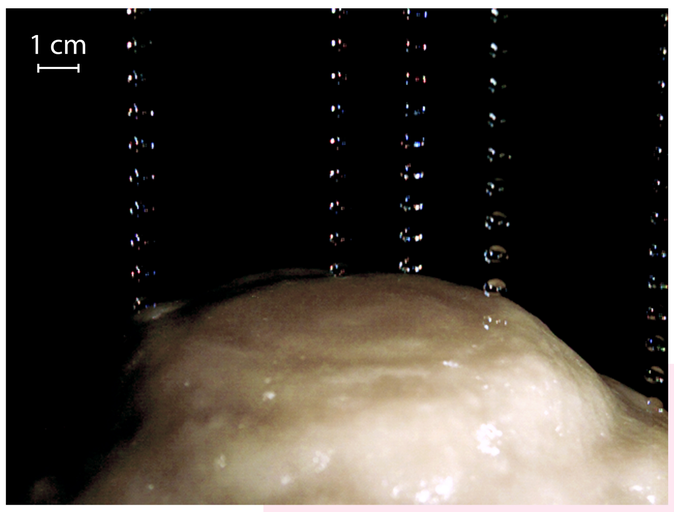
How do stalagmites grow?

1 Drop detachment from stalactite

2 **Free fall**

3 Splash:
a. **Lamella spreading**
b. Ejected droplets

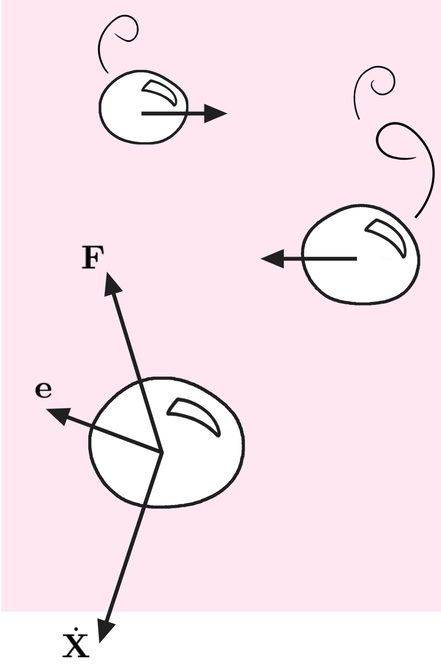
4 a. Calcite precipitation and deposition
b. Gravity-driven drainage of residual film



Drop impact point dispersal

Drop free fall
- Not a straight line
- Scattered sometimes over several cm
- Even with **no external** cause

Interaction between the drop and its wake
- Vortices emitted at frequency St
- Each vortex pushes the drop in a **random** direction $\mathbf{e}$
- Overall, drop deviated from its initial horizontal position
- Standard deviation Δ of impact position



Model
- Newton's law applied to the drop, integrated between two successive vortices $\Rightarrow$ difference equations $\Rightarrow$ random walk
- Aerodynamic force $\mathbf{F}$: randomly oriented lift due to the vortices + drag

$$\mathbf{F} = -C_L \frac{\rho_a S \left|\dot{\mathbf{X}}\right|}{2} \mathbf{e} \times \dot{\mathbf{X}} - C_D \frac{\rho_a S \left|\dot{\mathbf{X}}\right|}{2} \dot{\mathbf{X}}$$

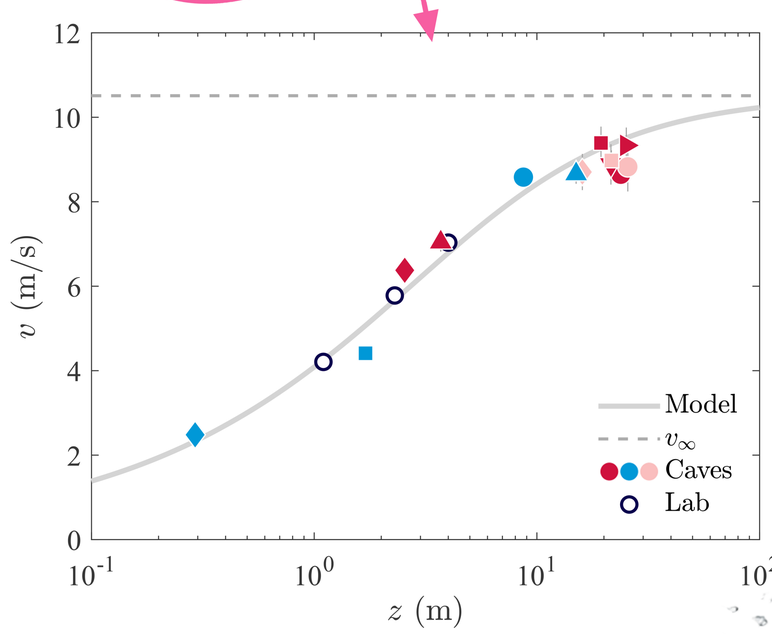
Vertical projection

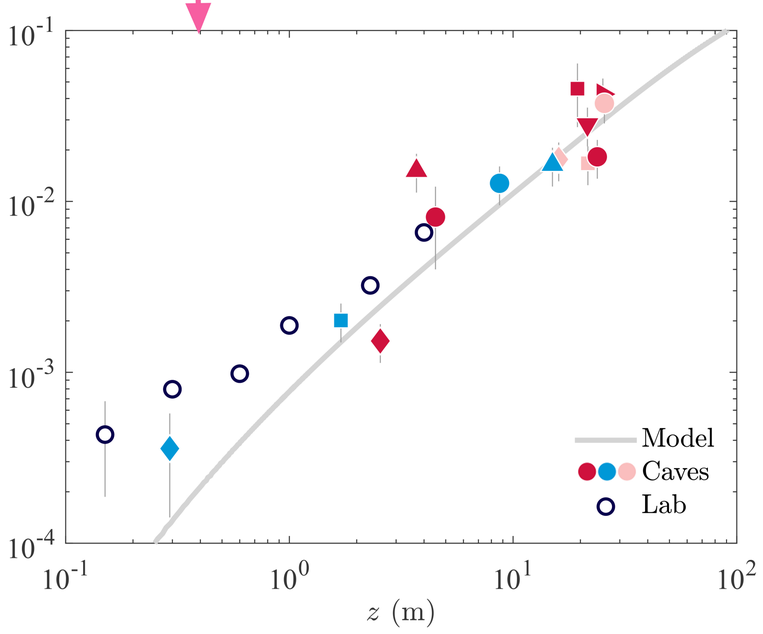
$$S = \pi R^2 + 2\pi c \frac{\rho_a R^3 v^2}{\sigma} \qquad C_D(\mathrm{Re}) \Rightarrow v(z)$$

Horizontal projection

$C_D(\mathrm{Re})$
$C_L(\mathrm{Re})$
$\mathrm{St}(\mathrm{Re})$

$\Rightarrow \Delta(z)$

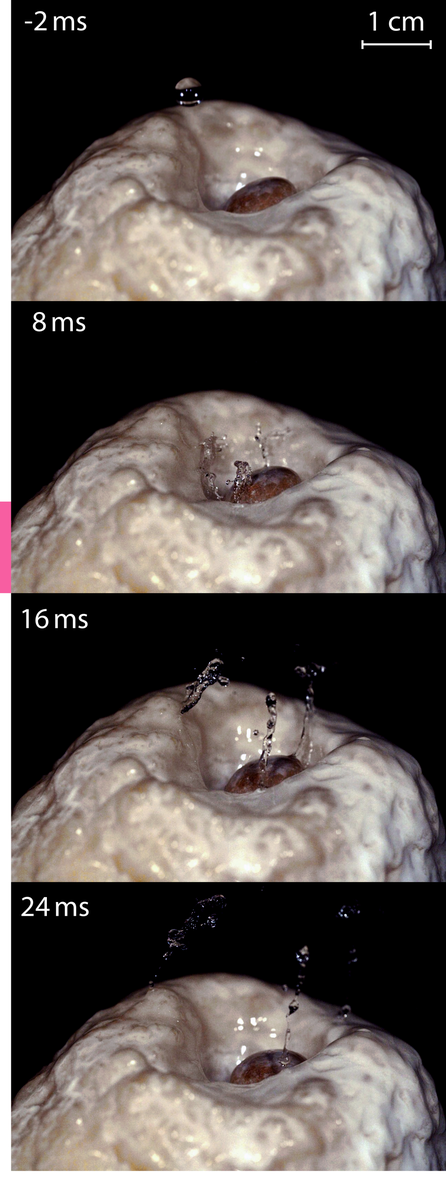




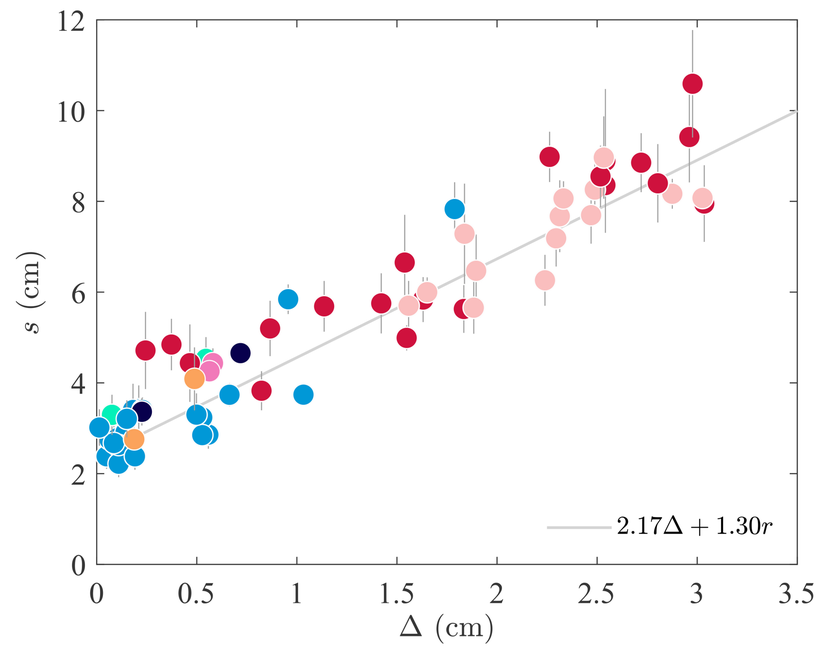
Methods

Dataset
- Pictures: 7 caves, 65 stalagmites
- High-speed videos: 3 caves, 14 stalagmites
- Additional lab videos

Measurements
- Laser telemetry: falling height z
- Image analysis: drop radius R, drop velocity v, lamella radius r, impact point dispersal Δ, stalagmite radius s

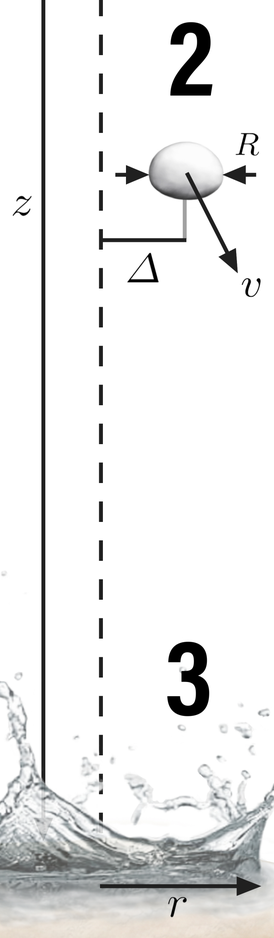


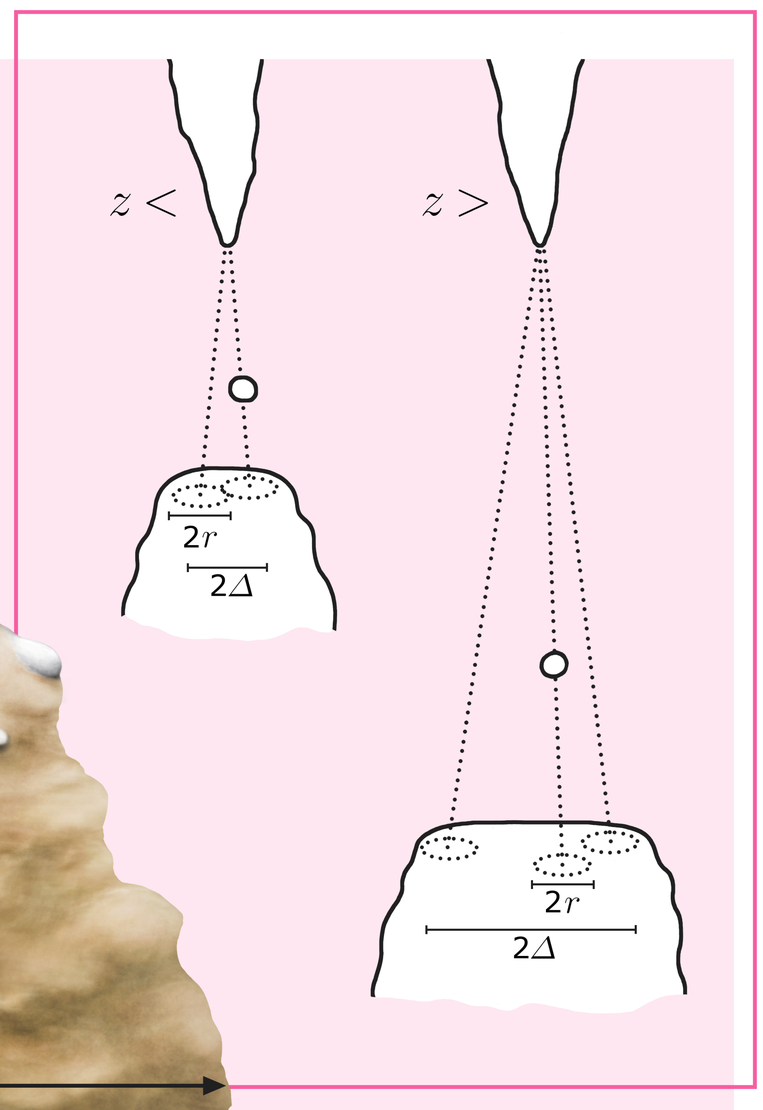
Link with stalagmite radius



- Maximal extension of the lamella spreading after impact r, **weakly** dependent on v
- Stalagmite radius averaged over the entire stalagmite s

$\Rightarrow$ *s linear combination of Δ and r*





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