Multiple Timescale Spectral Analysis of floating structures

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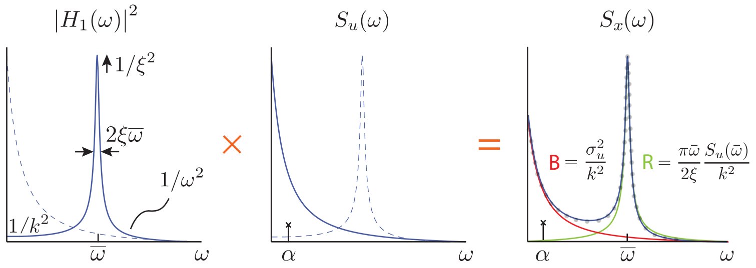
1. Background/Resonant Decomposition $\sigma_x^2 = \int S_x(\omega)\mathrm{d}\omega \approx$ B + R



Drastically decreases the number of integration points required, thus CPU time

2. The MTSA generalizes it for higher order statistics

The method is well developed for buffeting wind-loaded structures, for the variance, covariance, skewness and kurtosis of modal responses, for the variance of MDOF structural responses & for small nonlinearities in the loading or the structural behaviour

The turbulence intensity of wind buffeting is low $\sigma_u \ll U$

& small parameters appear as timescales are split $\begin{cases} \alpha \ll \overline{\omega} : \text{quasi-static loading} \\ \xi \ll 1 : \text{low damping ratio} \end{cases}$

Perturbation methods are then able to provide analytical approximations, with controllable discrepancy, which reduce the order of integration by one, at least, and makes the computation of statistics 100 times faster.

3. Resolution in the frequency domain

1) Stochastic polynomialisation of the nonlinear loading
2) Expansion of the nonlinear system in Volterra series : $x = \tilde{x}_1 + \tilde{x}_2 + \ldots$
3) Truncation of the series at a given order (here n = 2) : $\tilde{x} = \tilde{x}_1 + \tilde{x}_2$
4) Evaluation of the Volterra Frequency Response Functions $H_n(\boldsymbol{\omega})$
5) Integration of the spectra to obtain the statistics

Power Spectral Density

$$S_{\tilde{x}}(\omega) = S_{\tilde{x}_1}(\omega) + S_{\tilde{x}_2}(\omega)$$
$$\sigma_{\tilde{x}}^2 = \int S_{\tilde{x}}(\omega)\mathrm{d}\omega$$

Bispectrum

$$B_{\tilde{x}}(\omega_1,\omega_2) = 3B_{\tilde{x}_{112}}(\omega_1,\omega_2) + B_{\tilde{x}_2}(\omega_1,\omega_2)$$
$$m_{3,\tilde{x}} = \iint B_{\tilde{x}}(\omega_1,\omega_2)\mathrm{d}\omega_1\mathrm{d}\omega_2$$

4. Main differences between wind and wave loadings

$$f_u = k_d(U + u - \dot{x})^2$$
$$(U + u - \dot{x})^2 = U^2 + 2U(u - \dot{x}) + (u - \dot{x})^2 = \sum_{r=0}^{2} a_r (u - \dot{x})^r$$

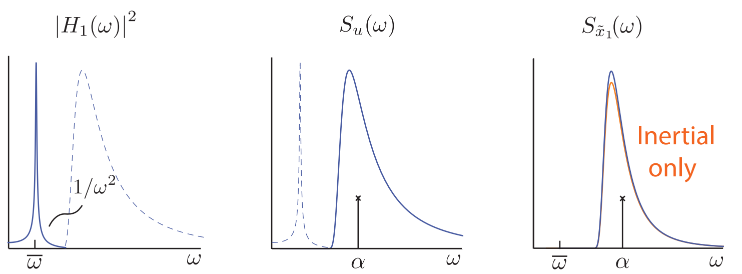
$$f_u = k_m\dot{u} + k_d(U + u - \dot{x})|U + u - \dot{x}|$$ more nonlinearities

$$(U + u - \dot{x})|U + u - \dot{x}| \approx \sum_{r=0}^{n} a_r (u - \dot{x})^r$$ with n not limited to 2

In some cases :
- the inertial components of the response are not negligible, when $\alpha \gg \overline{\omega}$
- the turbulence intensity of the wave speed is higher as $U_{\text{wave}} < U_{\text{wind}}$

HOW TO EXTEND THE MTSA FOR THE RESPONSE STATISTICS OF WAVE-LOADED FLOATING STRUCTURES ?

5. Variance of the surge response of a SDOF structure



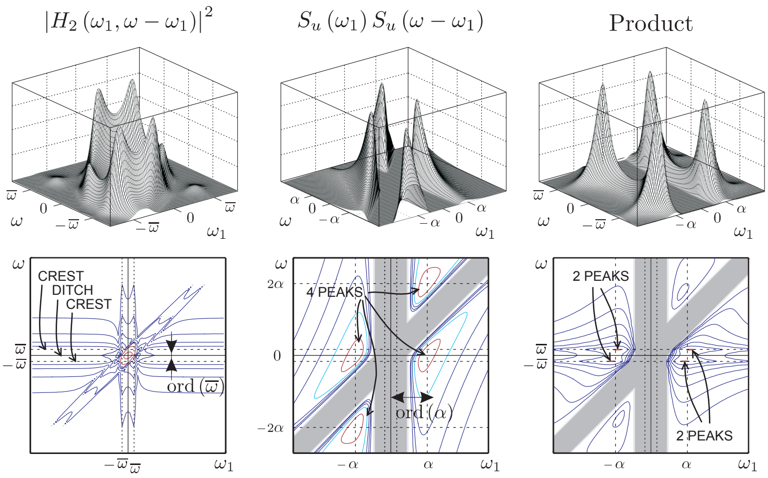
1[st] order response $S_{\tilde{x}_1}(\omega) = |H_1(\omega)|^2 S_u(\omega) \approx \frac{\kappa_m^2}{\omega^2} S_u(\omega)$

& $\sigma_{\tilde{x}_1}^2 \approx \kappa_m^2 \sigma_{fu}^2$ provided $\frac{a_1 k_d}{k_m} \ll \alpha$ & $2\xi\overline{\omega} \ll \alpha$

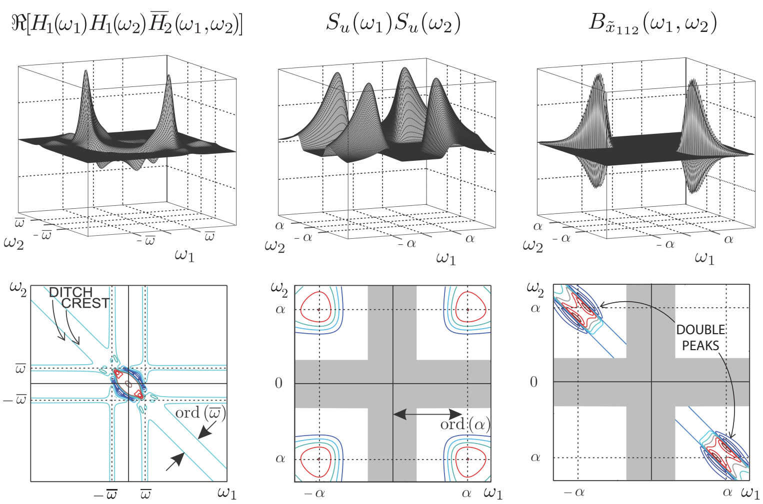
2[nd] order response $S_{\tilde{x}_2}(\omega) = 2\int_{\mathbb{R}} |H_2(\omega_1, \omega - \omega_1)|^2 S_u(\omega_1) S_u(\omega - \omega_1)\mathrm{d}\omega_1$

& $\sigma_{\tilde{x}_2}^2 \approx a_2^2 (1 - \kappa_m)^4 \frac{\pi\overline{\omega}}{\xi} S_{u^2}(\overline{\omega})$ if $\frac{a_1 k_d}{k_m} \ll \alpha$ & $2\xi\overline{\omega} \ll \alpha$

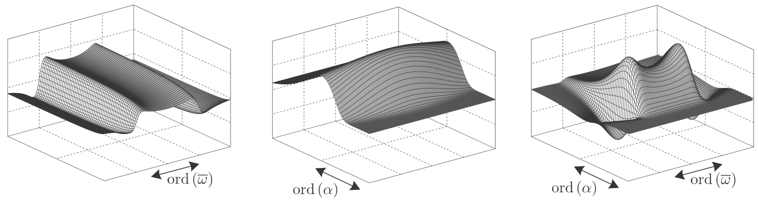
Order of integration ↘ by 2 ! ⟶ CPU time ↘ by 2 orders of magnitude, at least !



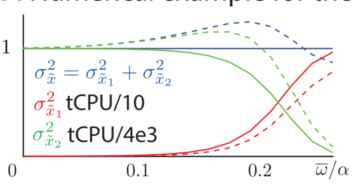
6. Skewness of the surge response of a SDOF structure



The first order response does not contribute to the skewness as it is fully linear. If we focus on one double peak and replace $H_1(\omega_1)H_1(\omega_2)$ by a constant, the remaining function is not integrable ⟶ work is still in progress...



7. Numerical example for the computation of the variance



—— exact - - - MTSA

The MTSA is a powerful tool :
- Small loss of accuracy
- Large reduction of CPU time
- Meaningful analytical approximations

Denoel, V. (2015). Multiple timescale spectral analysis. Probabilistic Engineering Mechanics., 39, 69-86 ; Denoel, V., & Carassale, L. (2015). Response of an oscillator to a random quadratic velocity-feedback loading. Journal of Wind Engineering & Industrial Aerodynamics, 147, 330-344
Denoel, V. (2009). Estimation of modal correlation coefficients from background and resonant responses. Structural Engineering and Mechanics, 32(6), 725-740 ; Carassale, L., & Kareem, A. (2010). Modeling non linear systems by Volterra series. Journal of Engineering Mechanics, 136(6)