Qualitative Spatial Reasoning

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KEYWORDS

Qualitative reasoning, qualitative spatial reasoning, qualitative spatial representation, calculus, topology, topological relationships, orientation, shape, composition table, conceptual neighbourhood, conceptual neighbourhood diagram, qualitative distance

GLOSSARY

Allen's Interval logic: It is a calculus for temporal reasoning that defines possible relations between time intervals (e.g. "*X* starts with *Y*" or "*X* takes place before *Y*") and provides a composition table that can be used as a basis for reasoning about temporal descriptions of events.

Artificial Intelligence: It is the study and design of intelligent agents where an intelligent agent is a system that perceives its environment and takes actions which maximizes its chances of success.

Convex hull: In mathematics, the convex hull or convex envelope for a set of points *X* in a real vector space *V* is the minimal convex set containing *X*.

Homeomorphism: In the mathematical field of topology, a homeomorphism is a special isomorphism between topological spaces which respects topological properties. Roughly speaking, a homeomorphism is a continuous stretching and bending of an object into a new shape which respects topological properties (e.g. a cube may be stretched into a ball, but can not be stretched into a donut).

Map Algebra: It is a set based algebra (primitive operators) to manipulate geographic data. Depending on the spatial neighborhood, operators are categorized into four groups: local, focal, zonal, and incremental. The input and output for each operator being map, the operators can be combined into a procedure to perform complex tasks.

Mereotopology: It combines mereology and topology. It is a first order theory of the topological relationships among wholes, parts, parts of parts, and the boundaries between parts.

SYNOPSIS

Spatial reasoning can be seen as any deduction of knowledge from a situation having spatial properties. In our every day interaction with the physical world, spatial reasoning is mostly driven by qualitative information rather than quantitative information. Inferring new knowledge from this qualitative spatial information is called Qualitative Spatial Reasoning (QSR). In this document, fundamentals of QSR are presented. If an important part of the research has been dedicated to qualitative spatial representation (basic spatial entities, relationships between entities, etc.), the biggest challenge remains the development of calculi, allowing reasoning about objects in multidimensional space. The two main prevalent forms of qualitative reasoning are Composition Table (based on transitivity between relationships) and Conceptual Neighbourhood diagram (based on a continuity concept between relationships). Some of the most achieved models are presented, especially models dealing with topological information and orientation information. Promising current researches are also sketched giving a wide view on this new research domain.

1 INTRODUCTION

"Saying that Alaska is 1 518 800 km² is sufficiently exact quantitative information about size and distances in Alaska but very likely it is not meaningful in relation to the spatial knowledge of the average listener. On the other hand saying that Alaska alone is bigger than all the states of the East coast from Maine to Florida is cognitively more immediate".

Spatial reasoning can be seen as any deduction of knowledge from a situation having spatial properties. It goes from simple calculation of spatial attributes (e.g. the area of a region) and the description of spatial relations (e.g. two polygons are disjoint) to all operations of map algebra. More complex tasks, such as planning of routes, answering to shortest path problems, and allocation of resources, can be categorised in the broad area of spatial reasoning as well. Reasoning can be performed on quantitative as well as on qualitative information. Typically when working with quantitative information, a predefined unit of a quantity is used. For example, one could say that the distance between Ghent and Liège is 150 kilometres. In the qualitative approach, continuous information is being quantised or qualitatively discretised by landmarks, depending on their relevance to the current research context. In this respect and in Belgian context, Liege is far from Gent (as opposed to “close to”), Brussels is closer from Ghent than Liege, Brussels is between Gent and Liege, Liege is east of Gent, etc.

Spatial reasoning in our every day interaction with the physical world is mostly driven by qualitative abstractions of the quantitative space. Of course, numerical representations may be well suited, in particular, where precise spatial information of a definite situation is available, and if the output required from the system is itself primarily numerical. However, such quantitative information is often too precise for the given spatial context. For example, if we show a person the way to get to the post office, we do not need to be more precise than the streets he has to follow. Inferring new knowledge from this qualitative spatial information is called Qualitative Spatial Reasoning – QSR.

QSR is a branch of Artificial Intelligence (AI), linked to Spatial Cognition, highly related to Geographical Information Science (GISc) and by extension to most aspects of geography. It is a particular (through its spatial component) sub-field of Qualitative Reasoning (QR) and is often associated with Spatio-Temporal (Qualitative) Reasoning when time is considered. In short, Qualitative Reasoning is an approach for dealing with commonsense knowledge of the physical world without numerical computation. Its main goal is to make this knowledge explicit, so that given appropriate reasoning techniques, a computer could make predictions, diagnose and explain the behaviour of physical systems in a qualitative manner. QSR is the part of QR dealing with spatial concepts. Due to space complexity, most work in qualitative spatial reasoning has focused on single aspects of space. The most important aspects of space are topology, orientation, and distance. As shown in psychological studies, this is also the order in which children acquire spatial notions. Other aspects of space include size, shape, morphology, and spatial change (motion). Beside geography, other domains interested in qualitative spatial reasoning are, for example, robotics, linguistics, experimental psychology and qualitative physics.

In this chapter, we present fundamentals of QSR, and some of the major current approaches and models in the domain. Afterwards, we point out some of the most promising research directions.

2 FUNDAMENTALS

First, it is necessary to choose the basic entities on which qualitative representation and reasoning will be based. This ontological choice in the QSR research community is often reduced to two possibilities: points or regions (being extended spatial entities). While it is easier to deal with points rather than with regions in a computational framework, taking regions as the basic entities is certainly more adequate for commonsense reasoning. Furthermore, if points are required, they can be generated from regions. Another ontological distinction is the nature of the embedding space. The most common notion of space is *n*-dimensional continuous space (R^n). But, there are also approaches which consider discrete or finite space.

Much of the work on QSR has concentrated on representational aspects, e.g. how to handle or characterize region's topological properties and relationships, how to characterize region's shape, etc. Talking about relationships, they could refer to two regions (binary relationships), three regions or two regions together with an external frame of reference (ternary relationships), etc. Examples of qualitative spatial representations are presented in next section.

However, the biggest challenge is to provide calculi, allowing reasoning about objects in multidimensional space without the use of traditional quantitative knowledge and techniques. The most popular reasoning methods used in qualitative spatial reasoning are constraint-based methods. Shortly, they are based on constraint satisfaction which is the process of finding a solution to a set of constraints, known as the constraint satisfaction problem (CSP). To be applied to spatial qualitative relationships based on particular aspects of space such as topology, direction, and distance such relationships must have the characteristic to be jointly exhaustive and pairwise disjoint (JEPD). Qualitative binary relations are jointly exhaustive and pairwise disjoint if between any two spatial entities exactly one of the basic relations holds. The same goes for *n*-ary relations, but between *n* spatial entities. Being JEPD, the set of all possible relations is then the set of all possible unions of the basic relations.

The most prevalent form of qualitative reasoning is based on the Composition Table (CT). The underlying idea of a CT (also known as inference table, formerly known as transitivity table) is to compose a finite set of new facts and rules from existing ones, i.e. if two existing relations ($R_1(a,b)$ and $R_2(b,c)$) share a common object (b) they can be composed into a new relation set $R_3(a,c)$. Such a composition is also called an inference (here symbolised by ⊗). Thus:

$$R_1(a,b) \otimes R_2(b,c) = R_3(a,c)$$

Compositions of relations are usually pre-computed and stored in the CT. CT[s] originate from the domain of temporal reasoning. An example of a temporal composition is:

$$(interval_1 \text{ before } interval_2) \otimes (interval_2 \text{ meets } interval_3) = (interval_1 \text{ before } interval_3)$$

Going beyond the domain of purely static spatial calculi, it is possible to reason about spatial changes. Spatial changes may occur on a specific entity (change of shape, position, etc.) and may have some impacts on its relationships with the environment. Assuming that changes are continuous, another major qualitative reasoning technique is the Conceptual Neighbourhood Diagram (CND). Continuous means that things change smoothly. A simple consequence of continuity, respected by all systems of qualitative physics, is that, in changing, a quantity must pass through all intermediate values. That is, if $A < B$ at time t_1, then it cannot be the case that at some later time t_2 $A > B$ holds, unless there was some time t_3 between t_1 and t_2 such that $A = B$. Following this continuity assumption, we could say that two relations between pairs of events are conceptual neighbours, if they can be directly transformed into one another by continuously deforming (i.e. shortening, lengthening, moving) the events (in a topological sense) and that a set of relations between pairs of events forms a conceptual neighbourhood if its elements are path-connected through conceptual neighbour relations. CND[s] describe all the physical possible transitions between relations for a given model. Such diagrams allow inferring information; for example, if relation R2 is conceptual neighbour of relation R1 and R3 and if R1 and R3 are not conceptual neighbours, then evolving from relation R1 between entities A and B to relation R3 is only possible through an intermediate stage R2 between entities A and B.

Keeping this framework, from representation to reasoning methods, in mind, we may move on to major approaches and models in QSR.

3 MAJOR APPROACHES AND MODELS

3.1 Topology

The properties of geometric shapes and geometric relations between spatial objects that are invariant under homeomorphisms such as translation, rotation and scaling are respectively topological properties and topological relations. Thus, a first group of stretching, bending, twisting, or compressing a figure, are transformations which do not affect topological properties and relations. However, a second group of tearing, puncturing, or inducing self-intersection do affect topological properties and relations. In other words, topological properties of a drawing on a rubber sheet are preserved when the rubber sheet is deformed according to the above-mentioned first group of transformations. For this reason, topology has also been called the geometry of the rubber sheet.

Generally, topology can be seen as the most developed area of qualitative spatial representation and reasoning, resulting in a variety of topological approaches that can be found in literature. We describe two standard works: the Region Connection Calculus (RCC) and the 9-Intersection Model. Both theories were developed independently from each other at the beginning of the 90s. Although the 9-Intersection Model originates from the domain of database theory and RCC is from the field of qualitative reasoning related to artificial intelligence, both have as a major conclusion that there are eight topological relations between two regions in 2D space.

3.1.1 The Region Connection Calculus (RCC)

At the end of the 80s and the beginning of the 90s, Randell, Cui and Cohn transformed Clarke's Interval Logic for reasoning about space and space-time, which is the spatial counterpart of the Interval Calculus, into RCC (Region Connection Calculus).

The basic idea is a primitive binary relation $C(x,y)$ read as x connects with y, which is defined on non-null regions, being the space that physical entities occupy at any given time. Using the relation $C(x,y)$, a set of eight JEPD relations has been worked out, resulting in RCC-8. These relations describe differing degrees of connection between regions: is disconnected from (DC), is externally connected to (EC), partially overlaps (PO), is identical with (EQ), is a tangential proper part of (TPP), and is a nontangential proper part of (NTPP). All except the last two are self-inverse. Taking the inverse of the last two results in the inverse of a tangential proper part of (TPPI) and the inverse of a nontangential proper part of (NTPPI).

The set of eight RCC-8 relations can be reduced to five by taking the disjunction of DC and EC (resulting in DR, meaning x is discrete from y), the disjunction of TPP and NTPP (resulting in PP), and the disjunction of TPPI and NTPPI (resulting in PPI). This set of five relations is known as RCC-5. RCC-23 is a result of the introduction of an additional primitive function $conv(r_1)$: the convex hull of r_1. By determining whether a region r_1 is inside (I), partially overlaps with (P), or is outside (O) the convex hull of another region r_2, EC and DC are replaced by more specialised relations, resulting in a set of twenty-three base relations: RCC-23. Ignoring the difference between DC and EC results in a set of fifteen base relations: RCC-15.

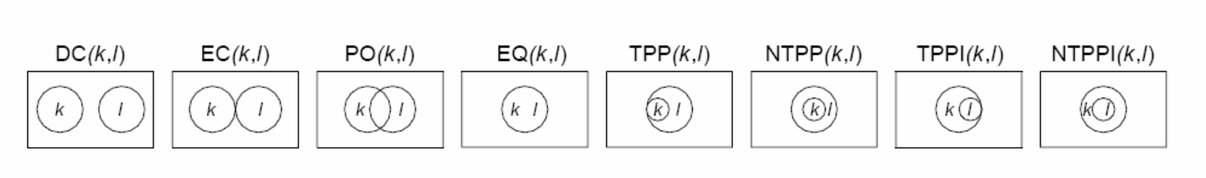

Figure 1

$R_1 \otimes R_2$	DC	EC	PO	TPP	NTPP	TPPI	NTPPI	EQ
DC	no info	DR,PO,PP	DR,PO,PP	DR,PO,PP	DR,PO,PP	DC	DC	DC
EC	DR,PO,PPI	DR,PO,TPP,TPI	DR,PO,PP	EC,PO,PP	PO,PP	DR	DC	EC
PO	DR,PO,PPI	DR,PO,PPI	no info	PO,PP	PO,PP	DR,PO,PPI	DR,PO,PPI	PO
TPP	DC	DR	DR,PO,PP	PP	NTPP	DR,PO,TPP,TPI	DR,PO,PPI	TPP
NTPP	DC	DC	DR,PO,PP	NTPP	NTPP	DR,PO,PP	no info	NTPP
TPPI	DR,PO,PPI	EC,PO,PPI	PO,PPI	PO,TPP,TPI	PO,PP	PPI	NTPPI	TPPI
NTPPI	DR,PO,PPI	PO,PPI	PO,PPI	PO,PPI	PO	NTPPI	NTPPI	NTPPI
EQ	DC	EC	EC	TPP	NTPP	TPPI	NTPPI	EQ

Figure 2

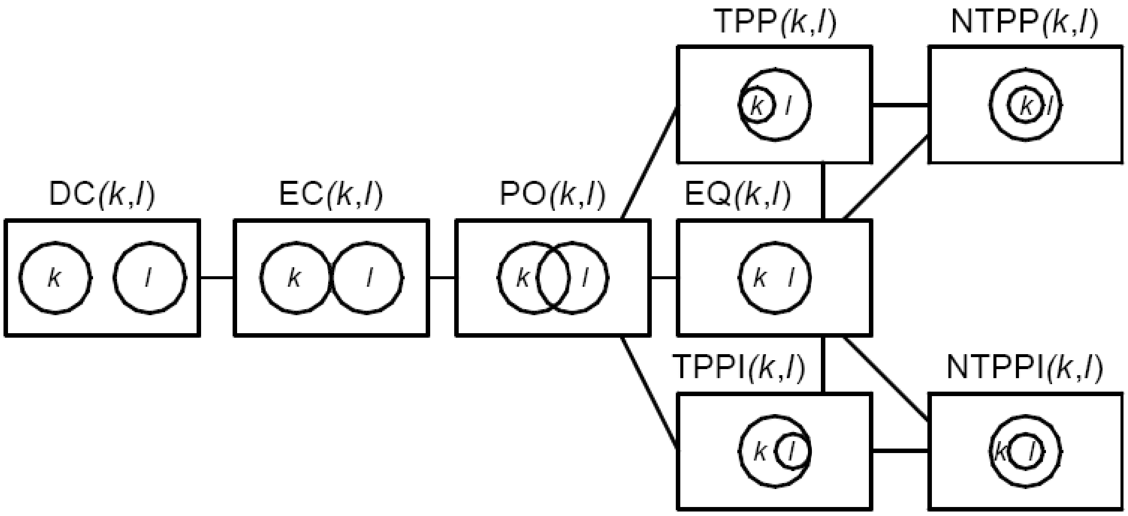

Figure 3

3.1.2 The 9-Intersection Model

Egenhofer and Franzosa present the same set of relations as those presented in RCC, but these were generated in a different manner. Originally, the 4-Intersection Model was developed. Here, two sets of points are associated with every region in R^2, its interior and its boundary. Only regions without holes, meaning 2D connected objects with connected boundaries, are considered.

- The interior of *A*, is defined to be the union of all open sets that are contained in *A*, i.e. the interior of *A* is the largest open set contained in *A*.
- The closure of *A* is defined to be the intersection of all closed sets that contain *A*, i.e. the closure of A is the smallest closed set containing *A*.
- The boundary of *A*, is the intersection of the closure of *Y* and the closure of the complement of *Y*. The boundary is a closed set.

Considering empty and non-empty as the values of the intersections between the boundaries and interiors of two sets, a total of sixteen topological relations can be characterised, each relation represented by a 2×2 matrix. If the sets are restricted to spatial regions in R^2, then the number of relations is reduced to eight: disjoint, overlaps, meets, equals, inside, contains, covers, and covered-by.

The 4-Intersection Model was extended to the 9-Intersection Model, in which a third set of points is associated with every region, being the region's complement. This approach resulted in 512 (2^9) matrices having three columns and three rows. However, after taking into account all assumptions about the nature of regions in 2D space, only eight matrices remain. These correspond once more to the RCC relations, with: disjoint = is disconnected from (DC), overlaps = partially overlap (PO), meets = is externally connected to (EC), equals = is identical with (EQ), inside = is a nontangential proper part of (NTPP), contains = is the inverse of a nontangential proper part of (NTPPI), covers = is a tangential proper part of (TPP), and covered-by = is the inverse of a tangential proper part of (TPPI).

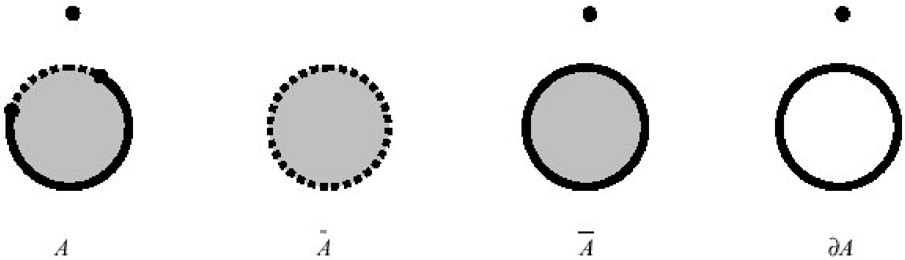

Figure 4

$$\begin{bmatrix} \partial A \cap \partial B & \partial A \cap \mathring{B} & \partial A \cap \overline{B} \\ \mathring{A} \cap \partial B & \mathring{A} \cap \mathring{B} & \mathring{A} \cap \overline{B} \\ \overline{A} \cap \partial B & \overline{A} \cap \mathring{B} & \overline{A} \cap \overline{B} \end{bmatrix}$$

Figure 5

The 9-Intersection model and also the Calculus based method (CBM) of Clementini, which is not presented here, are both Open GIS consortium (OGC) standards models for topological relationships. Such models have been widely extended to other geographical primitives; points, lines and bodies (object with a 3D extension) in R^2 and in R^3. Extensions have been made based on the dimension of objects intersections, number of intersections, etc. Other interesting work is the comparison of topological relations in a continuous (vector) space of R^2 and a discrete (raster) space of $\mathbb{Z}^2$, applying the 9-Intersection Model in a raster model.

3.2 Orientation

Topology may be regarded as the most abstract spatial structure or the weakest geometry, which makes clear that there are many situations where topological information alone is insufficient. This section explores an important complementary qualitative relationship, namely orientation: orientation relations describe where the objects are placed relative to one another and can be defined in terms of three basic concepts: the primary object, the reference object and the frame of reference. The orientation of the primary object is then expressed with respect to the reference object as it is determined by the frame of reference. Thus, unlike the formerly presented topological relations, orientation is not a binary relation, i.e. if we want to specify the orientation of a primary object with respect to a reference object, a frame of reference is necessary. The choice of reference system depends on the calculus one works in. Also due to the context of the spatial situation, one reference system can be more appropriate than another. When there is a sudden change in context, it appears to be preferable to use multiple frames of reference. An example situation occurs when a driver gets off a highway to enter a local street network. This is reflected in the use of maps having a different scale.
There are two main groups of qualitative orientation calculi: the first one having an extrinsic frame of reference and the second one having an explicit triadic relation. In the first approach,

the orientation of objects is obtained by using orthogonal or non-orthogonal projections of objects onto external axes, and by subsequent 1D reasoning mostly using Allen's temporal logic. An example of this is Guesgen's approach. The major theories of the second group are Frank's approach (cardinal direction calculus), Hernandez' approach, and the Freska's Double-Cross Calculus. Frank discusses the use of cardinal directions for spatial representation and reasoning. In order to determine each orientation relation, 2D space is segmented around the reference object, by the use of the eight well-known standard directional symbols (north, northeast, east, southeast, south, southwest, west, and northwest) resulting in the cone-shaped and the projection-based models. The eight cardinal directions of the cone-shaped approach are extended with the qualitative value `0`, representing an undetermined direction between points that are too close to each other to be able to give a cardinal direction. The eight cardinal directions of the projection-based model are also extended with the qualitative value `0`, standing for the so-called neutral zone. These extensions greatly increase the power of the calculus, which is exemplified by typical qualitative reasoning methods such as the creation of composition tables (CT^s). This approach could be combined with qualitative distances, such as far and farthest.

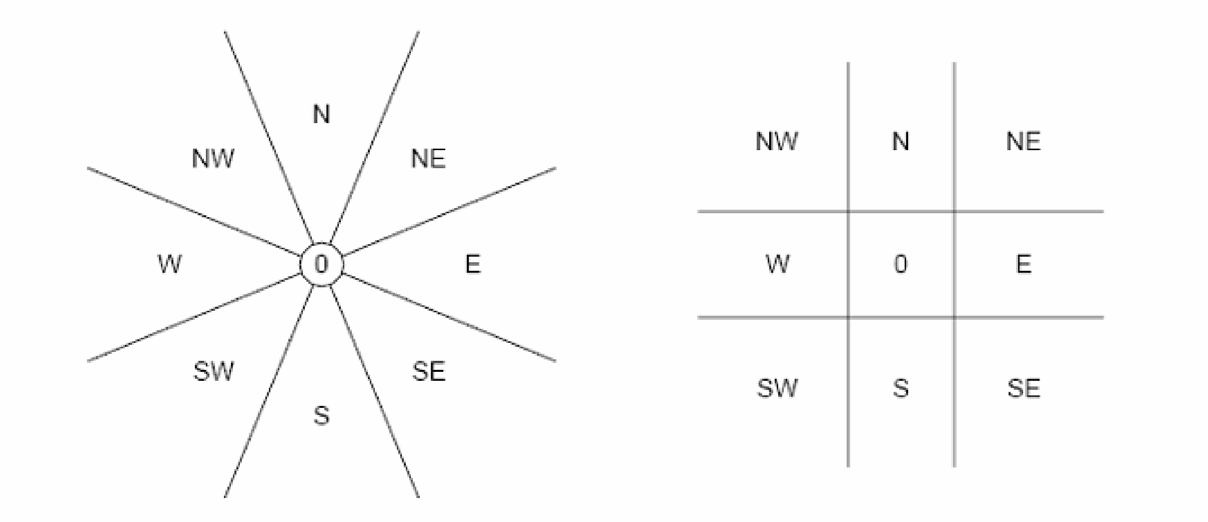

Figure 6

3.3 Other developments

Although topology and orientation are the most covered domains, there are other models and approaches in QSR. Without being exhaustive, we wish to show some of them to demonstrate the wide range of the field and its florishing research activities.

First of all, regions have been so far considered as having crisp boundaries. Introducing some vagueness in object's boundary definition, some of the previously mentioned models have been extended to explicitly represent and reason about uncertain information. For instance, RCC postulates the existence of non crisp regions in addition to crisp regions and then adds another binary relation meaning that "x is crisper than region y". This extension is known as the "egg-yolk" calculus, which essentially models regions with intermediate boundaries as a pair of regions. A rather similar extension of the 9-intersection model has been also developed, and refers to regions with broad boundaries. This calculus can also be specialized to cover other kinds of regions including convex hulls, minimum bounding rectangles, buffer zones and rasters.

Distance is one of the most important aspects of space. When communicating about distances, people usually use qualitative categories like “A is close to B” or qualitative distance comparatives like “A is closer to B than to C”. If quantitative information can also be used when considering distance between two entities (absolute distance) like “A is about one meter away from B”, comparative distances are purely qualitative. Most approaches to qualitative distance consider points as the basic entities. Absolute distance relations are obtained, e.g., by dividing the real line into several sectors such as “very close”, “close”, “far”, etc. depending on the chosen level of granularity. Relative distance can be obtained by comparing the distance to a given reference distance. However, such concepts lead to several reasoning difficulties, e.g., transitivity of relationships may depend on the orientation, position of the points. This is why distance is often combined with orientation to get positional information. Several models have been developed, for example one combining a cone-based orientation approach with absolute distance relations (Clementini)

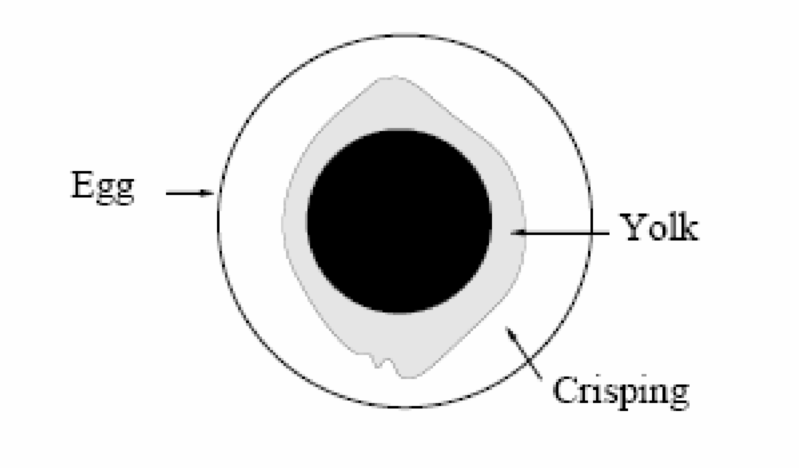

Figure 7

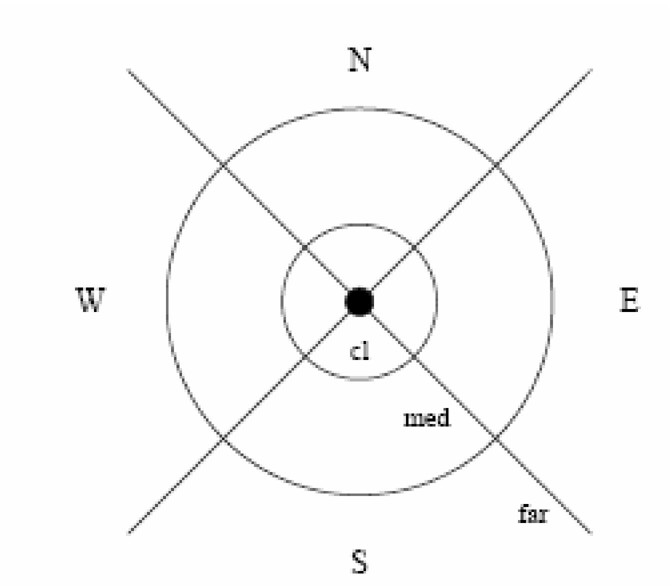

Figure 8

As it has been said when introducing orientation calculus, topological information is not always sufficient to deal with all the qualitative spatial situations. Between topological space and metric space, there are other mathematical spaces which can be used in QSR. One of them is the projective space, whose main invariant is collinearity (three collinear points remain collinear under projective transformation). Such property is the base of another type of relationships; ternary projective relationships (Billen and Clementini). Contrary ternary relationships mentioned in the orientation calculus section, these ternary relationships do not need an external frame of reference, but only a primary object and two reference objects. The space is partitioned into five zones based on the two references objects shapes. The relative position of the primary object against the two reference objects is determined by the primary object's intersections with the zones. Such approach is another way to model concept such as being on the left, on the right, between, e.g. an object A is left of objects B and C if A intersect the zone left(B,C).

Finally, we wish to mention time as a complementary concept of space. QSR is strongly link with temporal reasoning, resulting on the development spatio-temporal reasoning. Recently, spatio-temporal calculi have been designed. For example, the Spatio-temporal Constraint Calculus (STCC) by Gerevini and Nebel combines Allen's interval algebra with RCC-8. Moreover, the Qualitative Trajectory Calculus (QTC) allows for reasoning about moving objects (Van de Weghe).

4 CONCLUSIONS

Qualitative spatial representation and reasoning is so close to human spatial cognition processes that it should take a bigger place in geography in the future. It is (should be) of prime importance in naïve geography, mental mapping, landscape analysis, and also in analysing human behaviour (with time, allowing analysis of spatio-temporal history), etc.
However, it is a new domain, and there are still a lot of things to be developed; some of space aspects still have to be identified, formalised and if possible associated to reasoning processes. Furthermore, such models, even if they are based on strong mathematical concepts, can be meaningless if not related to some human cognitive perceptions; Cognitive validity of QSR models are one of the biggest challenge of this promising scientific domain.

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