IX ECCOMAS Thematic Conference on Smart Structures and Materials
SMART 2019
A. Benjeddou, N. Mechbal and J.F. Deü (Eds)

HYBRIDIZATION OF ACTIVE CONTROL AND PASSIVE RESONANT SHUNT

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Key words: Vibration damping, Hybridization, Smart Structures, Electromagnetic transducer

Abstract. Resonant shunt circuit applied on a transducer like piezoelectric or voice coil is widely used as a passive control technique to damp a specific vibration mode. While very simple and robust, the damping capability is proportional to the electromechanical coupling of the transducer. On the other hand, one of the major difficulties that arises in practical implementation of purely active vibration control is the power consumption required for conditioners and control units. The idea of using hybrid control system is proposed in this study to combine the passive shunt device with an active control in order to improve the performance with low power consumption. Both active voltage source and active current source are proposed and compared. The method of maximum damping, i.e. maximizing the exponential time-decay rate of the response subjected to the external impulse forcing function, is used to optimize the parameters of the passive and hybrid control systems. The advantage of using hybrid control configuration in comparison with purely active control system is also investigated in terms of power consumption.

1 INTRODUCTION

Recently, the electromagnetic shunt damper has been proposed as a very simple and effective passive control technique. The key idea of this technique is to connect a capacitor of capacitance (C) and a resistor of resistance (R) to the electromagnetic transducer of inductance (L) to form the resonant R-L-C circuit. The absorber dissipates the vibrational energy by the resistor when the resonance of the circuit is tuned close to the resonance of the primary system thanks to the tuned capacitor [1]. Many optimization methods have been proposed to optimize the parameter of R and C. de Marneffe [2] optimized the parameters through the root-locus analysis and H_∞ minimization when the system is under the base excitation. He has also compared the resonant R-L-C shunt with a resistive shunt. Inoue et al. [3] derived the optimal parameters by using the fixed point theory for the mechanical vibration absorber proposed by Den Hartog [4]. In [5], the optimal parameters have been obtained analytically using both H_2 and H_∞ optimization methods which are to minimize the root-mean-square (RMS) vibration under random excitation and the peak amplitude in the frequency domain, respectively. Moreover, Zhu et al. [6] studied the analogy between the electromagnetic shunt damper and a tuned mass damper (TMD).

Then, the optimal parameters of electromagnetic shunt have been adapted from the optimal parameters of the TMD (obtained by Ormondroyd and Den Hartog [7]) by using an equivalent mass, stiffness and damping coefficient for the electromagnetic shunt damper. The main shortcoming of this method is that the optimal parameters could be used only when the equivalent mass ratio is small enough because a full dynamic analogy was not given.
The hybrid control system may be an effective control configuration by combining the advantages of both active and passive control systems [8]. In other words, the active part of a hybrid system requires much less power than a similar purely active system, while providing better vibration suppression than the passive system alone. Despite this interest, no one to the best of our knowledge has studied the hybridization of the passive electromagnetic shunt damper with an active control system for the purpose of vibration damping improvement.
In the literature, there are a number of studies on active-passive hybrid piezoelectric network (APPN). In 1994, Agnes [9] proposed the concept of APPN and Tsai et al [10] presented more insight and fundamental understandings to the APPN configuration. Basically, the APPN integrates piezoelectric shunt damping with an active voltage or charge source to improve the control performance of the structure [11]. Morgan et al. [12] used active coupling feedback to enhance the electromechanical coupling of the transducer. In most studies about the APPN, a collocated piezoelectric sensor has been used to generate the feedback signal. However, MingMing el al. [13] employed a velocity feedback control for the application of the APPN by using a displacement sensor. The optimal values of the resistance and inductance could be quite different from those of the purely passive system in the case of APPN. Therefore, Tsai el al. [14] proposed a methodology to determine the optimal values of the resistor and inductor simultaneously with the control law.
In the present study, the active-passive hybrid electromagnetic shunt damper is proposed. This paper is organized as follows. First, a purely passive electromagnetic shunt damper is studied and the parameters of the R and C are optimized according to the method of maximum damping in Section 2. Then in Section 3, the hybrid configuration is modeled by combining both active voltage source in series and current source in parallel with the designed RC circuit, respectively. The power which flows between the structure and the transducer thanks to the application of active control system of hybrid configuration is discussed in Section 4. The conclusions are drawn in Section 5.

2 PASSIVE CONTROL SYSTEM

Figure 1 shows the system under consideration. It is an undamped single-degree-of-freedom (SDOF) oscillator with a mass m, spring k and a electromagnetic device connected to a resistor of resistance R and a capacitor of capacitance C. The system is excited under the disturbance force f_d; in particular, the impulse forcing amplitude is examined. The electromagnetic device which is made of a permanent magnet and a coil has the following parameters: coupling constant T, coil inductance L and coil resistance R_{coil}. It can generate a force f_a which is proportional to the current i flowing inside the coil. In addition, the voltage across the transducer V is proportional to the velocity of the mass. The governing equations of motion are written as:

$$m\ddot{x} + kx = f_d + f_a \tag{1a}$$

$$f_a = -Ti = -T\dot{q} \tag{1b}$$

$$V = L\ddot{q} + R\dot{q} + \frac{1}{C}q = T\dot{x} \tag{1c}$$

where q is the charge flowing inside the coil. For the sake of simplicity, R is considered as the total resistance of the circuit (Figure 1b). The above equations are normalized with respect to the dimensionless time $\tau = \omega_0 t$ where $\omega_0 = \sqrt{k/m}$ as below:

$$x_1'' + x_1 = f - \beta_1 \omega_0 x_2' \tag{2a}$$

$$x_2'' + 2\xi\alpha x_2' + \alpha^2 x_2 = \beta_2/\omega_0 x_1' \tag{2b}$$

where the normalized parameters are:

$$\begin{gathered} \tau = \omega_0 t,\ x_1(\tau) = x(t),\ x_2(\tau) = q(t),\ \Omega = \omega/\omega_0,\ f = \frac{1}{k} f_d,\ \beta_1 = \frac{T}{k} \\ \beta_2 = \frac{T}{L},\ \beta = \beta_1\beta_2,\ \omega_f = \frac{1}{\sqrt{LC}},\ \alpha = \frac{\omega_f}{\omega_0},\ \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \end{gathered} \tag{3}$$

The transfer function of the system from the normalized external force f to the normalized displacement of the mass x_1 is then given by:

$$\frac{x_1}{f} = \frac{s^2 + 2\xi\alpha s + \alpha^2}{(s^2+1)(s^2 + 2\xi\alpha s + \alpha^2) + \beta s^2} \tag{4}$$

where $s = j\Omega$ is the Laplace variable. According to Eq. (4), the passive control system adds another DOF to the system which makes the closed-loop response having two complex poles. In order to obtain the optimal parameters, the method of maximum damping, i.e. minimizing the settling time of the transient response of the system to the impulse disturbance, is used. This can be achieved by realizing equal damping ratio and tuning frequency for both poles of the closed-loop response. In other words, two poles are merged together. In this case, the normalized transfer function can be simplified as:

$$\frac{x_1}{f} = \frac{s^2 + 2\xi\alpha s + \alpha^2}{(s^2 + 2\eta\gamma s + \gamma^2)^2} \tag{5}$$

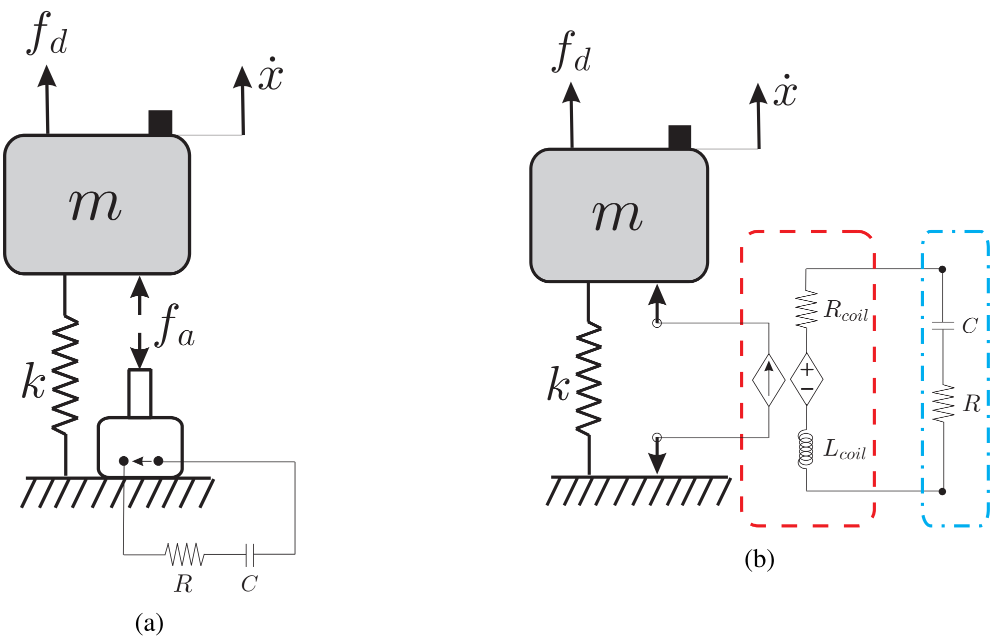


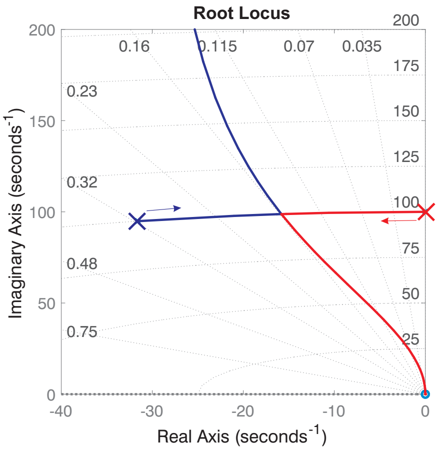
Figure 1: Single-degree-of-freedom (SDOF) oscillator combined with the electromagnetic shunt damper. (a) Mechanical model and (b) Electrical equivalent model of the transducer

where η is the damping ratio, ω_c, and $\gamma = \omega_c/\omega_0$ are the resonance frequency and the normalized resonance frequency of the closed-loop response function, respectively. By matching the polynomial coefficients of the denominators of Eq. 4 and Eq. 5, the set of equations can be obtained as below:

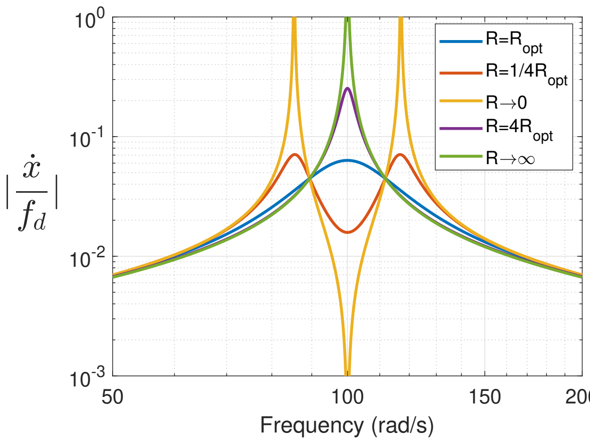
$$4\eta\gamma = 2\xi\alpha \tag{6a}$$
$$4\eta^2\gamma^2 + 2\gamma^2 = \alpha^2 + \beta + 1 \tag{6b}$$
$$4\eta\gamma^3 = 2\xi\alpha \tag{6c}$$
$$\gamma^4 = \alpha^2 \tag{6d}$$

From Eqs. (6a), (6c) and (6d), it can be concluded that

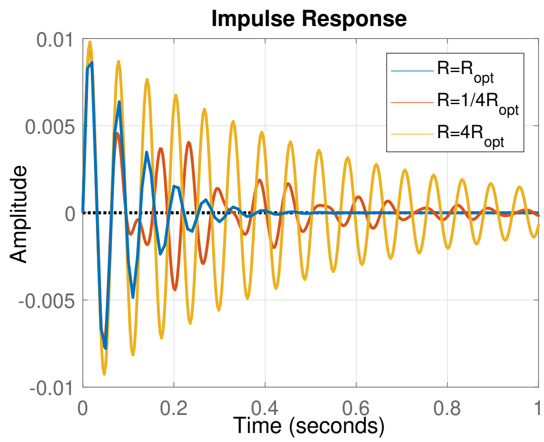
$$\gamma_{opt} = \alpha_{opt} = 1 \tag{7}$$



(a)



(b)



(c)

Figure 2: For the attached passive RC circuit, (a) root-locus of the system for a specific value of the resistance R and capacitance C, (b) frequency response as well as (c) the impulse response for different values of R

which means that the optimal frequency of the circuit and the closed-loop resonance frequency of the system are equal to the resonance frequency of the primary system. Considering the above equation and Eq. (6b) yields:

$$\xi_{opt} = \sqrt{\beta} \tag{8}$$

As a consequence, the optimal parameters of resistance and capacitance can be obtained as:

$$C_{opt} = \frac{1}{L\omega_0^2} \tag{9a}$$

$$R_{opt} = \frac{2T}{\sqrt{kC_{opt}}} \tag{9b}$$

In the remaining of the paper, the following numerical values have been used: m=1kg, $k = 10^4$N/m, T=1N/Amp, $L = L_{coil} = 10^{-3}$H, and $R_{coil} = 0\Omega$ (the resistance of the coil is included in R). The root-locus of the system with an attached passive RC shunt circuit is shown in Figure 2a. The locus consists of two loops (red and blue) starting from the pole of the primary system (red pole) and the pole of the resonant shunt (blue pole), respectively. One of the loop goes to origin and the other one goes to infinity. Both loops are intersecting at one point thanks to employing the optimal values of resistance R_{opt} and capacitance C_{opt}. Figure 2b shows the frequency response for five different values of the resistance R. All the curves are intersecting at two points which are called fixed-points. For $R < R_{opt}$, two resonances with equal peaks appear in the vicinity of the resonance frequency of the primary system. The controller is no longer effective in the terms of amplitude reduction when $R \to 0$. In addition, the performance degradation can also be observed when $R > R_{opt}$. Especially when $R \to \infty$, the controlled system acts like a primary system with no additional damping. The impulse response in time domain is also shown in Figure 2c for three different values of R. As it can be seen, the minimum settling time could be achieved by considering the designed optimal value of R obtained in Eq. (9b)

3 HYBRID CONTROL SYSTEM

From Eqs. (6)-(8) and (3), it can be concluded that the optimal closed-loop damping is $\eta_{opt} = T/(2\sqrt{kL})$. This shows that the stiffness of the structure as well as the coupling constant and the inductance of the transducer limit the maximum achievable damping obtained by the passive control system. The potential of using active control to improve the control performance of the system in the terms of magnitude of the response is investigated.

In the present study, two different configurations for the hybridization of the passive resonant shunt with an active control are considered (Figure 3). In the first configuration shown in Figure 3a, the electromagnetic transducer is connected in series with RC elements and an active voltage source. The active voltage source is proportional to the velocity of the structure. The total voltage across the transducer (V) is then obtained by the summation of the active input voltage (V_{in}) and the voltage across the RC shunt circuit (V_{shunt}). Furthermore in the second configuration shown in Figure 3b, the electromagnetic transducer is connected in series with RC circuit, and the active current source is in parallel with the shunting elements. Considering this configuration, the total current flowing in the transducer (i) is given by the summation of the input active current (i_{in}) and the current flowing inside the RC shunt circuit ($i_{shunt} = \dot{q}_{shunt}$).

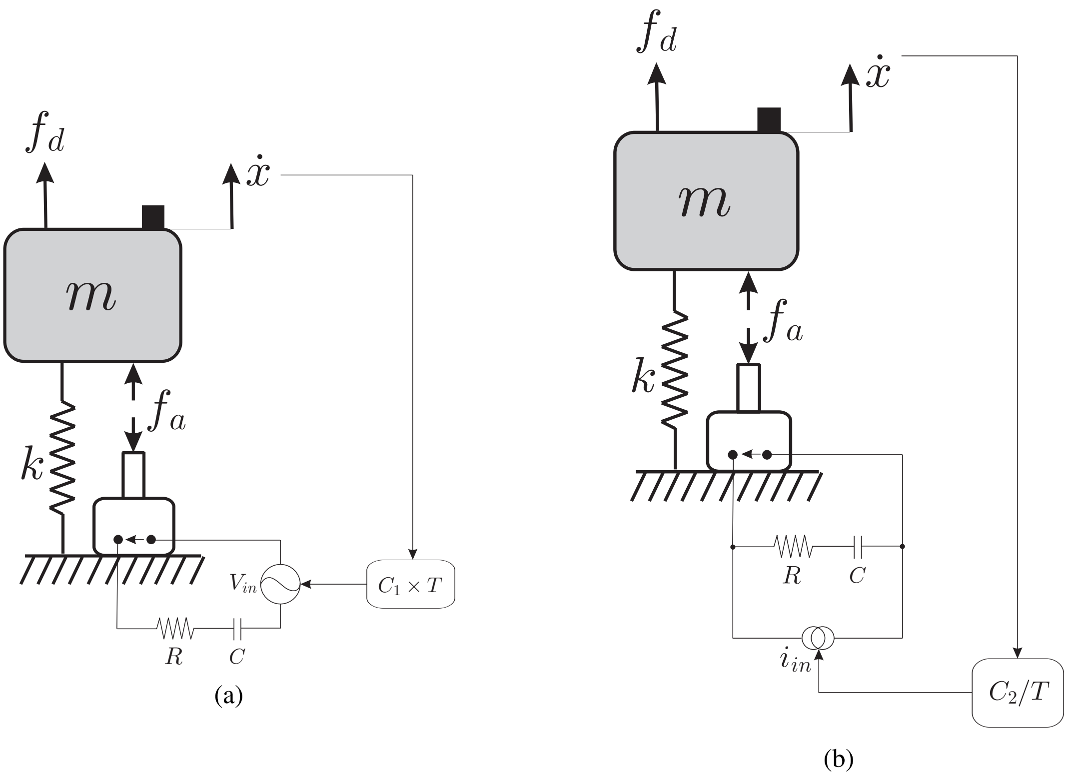


Figure 3: Schematic of a SDOF oscillator attached to an hybrid control system consisting of an electromagnetic transducer connected to the passive RC shunt (a) in series with an active voltage source and (b) in parallel with an active current source

3.1 Hybridization with an active voltage source

Considering the active voltage source, the equations of the motion are modified as:

$$m\ddot{x}+x=f_d+f_a \tag{10a}$$

$$f_a=-T^*\dot{q} \tag{10b}$$

$$L\ddot{q}+R\dot{q}+\frac{1}{C}q+V_{in}=T\dot{x} \tag{10c}$$

$$V_{in}=-C_1T\dot{x} \tag{10d}$$

By substituting Eq. (10d) into Eq. (10c), it can be seen that the active voltage source directly affects the effective coupling constant of the transducer. It is assumed that the transducer is ideal and there is a perfect balance between the electrical energy and the mechanical energy which means there is no energy to be stored in the transducer [15]. According to the concept of energy conservation principle [16], the variation of the stored energy is the sum of the external power input and the internal power generation. This concept can be written for the electromagnetic transducer in the presence of the active voltage source when there is no shunt as:

$$dW=Vdq+f_adx=T(1+C_1)\frac{dx}{dt}idt+T^*idx \tag{11}$$

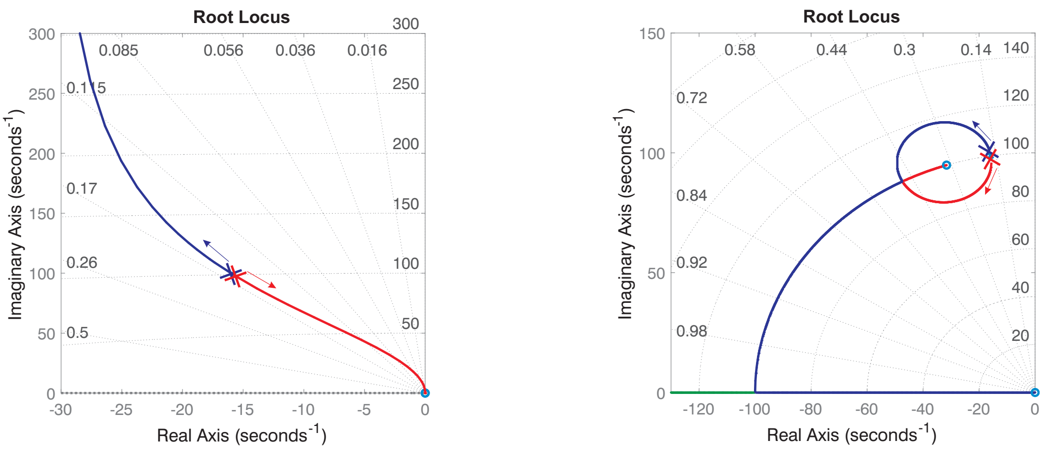
where d and W are the derivative operator and the stored energy, respectively. By equating the above equation to zero, T^* can be obtained as:

$$T^* = T(1 + C_1) \tag{12}$$

Figure 4a shows the root locus of the system, shunted with RC circuit and the active voltage source in series, for the variation of feedback gain C_1. One sees that the system is stable because the poles are always placed in the left half plane of the locus for all values of the feedback gain C_1. The locus has two complex poles and two zeros at the origin. By increasing the value of the feedback gain C_1, one pole goes toward the origin and the other one goes to infinity. It makes the system having two different resonances in the vicinity of the primary ones with the lower values of the damping than the damping of the passive control system. According to the Eq. (9b), the optimal values of the resistance R is proportional to the coupling constant of the transducer. Considering the new constant of the transducer Eq. (12), the optimal value of the resistance can be modified as:

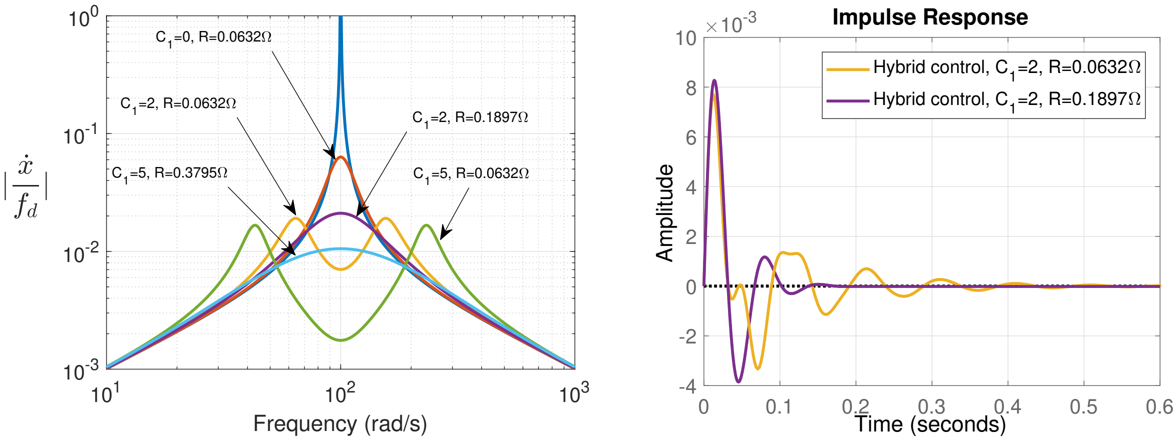
$$R^*_{opt} = \frac{2T}{\sqrt{kc}}(1 + C_1) \tag{13}$$

Figure 5a compares the frequency response of the system with the passive control system combined with the active voltage source for two different values of the feedback gain C_1. For each value of C_1, the result is shown with and without correction of the resistance R according to Eq. (13) and Eq. (9b), respectively. By updating the value of the resistance in this case, more energy can be dissipated in the resistor which leads to increase the damping of the system. The greater value of the resistance is required by the application of the active voltage source than the purely passive system. In order to have a fail-safe and optimum design, the value of resistance should be changed to the lower one when the active control is turned off. For a specific value of the feedback gain C_1, the impulse response is shown in Figure 5b when the value of the resistance R is modified based on Eq. (13). It can be seen that the exponential time-decay rate is maximized by updating the value of the resistance.



(a) (b)

Figure 4: Root-locus of the system shunted with RC circuit combined with (a) active voltage source under and (b) active current source



(a) (b)

Figure 5: With the application of the passive control system combined with the active voltage source, (a) the frequency response of the system for different values of the feedback gain C_1 as well as the resistance R and (b) the impulse response with and without the correction of the resistance R

3.2 Hybridization with an active current source

The equations of the motion with the application of the active current source read:

$$m\ddot{x} + kx = f_d + f_a \tag{14a}$$
$$f_a = -T(\dot{q} + i_{in}) \tag{14b}$$
$$i_{in} = \frac{C_2}{T}\dot{x} \tag{14c}$$
$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = T\dot{x} \tag{14d}$$

The corresponding root locus of the system is shown in Figure 4b for the variation of feedback gain C_2. The system is always stable by the application of the active current source because the closed-loop poles are always in the left half plane. Two poles are moving on the blue and red branches. The optimal feedback gain C_2 can be obtained when the two loops are intersecting at one point. Considering $\lambda = C_2\omega_0/k$ and Eqs. (3), (7), as well as (8), the normalized equations of motion are written as:

$$x_1'' + x_1 = f - \beta_1\omega_0 x_2' - \lambda x_1' \tag{15a}$$
$$x_2'' + 2\sqrt{\beta}x_2' + x_2 = \beta_2/\omega_0 x_1' \tag{15b}$$

The closed-loop transfer function from the normalized disturbance force f to the normalized displacement x_1 is obtained as:

$$\frac{x_1}{f} = \frac{s^2 + 2\sqrt{\beta}s + 1}{(s^2 + \lambda s + 1)(s^2 + 2\sqrt{\beta}s + 1) + \beta s^2} \tag{16}$$

When the two poles of the system have the same damping μ and normalized resonance frequency δ, the closed-loop transfer function can be re-written as:

$$\frac{x_1}{f} = \frac{s^2 + 2\sqrt{\beta}s + 1}{(s^2 + 2\mu\delta s + \delta^2)^2} \tag{17}$$

The following equations are obtained by equating the polynomial coefficients of the denominator of the fraction on the right hand side of Eqs. (16) and (17):

$$4\mu\delta = 2\sqrt{\beta} + \lambda \tag{18a}$$

$$4\mu^2\delta^2 + 2\delta^2 = 2\sqrt{\beta}\lambda + \beta + 2 \tag{18b}$$

$$4\mu\delta^3 = 2\sqrt{\beta} + \lambda \tag{18c}$$

$$\delta^4 = 1 \tag{18d}$$

The optimal value of the normalized tuning frequency δ is obtained from Eq. (18d) as:

$$\delta_{opt} = 1 \tag{19}$$

which shows that the closed-loop system has the same resonance frequency as the resonance frequency of the primary one. The optimal value of the normalized feedback gain can be realized by substituting the damping ratio μ obtained from Eq. (18a) and Eq. (19) into Eq. (18b):

$$\lambda_{opt} = 4\sqrt{\beta} \tag{20}$$

which yields:

$$C_2^{opt} = 4\frac{T}{\omega_0}\sqrt{\frac{k}{L}} \tag{21}$$

It should be mentioned that the parameters of the passive RC circuit do not change. Figure 6a compares the frequency response of the passive control system combined with the active current source for two different values of the feedback gain C_2. The optimal value is first used for the feedback gain $C_2 = 126.49$ and then it is increased to $C_2 = 200$ when a closed-loop pole touches the real axis and the other one merge with the zero. In this case, the zero canceled one of the poles and the other one adds damping to the system. This gain ($C_2 = 200$) can be defined as the second optimal value due to the fact that it has almost

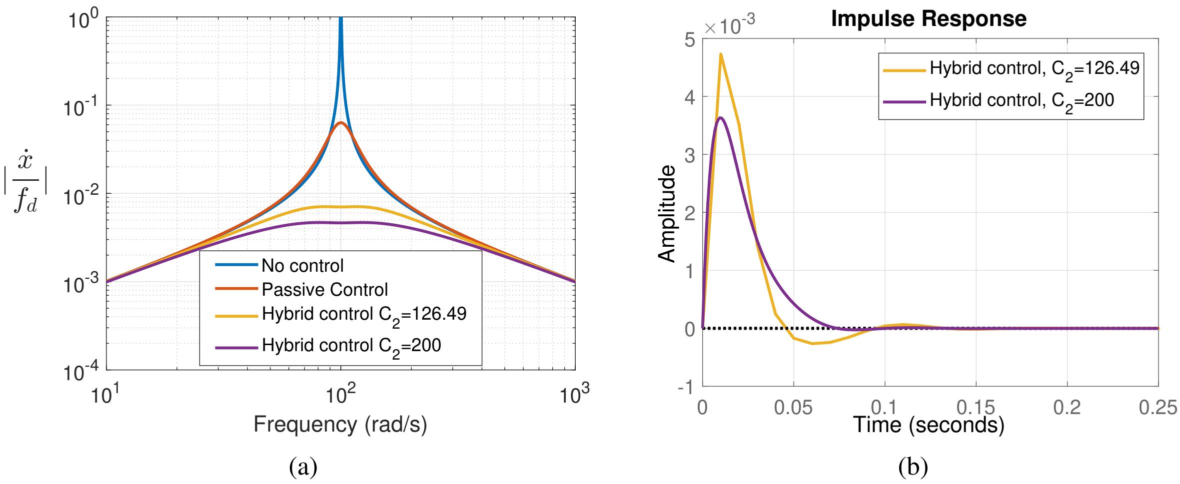


Figure 6: With the application of the passive control system combined with the active current source, (a) the frequency response and (b) the impulse response of the system for two different values of the feedback gain C_2

the same settling time (Figure 6b) as the one corresponding to the first optimal value. For more than this value of the gain, the settling time is no longer minimized although it might realize lower magnitude of response than the magnitude of response obtained with the optimal values of the feedback gain. It should be mentioned that by the application of the active current source, the hybrid control system is fail-safe because it will continue to behave as the passive electromagnetic shunt damper described in Section 2 when the controller is turned off. The controller behaves like a direct-velocity-feedback (DVF) which is able to damp several modes as well. This topic is proposed for the future study.

4 POWERFLOW ANALYSIS

In the previous section, it was shown that the hybrid control system can improve the control performance of the system in terms of the amplitude of response at the resonance frequency. A question may arise here is what is the main advantage of using hybrid control system compared to the purely active control system. To answer this question, it is proposed to analyze the powerflow between the device and the primary structure due to the fact that an active control system requires an external power source for its operation. The less external power required, the better. The power which flows at the interface of the structure and the actuator device can be written as:

$$P(s) = F_{aa}(s) \times \dot{x}^*(s) \tag{22}$$

where P, F_{aa} and $\dot{x}^*$ are the power, the control force applied by the active control system, and the complex conjugate of the velocity of the mass, respectively. Considering $s = j\omega$, the real part of P is called the active power which corresponds to the dissipative behavior and the imaginary part is named the reactive power which corresponds to the energy exchanging between the device and the structure [17].
The average active power P_{ac} and the reactive power P_{re} can be written as:

$$P_{ac} = 1/2\Re(P(s)) = \frac{f_d^2}{2}\Re(G_f(s) \times G_{CL}(s) \times G_{CL}^*(s)) \tag{23a}$$

$$P_{re} = 1/2\Im(P(s)) = \frac{f_d^2}{2}\Im(G_f(s) \times G_{CL}(s) \times G_{CL}^*(s)) \tag{23b}$$

where $G_f(s)$ and $G_{Cl}(s)$ are the transfer function from the velocity of the structure to the control force and the transfer function from the input force disturbance to the velocity. The superscripts $''*''$ represents the complex conjugate transpose.
For a unit forcing amplitude, Figure 7a compares the active and reactive power at a specific value of the closed-loop damping for purely active control system and the hybrid control system using active voltage source and current source, separately. DVF is used for the purely active control system. One can be observed that the active power for all configurations is always positive through the entire frequency range. This means that the device does not deliver energy in the system and it is *hyperstable*. The reactive power for hybrid control system when active voltage source is applied is positive before the resonance frequency and negative after that. This shows the amount of energy exchanged between the structure and the transducer. The total positive and negative reactive power is almost zero.
Figure 7b compares the H_2 norm of the active power as a function of the closed-loop damping ratio. One sees that the active power is zero for 16% damping ratio and below. This is because the passive control system is doing the job. While there is no external power required to realize 16% damping ratio by the passive control system, a similar purely active control system requires a large amount of external power. In addition, the active power for the hybrid control systems is always less than purely active control system. However, for high value of the closed-loop damping, the active power for the hybrid

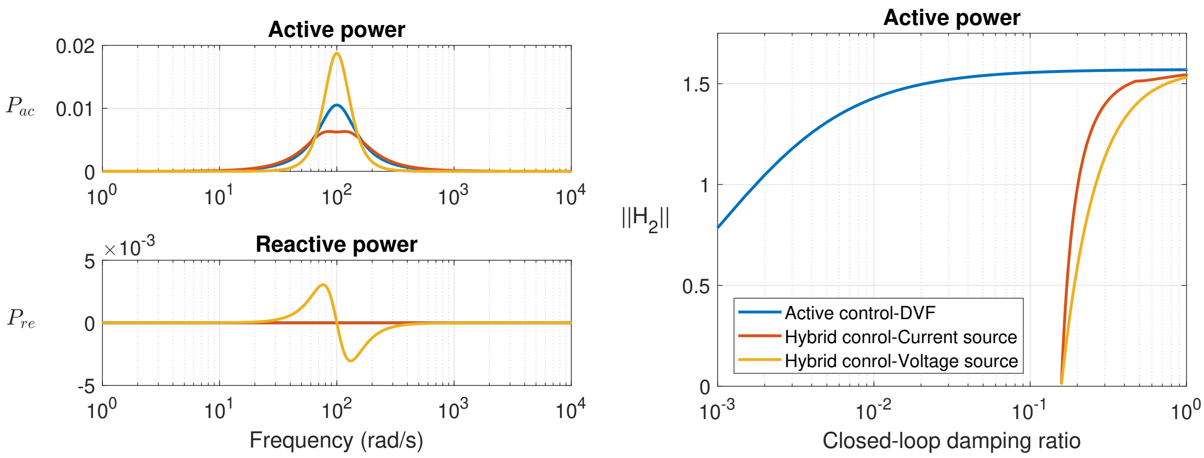


Figure 7: Comparison between the purely active control system (DVF) and Hybrid control systems using active voltage source and current source separately. (a) active and reactive power for a specific closed-loop damping ratio, (b) the H_2 norm of the active power

configurations is close to the active power for purely active control system. The is because most of the work is done by the active portion of the hybrid systems.

5 CONCLUSIONS

The hybridization of the passive electromagnetic shunt damper with the active control systems has been proposed and analyzed in details. The RC shunt has been used in series with an electromagnetic transducer as the passive control system and its parameters has been optimized based on the method of maximum damping. Both the active voltage source in series with RC elements and the active current source in parallel with it have been proposed for the hybrid configurations. It was shown that the system 'electromagnetic device + active voltage source' can be seen as an equivalent transducer with enhanced coupling constant T; equivalence formulas have been presented. In this case, it was shown that the optimal value of the resistance R is modified to a greater value as a function of the feedback gain C_1 in order to improve the damping of the system. In addition, it was shown that the active current source behaves like a DVF. In this case, the feedback gain C_2 has been optimized based on the method of maximum damping while there is no need to change the parameters of passive RC circuit. Moreover, the power consumption was highlighted to compare the hybrid control systems and purely active control system (using DVF). As a consequence, it has been demonstrated that both hybrid control systems can improve the control performance of the purely passive system while it has less power consumption in comparison with purely active control system.

Acknowledgment

The authors are grateful to the financial support of MAVERIC (Wal'innov project 1610122).

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