Comparison of Finite Element Formulations for the Modelling of High-Frequency Effects in Electromagnetic Coils

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The accurate prediction of coupled inductive and capacitive effects in electromagnetic coils is becoming of crucial importance in several industrial applications, either due to the increase of operating frequency or to the increase of voltage levels. In this paper we review and compare 2D and 3D finite element formulations of electromagnetic coils able to predict both inductive and capacitive effects, in order to design approximate 2D models whose computational complexity is compatible with current industrial design practice.

***Index Terms*—Finite element method, electromagnetic coils, high-frequency effects.**

I. Introduction

HIGH frequency effects, i.e. the coupling of inductive and capacitive effects, could long be ignored in the design of electromagnetic coils for electrotechnical applications. With the advent of extremely fast switching electronic components based on silicon carbide or gallium arsenide, this coupling can however not be neglected anymore when designing the magnetic components (inductors and transformers) of either extremely fast-switching, or very high-voltage, power electronic circuits. Resonances due to parasitic capacitances cannot be neglected at the considered frequencies, in particular for electromagnetic compatibility issues.

While the design of such magnetic components has traditionally been (and for the most part still is) carried out analytically, the last decade has increasingly seen the adoption in engineering offices of finite element software tools. To keep the computational cost acceptable, only 2D models are commonly used. Moreover, these models usually consider decoupled magnetic and electric phenomena [1,2].

This paper follows up on the comparison carried out in [3] for a multi-turn coreless inductor, which in addition to fully decoupled magnetic and electric formulations considered a weak (one-way) 3D coupling [4,5]. Here we compare both decoupled and fully-coupled models, and propose 2D approximations of the fully coupled model that aim at bringing down the computational cost to a level acceptable for current industrial design practice.

II. Finite Element Formulations for Electromagnetic Coils

The coupling level between inductive and capacitive effects occurring in an electromagnetic coil depends on the ratio between its length and the excitation frequency. For this reason, three main approaches are possible, which consider decoupled, weakly coupled or fully coupled formulations [3]. Fig. 1 and 2 illustrate the main characteristics of these methods and the results they provide on a simple transformer made of two circular coils, respectively.

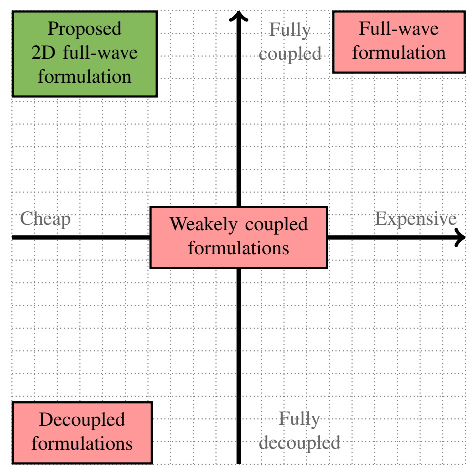


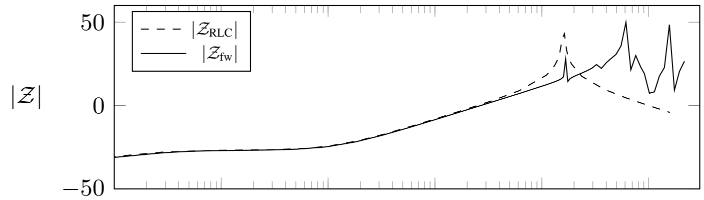
Fig. 1. Coupling level of the three approaches vs computing cost

In the simplest approach, in which the electric and magnetic fields are considered to be totally decoupled, a magnetodynamic formulation is used to evaluate the resistive and inductive effects whereas the capacitive effects are dealt using an electrostatic formulation [1,2]. Note that the source of the latter can be either a linear distribution of the scalar electric potential along the winding (2D case) [5] or come from a previous solution of the considered problem using an electrokinetic formulation (3D case).

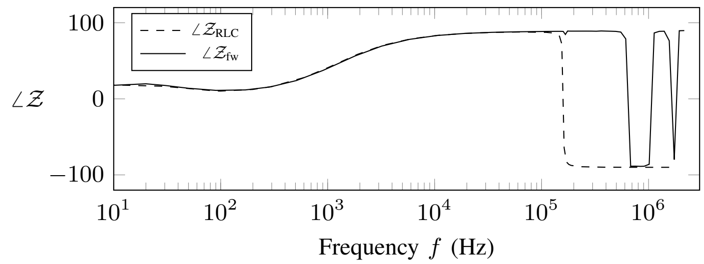
A weak coupling between the magnetodynamic and electrostatic formulations can also be considered, where the source term of the electrostatic formulation originates from the electric field obtained by the magnetodynamic computation inside the conductors [4]. This has the advantage to better capture the first resonance, but is only applicable in 3D [3].

Fully coupled formulations lead to the classical full-wave formulation of Maxwell's equations.

Those three approaches are represented in terms of coupling level and computing cost in Fig. 1. Decoupled and full-wave formulations are compared in Fig. 2. These preliminary results illustrate the numerous high frequency resonances not captured by the decoupled model.



(a) Modulus



(b) Argument

Fig. 2. Bode plots of the transformer impedance with full-wave and decoupled formulations

III. Towards Cheaper Fully-Coupled Formulations

While the full-wave formulation takes all physical effects into account, it has two main drawbacks. First, as is well known, the classical full-wave formulation is unstable in the low frequency regime, which is however crucial for power electronics applications where the whole spectrum (from DC to high frequencies) is of practical interest. This loss of stability can be overcome thanks to various stabilization strategies proposed over the last decade [6,7].

Second, the full-wave formulation (like the weakly-coupled formulation) is intrinsically 3D, and thus leads to a computational cost that is incompatible with industrial R&D. To overcome this second drawback, we propose a family of 2D full-wave formulations, which do not assume any weak coupling, and can be derived as follows. Assuming for simplicity a Cartesian coordinate system (x, y, z) and a geometrical invariance along z (axisymmetric configurations could be considered as well), we suppose that the electric and magnetic fields are of the form

$$\mathbf{E}(x,y,z,t) = \Re(\mathbf{e}(x,y)e^{-i(\omega t - i\gamma z)}),$$
$$\mathbf{H}(x,y,z,t) = \Re(\mathbf{h}(x,y)e^{-i(\omega t - i\gamma z)}),$$

where ω is the pulsation and γ a propagation constant along the invariance direction. Faraday's and Maxwell-Ampère's equations can thus be written respectively as

$$\mathbf{curl}_{\gamma}\mathbf{e} = i\omega\mu\mathbf{h} \quad \text{and} \quad \mathbf{curl}_{\gamma}\mathbf{h} = (\sigma - i\omega\epsilon)\mathbf{e},$$

where $\mathbf{curl}_{\gamma}\cdot := \mathbf{curl}\,(\cdot\, e^{i\gamma z})e^{-i\gamma z}$ and where μ, σ and ϵ are the magnetic permeability, electric conductivity and permittivity, respectively. Developing $\mathbf{e}(x, y)$ in its transverse and longitudinal components $\mathbf{e}_t(x, y)$ and $e_z(x, y)$, such that $\mathbf{e} := \mathbf{e}_t + e_z\mathbf{1}_z$, we get

$$\mathbf{curl}_{\gamma}\mathbf{e} = \mathbf{curl}\,\mathbf{e}_t + (\mathbf{grad}\,e_z - i\gamma\mathbf{e}_t) \times \mathbf{1}_z,$$

which leads to the following weak formulation [8]: find $\mathbf{e}_t$ and e_z satisfying appropriate boundary conditions such that

$$\begin{aligned}(\mathbf{curl}\,\mathbf{e}_t, \mathbf{curl}\,\bar{\mathbf{e}}_t') &+ (\mathbf{grad}\,e_z, \mathbf{grad}\,\bar{e}_z') \\ &- (i\gamma\mathbf{e}_t, \mathbf{grad}\,\bar{e}_z') + (i\gamma\mathbf{grad}\,e_z, \bar{\mathbf{e}}_t') + (\gamma^2\mathbf{e}_t, \bar{\mathbf{e}}_t') \\ &= ((i\omega\mu\sigma + w^2\mu\epsilon)\mathbf{e}_t, \bar{\mathbf{e}}_t') + ((i\omega\mu\sigma + w^2\mu\epsilon)e_z, \bar{e}_z')\end{aligned}$$

holds for appropriate test functions $\mathbf{e}_t'$ and e_z', where $(\cdot,\cdot)$ represents the integral over the domain of study of the scalar product of its arguments. This 2D formulation involves both components of the electric field in and perpendicular to the plane of study and is fully coupled, and can be readily discretized using 2D edge finite elements for $\mathbf{e}_t$ and nodal elements for e_z. For small coils compared to the wavelength, it decouples and leads to two uncoupled wave systems.

The full paper will detail this formulation and analyse its performance on test-cases (like the one from Figure 2) for which it does not introduce any approximation compared to the 3D formulation. We will then investigate how, in the case of multi-turn windings, several slices can be coupled through circuit equations, leading to an alternative to more classical transmission line matrix coil modelling techniques [9].

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