A Robust FRF-based technique for Model Updating

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Abstract

Frequency Response Functions (FRFs) residues have been widely used in time to update Finite Element models [2, 3, 4, 5, 6, 11, 12, 14, 15, 16]. Major reasons for this is that FRFs are very sensitive to damping properties at resonance peaks, local modes influence is included and no modal analysis is required. Nevertheless, it is well known that due to their nature, the frequency responses may change its order of magnitude very rapidly for small parameter or frequency changes. This situation may cause serious discontinuities in the topology of the objective function, causing the updating strategy to diverge or to find a local non-physical minimum [5, 15, 18]. A primary tool for the correlation of FRFs is the Frequency Domain Assurance Criterion [19]. This technique introduces the concept of frequency shift between the frequency response shapes of a reference model (the experimental structure) and a perturbed model (an initial non-updated FE model). Such a concept opens the way for using residues at different frequencies. For instance, in reference [6] the residue is composed by point FRFs at anti-resonances. This paper introduces a general FRF-based model-updating technique, which is focused in using stable residues during the interactive optimization procedure. A benchmark case from the Cost F3 action is used to assess the goodness of the method compared to other well known methods.

1 Introduction

In recent years, a significative amount of work in the field of structural dynamics has dealt with evaluating and reducing the distance between numerical models and experimental structures, in terms of their dynamic signature. Generally speaking, finite element model updating may be considered as an optimization problem and from this point of view, it is important to:

- formulate appropriate cost functions showing a smooth topological search space with no local minima around the global minimum.
- guarantee sufficient sensitivity of the objective with respect to the design parameters.
- select the minimum amount of design parameters to avoid prohibitive computation times and to increase the conditioning of the problem.
- scale the design parameters in order to avoid numerical problems.

The use of the forced dynamic responses of a system in the updating procedure seems attractive since:

- direct measures have a good level of confidence,
- the modal identification effort is avoided, as well as the errors it produces.
- the redundancy of information may be exploited to reduce noise effects,
- the operating deflection shapes show the influence of the local modes, which are known to be more sensitive to the presence of damage,
- damping effects are readily observed,
- operating deflection shapes are a property of linear systems, so averaging may be used.

In this paper we investigate an updating strategy that looks forward to alleviate problems that appear when using frequency response functions as updating residue. First we discuss the state-of-the art on the correlation issues and then we explore current updating techniques. This lead directly to a method which shows very good convergence properties.

2 Previous work

2.1 Correlation

Correlation in the field of model updating is understood as the set of techniques that allows us to measure the differences that exist between the responses of the model and those of the experimental structure. A primary tool for correlation of the operating deflection shapes is the Frequency Domain Assurance Criterion [19], which is presented in what follows.

Let us consider a reference system with stiffness matrix $\mathbf{K}$ and mass matrix $\mathbf{M}$; and a perturbed system with matrices $\mathbf{K}^*$, $\mathbf{M}^*$ which follow:

$$\begin{aligned} \mathbf{K}^* &= \alpha \mathbf{K} \\ \mathbf{M}^* &= \mathbf{M} \end{aligned} \tag{1}$$

where $\alpha > 0$ is a perturbing parameter (termed as *stiffness factor* [15]).

It can be easily proven that:

$$\mathbf{H}_i^* \left(\sqrt{\alpha}\omega_i\right) = \frac{1}{\alpha}\mathbf{H}_i\left(\omega_i\right) \tag{2}$$

where $\mathbf{H}$ and $\mathbf{H}^*$ are the reference and perturbed dynamic flexibility matrices, respectively:

$$\mathbf{H}_i = \left(-\omega_i^2\mathbf{M} + j\omega_i\mathbf{C} + \mathbf{K}\right)^{-1} = \mathbf{Z}_i^{-1} \tag{3}$$

Equation (2) shows that a shift in frequency

$$\left(\sqrt{\alpha} - 1\right)\omega_i$$

and a new scale factor appears on all the operating deflection shapes (each column in $\mathbf{H}_i$) and, what is more important, a direct correlation exists for both operating deflection shapes if this *frequency shift* is taken into account.

In a general situation, and referring to figure (1), it makes sense to compare the operating deflection shapes that show the best mutual agreement, considering frequency shift. It should be pointed out that usually, as several parameters at elementary levels are perturbed, each one of these parameters will shift the eigenfrequencies in different directions so that an average frequency shift exists for each operating deflection shape. Correlation methods based on modal information (like MAC [1] for instance) implicitly take the frequency shift into account, since they pair mode shapes at different frequencies: referring again to figure (1), the correlated mode pairs associated are not at the same eigenfrequencies.

In reference [17] it is proposed to measure the correlation between two operating deflection shapes at the same frequencies. In this case, a vector of correlation is obtained.

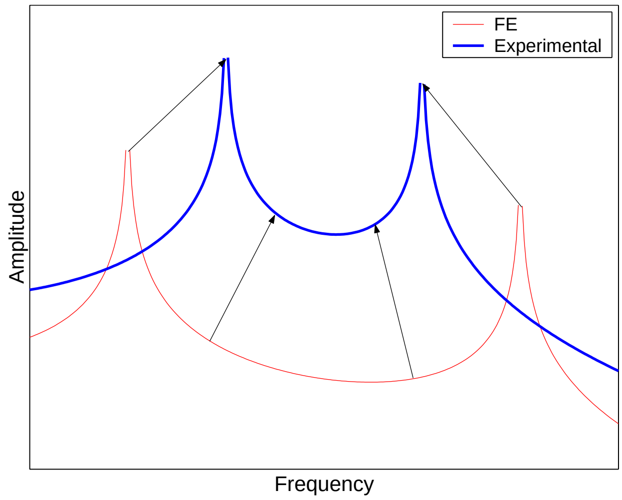

Figure 1: Frequency response function of the reference and perturbed systems

Based on the frequency shift mentioned above, we may measure the closeness between measured and synthesized operating deflection shapes by using the following correlation criterion:

$$FDAC(\omega_i, \omega_j) = \pm\sqrt{\frac{\left(\mathbf{h}_j^H\bar{\mathbf{h}}_i\right)^H\left(\mathbf{h}_j^H\bar{\mathbf{h}}_i\right)}{\left(\mathbf{h}_j^H\mathbf{h}_j\right)\left(\bar{\mathbf{h}}_i^H\bar{\mathbf{h}}_i\right)}} \tag{4}$$

where:

^{H} indicates the conjugate transposed,

ω_j corresponds to the frequency at which the numerical operating deflection shape $\mathbf{h}_j$ is calculated,

ω_i corresponds to one frequency at which the experimental operating deflection shape $\bar{\mathbf{h}}_i$ was measured experimentally (the bar indicates that it is measured),

and $\pm$ indicates the relative phase of the operating deflection shapes.

The so-called Frequency Domain Assurance Criterion (FDAC) can be regarded as equivalent to the MAC in the frequency domain. It follows from equation (4) that FDAC values are limited to the interval $[-1, 1]$. A value of 1 means perfect correlation, 0 no correlation at all, and -1 perfect correlation but a relative phase of 180^o.

From FDAC, it is possible to define the *frequency shift* residue:

$$\Delta\omega(\omega_j) = \omega_i^j - \omega_j \tag{5}$$

to measure the distance between the model and the test structure. ω_j represents the selected experimental

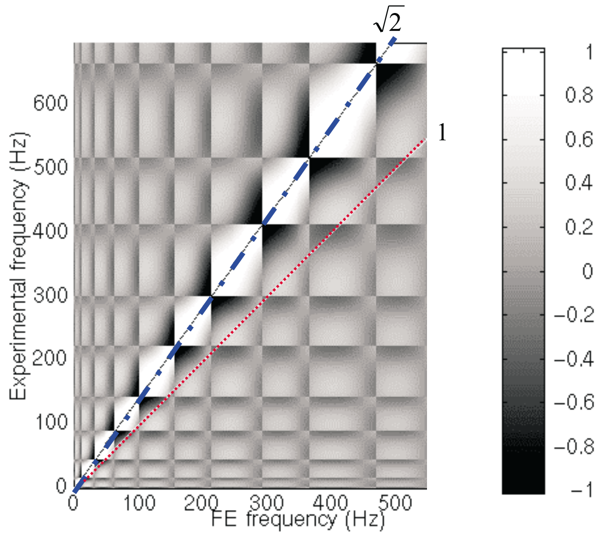

Figure 2: FDAC for a stiffness factor equal 2

frequency, and ω_i^j is the frequency at which FDAC reaches its maximum for all frequencies.

The evaluation of equation (4) for a given set of analytical frequencies and measured frequencies results in the FDAC matrix. A perfectly updated model will have only positive unitary values on the axis $\omega_i = \omega_j$. The special case considered in equation (1) appears clearly in the FDAC matrix as shown in figure (2). The *stiffness factor* may be estimated easily from the slope of the line (ω_i^j, ω_j). The lines of discontinuity in each frequency axis represent the eigenfrequencies. Thus, it may also be utilized for modal identification purposes.

Several other works deal with correlation issues using forced responses: [8] introduces a technique where the criterion mixes operating deflection shapes and mode shapes to obtain directly the so called correlated mode pairs between the model responses and the experimental data. [11] uses a version of FDAC at resonances that uses only the imaginary part of the operating deflection shapes; in this way an increased sensitivity to damping is obtained.

2.2 Model Updating

It is well known that dynamic flexibility is a function of frequency which readily changes in amplitude. Therefore, a residue based on the difference between experimental and numerical operating deflection shapes may be unstable, leading to numerical difficulties as reported in [5, 15]. References [2, 4, 10] avoid the problem by considering a residue using a log-least squares residue between test and model frequency response functions. Other methods use the forced responses directly [15, 16].

All methods use residues between the experimental and analytical operating deflection shapes *at the same frequency(ies)*. In what follows, an improvement to these approaches will be introduced.

Let us assume the existence of the system matrices $\mathbf{K}^*$, $\mathbf{C}^*$ and $\mathbf{M}^*$ with the same properties as the corresponding analytical matrices. The dynamic equilibrium equation in the frequency domain is written as:

$$\left(-\omega_i^2\mathbf{M}^* + j\omega_i\mathbf{C}^* + \mathbf{K}^*\right)\mathbf{h}_i^* = \mathbf{f} \tag{6}$$

and for the finite element model:

$$\begin{aligned} \left(-\omega_i^2\mathbf{M} + j\omega_i\mathbf{C} + \mathbf{K}\right)\mathbf{h}_i &= \mathbf{f} \\ \mathbf{Z}_i\mathbf{h}_i &= \mathbf{f} \end{aligned} \tag{7}$$

where $\mathbf{h}_i^*$ is the assumed operating deflection shape of the actual structure; $\mathbf{h}_i$ is the operating deflection shape of the model, and $\mathbf{f}$ is the unitary vector of excitation, $\mathbf{f} = \mathbf{e}_k$. For sake of conciseness the index k has been dropped from $\mathbf{h}_i$.

Keeping model updating in mind, a natural objective function to be minimized is the output residue computed at one or several frequency(ies) ω_i:

$$\min_{\mathbf{p}} \|\varepsilon_{d_i}\|^2 \tag{8}$$

with

$$\varepsilon_{d_i} = \mathbf{h}_i - \mathbf{h}_i^* \tag{9}$$

Note that for the moment we do not handle the frequency shift between the operating deflection shapes.

In order to solve problem (8), the forced response of the model may be approximated with a truncated Taylor series expansion in terms of the parameter correction vector $\mathbf{p}$ (with n_p parameters):

$$\mathbf{h}_i(\mathbf{p}) = \mathbf{h}_i + \sum_{j=1}^{n_p} \frac{\partial \mathbf{h}_i}{\partial p_j} p_j \tag{10}$$

The derivative of the forced response $\frac{\partial \mathbf{h}_i}{\partial \mathbf{p}}$ is a highly discontinuous, non-monotonous function. This limits the approximation severely as it is reported in references [5, 15]. We can observe the discontinuity in the derivative of $\mathbf{H}_i$ by looking at the computations needed to obtain it:

$$\frac{\partial \mathbf{H}_i}{\partial p_j} = -\mathbf{H}_i \frac{\partial \mathbf{Z}_i}{\partial p_j} \mathbf{H}_i \tag{11}$$

As can be seen in equation (11), the derivative depends twice on $\mathbf{H}_i$. $\mathbf{H}_i$ presents a discontinuity at each resonance, and so does its derivative. Updating methods based on equations (9 and 10) will be highly unstable:

$$\sum_{j=1}^{n_p} \frac{\partial \mathbf{h}_i}{\partial p_j} p_j = \mathbf{h}_i - \mathbf{h}_i^* \tag{12}$$

An alternative objective function that avoids the computation of equation (11), may be based on the minimization of the input errors (or force errors) in the equation of motion, that is:

$$\min_{\mathbf{p}} \|\varepsilon_{f_i}\|^2 \tag{13}$$

with

$$\varepsilon_{f_i} = \mathbf{Z}_i \varepsilon_{d_i} = \mathbf{e}_k - \mathbf{Z}_i \mathbf{h}_i^* \tag{14}$$

Similarly, the Taylor series expansion of $\mathbf{Z}_i$ is used to solve the problem (13):

$$\mathbf{Z}_i(\mathbf{p}) = \mathbf{Z}_i + \sum_{j=1}^{n_p} \frac{\partial \mathbf{Z}_i}{\partial p_j} p_j \tag{15}$$

This leads to the following updating equation:

$$\left(\sum_{j=1}^{n_p} \frac{\partial \mathbf{Z}_i}{\partial p_j} p_j \right) \mathbf{h}_i^* \approx \mathbf{e}_k - \mathbf{Z}_i \mathbf{h}_i^* \tag{16}$$

An approximative equality has been written because the design parameters vector $\mathbf{p}$ may be an incomplete list (and also biased) of the true parameters $\mathbf{p}^*$. Note that the measured $\bar{\mathbf{h}}_i$ always include some noise $\mathbf{n}$:

$$\bar{\mathbf{h}}_i = \mathbf{h}_i^* + \mathbf{n} \tag{17}$$

that will inevitably perturb the updating equation, so that it is more correct to write:

$$\left(\sum_{j=1}^{n_p} \frac{\partial \mathbf{Z}_i}{\partial p_j} p_j \right) \bar{\mathbf{h}}_i \approx \mathbf{e}_k - \mathbf{Z}_i \bar{\mathbf{h}}_i \tag{18}$$

The noise term $\mathbf{n}$ appears on both sides of the equation multiplied by $\frac{\partial \mathbf{Z}_i}{\partial \mathbf{p}}$ and $\mathbf{Z}_i$ respectively. This couples the noise in the updating equation and may induce biased and unstable results. To overcome the situation, a convenient weighting matrix may be employed. If equation (18) is pre-multiplied by $\mathbf{H}_i$, it becomes:

$$\mathbf{H}_i \left(\sum_{j=1}^{n_p} \frac{\partial \mathbf{Z}_i}{\partial p_j} p_j \right) \bar{\mathbf{h}}_i \approx \mathbf{h}_i - \bar{\mathbf{h}}_i \tag{19}$$

which can be expressed as:

$$\mathbf{A}\mathbf{p} = \mathbf{b} \tag{20}$$

with

$$\begin{aligned} \mathbf{A} &= \mathbf{H}_i \left[\begin{array}{ccc} \frac{\partial \mathbf{Z}_i}{\partial p_1} \bar{\mathbf{h}}_i, & \ldots, & \frac{\partial \mathbf{Z}_i}{\partial p_{n_p}} \bar{\mathbf{h}}_i \end{array} \right] \\ \mathbf{b} &= \mathbf{h}_i - \bar{\mathbf{h}}_i \end{aligned}$$

Note that the right hand side in equation (19) is the output residue that also appears in equation (12). The improvement is that the use of $\frac{\partial \mathbf{h}_i}{\partial \mathbf{p}}$ is avoided. $\mathbf{Z}_i$ is a *linear or quasi-linear explicit* function of the parameters [15] so its derivative $\frac{\partial \mathbf{Z}_i}{\partial \mathbf{p}}$ shows a smooth behavior; and its computation is straightforward. The introduction of $\mathbf{H}_i$ reduces the effects of the noise term and improves the conditioning of the formulation [9]. Equation (19) may be evaluated for a set of frequencies, in order to obtain the corrections $\mathbf{p}$. These will define a new model, which will also be updated until the residue $\mathbf{h}_i - \bar{\mathbf{h}}_i$ stabilizes at a minimum. Note also that each column in the matrix of coefficients $\mathbf{A}$ corresponds to the shape adopted by the structure according to the model for the external force configuration:

$$\frac{\partial \mathbf{Z}_i}{\partial p_k} \bar{\mathbf{h}}_i \tag{21}$$

The force (21) only operates in the degrees of freedom associated to the substructure k. If the substructures being updated are close in space, their effects on the global dynamic behavior will be similar. This may generate a set of columns almost linearly dependent in the left hand side of equation (19), and the problem of finding the right $\mathbf{p}$ becomes ill-conditioned. To alleviate the problem, parameters should actuate on larger substructures or *macro-elements* instead of single finite elements.

Model updating using equation (19) does, however have its own difficulties. If for instance:

$$\|\mathbf{h}_i\| >> \|\bar{\mathbf{h}}_i\| \tag{22}$$

occurs (and this may happen easily during the iterations due to the nature of the dynamic flexibility, see figure 3), then the right hand side of (19) becomes *unbalanced*. This may not be the case at the first iteration if the user chooses the adequate updating frequencies. But, at intermediary iterations steps, $\mathbf{h}_i$ may become close to a resonance frequency so that its amplitude increases drastically. Some common practice rules are given in references [15, 20]:

- choose updating frequencies at the foot of experimental peaks, and
- avoid frequencies between corresponding analytical and experimental resonances.

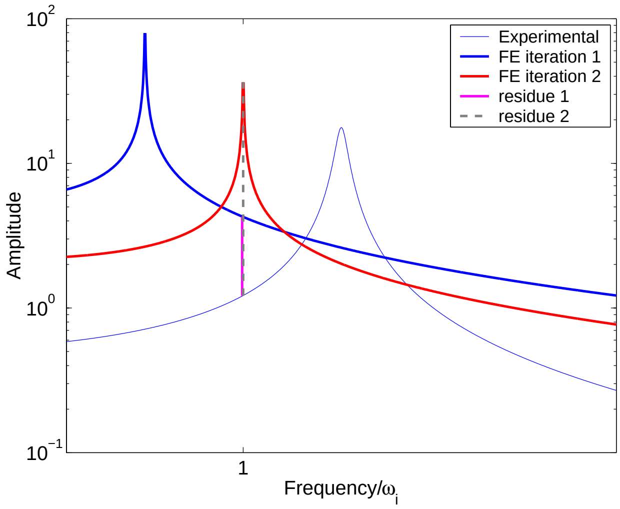

Figure 3: Problems during the iterations

The first rule assures a good sensitivity, according to equation (11). The second one avoids the problem of amplitude differences, but implies that a modal analysis has been performed. The implementation *must* revise at each iteration step to check that the chosen frequencies comply with these rules.

Other current approaches are presented in references [6] and [11] respectively. The first one presents a method where a residue is built between numerical and experimental anti-resonances; thus considering the frequency shift for each degree of freedom. The second reference introduces a strategy that defines a residue between numerical and experimental operating deflection shapes at different frequencies (at the resonances). The residue is built using the FDAC technique. They also use the imaginary part of the shapes in order to improve the updating of damping parameters.

3 Improved Model Updating

Taking into account the considerations described in §2.1, instead of comparing measured and analytical operating shapes at the same frequency ω_i, the idea developed here is to define a more balanced residue. For this purpose, let us use the following residue:

$$\min_{\mathbf{p}} \left\| \varepsilon'_{d_i} \right\|^2 \tag{23}$$

with

$$\varepsilon'_{d_i} = \mathbf{h}_j - \mathbf{h}_i^* \tag{24}$$

where *two* different frequencies are considered: ω_i for the experimental operating deflection shape and ω_j for the numerical operating deflection shape.

It is easy to show that:

$$\mathbf{Z}_i^* - \mathbf{Z}_j = \mathbf{\Delta Z}_i^* + j\left(\omega_i - \omega_j\right)\mathbf{C} - \left(\omega_i^2 - \omega_j^2\right)\mathbf{M} \tag{25}$$

This allows us to express a new force residue as:

$$\begin{aligned}\varepsilon'_{f_i} &= \mathbf{Z}_i \varepsilon'_{d_i} \\ &= -\mathbf{\Delta Z}_i^* \mathbf{h}_i^* + j\left(\omega_i - \omega_j\right)\mathbf{C}\mathbf{h}_i^* - \\ &\quad \left(\omega_i^2 - \omega_j^2\right)\mathbf{M}\mathbf{h}_i^*\end{aligned}$$

taking into account noise effects:

$$\begin{aligned}\varepsilon'_{f_i} &\approx -\mathbf{\Delta Z}_i^* \bar{\mathbf{h}}_i + j\left(\omega_i - \omega_j\right)\mathbf{C}\bar{\mathbf{h}}_i - \\ &\quad \left(\omega_i^2 - \omega_j^2\right)\mathbf{M}\bar{\mathbf{h}}_i\end{aligned}$$

and to alleviate the noise coupling, $\mathbf{H}_j$ is used:

$$\begin{aligned}\mathbf{H}_j\varepsilon'_{f_i} &\approx -\mathbf{H}_j\mathbf{\Delta Z}_i^* \bar{\mathbf{h}}_i + j\left(\omega_i - \omega_j\right)\mathbf{H}_j\mathbf{C}\bar{\mathbf{h}}_i - \\ &\quad \left(\omega_i^2 - \omega_j^2\right)\mathbf{H}_j\mathbf{M}\bar{\mathbf{h}}_i\end{aligned}$$

or

$$\begin{aligned}\mathbf{h}_j - \bar{\mathbf{h}}_i &\approx \mathbf{H}_j\mathbf{\Delta Z}_i^* \bar{\mathbf{h}}_i + j\left(\omega_i - \omega_j\right)\mathbf{H}_j\mathbf{C}\bar{\mathbf{h}}_i - \\ &\quad \left(\omega_i^2 - \omega_j^2\right)\mathbf{H}_j\mathbf{M}\bar{\mathbf{h}}_i\end{aligned}$$

Reordering

$$\begin{aligned}\mathbf{H}_j\mathbf{\Delta Z}_i^* \bar{\mathbf{h}}_i &\approx \left(\mathbf{h}_j - \bar{\mathbf{h}}_i\right) - j\left(\omega_i - \omega_j\right)\mathbf{H}_j\mathbf{C}\bar{\mathbf{h}}_i \\ &\quad + \left(\omega_i^2 - \omega_j^2\right)\mathbf{H}_j\mathbf{M}\bar{\mathbf{h}}_i\end{aligned}$$

and substituting the approximation to the exact $\mathbf{\Delta Z}_i^*$:

$$\begin{aligned}\mathbf{H}_j\left(\sum_{j=1}^{n_p}\frac{\partial \mathbf{Z}_i}{\partial p_j}p_j\right)\bar{\mathbf{h}}_i &\approx \left(\mathbf{h}_j - \bar{\mathbf{h}}_i\right) - \\ &\quad j\left(\omega_i - \omega_j\right)\mathbf{H}_j\mathbf{C}\bar{\mathbf{h}}_i + \\ &\quad \left(\omega_i^2 - \omega_j^2\right)\mathbf{H}_j\mathbf{M}\bar{\mathbf{h}}_i\end{aligned} \tag{26}$$

The right hand side of equation (26) is composed of a residue representing the difference in the operating shapes plus the terms penalizing their difference in frequency. Note that if $\omega_i = \omega_j$, equation (26) is identical to equation (19).

3.1 Model reduction

The problem of model matching is addressed here by using dynamic reduction of the model matrices at each frequency. The reduced matrices are obtained through:

$$\tilde{\mathbf{M}}_i = \mathbf{T}_i^T\mathbf{M}\mathbf{T}_i \tag{27}$$

$$\tilde{\mathbf{Z}}_i = \mathbf{T}_i^T\mathbf{Z}_i\mathbf{T}_i \tag{28}$$

where $\mathbf{T}_i$ corresponds to the operator of the dynamic expansion technique [13] (superscript $\tilde{}$ is used for all variables related to the reduced model).

An advantage of this reduction approach is that it avoids approximations in the dynamic response of the reduced system:

$$\tilde{\mathbf{H}}_i = \mathbf{H}_{mm(i)} \tag{29}$$

where $_{mm}$ represents the measured partition of degrees of freedom.

In order to use equation (26) we need to find also $\frac{\partial \tilde{\mathbf{Z}}_i}{\partial p_k}$. The dynamic flexibility matrix of the reduced model is by definition:

$$\tilde{\mathbf{H}}_i = \tilde{\mathbf{Z}}_i^{-1} \tag{30}$$

and the sensitivities of the reduced dynamic stiffness with respect to the design parameters are obtained through the identity relationships:

$$\frac{\partial \mathbf{H}_i}{\partial p_k} = -\mathbf{H}_i \frac{\partial \mathbf{Z}_i}{\partial p_k} \mathbf{H}_i = \begin{bmatrix} \delta \mathbf{H}^k_{mm(i)} & \delta \mathbf{H}^k_{mo(i)} \\ \delta \mathbf{H}^k_{om(i)} & \delta \mathbf{H}^k_{oo(i)} \end{bmatrix} \tag{31}$$

$$\frac{\partial \mathbf{Z}_i}{\partial p_k} = -\mathbf{Z}_i \frac{\partial \mathbf{H}_i}{\partial p_k} \mathbf{Z}_i \tag{32}$$

where $_o$ represents the unmeasured partition of degrees of freedom.

Recalling that

$$\delta \mathbf{H}^k_{mm(i)} = \frac{\partial \tilde{\mathbf{H}}_i}{\partial p_k} \tag{33}$$

leads to:

$$\frac{\partial \tilde{\mathbf{Z}}_i}{\partial p_k} = \tilde{\mathbf{Z}}_i \, \delta \mathbf{H}^k_{mm(i)} \, \tilde{\mathbf{Z}}_i \tag{34}$$

thus, problem (19) is implemented using the following equation:

$$\tilde{\mathbf{H}}_i \left(\sum_{j=1}^{n_p} \frac{\partial \tilde{\mathbf{Z}}_i}{\partial p_j} p_j \right) \bar{\mathbf{h}}_i \approx \tilde{\mathbf{h}}_i - \bar{\mathbf{h}}_i \tag{35}$$

The improved method (26) is implemented using:

$$\begin{aligned} \tilde{\mathbf{H}}_j \left(\sum_{j=1}^{n_p} \frac{\partial \tilde{\mathbf{Z}}_i}{\partial p_j} p_j \right) \bar{\mathbf{h}}_i \quad \approx \quad & \left(\tilde{\mathbf{h}}_j - \bar{\mathbf{h}}_i \right) - \\ & j \left(\omega_i - \omega_j \right) \tilde{\mathbf{H}}_j \tilde{\mathbf{C}} \bar{\mathbf{h}}_i + \\ & \left(\omega_i^2 - \omega_j^2 \right) \tilde{\mathbf{H}}_j \tilde{\mathbf{M}} \bar{\mathbf{h}}_i \end{aligned} \tag{36}$$

A drawback of any method based on the reduction (27) is that it is valid only at one frequency ω_j; and several frequencies may be used to handle noise.

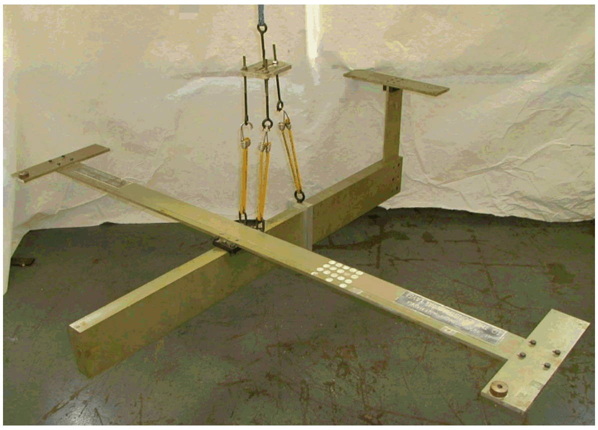

Figure 4: The GARTEUR structure

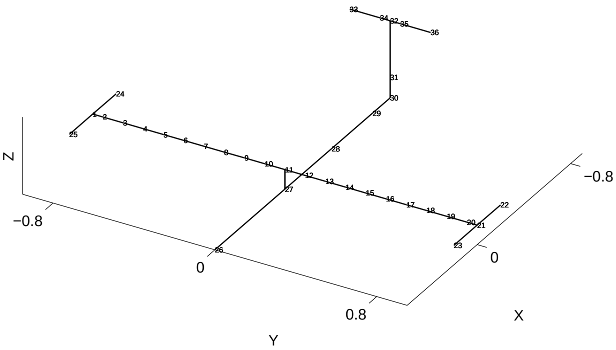

Figure 5: Beam elements model

4 Example

The model updating method proposed here is tested with a benchmark from the COST F3 Group[1]. The GARTEUR[2] structure is shown in figure 4. It consists of 6 aluminium beams with rectangular cross sections which represent a typical aircraft design. The total mass of the structure is 44 Kg. In order to increase damping a viscoelastic tape bounded the wing upper surface and was covered by a thin aluminium constraining layer. The structure was suspended with soft cords in order to assure correct free-free boundary conditions. The FE model is shown in figure 5 and considers 35 Euler-Bernoulli elements (216 degrees of freedom). 24 experimental FRFs were used as input for the model updating procedure. Excitation actuated at the end of the right wing.

[1]European COoperative on Science and Technology

[2]Group for Aeronautical Research and Technology in EURope. See ref. [7].

Mode n^o	Exp (Hz)	FE (Hz)	MAC	df %
1	6.38	5.80	0.995	-8.94
2	16.10	15.38	0.988	-4.51
3	33.13	31.94	0.679	-3.57
4	33.53	32.75	0.847	-2.32
5	35.65	33.80	0.670	-5.19
6	48.38	45.11	0.964	-6.76
7	49.43	49.70	0.968	0.53
8	55.08	56.66	0.966	2.88
9	63.04	59.40	0.958	-5.78
10	66.50	62.48	0.998	-6.04
		mean	**0.903**	**4.65**

Table 1: Initial corrrelation

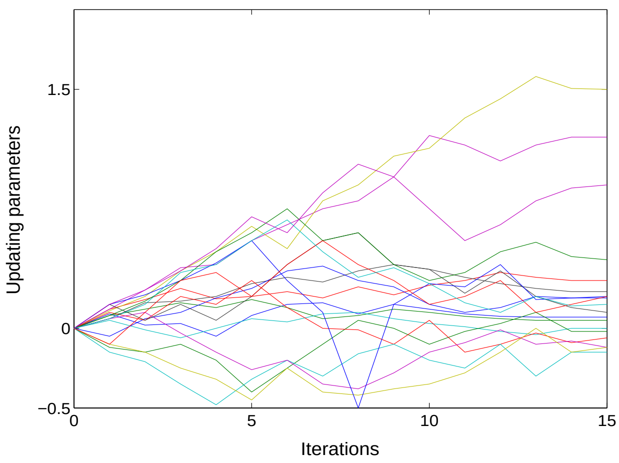

Figure 6: Updating history

Location	Parameter	Correction %
Right wing	EI_{xx}	22
	EI_{yy}	0
	EI_{zz}	-15
Left wing	I_{xx}	17
	EI_{yy}	-1
	EI_{zz}	-12
Connection, right wing	I_{xx}	7
	GI_{yy}	118
	EI_{zz}	30
Connection, left wing	EI_{xx}	5
	GI_{yy}	149
	EI_{zz}	0
Fuselage/wings connection	EI_{xx}	90
	GI_{zz}	15
Fuselage/vertical tail connection	EI_{xx}	21
	GI_{zz}	43
Connection, horizontal tailplane	EI_{xx}	19
Fuselage	GI_{xx}	-20
	EI_{yy}	10

Table 2: Updating parameters

The initial correlation of experimental and numerical mode shapes is listed in table 1.

A preliminary comparison between correlated mode pairs allowed a selection of the design parameters listed in table 2 where we also present the updated values referred to the initial values. Figure 6 shows the evolution of the updating parameters. The corrections are physically significant and lead to a notorious improvement of the correlation between the natural frequencies and mode shapes (table 3), and also with FRFs (figure 7). In this last figure we have marked the frequency shift proposed by FDAC (see the connecting line at 30 Hz).

5 Conclusions

A model updating method which uses response measurements as input has been presented. Its main advantage over current methods resides in its ability to avoid the numerical difficulties induced by the discontinuities in the frequency response functions. This

Mode n^o	Exp (Hz)	FE (Hz)	MAC	df %
1	6.38	6.23	0.994	-2.36
2	16.10	16.50	0.984	2.44
3	33.13	32.70	0.982	-1.30
4	33.53	33.30	0.982	-0.68
5	35.65	35.63	0.953	-0.04
6	48.38	48.49	0.971	0.23
7	49.43	49.59	0.972	0.32
8	55.08	54.68	0.963	-0.71
9	63.04	62.85	0.925	-0.30
10	66.50	66.49	0.996	-0.01
			0.970	**0.84**

Table 3: Updated correlation

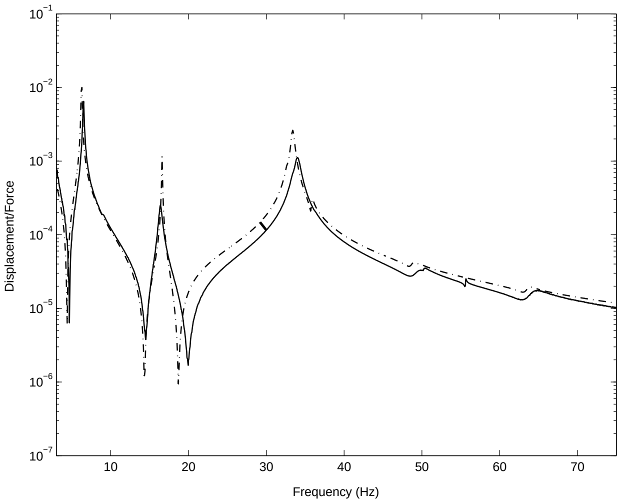

Figure 7: Typical FRF matching

is achieved by tracking the numerical operating deflection shapes that show the best correlation with the selected experimental shapes. Each selected frequency provides a number of equations equal to the number of measured degrees of freedom; so a large over-determined system of equations for the updating parameters is found. The procedure was successfully applied to a well-known experimental benchmark.

The combined used of the improved model updating strategy with the Frequency Domain Assurance Criterion represents a powerful tool that can be used advantageously in the context of model updating. As a global correlation tool, FDAC evaluates quantitatively the closeness between measured and numerical operating deflection shapes. This information is very helpful for the engineer, who is frequently asked to reduce vibrations in terms of the dynamic responses. Frequency zones where the model shows poor results are easily detected. No identification is needed, since the measurements are used directly.

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