An augmented Lagrangian frictional contact formulation for nonsmooth multibody systems

Javier Gálvez*, Alberto Cardona**, Federico Cavalieri**, Olivier Brüls*
**Department of Aerospace and Mechanical Engineering, University of Liège, Liège, Belgium*
*** Centro de Investigación de Métodos Computacionales (CIMEC), Universidad Nacional del Litoral-CONICET, Santa Fe, Argentina*

Summary. This work presents a frictional contact formulation for the transient simulation of non-smooth dynamic mechanical systems composed of rigid bodies, kinematic joints and contact conditions. More precisely, we extend the non-smooth generalized$-\alpha$ scheme to account for frictional contacts. The non-smooth generalized-alpha method imposes the constraints at position and velocity levels, preventing any non-physical penetration. The formulation and the numerical behaviour of the method for frictional contact problems are studied. Finally, its performance is evaluated for several numerical examples.

Various numerical strategies are available to deal with the dynamics of multibody systems with contacts. Regularization techniques and nonsmooth techniques are the two main approaches to deal with contact problems [1]. In the present work we focus on nonsmooth and implicit techniques, initially introduced by Moreau and Jean [4, 5]. The interaction laws are written as multi mappings relating contact unknowns (impulses and relative velocities). A fundamental property of the Moreau-Jean scheme is that the unilateral constraints are imposed at velocity level. As recognized by many authors, this choice leads to interesting consistency and stability properties of a simulation algorithm for dynamic contact analysis. As the unilateral constraints are not imposed at position level, some penetration can be observed in the simulated results. Another property of the Moreau-Jean algorithm is that the complete system is integrated in time using a method which is only first-order accurate. To improve the quality of the numerical results the Moreau-Jean scheme is proposed in [3] which enforces the constraint at velocity level, but also at position level. As a result, penetration is avoided. Also in this scheme the terms of the motion equation are split in to two parts, on the one hand the smooth contribution and on the other the nonsmooth contribution. This splitting makes it possible to integrate in the time domain the smooth part using a higher-order scheme, in that case the generalized$-\alpha$ method was used.
The present work extends this technique to include friction following the idea proposed by Alart and Curnier [2]. This method leads to an implicit formulation of the contact problem which can be solved at every time step using a Newton semi-smooth algorithm. This means that the contact status, which can be defined as gap (no contact between bodies), slip (contact between bodies with relative velocity at the contact point) and stick (contact between bodies without relative velocity at the contact point), is updated at every iteration of the Newton process according to an active set method. Following the idea proposed in [3], the discrete model is based on an interplay between the constraints enforced at time t_{n+1} and the integrals of the contact reaction forces defined as follows

$$\Lambda_N(t_n;t) = \int_{(t_n,t]} \mathrm{d}i_N \qquad \mathbf{\Lambda}_T(t_n;t) = \int_{(t_n,t]} \mathrm{d}\mathbf{i}_T \tag{1}$$

$$\nu_N(t_n;t) = \int_{(t_n,t]} \Lambda_N(t_n;\tau)\,\mathrm{d}\tau \qquad \boldsymbol{\nu}_T(t_n;t) = \int_{(t_n,t]} \mathbf{\Lambda}_T(t_n;\tau)\,\mathrm{d}\tau \tag{2}$$

where $\mathrm{d}i_N$ and $\mathrm{d}\mathbf{i}_T$ are the impulse measures of the contact forces in the normal and tangent direction, respectively.
The formulation of the frictional contact problem by Alart and Curnier [2] relies on a mixed penalty-duality approach, which involves the definition of augmented Lagrange multipliers. In order to adapt this strategy to the two-level formulation presented in [3], two sets of augmented Lagrangian multipliers are introduced as follows

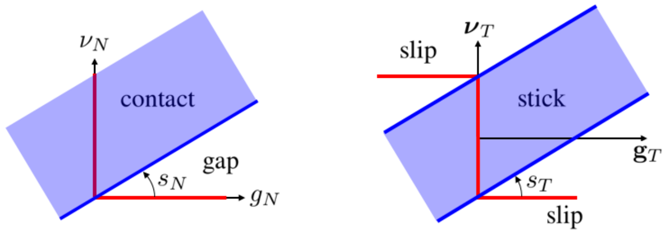
$$\boldsymbol{\sigma}_{n+1} = \sigma_{N,n+1}\mathbf{n}_n + \boldsymbol{\sigma}_{T,n+1} \qquad \boldsymbol{\xi}_{n+1} = \xi_{N,n+1}\mathbf{n}_n + \boldsymbol{\xi}_{T,n+1} \tag{3}$$

$$\sigma_{N,n+1} = \Lambda_{N,n+1} - r_N\,(\dot g_{N,n+1} + e_N\,\dot g_{N,n}); \qquad \boldsymbol{\sigma}_{T,n+1} = \mathbf{\Lambda}_{T,n+1} - r_T\,(\dot{\mathbf{g}}_{T,n+1} + e_T\,\dot{\mathbf{g}}_{T,n}) \tag{4}$$

$$\xi_{N,n+1} = \nu_{N,n+1} - s_N\,g_{N,n+1}; \qquad \boldsymbol{\xi}_{T,n+1} = \boldsymbol{\nu}_{T,n+1} - s_T\,\mathbf{g}_{T,n+1} \tag{5}$$

where r_K and s_K are numerical coefficients, $\mathbf{n}_n$ is the normal vector to the contact surface, $\mathbf{\Lambda}_{n+1} = \mathbf{\Lambda}(t_n;t_{n+1})$, $\boldsymbol{\nu}_{\boldsymbol{n+1}} = \boldsymbol{\nu}(t_n;t_{n+1})$, $\boldsymbol{\sigma}$ and $\boldsymbol{\xi}$ are the augmented Lagrangian multipliers, g_N is the gap, $\mathbf{g}_T$ is the tangential displacement, e_N and e_T are the restitution coefficients in the normal and tangential directions and μ is the friction coefficient of the Coulomb model.
To define the activation status three criteria are defined, at position and velocity levels. Then, we have at position level

- in gap situation, $\xi_{N,n+1} < 0$ and $\nu_{N,n+1} = 0; \boldsymbol{\nu}_{T,n+1} = \mathbf{0}$;
- in slip situation, $\xi_{N,n+1} \geq 0$, $\|\boldsymbol{\xi}_{T,n+1}\| > \mu\,\xi_{N,n+1}$, $g_{N,n+1} = 0$ and $\boldsymbol{\nu}_{T,n+1} = -\mu\,\xi_{N,n+1}\,\frac{\dot{\mathbf{g}}_{T,n+1}}{\|\dot{\mathbf{g}}_{T,n+1}\|}$;
- in stick situation, $\xi_{N,n+1} \geq 0$, $\|\boldsymbol{\xi}_{T,n+1}\| \leq \mu\,\xi_{N,n+1}$, $g_{N,n+1} = 0$ and $\mathbf{g}_{T,n+1} = \mathbf{0}$.



(a) gap-contact (b) stick-slip

Figure 1: Activation criteria at position level (a) gap-contact; (b) stick-slip.

At velocity level, we have

- in gap situation, ($\xi_{N,n+1} < 0$ or ($\xi_{N,n+1} \geq 0$ and $\sigma_{N,n+1} < 0$)), $\Lambda_{N,n+1} = 0$; $\mathbf{\Lambda}_{T,n+1} = \mathbf{0}$;
- in slip situation, $\xi_{N,n+1} \geq 0$, $\sigma_{N,n+1} \geq 0$, $\|\boldsymbol{\sigma}_{T,n+1}\| > \mu\,\sigma_{N,n+1}$, $\dot{g}_{N,n+1} + e_N\,\dot{g}_{N,n} = 0$ and $\mathbf{\Lambda}_{T,n+1} = -\mu\,\xi_{N,n+1}\,\frac{\dot{\mathbf{g}}_{T,n+1}}{\|\dot{\mathbf{g}}_{T,n+1}\|}$;
- in stick situation, $\xi_{N,n+1} \geq 0$, $\sigma_{N,n+1} \geq 0$, $\|\boldsymbol{\sigma}_{T,n+1}| \leq \mu\,\sigma_{N,n+1}$, $\dot{g}_{N,n+1} + e_N\,\dot{g}_{N,n} = 0$ and $\dot{\mathbf{g}}_{T,n+1} + e_T\,\dot{\mathbf{g}}_{T,n} = \mathbf{0}$.

Graphically these conditions are illustrated in the figure 1 at the position level, and could be shown comparably at the velocity level. Both at position and velocity levels, the three subsets defined by the above inequalities are disjoint and cover the full space. Thus, for arbitrary (even non-physical) values of the kinematic variables and reaction impulses, the constraint status is well-defined. In each situation, two equality equations are obtained: one in the normal direction and another one in the tangential direction. Finally, to demonstrate the capabilities of the method several simple examples involving frictional contact are studied.

References

[1] F. Gholami, M. Nasri, J. Kövecses and M. Teichmann. (2016) A linear complementarity formulation for contact problems with regularized friction. *Mechanism and Machinery Theory* 105:568-582

[2] P. Alart and A. Curnier. (1991) A mixed formulation for frictional contact problems prone to Newton like solution methods. *Computer Methods in Applied Mechanics and Engineering* 92:353-375.

[3] O. Brüls, V. Acary, and A. Cardona. (2014) Simultaneous enforcement of constraints at position and velocity levels in the nonsmooth generalized-α scheme. *Computer Methods in Applied Mechanics and Engineering* 281:131−161.

[4] J.J. Moreau. (1988) Unilateral contact and dry friction in finite freedom dynamics. *International Center for Mechanical Sciences, Courses and Lectures.* pp.1−82, J.J. Moreau, P.D. Panagiotopoulos, Springer, Vienna.

[5] M. Jean. (1999) The Non Smooth Contact Dynamics Method. *Computer Methods in Applied Mechanics and Engineering, special issue on computational modelling of contact and friction.* J.A.C. Martins, A. Klarbring (ed), 177, pp. 235−257, 1999