Abstract

In this work, we propose a new methodology for jointly sizing a dynamical system and designing its control law. First, the problem is formalized by considering parametrized reinforcement learning environments and parametrized policies. The objective of the optimization problem is to jointly find a control policy and an environment over the joint hypothesis space of parameters such that the sum of rewards gathered by the policy in this environment is maximal. The optimization problem is then addressed by generalizing the direct policy search algorithms to an algorithm we call Direct Environment Search with (projected stochastic) Gradient Ascent (DESGA). We illustrate the performance of DESGA on two benchmarks. First, we consider a parametrized space of Mass-Spring-Damper (MSD) environments and control policies. Then, we use our algorithm for optimizing the size of the components and the operation of a small-scale autonomous energy system, i.e. a solar off-grid microgrid, composed of photovoltaic panels, batteries, etc. On both benchmarks, we compare the results of the execution of DESGA with a theoretical upper-bound on the expected return. Furthermore, the performance of DESGA is compared to an alternative algorithm. The latter performs a grid discretization of the environment’s hypothesis space and applies the REINFORCE algorithm to identify pairs of environments and policies resulting in a high expected return. The choice of this algorithm is also discussed and motivated. On both benchmarks, we show that DESGA and the alternative algorithm result in a set of parameters for which the expected return is nearly equal to its theoretical upper-bound. Nevertheless, the execution of DESGA is much less computationally costly.

1. Introduction

Problems where one has to design a system that has to be controlled afterwards are ubiquitous in the field of engineering. Common examples include the combined design and control of a robotic arm for achieving a specific goal (Castejón et al., 2010; Ajwad et al., 2018) or the sizing and the operation of a microgrid to minimize electricity costs (François-Lavet et al., 2016). System performance depends on both the system parameters and the method by which it is operated, and the interplay between the two should be properly accounted for when designing them (Brekken et al., 2010).

This type of joint design and control problems can often be cast as multi-step optimization problems under uncertainty (Bakker et al., 2020). Roughly speaking, in this framework, an
agent must take a decision at every step of a discretised time horizon in order to optimize a pre-specified criterion. Information about the underlying system is typically available in the form of a state-space representation, whose transition dynamics may be constrained and/or stochastic. Uncertainty is represented by stochastic processes, the outcomes of which may be conditioned on both states and decisions and usually become known immediately after decisions have been taken at every step of the time horizon. A reward (resp. cost) is associated with each pair of realizations and decisions, and solving the problem essentially consists in selecting a sequence of decisions maximizing (resp. minimizing) some function of the sum of rewards (resp. costs) collected at every step (e.g., its expectation). In our design and control problem, the first stages consist of the decisions regarding the design of the system and the following stages are concerned with its control over its lifetime. A variety of methods have been deployed to tackle such problems, as discussed next.

Firstly, multi-stage stochastic programming, which forms a subset of mathematical programming, has been widely used in the literature (Wallace & Fleten, 2003). In this context, a mathematical model of the system is assumed to be available, in which the design and operational decisions as well as the system states are represented as optimization variables. Some model parameters are assumed to be uncertain and are represented as realizations of a stochastic process whose probability distribution is assumed to be known (Birge & Louveaux, 2011). Moreover, the latter is usually assumed to be independent of decision variables, in which case the uncertainty is said to be exogenous. Conversely, the uncertainty may be endogenous, which implies that decisions have an influence on its probability distribution. In addition to constraints representing the dynamics and control of the system, non-anticipativity constraints are added to define the temporal structure of the uncertainty and specify how it is revealed over time. The main computational approach to solving multi-stage stochastic programming problems consists in approximating the uncertainty by a discrete stochastic process exhibiting a tree structure (resulting in a so-called scenario tree), and solving all scenarios at once via a large-scale mathematical program (Heitsch & Roemisch, 2009). Clearly, the number of scenarios increases with the number of stages and the number of realizations required at each stage to properly approximate the original probability distribution. This can quickly lead to intractable problems and scenario tree reduction techniques are often used in practice (Heitsch & Roemisch, 2009). Furthermore, considering nonlinear transition dynamics and control laws usually results in nonconvex optimization problems, which are notoriously difficult to solve to optimality (Nemirovsky & Yudin, 1983). Taking endogenous uncertainty into account usually involves additional nonconvexities (Goel & Grossmann, 2006), which further complicate matters. Hence, in practice, system design problems are often approximated using two-stage stochastic programs (possibly with recourse) (Wallace & Fleten, 2003; Marufuzzaman et al., 2014; Schwele et al., 2020). In this setup, the first stage typically represents the design stage, while the second stage models system operation over its lifetime (or a representative truncated time horizon). This approach therefore reduces to having a star-shaped scenario tree, which limits the ability of these methods to properly represent short-term uncertainty and its impact on system operation. Once a system design has been identified, real-time operation is usually conducted using receding horizon control strategies such as model predictive control (MPC) (Camacho & Alba, 2013). In MPC, an optimization model representing short-term system operation is initialised with the current system state and solved online in order to
identify a sequence of optimal (open-loop) control actions. A subset of these actions is then applied to the system before recovering the system state, and repeating the procedure. In other words, in such approaches, the original design and control problem is split into two separate sub-problems that are solved virtually independently.

A different approach proposed in the literature consists in specifying a control law \textit{a priori}, selecting the system configuration and simulating system behaviour under this control law. During simulation, the system configuration is typically specified by a model whose parameters remain fixed. In addition, in order to perform these simulations, the uncertainty may be specified via its probability distribution or may be revealed through an oracle, which are sampled or queried online. Different system configurations can be tested in such fashion, and the configuration yielding the most desirable outcome is selected. To this end, derivative-free optimization methods and evolutionary algorithms are typically employed to this end. Such methods have been applied to the design of electrical microgrids (Zhao \textit{et al.}, 2014; Zhang \textit{et al.}, 2017; Dakir \textit{et al.}, 2020), where a rule-based controller is used and the system parameters are selected to minimize the expected cost over different operational scenarios. In some cases, the pre-specified control law may be defined implicitly by solving an optimization problem online, similarly to traditional MPC. In particular, an application to the design of smart buildings is given in (Baniasadi \textit{et al.}, 2020). Compared with applied multi-stage stochastic programming approaches, such methods are capable of better representing the uncertainty and its impact on system operation, since no \textit{a priori} approximate representation of the uncertainty (in the form of a reduced scenario tree) is required in practice. However, the derivative-free strategies used to explore the space of system configurations can be ineffective and time consuming, especially in high dimensional spaces (Jamieson \textit{et al.}, 2012). In addition, the fact that control laws are selected \textit{a priori} may limit the ability of such methods to effectively capture the interplay between system configuration and control, and eventually result in system designs with lower performance.

This crucial insight was made clear in (Brekken \textit{et al.}, 2010), where a first attempt to address the issue was made by defining a parametric policy (e.g., in the form of a neural network) whose parameters were then jointly optimized with system parameters. This method was then applied to electrical energy storage system design and control. A genetic algorithm was used for the optimization, which suffers from the same drawbacks as the derivative-free methods discussed above (Oliveto & Witt, 2015) and has therefore commonly been substituted by derivative-based methods in machine learning applications (Bottou, 2010).

On the other hand, reinforcement learning (RL) provides effective tools to design complex control policies adaptively while properly accounting for uncertainty, both endogenous and exogenous. In this setup, an active decision-making agent attempts to learn a policy in order to maximize its so-called value function through interaction with its environment (Kaelbling \textit{et al.}, 1996). During this interaction, the agent gathers experience that is used to improve its performance over time. The goal of the agent is defined by the reward signal collected after each interaction with the environment, and the value function is typically taken as the expected sum of rewards collected over the entire time horizon. In recent years, the subclass of solution methods known as direct policy search techniques have met with considerable success. These techniques essentially parametrize the policy and navigate in the space of candidate policies towards a (locally) optimal one by processing the information contained
in trajectories generated throughout the optimization process. Typically, two main classes of
direct policy search techniques can be distinguished, namely gradient-free and gradient-based
methods. The first class uses derivative-free optimization techniques, e.g. the covariance
matrix adaptation (CMA) (Hansen & Ostermeier, 1996) and the cross-entropy method
(CEM) (Szita & Lörincz, 2006; Buşoniu et al., 2011). The latter class of methods moves
from one point to the next, in the space of candidate policies, through the reconstruction
of a gradient of the objective from information contained in trajectories. Derivative-free
methods are known to scale unfavourably with the number of policy parameters and do not
perform well on large-scale problems (Schulman et al., 2015). On the other hand, gradient
descent (or ascent) methods have been very successful at learning function approximators
for supervised learning tasks with a large number of parameters (Bottou, 2010; Kingma &
Ba, 2014). Gradient-based direct policy search methods extend these ideas to reinforcement
learning and allow for efficient training of complex and powerful policies (Grondman et al.,
2012).

In the standard reinforcement learning setup, the environment is fixed and the agent
merely seeks to learn an optimal control policy. From a modelling perspective, in order to
extend reinforcement learning methods to joint design and control problems, the configuration
of the system that an agent seeks to control may be encapsulated in the environment it
faces. In this paper, we explore this idea and extend the standard deep RL framework
by considering that, in addition to the policy, the environment (transition dynamics and
reward signal) can be parametrized. The objective of our approach is to jointly optimize
the environment and policy parameters in order to maximize the total expected cumulative
rewards received. Our algorithm works as follows. Given an initial set of parameters,
we compute the gradient of the expected cumulative rewards and perform a projected
gradient ascent step in the space of environment and policy parameters. This procedure is
then repeated a fixed number of times. We call this algorithm Direct Environment Search
with (projected stochastic) Gradient Ascent (DESGA). Compared with methods previously
introduced for solving joint design and control problems, this approach has several key
advantages. It accurately represents uncertainty and its impact on system operation, allows
for the definition of complex policies, and naturally accounts for the interplay between
system configuration and control. Furthermore, it exploits gradients to explore the joint
design and control hypothesis space, which have been shown to be very efficient on complex
machine learning tasks (Bottou, 2010).

The DESGA algorithm can be interpreted as an extension of gradient-based direct policy
search techniques and more particularly the REINFORCE algorithm (Williams, 1992). Our
method also shares some similarities with model-based reinforcement learning algorithms
(Moerland et al., 2020). In this sub-field of RL, a parametric model of a physical environment
is learned from the trajectories collected from this environment. The models used range
from parametrized stochastic processes to neural networks and parametrized dynamical
systems. It is then possible to infer a control policy from the learned model. The latter
class of methods has been successively applied on diverse problems (Serban et al., 2020
Bechtle et al., 2020; Wu et al., 2020). However, in the DESGA algorithm, the environment
parameters are learned to maximize the rewards collected by an optimal policy in this
environment.
The rest of the paper is organized as follows. In Section 2, we present the theoretical background and the problem statement of optimizing over the joint environment and policy parameter space. In Section 3, the proposed methodology as well as the algorithmic implementation for direct environment search with gradient ascent (DESGA) are described. The experimental protocol for the evaluation of the proposed algorithm is introduced and the results are demonstrated in Section 4. Finally, the conclusions and certain considerations for future work are discussed in Section 5.

2. Theoretical background and problem statement

In this section, we provide a generic formulation for the optimal control problem of a discrete-time dynamical system with a finite-time optimization horizon. Then, we introduce a parametrization of both the dynamical system and the policy spaces. Subsequently, we formulate the problem of jointly optimizing the vector of parameters of the dynamical system and the policy with the goal to maximize the total expected rewards.

2.1 Discrete-time dynamical systems

Let us consider a discrete-time and time-invariant dynamical system defined as follows [Bertsekas 2005]. Let $T \in \mathbb{N}$ be the optimization horizon referring to the number of decisions to be taken in the control process. The system is defined by a state space $\mathcal{S}$, an action space $\mathcal{A}$, a disturbance space $\Xi$, a transition function $f : \mathcal{S} \times \mathcal{A} \times \Xi \rightarrow \mathcal{S}$, a bounded reward function $\rho : \mathcal{S} \times \mathcal{A} \times \Xi \rightarrow \mathbb{R}$ and a conditional probability distribution $P\xi$ giving the probability $P(\xi_t|s_t, a_t)$ of drawing a disturbance $\xi_t \in \Xi$ when taking an action $a_t \in \mathcal{A}$ while being in a state $s_t \in \mathcal{S}$. A probability measure $P_0$ yields the probability $P_0(s_0)$ of each state $s_0 \in \mathcal{S}$ to be the initial state. At time $t \in \{0, 1, \ldots, T-1\}$, the system moves from state $s_t \in \mathcal{S}$ to state $s_{t+1} \in \mathcal{S}$ under the effect of an action $a_t \in \mathcal{A}$ and a random disturbance $\xi_t \in \Xi$, drawn with probability $P\xi(\xi_t|s_t, a_t)$, according the transition function $f$:

$$s_{t+1} = f(s_t, a_t, \xi_t).$$ (1)

After each transition, a reward signal $r_t$ is collected from the reward function according to $r_t = \rho(s_t, a_t, \xi_t)$ with $|r_t| \leq r_{\text{max}}$. The different elements of this optimal control problem are gathered in a tuple $(\mathcal{S}, \mathcal{A}, \Xi, P_0, f, \rho, P\xi, T)$ referred to as the environment.\footnote{Let us note that from the environment $(\mathcal{S}, \mathcal{A}, \Xi, P_0, f, \rho, P\xi, T)$, we can define an equivalent Markov Decision Process (MDP) with horizon $T$, state space $\mathcal{S}$, action space $\mathcal{A}$, initial probability distribution $P_0$, reward probability distribution $r$ and transition probability distribution $p$ such that:

$$r(r_t|s_t, a_t) = \mathbb{E}_{\xi_t \sim P\xi(|s_t, a_t)} \{\delta\rho(s_t, a_t, \xi_t)(r_t)\}, \forall s_t, s_{t+1} \in \mathcal{S}, a_t \in \mathcal{A}, r_t \in \mathbb{R}$$ (2)

$$p(s_{t+1}|s_t, a_t) = \mathbb{E}_{\xi_t \sim P\xi(|s_t, a_t)} \{\delta f(s_t, a_t, \xi_t)(s_{t+1})\}, \forall s_t, s_{t+1} \in \mathcal{S}, a_t \in \mathcal{A},$$ (3)

where $\delta_y(x)$ is a function returning one if and only if $x$ equals $y$ and zero otherwise.}

We define a closed-loop policy $\pi \in \Pi$ as a function associating a probability distribution with support $\mathcal{A}$ to current state $s_t$ of the system at a decision stage $t = 0, \ldots, T-1$. Applying the policy to the dynamical system consists in sampling an action $a_t$ with probability $\pi(a_t|s_t, t)$ at each time $t$. A trajectory $\tau = (s_0, 0, a_0, 0, 1, a_1, \ldots a_{T-1}, x_{T-1})$ contains the information...
collected from executing policy $\pi$ over the horizon $T$. The cumulative reward $R(\tau)$ over trajectory $\tau$ can be computed as:

$$R(\tau) = \sum_{t=0}^{T-1} \rho(s_t, a_t, \xi_t),$$  \hspace{1cm} (4)

where $s_{t+1} = f(s_t, a_t, \xi_t)$. The expected cumulative reward associated to a policy $\pi$, and to a state $s_t \in S$ at time $t$, is called the return of the policy and is given by:

$$V^\pi(s_t, t) = \sum_{t'=t}^{T-1} E_{a_{t'} \sim \pi(\cdot | s_{t'}, t')} \{ \rho(s_{t'}, a_{t'}, \xi_{t'}) \}.$$ \hspace{1cm} (5)

Optimal policies are defined by the principle of optimality (Bertsekas, 2005). This principle states that a policy $\pi^* \in \Pi_A$, where $\Pi_A$ is the set of probability distribution functions with support $A$, is optimal in a state $s_t$ at a time $t$ if it maximizes the expected reward-to-go from that state at that time. An optimal policy $\pi^* \in \Pi$ is thus such that $\forall s_t \in S, \forall t = 0, \ldots, T - 1$:

$$\pi^* \in \arg\max_{\pi \in \Pi} \{ V^\pi(s_t, t) \}.$$ \hspace{1cm} (6)

2.2 Problem statement: optimizing over a set of environments

We consider the environment $(S, A, \Xi, P_0, f_\psi, \rho_\psi, P_\xi, T)$, as defined in Section 2.1, with continuous state space $S \subseteq \mathbb{R}^{d_S}$, action space $A \subseteq \mathbb{R}^{d_A}$, disturbance space $\Xi \subseteq \mathbb{R}^{d_\Xi}$, distribution $P_0$ over the initial states and horizon $T$; where $d_S, d_A, d_\Xi \in \mathbb{N}$. The state, action and disturbance spaces are assumed to be compact. The transition and reward functions are two parametric functions $f_\psi$ and $\rho_\psi$, parametrized by the vector $\psi$ defined over the compact $\Psi \subseteq \mathbb{R}^{d_\psi}$, with $d_\psi \in \mathbb{N}$. Both functions are assumed continuously differentiable with respect to their parameters and to the state space for every action in $A$ and every disturbance in $\Xi$.

Additionally, we consider the parametric function $\pi_\theta$ to be a policy parametrized by the real vector $\theta$ in the compact $\Theta \subseteq \mathbb{R}^{d_\theta}$, with $d_\theta \in \mathbb{N}$, and continuously differentiable with respect to its parameters $\Theta$ and to its domain $S$ for every action in $A$ and for every time $t$. We want to identify a pair of parameter vectors $(\psi, \theta)$ such that the policy $\pi_\theta$ maximizes the expected return, on expectation over the initial states, in the environment $(S, A, \Xi, P_0, f_\psi, \rho_\psi, P_\xi, T)$.

We thus want to solve the following optimization problem:

$$\psi^*, \theta^* \in \arg\max_{\psi \in \Psi, \theta \in \Theta} V(\psi, \theta)$$ \hspace{1cm} (7)

$$V(\psi, \theta) = \sum_{s_0 \sim P_0(\cdot)} \{ \sum_{t=0}^{T-1} r_t \}$$ \hspace{1cm} (8)

$$s_{t+1} = f_\psi(s_t, a_t, \xi_t)$$ \hspace{1cm} (9)

$$r_t = \rho_\psi(s_t, a_t, \xi_t).$$ \hspace{1cm} (10)
3. Direct environment search with gradient ascent

In this section, we address the problem defined in Section 2.2. First, we show in Section 3.1 that the expected cumulative reward is differentiable with respect to the parameters of the system and the policy if the different parametric functions and the disturbance probability function are continuously differentiable. In such a context, we derive an analytical expression of the gradient. The results are also extended for discrete action and disturbance spaces. We also derive the expression of an unbiased estimator of the gradient from the differentiation of a loss function built from Monte-Carlo simulations. In Section 3.2, we present our Direct Environment Search with (projected stochastic) Gradient Ascent (DESGA) algorithm that uses a projected stochastic gradient ascent for optimizing both the parameters of the environment and the policy.

3.1 Gradient for learning optimal environments

In Theorem 1, we first prove the differentiability of the expected cumulative reward with respect to the policy and the environment parameters, assuming the functions composing the environment and the policy are continuously differentiable. We then extend these results in a straightforward way to the case where \( A \) and/or \( \Xi \) are discrete in Corollary 1. Corollaries 2 and 3 finally give the expressions of the gradients.

**Theorem 1.** Let \((S, A, \Xi, P_0, f_\psi, \rho_\psi, P_\xi, T)\) and \(\pi_\theta\) be an environment and a policy as defined in Section 2.2. Additionally, let the functions \(f_\psi, \rho_\psi\) and \(P_\xi\) be continuously differentiable over their domain of definition. Let \(V(\psi, \theta)\) be the expected cumulative reward of policy \(\pi_\theta\), averaged over the initial states, for all \((\psi, \theta) \in \Psi \times \Theta\), as defined in Eqn. (8).

Then, the function \(V\) exists, is bounded, and is continuously differentiable in the interior of \(\Psi \times \Theta\).

**Corollary 1.** The function \(V\), as defined in Theorem 1, exists, is bounded, and is continuously differentiable in the interior of \(\Psi \times \Theta\) if \(A\) and/or \(\Xi\) are discrete.

**Corollary 2.** The gradient of the function \(V\) defined in Eqn. (8) with respect to the parameter vector \(\psi\) is such that:

\[
\nabla_\psi V(\psi, \theta) = \mathbb{E}_{\substack{s_0 \sim P_0(\cdot) \\
\alpha_t \sim \pi_\theta(\cdot|s, t) \\
\xi_t \sim P_\xi(\cdot|s, \alpha_t) \\
\sum t=0 \sum t=0}} \left\{ \left( \sum_{t=0}^{T-1} (\nabla_s \log \pi_\theta(\alpha_t|s, t)|_{s=s_t} + \nabla_s \log P_\xi(\xi_t|s, a_t)|_{s=s_t}) \cdot \nabla_\psi s_t \right) \right. \\
\times \left. \left( \sum_{t=0}^{T-1} r_t \right) + \left( \sum_{t=0}^{T-1} \nabla_\psi \rho_\psi(s, a_t, \xi_t)|_{s=s_t} + \nabla_s \rho_\psi(s, a_t, \xi_t)|_{s=s_t} \cdot \nabla_\psi s_t \right) \right\},
\]

where:

\[
\nabla_\psi s_t = (\nabla_s f_\psi)(s, a_{t-1}, \xi_{t-1})|_{s=s_{t-1}} \cdot \nabla_\psi s_{t-1} + (\nabla_s f_\psi)(s, a_{t-1}, \xi_{t-1})|_{s=s_{t-1}},
\]

with \(\nabla_\psi s_0 = 0\).
Corollary 3. The gradient of the function $V$, defined in Eqn. \[8\], with respect to the parameter vector $\theta$ is given by:

$$
\nabla_\theta V(\psi, \theta) = \mathbb{E}_{s_0 \sim P_0(\cdot), \ a_t \sim \pi_\theta(\cdot|s_t, t), \ \xi_t \sim P_\xi(\cdot|s_t, a_t)} \{ \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t, t)(\sum_{t=0}^{T-1} r_t) \} .
$$

(13)

Definition 1. Let $(S, A, \Xi, P_0, f, \rho, P_\xi, T)$ and $\pi$ be an environment and a policy, respectively, as defined in Section 2. We call a history $h$ of the policy in the environment, the sequence:

$$
h = (s_0, a_0, \xi_0, r_0, a_1, \xi_1, r_1, \ldots a_{T-1}, \xi_{T-1}, r_{T-1}) ,
$$

(14)

where $s_0$ is an initial state sampled from $P_0$, and where, at time $t$, $\xi_t$ is a disturbance sampled from $P_\xi$, $a_t$ is an action sampled from $\pi$, and $r_t$ is the reward observed.

For computing the gradients, our DESGA algorithm will exploit the following theorem that shows that an unbiased estimate of the gradients can be obtained by evaluating the gradients of a loss function computed from a set of histories. Automatic differentiation will later be used for computing these gradients in our simulations.

Theorem 2. Let $(S, A, \Xi, P_0, f_\psi, \rho, P_\xi, T)$ and $\pi_\theta$ be an environment and a policy, respectively, as defined in Section 2. Let $V(\psi, \theta)$ be the expected cumulative reward of policy $\pi_\theta$ averaged over the initial states, as defined in Eqn. \[8\]. Let $D = \{h^m|m = 0, \ldots, M - 1\}$ be a set of $M$ histories sampled independently and identically from the policy $\pi_\theta$ in the environment. Let $L$ be a loss function such that, $\forall(\psi, \theta) \in \Psi \times \Theta$:

$$
L(\psi, \theta) = -\frac{1}{M} \sum_{m=0}^{M-1} \left( \sum_{t=0}^{T-1} \log \pi_\theta(a^m_t|s^m_t, t) + \log P_\xi(\xi^m_t|s^m_t, a^m_t) \right) \\
\times \left( (\sum_{t=0}^{T-1} r^m_t) - B \right) + \left( \sum_{t=0}^{T-1} \rho_\psi(s^m_t, a^m_t, \xi^m_t) \right) ,
$$

(15)

where $B$ is a constant value called the baseline.

The gradients with respect to $\psi$ and $\theta$ of the loss function are unbiased estimators of the gradients of the function $V$ as defined in Eqn. \[8\] with opposite directions, i.e. they are such that:

$$
\mathbb{E}_{s_0 \sim P_0(\cdot), \ a_t \sim \pi_\theta(\cdot|s_t, t), \ \xi_t \sim P_\xi(\cdot|s_t, a_t)} \{ \nabla_\psi L(\psi, \theta) \} = -\nabla_\psi V(\psi, \theta)
$$

(16)

$$
\mathbb{E}_{s_0 \sim P_0(\cdot), \ a_t \sim \pi_\theta(\cdot|s_t, t), \ \xi_t \sim P_\xi(\cdot|s_t, a_t)} \{ \nabla_\theta L(\psi, \theta) \} = -\nabla_\theta V(\psi, \theta) .
$$

(17)

Corollary 4. The gradient of the loss function, defined in Eqn. \[15\] with respect to $\theta$ corresponds to the opposite of the update direction computed with the REINFORCE algorithm (Williams [1992]) averaged over $M$ simulations.

The proofs for the theorems and corollaries presented in this section are given in Appendix A.
3.2 Parameter optimization with projected stochastic gradient ascent

In the previous section, we have developed an analytical expression for the computation of the gradients of the expected cumulative reward with respect to the parameters of the environment and of the policy. In order to allow for the event where these parameters belong to a constrained set, our DESGA algorithm will use the projected gradient ascent method (Cohen et al., 2016).

Gradient ascent is an optimization technique where the optimized variables are updated at each iteration step \( k \), by a fixed-size step that is proportional to the gradient of the objective function with respect to these variables. The size of the update can be controlled by parameter \( \alpha \), called the learning rate. In the problem defined by Eqn. (7), we aim to find a parameter vector \( x = (\psi, \theta) \in X = \Psi \times \Theta \subseteq \mathbb{R}^{d\psi+d\theta} \) that maximizes the expected cumulative reward. Gradient ascent updates the parameter vector \( x_k \) at time \( k \) as:

\[
x_{k+1} \leftarrow x_k + \alpha \cdot \nabla_x V(x_k).
\] (18)

The new point \( x_{k+1} \) computed by simple gradient ascent according to Eqn. (18), may not belong to the constraint set \( X \). In projected gradient ascent, we choose the point nearest to \( x_{k+1} \), according to the Euclidean distance, that is located in the set \( X \) i.e., the projection of \( x_{k+1} \) onto the set \( X \). The projection \( \Pi_X \) of a point \( y \) onto a set \( X \) is defined as:

\[
\Pi_X(y) = \arg \min_{x \in X} \frac{1}{2} \| x - y \|_2^2.
\] (19)

Using projected gradient ascent, we first compute the update:

\[
y_{k+1} = x_k + \alpha \cdot \nabla_x V(x_k),
\] (20)

and then we project the new point \( y_{k+1} \) into the feasible set \( X \), according to:

\[
x_{k+1} \in \Pi_X(y_{k+1}).
\] (21)

The projected gradient descent (or ascent) shares the same convergence rate and guarantees as the unconstrained case, under specific conditions on the smoothness and the convexity of the objective function (Cohen et al., 2016). However, the computational cost of the projection operation depends on the characteristics of the constrained space \( X \). Let us also remark that, in practice, we assume the gradients to exist on the boundary of \( \Psi \times \Theta \). If this assumption does not hold, we can consider a compact subset \( K \) of the interior of \( \Psi \times \Theta \) such that Theorem 1 ensures the existence of the gradients on \( K \).

The DESGA algorithm will update the vector of parameters \( \psi \) and \( \theta \) according to Eqns. (20) and (21). In practice, the gradients are approximated using Theorem 2, such that projected stochastic gradient ascent is performed. Furthermore, we choose as the baseline the expected cumulative reward approximated by averaging the observed cumulative reward over the \( M \) histories \( h_m \) used for computing the loss function:

\[
B = \frac{1}{M} \sum_{m=0}^{M-1} \sum_{t=0}^{T-1} r_t^m.
\] (22)

The execution of projected stochastic gradient ascent algorithm for optimizing the objective in Eqn. (7) is fully detailed in Algorithm 1 in Appendix B.
4. Experiments

In this section, we first introduce the methodology used for assessing the performance of DESGA. Afterwards, we test the DESGA algorithm on two benchmarks, the Mass-Spring-Damper (MSD) environment and one related to the design of a solar off-grid microgrid. Both environments are fully described in Appendices C and D.

4.1 Methodology

When running the DESGA algorithm on a test problem, we will report the following results. First, at every iteration \( k \) of the algorithm we will compute the expected return of the policy on the environment for the current pair of parameter vectors \((\theta_k, \psi_k)\), that is \( V(\psi_k, \theta_k) \). This value is computed by running 100 Monte-Carlo simulations. Since the DESGA algorithm is stochastic, we will actually report the average of this value obtained over 20 runs (random seeds) of the algorithm. The standard deviation over the 20 runs of the algorithm will also be reported.

For every problem we will also compare the performance of DESGA with an algorithm based on a discretization \( \Psi_d \) of the environment’s hypothesis space \( \Psi \). This algorithm will run the REINFORCE algorithm for every value of \( \psi_d \in \Psi_d \) and compute the expected return of the policy obtained using 100 Monte-Carlo simulations. The process will be repeated five times to estimate the average expected return that could be obtained by a policy learned by the REINFORCE algorithm for each \( \psi_d \).

4.2 Mass-Spring-Damper environment

We consider here the MSD environment \( (S, A, \Xi, P_0, f_\psi, \rho_\psi, P_\xi, T) \) described in detail in Appendix C.

Hypothesis spaces. The environment is parametrized by the real vector \( \psi = (\omega, \zeta, \phi_0, \phi_1, \phi_2) \in \Psi = [0.1, 1.5] \times [0.1, 1.5] \times [-2, 2] \times [-2, 2] \times [-2, 2] \subseteq \mathbb{R}^5 \). We will constrain the hypothesis space for the policies to time-invariant policies, meaning \( \pi_\theta(a|s, t) = \pi_\theta(a|s, t'), \forall a \in A, \forall s \in S, \forall t, t' \in \{0, \ldots, T - 1\} \). Any of these policies is a multi-layer perceptron (MLP) with two inputs (one for each value of the state vector \( s \)), and with one hidden layer of 128 neurons with hyperbolic tangent activation functions. The MLP has five output neurons \(|A| = 5\) from which a probability distribution over \( A \) will be inferred using a softmax function. All the possible values for the parameters of the MLP define the policy’s hypothesis space \( \Theta \).

Parameters of the DESGA algorithm. The gradients are evaluated applying automatic differentiation on the loss function defined in Eqn. 15. Furthermore, the Adam algorithm is used for updating \((\psi, \theta)\). It is a variant on the vanilla stochastic gradient ascent given in Algorithm 1 which has proven to perform well on highly non-convex problems (Kingma & Ba, 2014). The gradients are estimated on batches of \( M = 64 \) trajectories and the stepsize \( \alpha \) of the Adam algorithm is chosen equal to 0.005. We keep the default values for the other parameters of the Adam algorithm. Furthermore, the states are z-normalized by an

---

2. The implementation of our algorithm and of the different benchmarks are provided in the following github repository: [https://github.com/adrienBolland/Direct-Environment-Search-with-Gradient-Ascent](https://github.com/adrienBolland/Direct-Environment-Search-with-Gradient-Ascent)
The average vector corresponding to the equilibrium position \((x_{eq}, 0)\) targeted by an optimal policy, as explained in Appendix C. The standard deviation of the scaling is chosen equal to \((0.005, 0.02)\), an approximation of the standard deviation vector of the states collected over high-performing trajectories.

**Performance of the DESGA algorithm.** Figure 1a shows the evolution of the expected return, estimated with 100 Monte-Carlo samples, averaged over 20 runs of the DESGA algorithm. The standard deviation between the different runs is illustrated by the shaded area around the mean. As we can see, the DESGA algorithm converges towards a maximal expected return almost equal to 100. We note that 100 is an upper-bound on the return that can only be reached if at each time-step \(t\), the position of the mass is at its equilibrium \(x_{eq}\). The standard deviation also strongly decreases as the iterations go on. We discovered that by using time-variant policies, better results could not be obtained for this problem. Furthermore, Fig 1b shows the average expected return of 5 policies computed by the REINFORCE algorithm for each \(\psi_d \in \Psi_d = \Omega_d \times Z_d \times \{c_0\} \times \{c_1\} \times \{c_2\}\) where \(\Omega_d = Z_d = \{0.1 + k \cdot \Delta|k = 1, \ldots, 15\}\) with \(\Delta = 0.082\). We note that, \(c_0, c_1\) and \(c_2\) correspond to an optimal triplet of values for \(\phi_1, \phi_2\) and \(\phi_3\), respectively, as described in Appendix C. The highest average expected return of the policies occurs for \((\omega, \zeta) = (0.5, 0.5)\). Finally, the average expected return of the policies identified by the REINFORCE algorithm, for this value of \(\psi = (0.5, 0.5, c_0, c_1, c_2)\), was almost identical to the expected returns obtained by the policies computed with the DESGA algorithm. We also note that the DESGA algorithm converged at every run towards a \(\psi\) whose \(\omega\) and \(\zeta\) components were both equal to 0.5 and whose triplet \((\phi_0, \phi_1, \phi_2)\) was always optimal, but not necessarily equal to \((c_0, c_1, c_2)\).

4.3 Sizing and operation of a solar off-grid microgrid

In this section, we consider the solar off-grid microgrid environment \((S, A, \Xi, P_0, f_\psi, p_\psi, P_\xi, T)\) presented in Appendix D.
Hypothesis spaces. The environment is parametrized by the real vector \( \psi = (\text{SoC}, \text{PV}) \in \Psi = [0, 200] \times [0, 300] \). We will constrain the hypothesis space for the policies to time-invariant Gaussian policies, i.e. policies such that \( \pi_\theta(a|s,t) = \mathcal{N}(a|\mu_\theta(s),\sigma_\theta(s)) \), \( \forall a \in A, \forall s \in S, \forall t \in \{0, \ldots, T - 1\} \) where \( \mu_\theta(s) \) and \( \sigma_\theta(s) \) are the expectation and the standard deviation of the normal distribution \( \mathcal{N} \) in function of the state \( s \) and of the parameter vector \( \theta \), respectively. A MLP with four inputs (one for each value of the state vector \( s \)), and with one hidden layer of 128 neurons with hyperbolic tangent activation functions, outputs the two values \( \mu_\theta(s) \) and \( \sigma_\theta(s) \). All the possible values for the parameters of the MLP define the policy’s hypothesis space \( \Theta \).

Parameters of the DESGA algorithm. The parameters related to the optimization process are the same as those used for the MSD environment in Section 4.2. The states are z-normalized by the average vector \((100, 12, 6.31, 6.48)\) and by the standard deviation vector \((50, 6, 8.9, 2)\). These values represent the mean and the standard deviation of the state vector for a microgrid configuration where \( \psi = (200, 300) \). The rewards collected are scaled linearly from the interval \([-5000, 0]\) to the interval \([0, 1]\). Moreover, the vector \( \psi \) is scaled from \([0, 200] \times [0, 300]\) to \([0, 1] \times [0, 1]\) in the interest of keeping the optimization variables in a small range.

Performance of the DESGA algorithm. Similar to Section 4.2 Figure 2a presents the evolution of the average expected (scaled) return collected in the solar off-grid microgrid environment, averaged over 20 runs of the DESGA algorithm. As we can see, the DESGA algorithm converges towards a maximal expected return that stands around a value of 100. We note that 120 is an upper bound on the expected return that can only be reached if, during the entire horizon \( T = 120 \), the instantaneous reward takes the value one. The standard deviation also strongly decreases as the iterations go on. Furthermore, Fig 2b shows the average expected return of five policies computed with the REINFORCE algorithm at each point \( \psi^d \in \Psi^d = \{0, 2, \ldots, 200\} \times \{0, 3, \ldots, 300\} \), where \( \Psi^d \) is a discrete subset of the hypothesis space \( \Psi \) that forms a mesh \( 100 \times 100 \). We also note that the DESGA algorithm is converging at every run towards a value of \( \psi = (\text{SoC}, \text{PV}) = (114, 165) \), which is very close to the value of \( \psi^d = (114, 166) \) that leads to the highest average expected return of policies computed with the REINFORCE algorithm.

4.4 Discussion on the alternative to the DESGA algorithm

In order to compare the DESGA algorithm, we have decided to discretize the hypothesis space of environments \( \Psi \) and to apply the REINFORCE algorithm on each environment \( \psi_d \) of the discretized set \( \Psi_d \). In the following, we will describe some implications of this choice and justify this procedure. First, we used the REINFORCE algorithm instead of any other policy gradient method. This choice is motivated by Corollary 4 stating that for a fixed environment, the DESGA algorithm is equivalent to the REINFORCE algorithm. The Figures 1b and 2a thus provide the best possible average performance of the DESGA algorithm assuming the discrete set \( \Psi_d \) is precise enough. Also, since this method enabled us to approach the theoretical (tight) upper bounds on the return of any policies in the environments for both benchmarks, it was not necessary to use any other policy gradient algorithm to provide a clear view on the maximal performance one could achieve without
the DESGA algorithm. This method based on a discretization has as only drawback that it is computationally inefficient and not scalable to larger problems.

5. Conclusions and Future Work

In this paper, we propose an algorithm that can jointly optimize an RL environment and a policy with maximal expected return over a joint hypothesis space of environments and policies. This algorithm is suited to cases in which the design of the environment and the applied policy are interdependent. We demonstrate the performance of DESGA on the design of an MSD environment and on the sizing of an autonomous energy system. The results show that the DESGA algorithm outputs a solution which is equivalent in terms of performance to the one obtained by the REINFORCE algorithm run for every element $\psi_d \in \Psi_d$. A magnified area of the original graph is presented, where the maximum values are located.

In this paper, the DESGA algorithm was designed in the context of jointly optimizing the design of a discrete-time dynamical system and its policy. This algorithm could be extended to the case where the environment is a finite-time Markov Decision Process (MDP) performing a similar development as the one presented in Section 3.1. The approach could also be extended to environments with infinite-time horizons.

Future work could also be directed on an approximation of the gradients. With the computational complexity of the automatic differentiation being proportional to the optimization horizon, the problem may become intractable for long horizons. An analytical bound on the error when performing this approximation would be valuable for striking a trade-off between computational efficiency and the quality of the solution.

Additionally, as future work, the proposed method could also be combined with recent research in gradient-based direct policy search. The use of actor-critic methods, proximal
policy optimization, etc., that are shown to result in stable learning and efficient exploration, could lead to better performance. This would come at the expense of involving the additional approximation architecture (set of parameters) of a value function.

Finally, in this paper we assumed that we have direct access to the parametrized dynamics of the system, the reward function, and the disturbance function. In the event these assumptions do not hold, we propose constructing an approximation of these functions by a differentiable function approximator as future work. This would introduce an additional learning step, in order to obtain a good approximation architecture from observations, which would then be used in the proposed algorithm.

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References


Appendices

Appendix A. Analytical derivation of the gradient for learning optimal environments

**Theorem 1.** Let \((S,A,\Xi,P_0,f_\psi,\rho_\psi,P_\xi,T)\) and \(\pi_\theta\) be an environment and a policy as defined in Section 2.2. Additionally, let the functions \(f_\psi,\rho_\psi\) and \(P_\xi\) be continuously differentiable over their domain of definition. Let \(V(\psi,\theta)\) be the expected cumulative reward of policy \(\pi_\theta\), averaged over the initial states, for all \((\psi,\theta) \in \Psi \times \Theta\) as defined in Eqn. (8).

Then, the function \(V\) exists, is bounded, and is continuously differentiable in the interior of \(\Psi \times \Theta\).

**Proof.** Let us first define the random variable associating the cumulative reward to a realization of a trajectory sampled from a policy in the environment for fixed parameter vectors \((\psi,\theta) \in \Psi \times \Theta\). We prove its expectation exists and is bounded for all \((\psi,\theta) \in \Psi \times \Theta\). Furthermore, \(V(\psi,\theta)\) is defined by a parametric integral which we prove to be continuously differentiable for all \((\psi,\theta) \in \Psi \times \Theta\).

Let \(R_{\psi,\theta}\) be the real random variable that associates to the realization of a trajectory given \(\psi \in \Psi\) and \(\theta \in \Theta\) its cumulative reward. Given a trajectory \(\tau\), the random variable \(R_{\psi,\theta}\) takes as values \(R_{\psi,\theta}(\tau)\) as defined in Eqn. (4). Let \(P_{R_{\psi,\theta}}\) be the induced probability of this random variable. We can write:

\[
P_{R_{\psi,\theta}} = P_{\psi,\theta}(s_0, a_0, \xi_0, a_1, \xi_1, \ldots, a_{T-1}, \xi_{T-1})
\]

\[
= P_0(s_0) \prod_{t=0}^{T-1} \pi_\theta(a_t|s_t,t)P_\xi(\xi_t|s_t,a_t) ,
\]

where \(s_{t+1} = f_\psi(s_t,a_t,\xi_t)\). The expected cumulative reward given in Eqn. (5) is the expectation of the random variables \(R_{\psi,\theta}\). If the expectation exists, it can therefore be written as:

\[
V(\psi,\theta) = \int (P_0(s_0) \prod_{t=0}^{T-1} \pi_\theta(a_t|s_t,t)P_\xi(\xi_t|s_t,a_t))
\]

\[
\sum_{t=0}^{T-1} \rho_\psi(s_t,a_t,\xi_t)) \ ds_0 da_0 \ldots da_{T-1} d\xi_0 \ldots d\xi_{T-1} ,
\]

or, more simply, as:

\[
V(\psi,\theta) = \int P_{R_{\psi,\theta}}(\tau) R_{\psi,\theta}(\tau) d\tau .
\]

The integration theory has shown that a measurable function upper-bounded in norm almost-everywhere by an integrable function on a domain is itself integrable on this domain. Moreover, a random variable is measurable by definition and the cumulative reward is such that:

\[
\int |P_{R_{\psi,\theta}} R_{\psi,\theta}(\tau)| d\tau \leq \int P_{R_{\psi,\theta}} T \tau_{max} d\tau \leq T \tau_{max} .
\]
The integral defined by Eqn. (26) thus exists and the function $V$ is bounded for all $(\psi, \theta) \in \Psi \times \Theta$.

As a corollary to the Leibniz integral rule, a function defined as in Eqn. (26) is continuously differentiable on the interior of the set $\Psi \times \Theta$ if $P_{\psi, \theta} R_{\psi, \theta}(\tau)$ is continuously differentiable on the compact $\Psi \times \Theta \times X$ where $X = S \times (A \times \Xi)^T$ is the set of all trajectories. The latter is true by hypothesis. Furthermore, it implies that the partial derivative of the integral equals the integral of the partial derivative of the integrand.

\[ \Box \]

**Corollary 1.** The function $V$, as defined in Theorem 1, exists, is bounded and is continuously differentiable on the interior of $\Psi \times \Theta$ if $A$ and/or $\Xi$ are discrete.

**Proof.** Let us write the expression of the expectation (8) in the three cases depending on whether $A$ and/or $\Xi$ are discrete and show that the different results of Theorem 1 are still valid.

1. If $A$ is discrete:

\[
V(\psi, \theta) = \int \sum_{(a_0, \ldots, a_{T-1}) \in A^T} (P_0(s_0) \prod_{t=0}^{T-1} \pi_{\theta}(a_t|s_t, t) P_\xi(\xi_t|s_t, a_t)) \left( \sum_{t=0}^{T-1} \rho_\psi(s_t, a_t, \xi_t) \right) ds_0 d\xi_0 \ldots d\xi_{T-1} . \quad (28)
\]

2. If $\Xi$ is discrete:

\[
V(\psi, \theta) = \int \sum_{(\xi_0, \ldots, \xi_{T-1}) \in \Xi^T} (P_0(s_0) \prod_{t=0}^{T-1} \pi_{\theta}(a_t|s_t, t) P_\xi(\xi_t|s_t, a_t)) \left( \sum_{t=0}^{T-1} \rho_\psi(s_t, a_t, \xi_t) \right) ds_0 da_0 \ldots da_{T-1} . \quad (29)
\]

3. If $A$ and $\Xi$ are discrete:

\[
V(\psi, \theta) = \int \sum_{(a_0, \ldots, a_{T-1}) \in A^T} \sum_{(\xi_0, \ldots, \xi_{T-1}) \in \Xi^T} (P_0(s_0) \prod_{t=0}^{T-1} \pi_{\theta}(a_t|s_t, t) P_\xi(\xi_t|s_t, a_t)) \left( \sum_{t=0}^{T-1} \rho_\psi(s_t, a_t, \xi_t) \right) ds_0 . \quad (30)
\]

In the three cases, we can still bound the integral as in Eqn. (27) and apply the corollary of the Leibniz integral rule if the integrand is continuously differentiable for all discrete values. Finally, by linearity of the differential operator, the operator can be distributed on the terms of the different sums when computing the derivative of the function $V$.

\[ \Box \]
Corollary 2. The gradient of the function $V$ defined in Eqn. [8] with respect to the parameter vector $\psi$ is such that:

$$
\nabla_\psi V(\psi, \theta) = \mathop{\mathbb{E}}_{s_0 \sim P_{0}(\cdot) \atop a_t \sim \pi_\theta(s_t, a_t) \atop \xi_t \sim P_\xi(\cdot|s_t, a_t)} \left\{ \left( \sum_{t=0}^{T-1} \nabla_s \log \pi_\theta(a_t|s_t, t)|_{s=s_t} + \nabla_s \log P_\xi(\xi_t|s_t, a_t)|_{s=s_t} \right) \cdot \nabla_\psi s_t \right\} \\
\times \left( \sum_{t=0}^{T-1} r_t \right) + \left( \sum_{t=0}^{T-1} \nabla_\psi \rho_\psi(s_t, a_t, \xi_t)|_{s=s_t} + \nabla_s \rho_\psi(s_t, a_t, \xi_t)|_{s=s_t} \cdot \nabla_\psi s_t \right),
$$

(31)

where:

$$
\nabla_\psi s_t = (\nabla_s f_\psi)(s, a_{t-1}, \xi_{t-1})|_{s=s_{t-1}} \cdot \nabla_\psi s_{t-1} + (\nabla_\psi f_\psi)(s, a_{t-1}, \xi_{t-1})|_{s=s_{t-1}},
$$

(32)

with $\nabla_\psi s_0 = 0$.

Proof. To compute this gradient, we first apply the product rule for gradients to Eqn. [8]. Afterwards, we exploit the equality $\nabla f = f \nabla \log f$ that holds if $f$ is a continuously differentiable function.

$$
\nabla_\psi V(\psi, \theta) = \int (\nabla_\psi P_{R_\psi, \theta}(\tau)) R_{\psi, \theta}(\tau) d\tau + \int P_{R_\psi, \theta}(\tau)(\nabla_\psi P_{R_\psi, \theta}(\tau)) d\tau
$$

(33)

$$
\quad = \int P_{R_\psi, \theta}(\tau)(\nabla_\psi \log P_{R_\psi, \theta}(\tau)) R_{\psi, \theta}(\tau) d\tau + \int P_{R_\psi, \theta}(\tau)(\nabla_\psi P_{R_\psi, \theta}(\tau)) d\tau
$$

(34)

$$
\quad = \mathop{\mathbb{E}}_{s_0 \sim P_0(\cdot) \atop a_t \sim \pi_\theta(s_t, a_t) \atop \xi_t \sim P_\xi(\cdot|s_t, a_t)} \left\{ \left( \nabla_\psi \log P_{R_\psi, \theta}(\tau) \right) R_{\psi, \theta}(\tau) + \left( \nabla_\psi P_{R_\psi, \theta}(\tau) \right) \right\} .
$$

(35)

By applying the logarithmic operator to both sides of Eqn. [24], we have:

$$
\log P_{R_\psi, \theta}(\tau) = \log P_0(s_0) + \sum_{t=0}^{T-1} \log \pi_\theta(a_t|s_t, t) + \sum_{t=0}^{T-1} \log P_\xi(\xi_t|s_t, a_t).
$$

(36)

Let $\cdot$ denote the dot product operator. Using the chain rule formula together with Eqn. [4], we can write:

$$
\nabla_\psi \log \pi_\theta(a_t|s_t, t) = \nabla_s \log \pi_\theta(a_t|s_t, t)|_{s=s_t} \cdot \nabla_\psi s_t
$$

(37)

$$
\nabla_\psi \log P_\xi(\xi_t|s_t, a_t) = \nabla_s \log P_\xi(\xi_t|s_t, a_t)|_{s=s_t} \cdot \nabla_\psi s_t
$$

(38)

$$
\nabla_\psi \rho_\psi(s_t, a_t, \xi_t) = \nabla_s \rho_\psi(s_t, a_t, \xi_t)|_{s=s_t} + \nabla_s \rho_\psi(s, a_t, \xi_t)|_{s=s_t} \cdot \nabla_\psi s_t,
$$

(39)

where:

$$
\nabla_\psi s_t = (\nabla_s f_\psi)(s, a_{t-1}, \xi_{t-1})|_{s=s_{t-1}} \cdot \nabla_\psi s_{t-1} + (\nabla_\psi f_\psi)(s, a_{t-1}, \xi_{t-1})|_{s=s_{t-1}},
$$

(40)

with $\nabla_\psi s_0 = 0$. 

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Finally, combining the previous results with Eqs. (35) and (36), we have:

\[ \nabla \psi V(\psi, \theta) = \mathbb{E}_{s_0 \sim P_0(\cdot)} \left\{ \left( \sum_{t=0}^{T-1} (\nabla_s \log \pi_\theta(a_t | s, t)|_{s=s_t} + \nabla_s \log P_\xi(\xi_t | s, a_t)|_{s=s_t}) \cdot \nabla_s s_t \right) \right. \\
\left. \times \left( \sum_{t=0}^{T-1} r_t \right) + \left( \sum_{t=0}^{T-1} \nabla_s \rho_\psi(s_t, a_t, \xi_t) \right) \right\} \right. \\
\left. \times \left( \sum_{t=0}^{T-1} r_t \right) + \left( \sum_{t=0}^{T-1} \nabla_s \rho_\psi(s_t, a_t, \xi_t) \right) \right\} . \] (41)

\[ \square \]

**Corollary 3.** The gradient of the function \( V \) defined in Eqn. (8) with respect to the parameter vector \( \theta \) is given by:

\[ \nabla_\theta V(\psi, \theta) = \mathbb{E}_{s_0 \sim P_0(\cdot)} \left\{ \left( \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t | s, t)|_{s=s_t} \right) \left( \sum_{t=0}^{T-1} r_t \right) \right\} . \] (42)

**Proof.** Using similar derivations as for the Corollary 1, we have for the gradient with respect to \( \theta \):

\[ \nabla_\theta V(\psi, \theta) = \mathbb{E}_{s_0 \sim P_0(\cdot)} \left\{ \left( \nabla_\theta \log P_r(\phi_\psi(\tau)) R_{\psi_\theta}(\tau) \right) \right\} \] (43)

\[ = \mathbb{E}_{s_0 \sim P_0(\cdot)} \left\{ \left( \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t | s, t)|_{s=s_t} \right) \left( \sum_{t=0}^{T-1} r_t \right) \right\} . \] (44)

\[ \square \]

**Theorem 2.** Let \((S, A, \Xi, P_0, f_\psi, \rho_\psi, P_\xi, T)\) and \(\pi_\theta\) be an environment and a policy as defined in Section 2.2. Let \(V(\psi, \theta)\) be the expected cumulative reward of policy \(\pi_\theta\) averaged over the initial states as defined in Eqn. [8]. Let \(D = \{h^m | m = 0, \ldots, M-1\}\) be a set of \(M\) histories sampled independently and identically from the policy \(\pi_\theta\) in the environment. Let \(\mathcal{L}\) be a loss function such that, \(\forall(\psi, \theta) \in \Psi \times \Theta:\)

\[ \mathcal{L}(\psi, \theta) = -\frac{1}{M} \sum_{m=0}^{M-1} \left( \sum_{t=0}^{T-1} \log \pi_\theta(a_t^m | s_t^m, t) \right) + \log P_\xi(\xi_t^m | s_t^m, a_t^m) \right) \] \[ \times \left( \sum_{t=0}^{T-1} r_t^m \right) - B + \left( \sum_{t=0}^{T-1} \rho_\psi(s_t^m, a_t^m, \xi_t^m) \right) , \] (45)

where \(B\) is a constant value called the baseline.
The gradients with respect to $\psi$ and $\theta$ of the loss function are unbiased estimators of the gradients of the function $V$ as defined in Eqn. (8), with opposite directions, i.e. such that:

$$
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\psi L(\psi, \theta) \} = -\nabla_\psi V(\psi, \theta) \tag{46}
$$

Moreover:

$$
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\theta L(\psi, \theta) \} = -\nabla_\theta V(\psi, \theta) . \tag{47}
$$

Proof. Let us first rewrite the loss function using the notations of Theorem 1. We have:

$$
L(\psi, \theta) = -\frac{1}{M} \sum_{m=0}^{M-1} \left( \log P_{\mathcal{R}, \theta}(\tau^m) - \log P_0(s_0^m) \right) \left( \sum_{t=0}^{T-1} r_t^m - B \right) + (R_{\psi, \theta}(\tau^m)) . \tag{48}
$$

The expectation of the gradient with respect to $\psi$ is given by:

$$
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\psi L(\psi, \theta) \} = \mathbb{E}_{s_0 \sim P_0(\cdot)} \left\{ -\frac{1}{M} \sum_{m=0}^{M-1} \nabla_\psi \left( \log P_{\mathcal{R}, \theta}(\tau^m) - \log P_0(s_0^m) \right) \right. \\
\times \left. \left( \sum_{t=0}^{T-1} r_t^m - B \right) + \nabla_\psi (R_{\psi, \theta}(\tau^m)) \right\} . \tag{49}
$$

Observing that every term in the sum has the same expectation and that $\nabla_\psi \log P_0(s_0^m) = 0$, we can write:

$$
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\psi L(\psi, \theta) \} = - \mathbb{E}_{s_0 \sim P_0(\cdot)} \left\{ \nabla_\psi \left( \log P_{\mathcal{R}, \theta}(\tau) \right) \times \left( \sum_{t=0}^{T-1} r_t - B \right) + \nabla_\psi (R_{\psi, \theta}(\tau)) \right\} . \tag{50}
$$

Moreover:

$$
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\psi \left( \log P_{\mathcal{R}, \theta}(\tau) \right) B \} = \nabla_\psi \mathbb{E}_{s_0 \sim P_0(\cdot)} \{ B \} = 0 , \tag{51}
$$

such that:

$$
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\psi L(\psi, \theta) \} = -\nabla_\psi V(\psi, \theta) . \tag{52}
$$

Equivalently, the expectation of the gradient with respect to $\theta$ is given by:

$$
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\theta L(\psi, \theta) \} = - \mathbb{E}_{s_0 \sim P_0(\cdot)} \left\{ \nabla_\theta \left( \log P_{\mathcal{R}, \theta}(\tau) \right) \right. \\
\times \left. \left( \sum_{t=0}^{T-1} r_t - B \right) + \nabla_\theta (R_{\psi, \theta}(\tau)) \right\} . \tag{53}
$$
The expectation of the term relative to the baseline is zero:

\[
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\theta \left( \log P_{\mathcal{R},\psi,\theta}(\tau) \right) \} B = \nabla_\theta \mathbb{E}_{s_0 \sim P_0(\cdot)} \{ B \} = 0 .
\] (54)

Furthermore, the gradient of the reward function with respect to \( \theta \) is zero:

\[
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\theta \left( R_{\psi,\theta}(\tau) \right) \} = \mathbb{E}_{s_0 \sim P_0(\cdot)} \{ (\sum_{t=0}^{T-1} \nabla_\theta \rho_{\psi}(s_t, a_t, \xi_t)) \} = 0 .
\] (55)

We thus have that:

\[
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\theta \mathcal{L}(\psi, \theta) \} = - \mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\theta \left( \log P_{\mathcal{R},\psi,\theta}(\tau) \right) \} \times \left( \sum_{t=0}^{T-1} r_t \right) \}
\] (56)

\[
= - \mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\theta \left( \sum_{t=0}^{T-1} \log \pi_{\theta}(a_t|s_t,t) \right) \} \times \left( \sum_{t=0}^{T-1} r_t \right) \} .
\] (57)

Finally, we have that:

\[
\mathbb{E}_{s_0 \sim P_0(\cdot)} \{ \nabla_\theta \mathcal{L}(\psi, \theta) \} = - \nabla_\theta V(\psi, \theta) .
\] (58)

\[\square\]

**Corollary 4.** The gradient of the loss function, defined in Eqn. 15, with respect to \( \theta \) corresponds to the opposite of the update direction computed with the REINFORCE algorithm (Williams, 1992) averaged over \( M \) simulations.

**Proof.** The gradient of the loss function with respect to \( \theta \) is given by:

\[
\nabla_\theta \mathcal{L}(\psi, \theta) = - \sum_{m=0}^{M-1} \left( \nabla_\theta \left( \log P_{\mathcal{R},\psi,\theta}(\tau^m) \right) \times \left( R_{\psi,\theta}(\tau^m) - B \right) \right) .
\] (59)

The gradient is the opposite of the average over \( M \) trajectories of the update direction of the REINFORCE algorithm (Williams, 1992).

\[\square\]
Appendix B. Direct environment search with (projected stochastic) gradient ascent

Algorithm 1 DESGA

function Optimize((S, A, Ξ, P₀, fₚ, ρₚ, Pₓ, T), πₜ, Πₚ, Πₓ)
Parameter Number of gradient steps N
Parameter Batch size M
Parameter Learning rate α
for all n ∈ {0, ..., N − 1} do
    for all m ∈ {0, ..., M − 1} do
        h = GenerateHistory((S, A, Ξ, P₀, fₚ, ρₚ, Pₓ, T), πₜ)
        Add h to the set D
    end for
    Compute the baseline using the histories
    B = \frac{1}{m} \sum_{m=0}^{M-1} \sum_{t=0}^{T-1} r_t
    Differentiate Eqn. (15) for estimating the gradients Eqns. (11) and (13) using D
    (ψ, θ) = VanillaGradientAscent(ψ, θ, α, \hat{∇}_ψ V(ψ, θ), \hat{∇}_θ V(ψ, θ))
    ψ ← Πₚ(ψ)
    θ ← Πₓ(θ)
end for
return (ψ, θ)

function GenerateHistory((S, A, Ξ, P₀, fₚ, ρₚ, Pₓ, T), πₜ)
Sample an initial state: s₀ ∼ P₀()
for all t ∈ {0, ..., T − 1} do
    aₜ ∼ πₜ(sₜ, t)
    ξₜ ∼ Pₓ(sₜ, aₜ)
    sₜ₊₁ = fₚ(sₜ, aₜ, ξₜ)
    rₜ = ρₚ(sₜ, aₜ, ξₜ)
end for
h = (s₀, a₀, ξ₀, r₀, a₁, ξ₁, ..., a_T−1, ξ_T−1, r_T−1)
return h

function VanillaGradientAscent(ψ, θ, α, \hat{∇}_ψ V(ψ, θ), \hat{∇}_θ V(ψ, θ))
ψ ← ψ + α · \hat{∇}_ψ V(ψ, θ)
θ ← θ + α · \hat{∇}_θ V(ψ, θ)
return (ψ, θ)
Appendix C. Mass-Spring-Damper environment

Let us consider a Mass-Spring-Damper (MSD) system defined as follows. A point mass $m$ is attached to a spring and a damper. The spring has a Hooke constant $k$ and the damping is proportional to the speed through the damping constant $b$. The damping force acts in the direction opposite to the motion. Furthermore, the system is subject to an external force $u$. Let $x$ denote the position of the mass. The continuous-time system dynamics is described by Newton’s second law as:

$$m \ddot{x} = -kx - b\dot{x} + u,$$  \hspace{1cm} (60)

which can equivalently be written as:

$$\ddot{x} + 2\zeta \omega \dot{x} + \omega^2 x = a,$$  \hspace{1cm} (61)

where:

$$\omega = \sqrt{\frac{k}{m}},$$  \hspace{2cm} (62)

$$\zeta = \frac{b}{2m\omega},$$  \hspace{2cm} (63)

$$a = \frac{u}{m}.$$  \hspace{2cm} (64)

The evolution of the position $x$ of the mass is thus described by the position itself and the speed $v$ as:

$$\left\{ \begin{array}{l}
\dot{x} = v \\
\dot{v} = a - 2\zeta \omega v - \omega^2 x.
\end{array} \right.$$  \hspace{1cm} (65)

**Optimization horizon.** The optimization horizon $T$ refers to the number of actions to be taken in the discrete process.

**State space.** The state is described at every time $t$ by two variables: the position $x_t$ and the speed $v_t$. The state space of the system is:

$$S = \mathbb{R}^2.$$  \hspace{1cm} (66)
**Initial state distribution.** The initial states $x_0$ and $v_0$ are uniformly drawn from the intervals $[x_{0,\text{min}}, x_{0,\text{max}}]$ and $[v_{0,\text{min}}, v_{0,\text{max}}]$.

**Action space.** In its most general setting, the system can be submitted to any external acceleration $a$. However, we will only consider a discrete action space defined as follows:

$$A = \{-0.3, -0.1, 0, 0.1, 0.3\}.$$  

**Disturbance space.** We will consider a stochastic version of the problem where a real disturbance $\xi_t$ is added to the action $a_t$ such that an acceleration $a_t + \xi_t$ is applied to the system. In such a context, we have:

$$\xi_t \in \Xi = \mathbb{R}.$$  

**Disturbance distribution.** The disturbance is sampled at time $t$ from a Normal distribution centred at the current position $x_t$, and whose standard deviation is a linear combination of the magnitude of the action $a_t$ and of the speed $v_t$:

$$P_\xi(\xi_t|s_t, a_t) = \mathcal{N}(\xi_t|x_t, 0.1 \times |a_t| + |s_t| + \epsilon),$$  

where $\epsilon$ is a constant equal to $10^{-6}$.

**Discrete dynamics.** The discrete-time process comes from a discretization of the continuous process defined by Eqn. (65) with a discretization time-step $\Delta = 50$ms. The discrete dynamics $f$ is the function computing the position and speed after a period $\Delta$ during which the constant acceleration $a_t + \xi_t$ is applied. The position $x_{t+1}$ and the speed $v_{t+1}$ can be computed from $x_t$ and $v_t$ using these analytical expressions:

$$x_{t+1} = g(x_t, v_t, a_t + \xi_t, \Delta),$$
$$v_{t+1} = \frac{\partial g}{\partial t}(x_t, v_t, a_t + \xi_t, t)|_{t=\Delta},$$

where:

$$g(x_t, v_t, a, t) = \frac{a}{\omega^2} + \exp(-\zeta\omega t) \times 
\begin{cases} 
(x_t - \frac{a}{\omega^2}) \cosh(\sqrt{\zeta^2 - 1}\omega t) + \frac{v_t + \zeta(x_t - \frac{a}{\omega^2})}{\sqrt{\zeta^2 - 1}} \sinh(\sqrt{\zeta^2 - 1}\omega t) & \text{if } \zeta > 1 \\
(x_t - \frac{a}{\omega^2}) + (v_t + \omega(x_t - \frac{a}{\omega^2}))t & \text{if } \zeta = 1 \\
(x_t - \frac{a}{\omega^2}) \cos(\sqrt{1 - \zeta^2}\omega t) + \frac{v_t + \zeta(x_t - \frac{a}{\omega^2})}{\sqrt{1 - \zeta^2}} \sin(\sqrt{1 - \zeta^2}\omega t) & \text{if } 0 < \zeta < 1. 
\end{cases}$$

**Reward function.** The reward function is defined as:

$$\rho(a_t, s_t, \xi_t) = \exp \left( -|x_t - x_{eq}| - (\omega - c_\omega)^2 - (\zeta - c_\zeta)^2 - \prod_{k=1}^{K}(\phi_k - c_k)^2 \right),$$

where $\omega$, $\zeta$ and $\phi_k$ are parameters of the system that need to be optimized. Furthermore $x_{eq}$, $c_\omega$, $c_\zeta$, $K$ and $c_k$ are constant values. Let us also remark that the reward function does not depend on the disturbance.
The first term of the exponential will be minimized if the mass is stabilized at the position $x_{eq}$. The second and third terms are minimized if the parameters $\omega$ and $\zeta$ are equal to $c_\omega$ and $c_\zeta$, respectively. The last term is a strictly positive function minimized if, at least one of the parameters $\phi_k$ equals the value $c_k$. Minimizing these terms results in maximizing the reward. Furthermore, since the the reward function is the exponential of a negative value, the reward is bounded by $r_{max} = 1$.

**Parametrized MSD environment.** A parametrized MSD environment is an environment $(S, A, \Xi, P_0, f_\psi, \rho_\psi, P_\xi, T)$ parametrized by the real vector $\psi = (\omega, \zeta, \phi_0, \phi_1, \phi_2) \in \mathbb{R}^5$.

**Numerical values.** In this work, we will consider the values given in Table 1 for the constant parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>$x_{0,\text{min}}$</td>
<td>0.198</td>
</tr>
<tr>
<td>$x_{0,\text{max}}$</td>
<td>0.202</td>
</tr>
<tr>
<td>$v_{0,\text{min}}$</td>
<td>-0.010</td>
</tr>
<tr>
<td>$v_{0,\text{max}}$</td>
<td>0.010</td>
</tr>
<tr>
<td>$x_{eq}$</td>
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</tr>
<tr>
<td>$c_\omega$</td>
<td>0.500</td>
</tr>
<tr>
<td>$c_\zeta$</td>
<td>0.500</td>
</tr>
<tr>
<td>$K$</td>
<td>3.000</td>
</tr>
<tr>
<td>$c_0$</td>
<td>0.500</td>
</tr>
<tr>
<td>$c_1$</td>
<td>-0.300</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.200</td>
</tr>
<tr>
<td>$T$</td>
<td>100</td>
</tr>
</tbody>
</table>

**Appendix D. Optimal design of a solar off-grid microgrid**

A solar off-grid microgrid is a small-scale electrical grid composed of photovoltaic (PV) panels (converting solar energy into electricity) and a battery for ensuring the supply of an electrical load. A schematic of the considered configuration is presented in Fig 4. The total cost of the microgrid is the sum of the investment costs and the penalties obtained for shedding the load if there is insufficient electricity available. In this section, we are interested in sizing the microgrid components, i.e. identifying the optimal investment in equipment that leads to the least total cost over the investment lifetime, assuming that the microgrid is operated in an optimal way.

This problem is therefore related to the one addressed in this paper, by noticing that finding the optimal investment (i.e., the size of the PV panels and the battery) is equivalent to optimizing both the "solar off-grid microgrid" environment and the policy at the same time. We note that the actions that can be taken by the policy are related to the charging/discharging power of the battery. An optimal policy should, in principle, charge the
battery when there is an excess of solar power generated by the PV panels, and discharge that power from the battery when the electrical demand cannot be fully covered by the PV panels.

We will now provide, hereafter, a formalization of this problem that exactly fits the generic problem tackled in this paper. We note that more generic formalizations may exist, as for example those where the load consumption and the PV production cannot be considered as variables fully conditioned on the hour of the day, as will be assumed here. Those stand beyond the scope of this paper, even if they could lead to other interesting problem statements. Before carefully defining this benchmark problem, let us emphasize that we will use the notation $[\cdot]$ to indicate the corresponding unit of the symbol preceding it. In this section, $[W]$ denotes instantaneous power production in Watts, $[W_p]$ denotes nameplate (manufacturer) power capacity, $[Wh]$ denotes energy in Watt-hours and $[Wh_p]$ denotes nameplate (manufacturer) energy capacity. We now define the different elements of this learning optimal environment type of problem.

**Optimization horizon.** The optimization horizon is denoted by the value $T$.

**State space.** The state of the system can be fully described by $s_t = (SoC_t, h_t, \bar{P}^{C,h}_t, \bar{P}^{PV,h}_t) \in S = [0, \bar{SoC}] \times \{0, \ldots, 23\} \times \mathbb{R}^{+} \times \mathbb{R}^{+}$, where, at time $t$:

- $SoC_t [Wh] \in [0, \bar{SoC}]$ denotes the state of charge of the battery. The installed capacity of the battery is denoted by $\bar{SoC} [Wh_p] \in \mathbb{R}^{+}$.
- $h_t [h] \in \{0, \ldots, 23\}$ denotes the hour of the day.
- $\bar{P}^{C,h}_t [W] \in \mathbb{R}^{+}$ denotes the expected value of the electrical consumption level during hour $h$ that is considered to be known.
- $\bar{P}^{PV,h}_t [W] \in \mathbb{R}^{+}$ denotes the expected value of the PV power generation during hour $h = h_t$ that is also considered to be known.
Initial state distribution. The initial state of charge $SoC_0$ is drawn uniformly from the interval $[0, SoC]$ and the initial hour $h_0$ takes the value zero with probability one. The initial value for $P_{0}^{C,h}$ is given by the first line of Table 3 in the corresponding column. Let $PPV[W_p] \in \mathbb{R}^+$ denote the capacity of PV panels installed, the column $\bar{p}^{PV,h}$ in Table 3 gives the average PV production per installed capacity ($\%$). Subsequently, the initial value for $P_{0}^{PV,h}$ is given by the product of $PPV$ and the first element of column $\bar{p}^{PV,h}$ in Table 3.

Action space. As previously described, the available actions correspond to defining the charging/discharging power of the storage system. The charging power is denoted by $P_B \in [-P_B, P_B]$, which will be positive during charging and negative during discharging. The charging/discharging limit $P_B \in \mathbb{R}^+$ is assumed to be a proportion $p$ ($\%$) of the battery capacity as $P_B = p \cdot SoC$.

We therefore consider the continuous action space:

$$A = [-P_B, P_B] .$$

Disturbance space. We consider as disturbance the variable $\xi_t = E_{C,h}^t \in \Xi \subseteq \mathbb{R}$, the stochastic deviation from the expected consumption for hour $h_t$.

Disturbance distribution. The disturbance is sampled at time $t$ from a Normal distribution centred at zero with standard deviation $\sigma_{C,h}$ depending on the hour $h = h_t$:

$$P_\xi(\xi_t|s_t, a_t) = \mathcal{N}(\xi_t|0, \sigma_{C,h}) .$$

The values of the standard deviations $\sigma_{C,h}$ are given in Table 3 for every hour $h$ of the day.

Transition function. We use a discretization time-step $\Delta t$ of one hour for defining the discrete-time dynamics. For the state variable $h$ we have therefore:

$$h_{t+1} = (h_t + 1) \mod 24 .$$

The state of charge of the battery is updated using a linear water tank model (Boukas et al., 2020). With this tank model, the value of $SoC_{t+1}$ at time $t + 1$, if there were no limits on it, would be equal to $A_{t+1}$ defined as follows:

$$A_{t+1} = SoC_t + \Delta t \cdot \begin{cases} \eta_{ch} \cdot P_B^t, & \text{if } P_B^t \geq 0 \\ P_B^t/\eta_{dis}, & \text{if } P_B^t < 0 , \end{cases}$$

where $\eta_{ch} \in [0, 1]$, $\eta_{dis} \in [0, 1]$ represent the charging and discharging efficiencies of the storage system. Given the fact that the state of charge of the battery lies within predefined limits, its state of charge at time $t + 1$ is therefore defined as:

$$SoC_{t+1} = \begin{cases} 0, & \text{if } A_{t+1} < 0 \\ SoC, & \text{if } A_{t+1} \geq SoC \\ A_{t+1}, & \text{otherwise} . \end{cases}$$
The variable $\bar{P}_{t+1}^{PV}$ takes the value reported in Table 3 at the line corresponding to the hour $h = h_{t+1}$. Finally, the variable $\bar{P}_{t+1}^{PV}$ is updated as:

$$\bar{P}_{t+1}^{PV} = \bar{p}^{PV} \cdot P^{PV},$$  \hspace{1cm} (79)$$

where $\bar{p}^{PV}$ take the values reported in Table 3 at the line corresponding to the hour $h = h_{t+1}$.

**Reward function.** The reward signal is, in this case, a cost function composed of two parts, namely the investment cost and the operational cost. The reward signal is given by:

$$r_t = \rho(s_t, a_t, \xi_t) = -(c_t^{fix} + c_t^{shed}),$$  \hspace{1cm} (80)$$

where $c_t^{fix} [\$] \in \mathbb{R}^+$ represents a fixed hourly payment for settling the initial investment cost and $c_t^{shed} [\$] \in \mathbb{R}^+$ corresponds to the cost of shedding load at each time-step $t$.

In order to compute the fixed cost term $c_t^{fix}$ we proceed as follows. Let $c^{PV} [\$/W_p] \in \mathbb{R}^+$ denote the cost per unit of PV capacity installed. The total installation cost for PV $I^{PV} [\$] \in \mathbb{R}^+$ is defined as:

$$I^{PV} = c^{PV} \cdot P^{PV}.$$  \hspace{1cm} (81)$$

Let $c^B [\$/Wh_p] \in \mathbb{R}^+$ denote the cost per unit of storage capacity installed. The total installation cost for battery storage $I^B [\$] \in \mathbb{R}^+$ is defined as:

$$I^B = c^B \cdot \overline{SoC}.$$  \hspace{1cm} (82)$$

The investment cost $I$ is the sum of the investment costs for each component of the microgrid defined as:

$$I = I^B + I^{PV}.$$  \hspace{1cm} (83)$$

This payment occurs once in the beginning of the investment. In this case, we assume this investment to be a loan in its entirety. A fixed yearly payment $P$ over the lifetime of the investment for settling the initial loan, is given by the following amortization formula:

$$P = I \frac{r(1 + r)^n}{(1 + r)^n - 1},$$  \hspace{1cm} (84)$$

where $n$ is the number of years considered for the lifetime of the investment and $r(\%)$ is the interest rate considered. By noting that a common (non-leap) year has 8760 hours, we define the fixed hourly cost as:

$$c_t^{fix} = \frac{P}{8760}.$$  \hspace{1cm} (85)$$

In order to compute the shedding cost term $c_t^{shed}$ we proceed as follows. The realization of the consumption $P_t^{C,h} [W] \in \mathbb{R}^+$, after an action is taken at each time-step $t \in T$, corresponds to the actual consumption level in the interval $(t, t + 1]$, i.e. for hour $h_t$. This variable takes the value:

$$P_t^{C,h} = \bar{P}_t^{C,h} + E_t^{C,h},$$  \hspace{1cm} (86)$$
where \( h = h_t \) is the hour of the day at time \( t \).

We denote by \( \tilde{P}_B^t \) the actual charging power that can be applied to the battery considering its limited capacity. Given an action to charge \( P_B^t \), the actual charge \( \tilde{P}_B^t \) is constrained by the battery capacity limit for charging the available energy stored in the battery for discharging, according to:

\[
\tilde{P}_B^t = \begin{cases} \frac{(SoC - SoC_t)}{\eta_{ch}}, & \text{if } P_B^t > \frac{(SoC - SoC_t)}{\eta_{ch}} \\ -(SoC_t) \cdot \eta_{dis}, & \text{if } P_B^t < -(SoC_t) \cdot \eta_{dis} \\ P_B^t, & \text{otherwise} \end{cases}
\]  

(87)

At each time-step \( t \) in the simulation horizon, there exists a power balance between the injections and the off-takes. The residual power resulting from the mismatch between production and consumption is curtailed \( P_{\text{curtail}}^t [W] \in \mathbb{R}^+ \). Formally the power balance is given by:

\[
P_{\text{curtail}}^t = \bar{P}^{PV,h}_t - \bar{P}^{C,h}_t - \tilde{P}_B^t.
\]  

(88)

If \( P_{\text{curtail}}^t \) is positive, the excess of generation is simply lost (curtailed). If \( P_{\text{curtail}}^t \) is negative, there is a lack of generation and a part of the load has to the shed. This is associated with a cost of shedding load \( c_{\text{shed}}^t [\$] \in \mathbb{R}^+ \) equal to:

\[
c_{\text{shed}}^t = -\min(0, P_{\text{curtail}}^t) \cdot \pi_{\text{shed}},
\]  

(89)

where \( \pi_{\text{shed}} [\$/W] \in \mathbb{R}^+ \) corresponds to the penalty per unit of power shed.

**Parametrized environment.** The off-grid microgrid environment \( (S, A, \Xi, P_0, f_\psi, \rho_\psi, P_\xi, T) \) will be parametrized by the vector \( \psi = (\bar{SoC}, \bar{P}^{PV}) \in \mathbb{R}^{+2} \).

**Numerical values.** Table 2 summarises the parameter values used in the experiments presented in this paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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<td>( \eta_{ch}, \eta_{dis} )</td>
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</tr>
<tr>
<td>( \sigma^C, \sigma^{PV} )</td>
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<td>Wh</td>
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<tr>
<td>( p )</td>
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<td>%</td>
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<tr>
<td>( \Delta t )</td>
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<td>hour</td>
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<tr>
<td>( c^{PV} )</td>
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<td>$/W_p</td>
</tr>
<tr>
<td>( c^B )</td>
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<td>$/W_p</td>
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<td>( r )</td>
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<td>%</td>
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<tr>
<td>( n )</td>
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</tr>
<tr>
<td>( \pi_{\text{shed}} )</td>
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<td>$/Wh</td>
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<tr>
<td>( T )</td>
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<td>hour</td>
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Table 2: Parameters for the solar off-grid microgrid.
Table 3: Electrical load consumption and PV production power factor data.

<table>
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<tr>
<th>Hour</th>
<th>$P_{C,h}^C$</th>
<th>$\sigma_{C,h}^2$</th>
<th>$\tilde{p}_{PV,h}^C$</th>
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<td>7.8</td>
<td>0.59</td>
<td>0.0</td>
</tr>
</tbody>
</table>