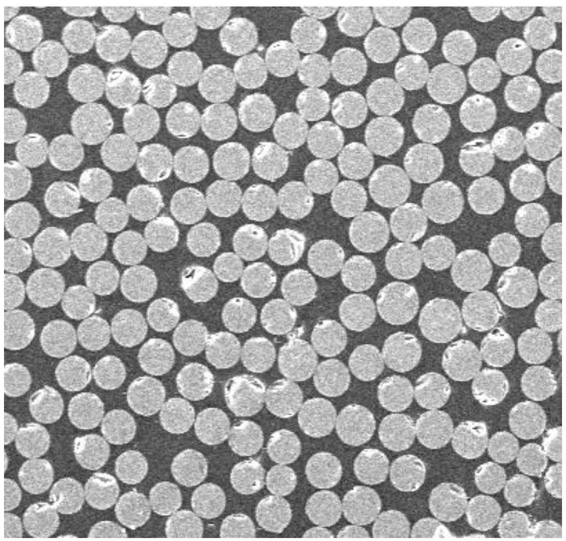
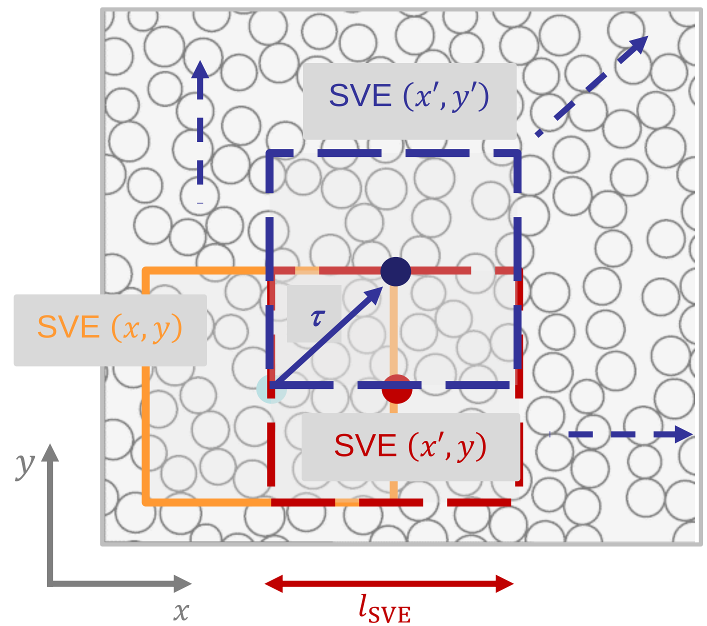
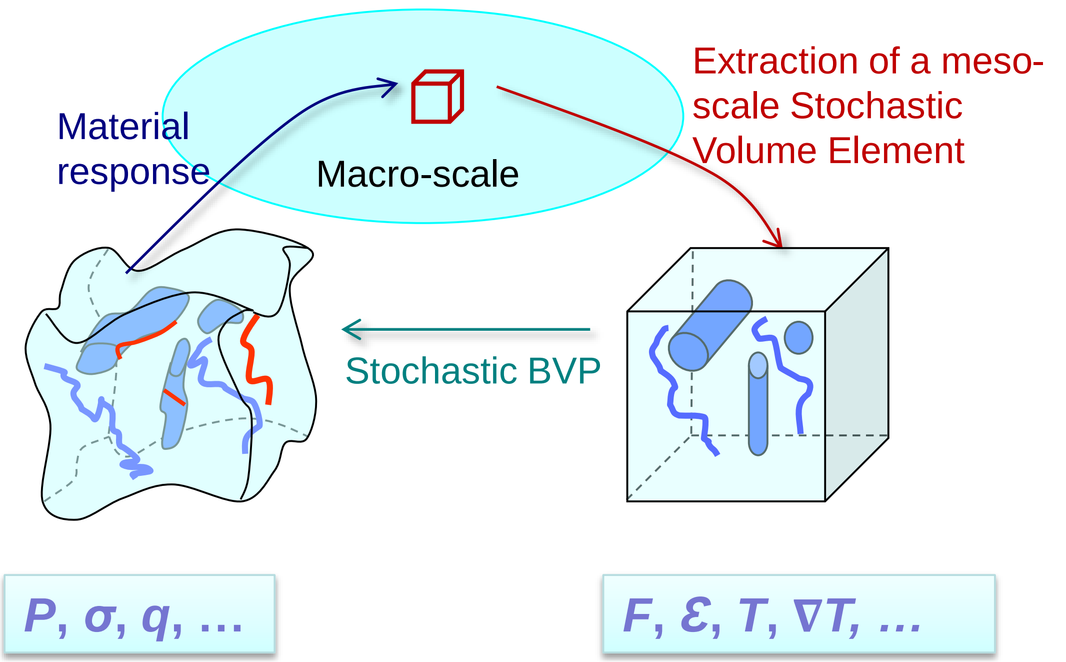
Multiscale stochastic simulations using a MFH model constructed from full-field SVE realizations





Objectives

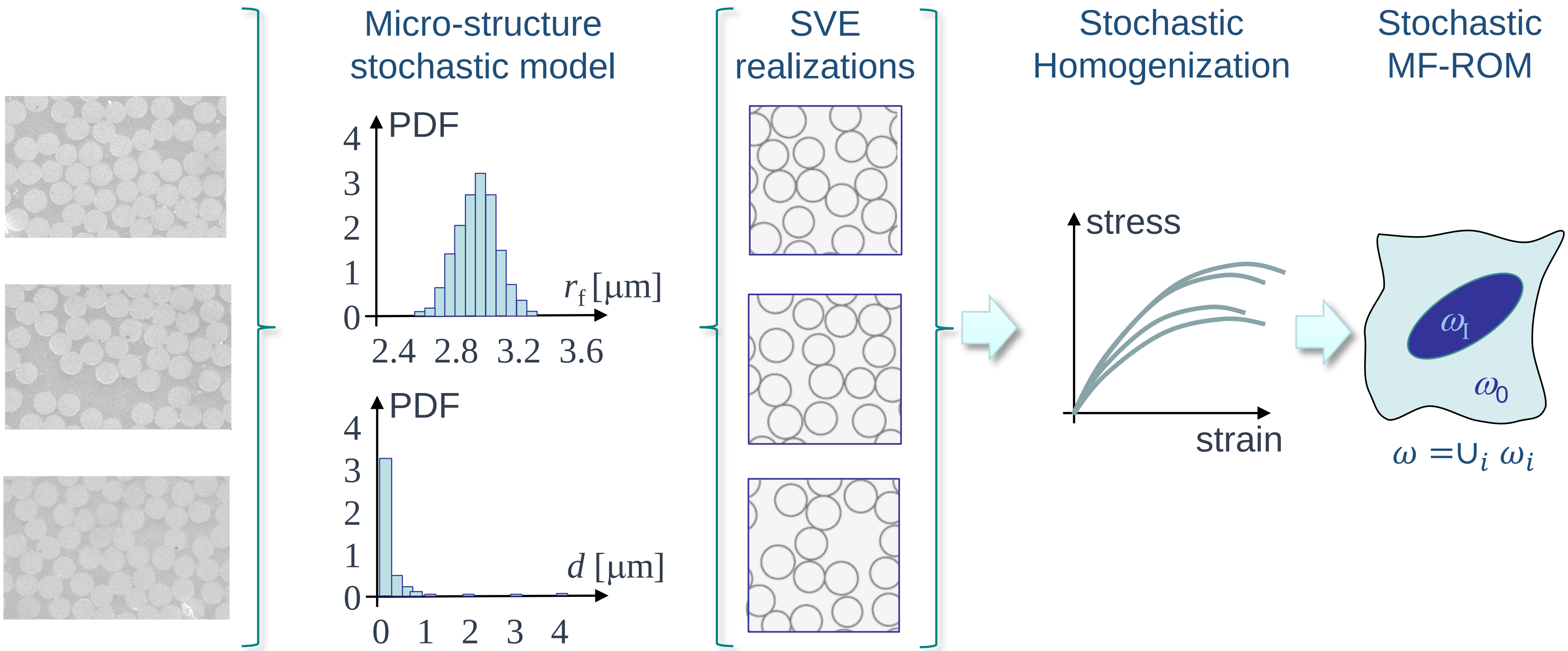
- Main objective
  - To develop an integrated stochastic multiscale approach to predict failure of composites



- Main deliverable
  - Development of a modelling methodology able to account for micro-scale uncertainties

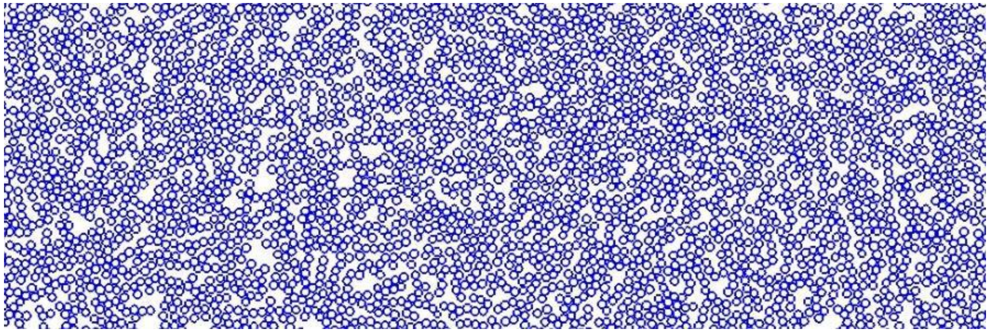
Methodology

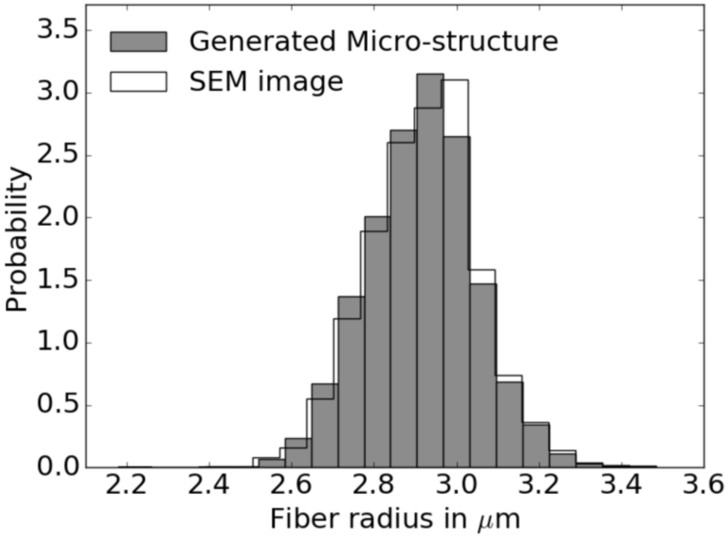
- Proposed methodology for material system:
  - To develop a stochastic Mean Field Homogenization method able to predict the probabilistic distribution of material response at an intermediate scale from micro-structural constituents characterization



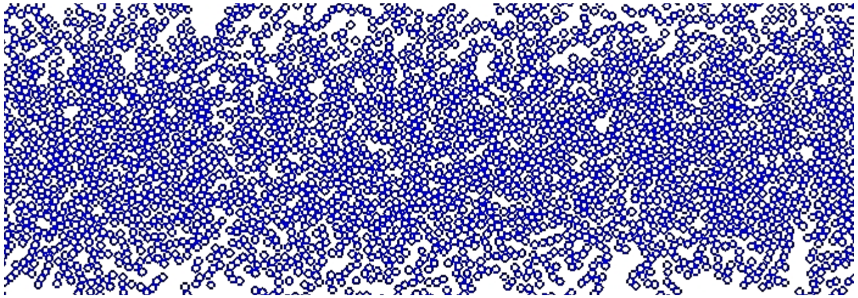
Micro-structure stochastic model

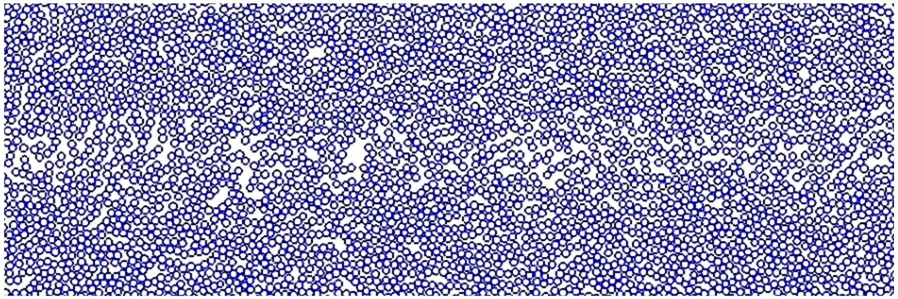
- Numerical micro-structures are generated by a fiber additive process
  - Arbitrary size
  - Arbitrary number





  - Possibility to generate non-homogenous distributions



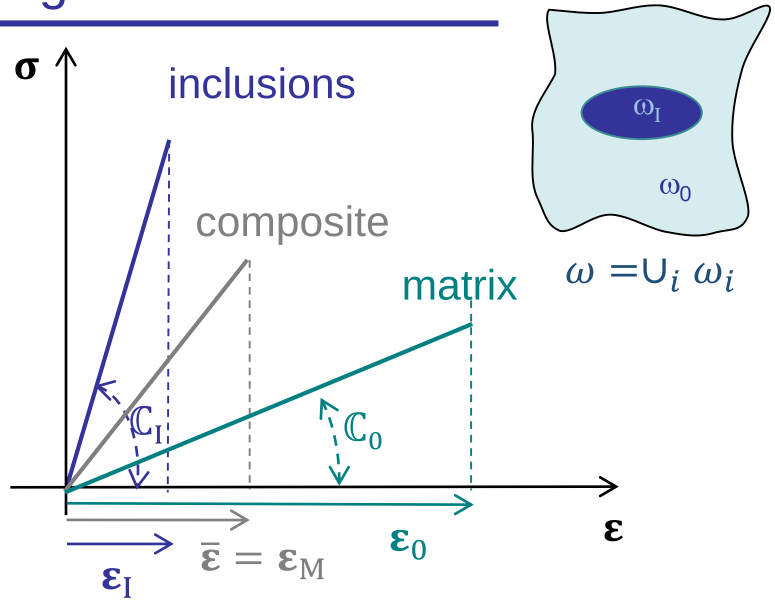


Stochastic Mean-Field Homogenization

- Mean-Field-Homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_M = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_I \boldsymbol{\sigma}_I \\ \boldsymbol{\varepsilon}_M = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_I \boldsymbol{\varepsilon}_I \\ \boldsymbol{\varepsilon}_I = \mathbb{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_I): \boldsymbol{\varepsilon}_0 \end{cases}$$

$$\Rightarrow \hat{\mathbb{C}}_M = \hat{\mathbb{C}}_M(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_I, v_I)$$



  - We use Mori-Tanaka assumption for $\mathbb{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_I)$
- Stochastic MFH
  - How to define random vectors $\boldsymbol{\mathcal{V}}_{MT}$ of $\mathrm{I}, \mathbb{C}_0, \mathbb{C}_I, v_I$?

L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

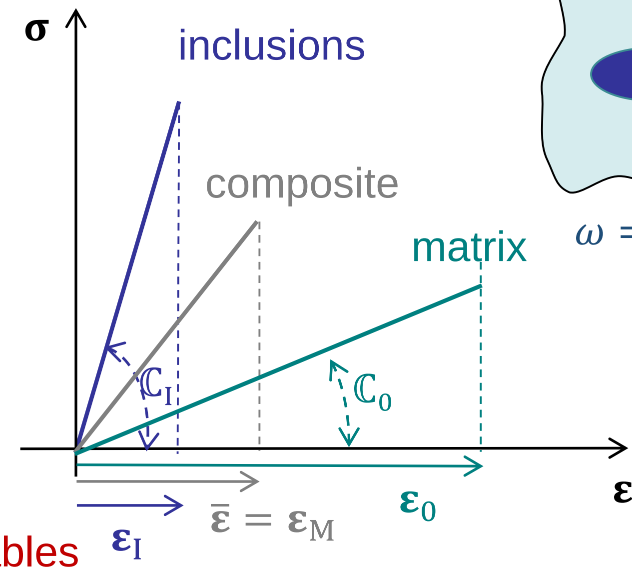
Stochastic Mean-Field Homogenization

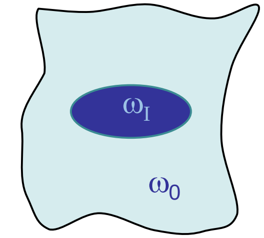
- Mean-Field-Homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{M}} = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbb{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}) : \boldsymbol{\varepsilon}_0 \end{cases}$$

$$\hat{\mathbb{C}}_{\mathrm{M}} = \hat{\mathbb{C}}_{\mathrm{M}}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}})$$

Defined as random variables

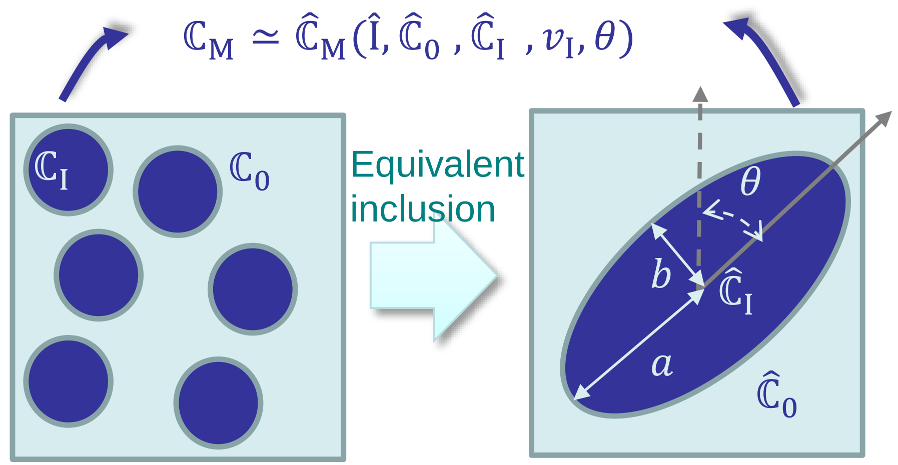




$\omega = \cup_i \, \omega_i$

- Consider an equivalent system
  - For each SVE realization i:
    - $\mathbb{C}_{\mathrm{M}}$ and v_{I} known
  - Anisotropy from $\mathbb{C}_{\mathrm{M}}^{i}$
    - θ is evaluated
  - Fiber behavior uniform
    - $\hat{\mathbb{C}}_{\mathrm{I}}$ for one SVE
  - Remaining optimization problem:

$$\mathbb{C}_{\mathrm{M}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}(\hat{\mathrm{I}}, \hat{\mathbb{C}}_0, \hat{\mathbb{C}}_{\mathrm{I}}, v_{\mathrm{I}}, \theta)$$

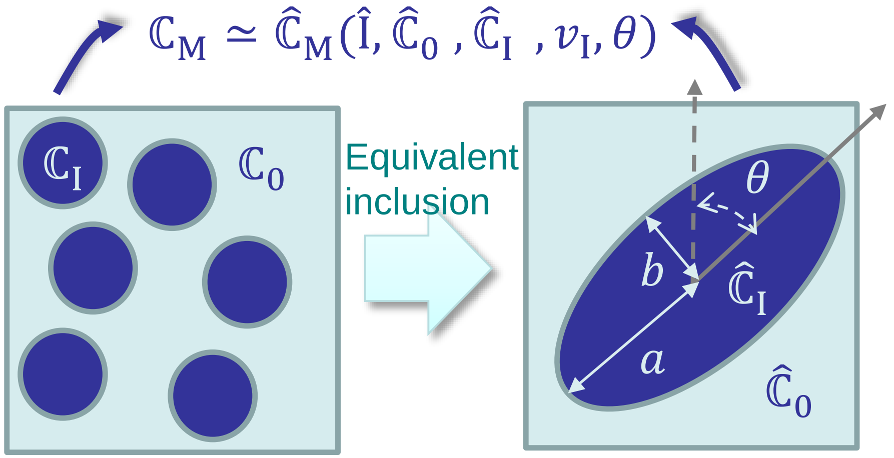


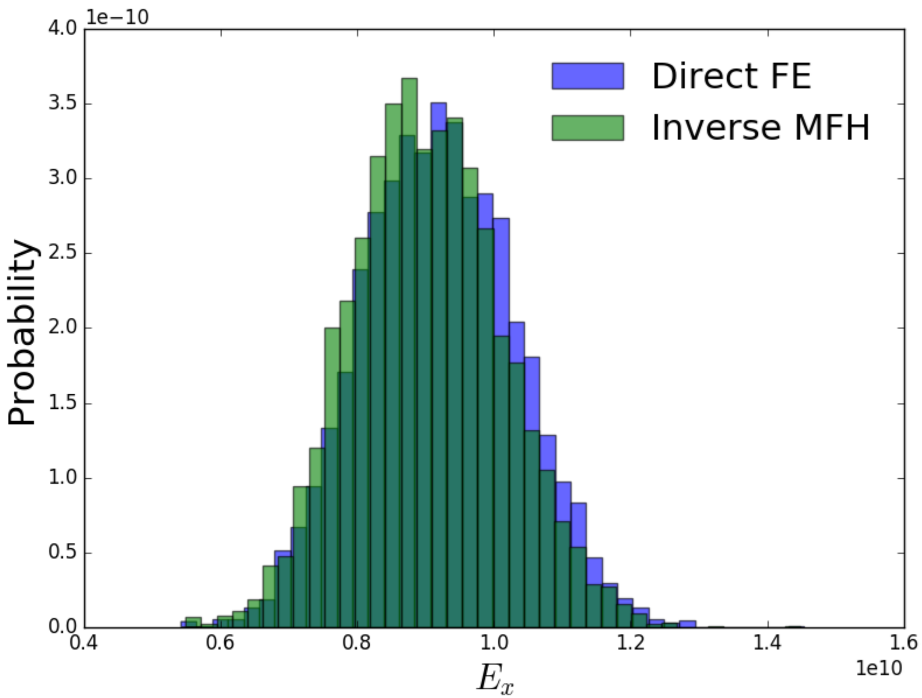
$$\min_{\frac{a}{b}, \hat{E}_0, \hat{\nu}_0} \left\| \mathbb{C}_{\mathrm{M}} - \hat{\mathbb{C}}_{\mathrm{M}}\left(\frac{a}{b}, \hat{E}_0, \hat{\nu}_0 ; v_{\mathrm{I}}, \theta, \hat{\mathbb{C}}_{\mathrm{I}}\right) \right\|$$

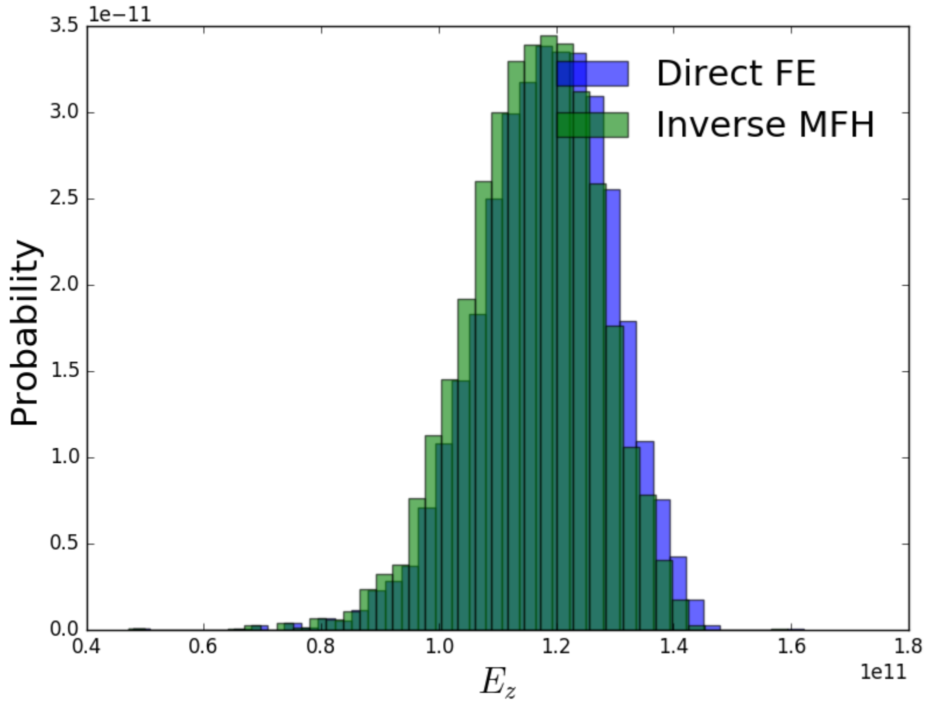
L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

Stochastic Mean-Field Homogenization

- Inverse stochastic identification
  - Comparison of homogenized properties from SVE realizations and stochastic MFH



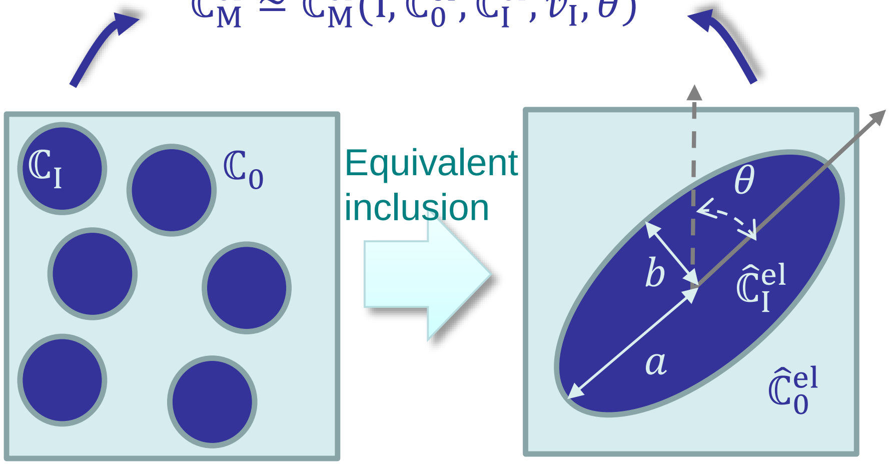


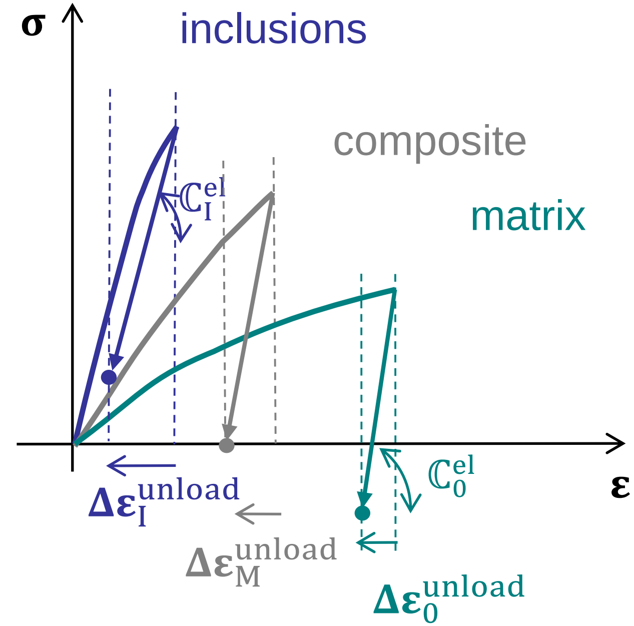


L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

- Damage-enhanced inverse identification
  - First step from elastic response
    - Before damage occurs

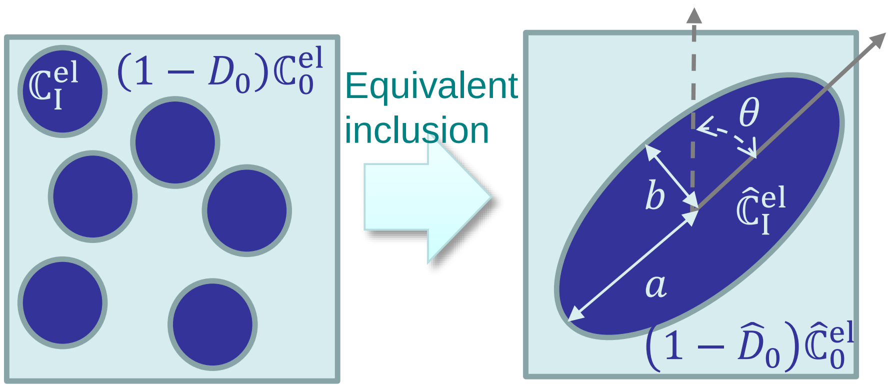
$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}(\hat{\mathrm{I}}, \hat{\mathbb{C}}_{0}^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta)$$

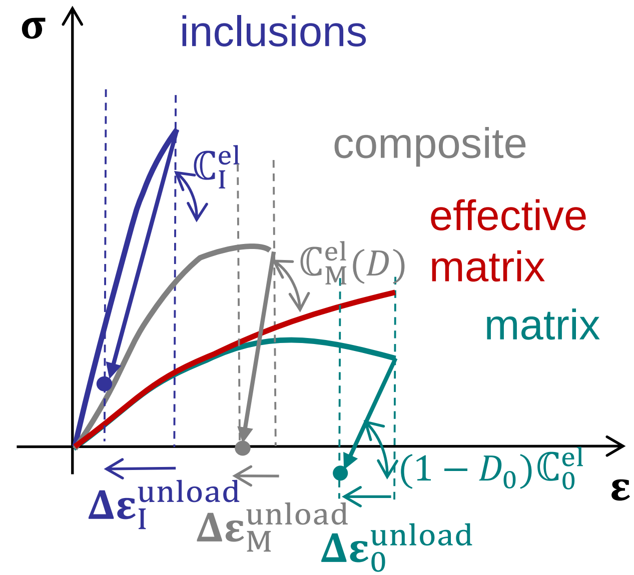




L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

- Damage-enhanced inverse identification
  - Second step: elastic unloading



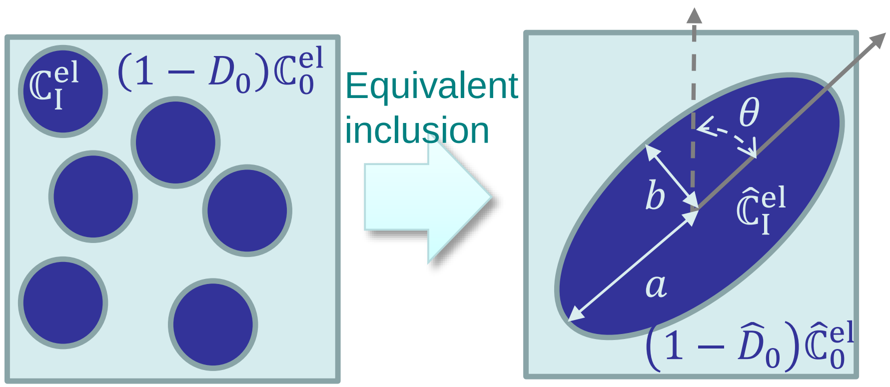


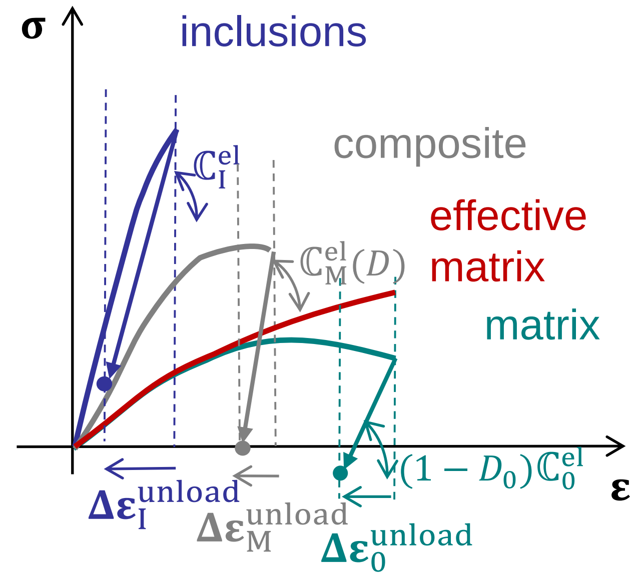
- Identify damage evolution $\hat{D}_0$

$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}}(D) \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}\left(\hat{\mathrm{I}}, \left(1-\hat{D}_0\right)\hat{\mathbb{C}}_0^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta\right)$$

L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

- Damage-enhanced inverse identification
  - Second step: elastic unloading



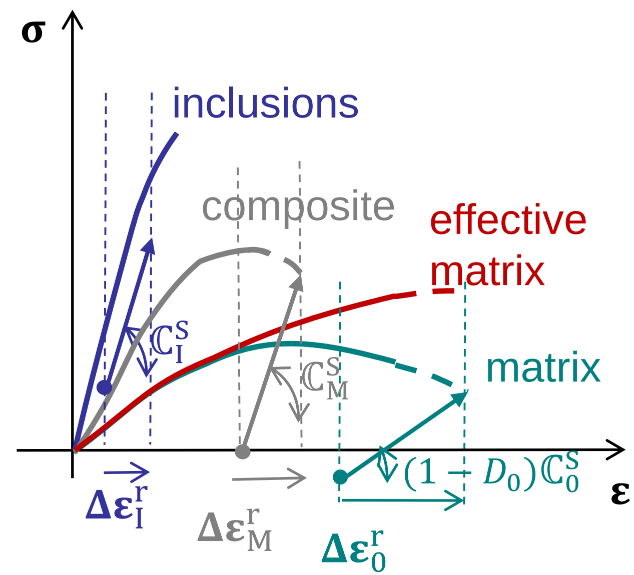


  - Identify damage evolution $\hat{D}_0$

$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}}(D) \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}\left(\hat{\mathrm{I}}, \left(1-\hat{D}_0\right)\hat{\mathbb{C}}_0^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta\right)$$

  - Third step from the LCC
    - $\boldsymbol{\Delta\sigma}_{\mathrm{M}} = \mathbb{C}_{\mathrm{M}}^{S}\left(\mathrm{I}, (1-D_0)\mathbb{C}_0^{S}, \mathbb{C}_{\mathrm{I}}^{S}, v_{\mathrm{I}}\right) : \boldsymbol{\Delta\varepsilon}_{\mathrm{M}}^{\mathrm{r}}$
    - Extract the equivalent hardening $\hat{R}(\hat{p}_0)$ & damage evolution $\hat{\mathrm{D}}_0(\hat{p}_0)$ from incremental secant tensor:

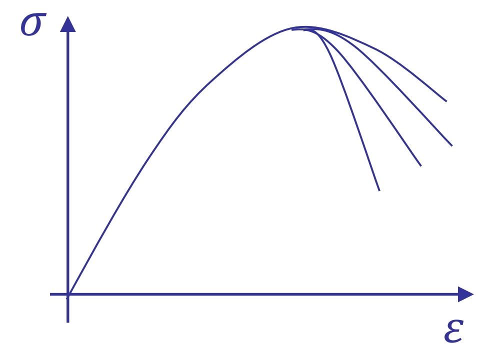
$$(1-\mathrm{D}_0)\mathbb{C}_0^{S} \simeq \left(1-\hat{\mathrm{D}}_0(\hat{p}_0)\right)\hat{\mathbb{C}}_0^{S}\left(\hat{R}(\hat{p}_0); \hat{\mathbb{C}}_0^{\mathrm{el}}\right)$$



L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

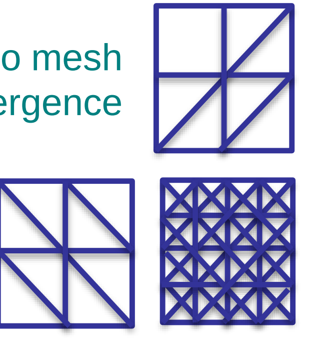
Micro-structural simulation of fiber-reinforced highly crosslinked epoxy

- UD Composites with RTM6 epoxy matrix
  - Identified matrix material behaviour
    - Hyperelastic viscoelastic-viscoplastic constitutive model enhanced by a multi-mechanism nonlocal damage model
      - To be used in micro-structural analysis
        - » Behaviour in composite is different
        - » Introduce a length-scale effect



- Resin model implementation:
  - Requires non-local form [Bažant 1988]
    - Introduction of characteristic length l_c
    - Weighted average: $\tilde{Z}(\boldsymbol{x}) = \int_{V_C} W(\boldsymbol{y};\boldsymbol{x},l_c)\ Z(\mathbf{y})d\boldsymbol{y}$
  - Implicit form [Peerlings et al. 1998]
    - New degrees of freedom: $\tilde{Z}$
    - New Helmholtz-type equations: $\tilde{Z} - l_c^2\ \Delta\tilde{Z} = Z$

Local: no mesh convergence



Micro-structural simulation of fiber-reinforced highly crosslinked epoxy

- UD Composites with RTM6 epoxy matrix
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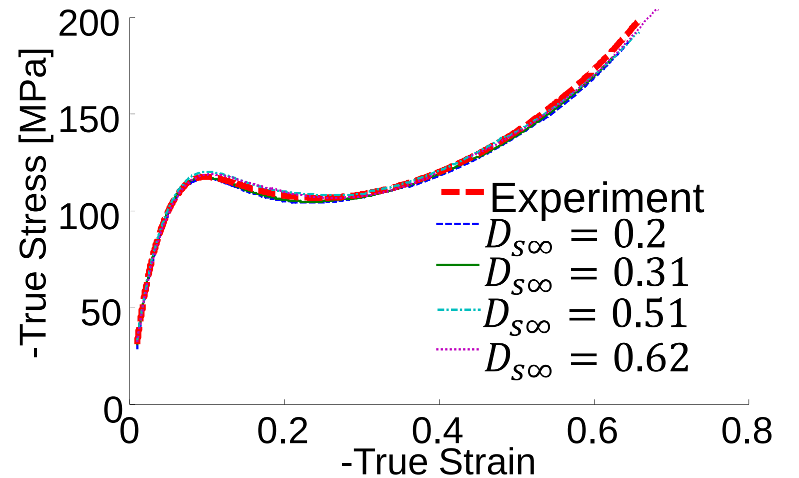
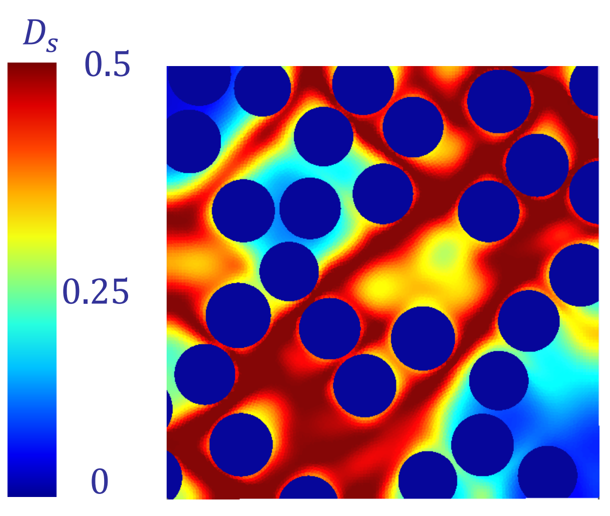
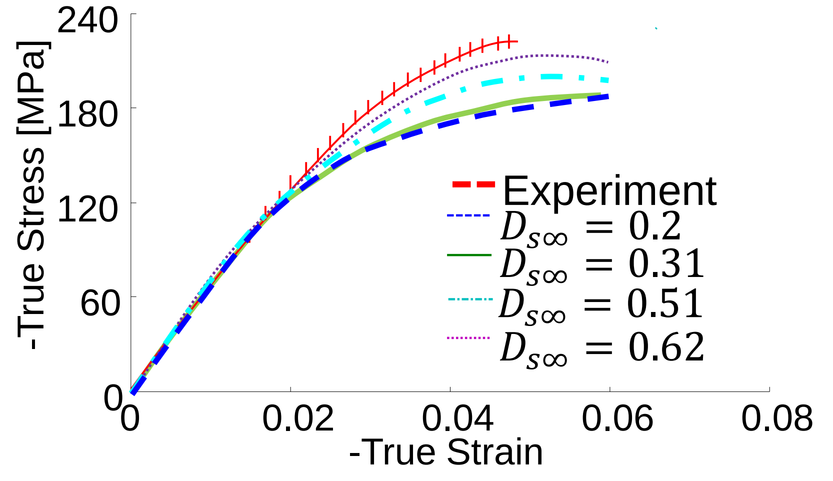
    - Hyperelastic viscoelastic-viscoplastic constitutive model enhanced by a multi-mechanism nonlocal damage model
- Resin model: saturated softening
  - Damage evolution

$$\begin{cases} \dot{D}_s = H_s(\chi_s - \chi_{s0})^{\zeta_s}(D_{s\infty} - D_s)\dot{\chi}_s \\ \chi_s = \max_\tau\left(\chi_{s0}, \tilde{\gamma}_s(\tau)\right) \\ \tilde{\gamma}_s - l_s^2\,\Delta\tilde{\gamma}_s = \gamma \end{cases}$$







V.-D. Nguyen, L. Wu, L. Noels, A micro-mechanical model of reinforced polymer failure with length scale effects and predictive capabilities. Validation on carbon fiber reinforced high-crosslinked RTM6 epoxy resin (2019)

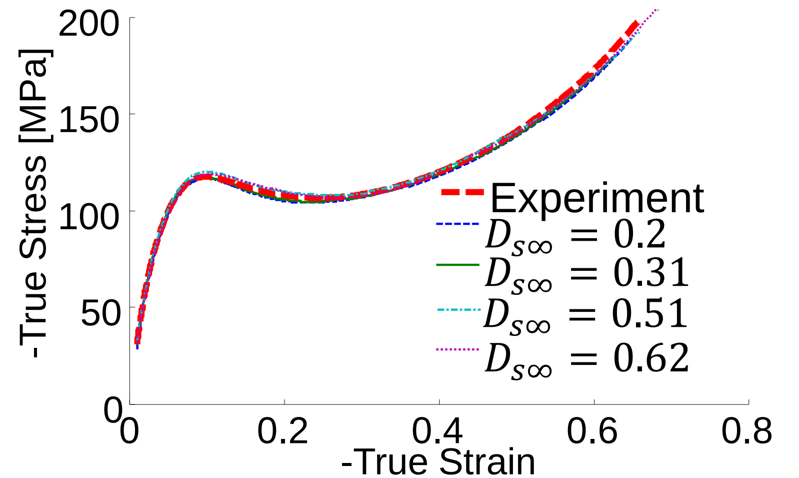
Micro-structural simulation of fiber-reinforced highly crosslinked epoxy

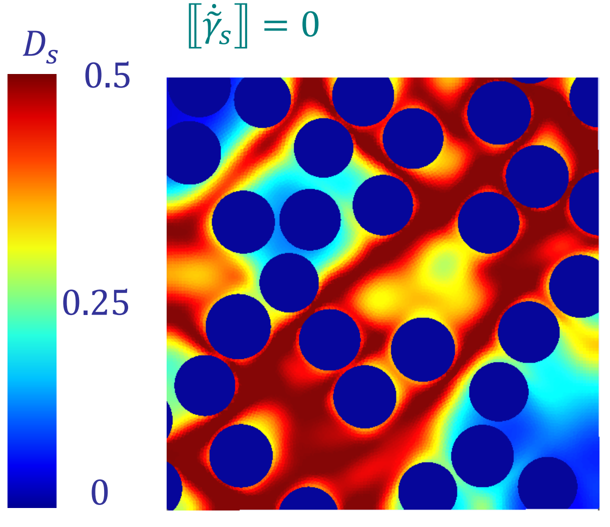
- UD Composites with RTM6 epoxy matrix
  - Identified matrix material behaviour
    - Hyperelastic viscoelastic-viscoplastic constitutive model enhanced by a multi-mechanism nonlocal damage model
  - Calibration
    - Several hardening/softening combinations
    - Requires composite material tests
      - Length scale effect

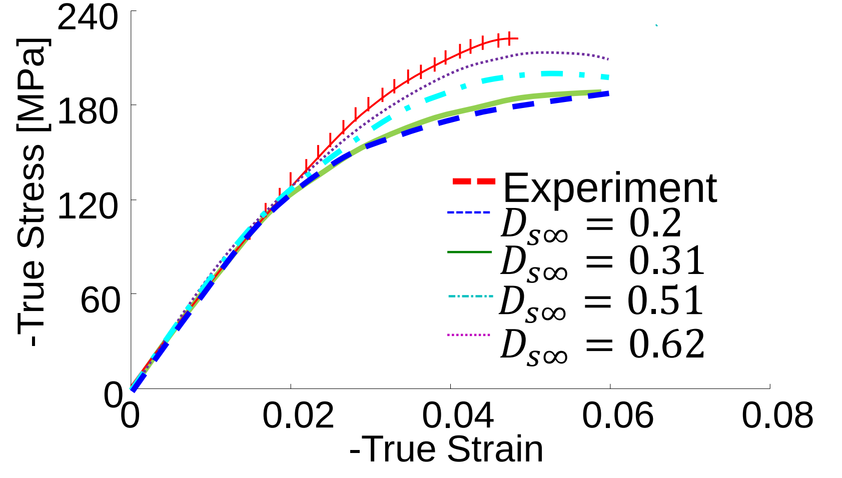
$$l_s = 3\mu\text{m}\left(1 - \frac{D_s}{D_{s\infty}}\right)$$

      - Non-local BC at fibre interface

$$[\![\dot{\gamma}_s]\!] = 0$$







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Micro-structural simulation of fiber-reinforced highly crosslinked epoxy

- Resin model: failure softening
  - Failure surface

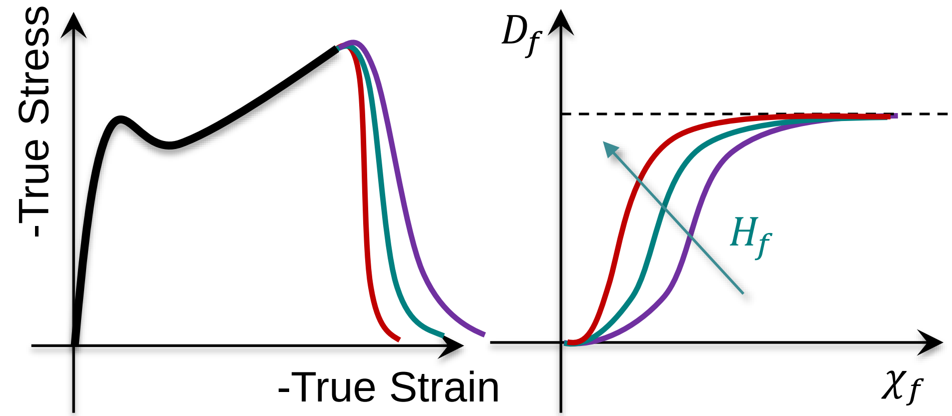
$$\begin{cases} \phi_f = \gamma - a \exp\left(-b \dfrac{\mathrm{tr}(\hat{\boldsymbol{\tau}})}{3\hat{\tau}^{eq}}\right) - c \\ \phi_f - r \leq 0;\ \dot{r} \geq 0; \text{and } \dot{r}\left(\phi_f - r\right) = 0 \\ \dot{\gamma}_f = \dot{r} \end{cases}$$

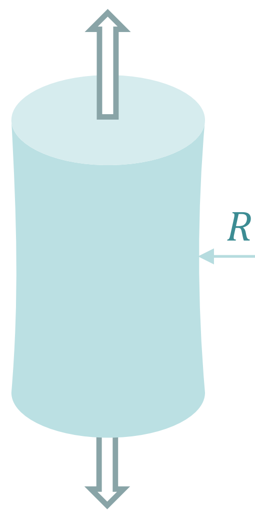
  - Damage evolution

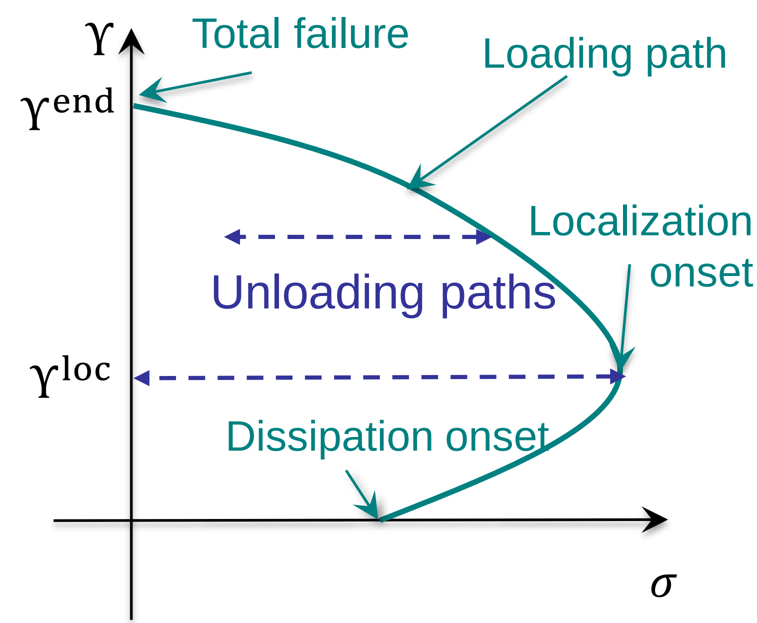
$$\begin{cases} \dot{D}_f = H_f(\chi_f)^{\zeta_f}(1 - D_f)^{-\zeta_d}\, \dot{\chi}_f \\ \chi_f = \max_{\tau}\left(\tilde{\gamma}_f(\tau)\right) \\ \tilde{\gamma}_f - l_f^2\, \Delta\tilde{\gamma}_f = \gamma_f \\ l_f = 3\ \mu\mathrm{m} \qquad \nabla_0\tilde{\gamma}_f \cdot \mathbf{N} = 0 \end{cases}$$

  - Affect ductility
  - Calibration
    - Recover the epoxy G_c
    - From localization simulation

$$G_c = \frac{\mathcal{D}_{V_0}^{\mathrm{end}} - \mathcal{D}_{V_0}^{\mathrm{loc}}}{A_0}$$







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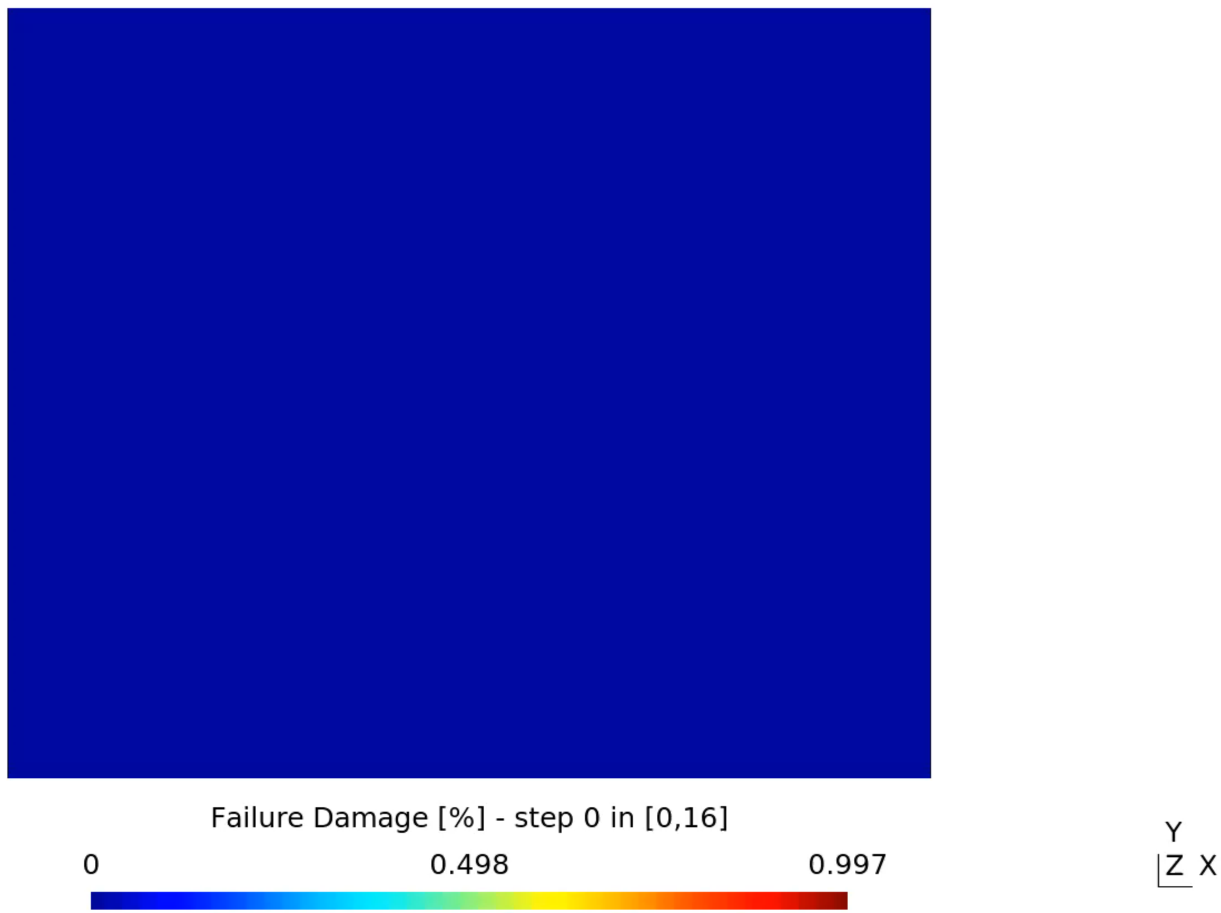
Full-Field UD composite simulations

- Non-linear SVE simulations



Full-Field UD composite simulations

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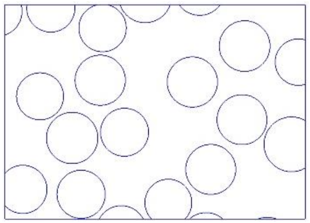
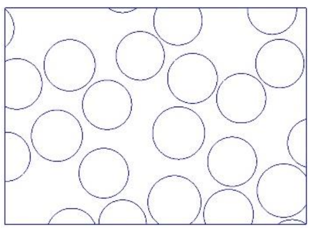
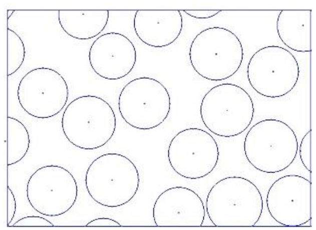
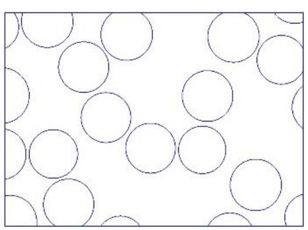
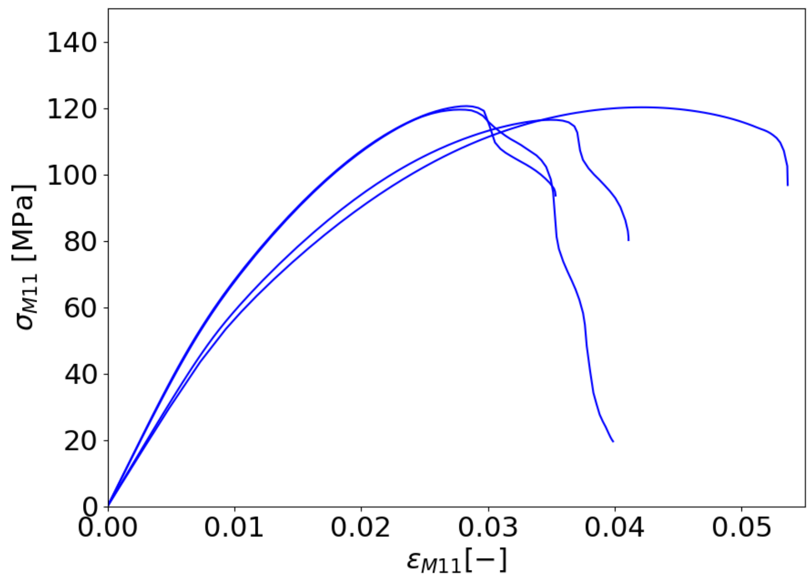





- Elastic Tensor $\mathbb{C}_{\mathrm{M}}^{i}$
- Strain-stress evolution
- Energy Release Rate (G_c)
- Fibre volume Fraction
- Maximum Attained Stress
- Strain at Maximum Stress



G_c [N/mm]

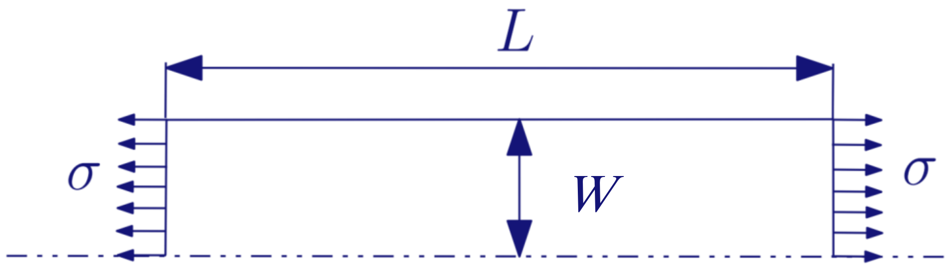
0.0408

0.0780

0.0654

0.1019

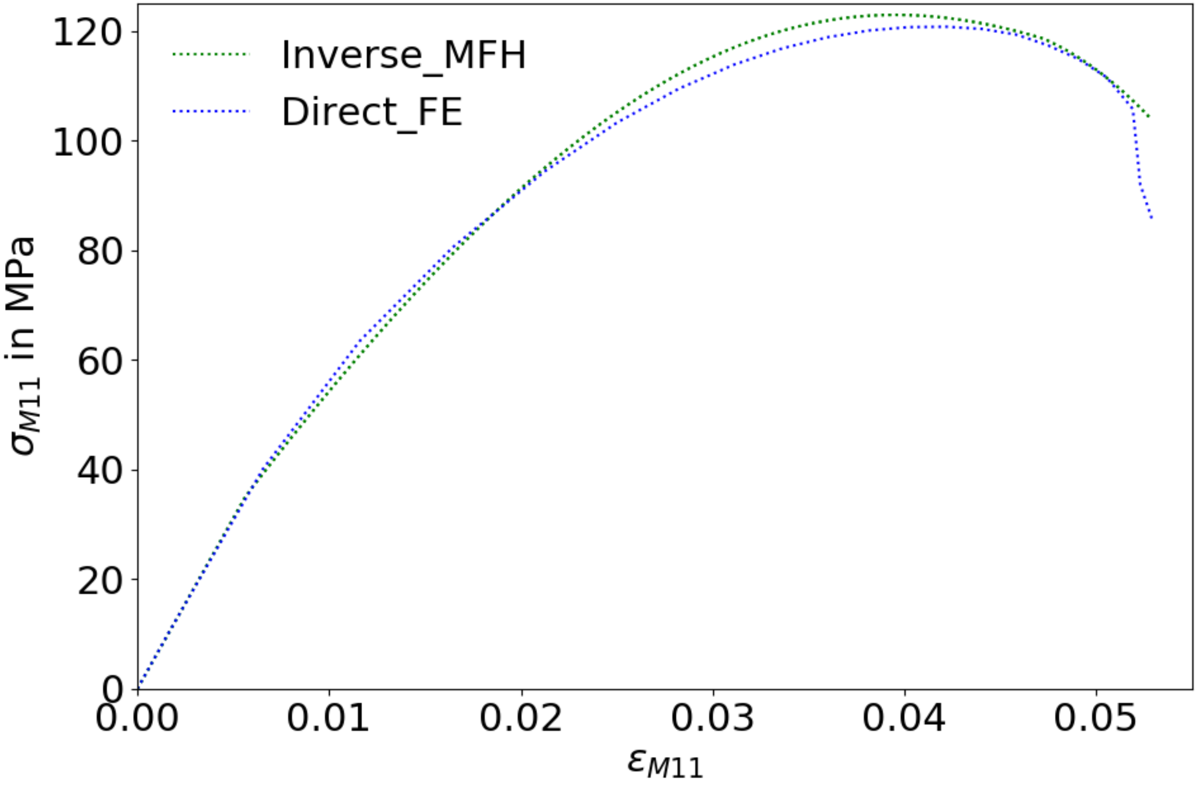
- Non-linear SVE simulations
  - Energy Release Rate (G_c) from MFH simulations
    - L >> l ; W << l to obtain non-dimensional G_c



- Damage evolution modelled with a two-part function:
  - Linear increase of damage up to onset point
    - Rate of the damage evolution depends on the damage and the accumulated plastic strain at onset point → All parameters obtained directly after inverse process
      - $\dot{\widetilde{D_0}} = \widetilde{D_0}_{Onset} / \widetilde{p_0}_{Onset} (\Delta\widetilde{p_0})$
  - After onset point, G_c becomes the only objective value. Objective function based on matching full-field G_c → Optimization process for parameter identification (α, β)
    - $\dot{\widetilde{D_0}} = \alpha(\widetilde{p_0} + \Delta\widetilde{p_0} - \widetilde{p_0}_{Onset})^{\beta}\, \Delta\widetilde{p_0}$

Non-linear stochastic Mean-Field Homogenization

- MFH model results:
  - The non-local length affect final G_c → Its value results from micro-scale simulations
    - Used non-local length of 5.5 μm



Method	G_c [N/mm]
Direct FE	0.0945
Inverse MFH	0.0932

- Good representation of SVE behavior up to failure point

Non-linear stochastic Mean-Field Homogenization

- On-Going work
  - Study of post-onset function parameter optimization process
  - Optimization of initial parameters to reduce iterations
  - Possibility of introducing a crack to obtain the desired G_c

- Next step
  - Perform multiple SVE simulations
    - Stochastic data for macro-scale simulations
  - Perform efficient macro scale simulations of a ply failure taking into account geometrical variabilities of the microstructure

Thank you for your attention!

Special thanks to:

Wallonie

STOMMMAC project

fnrs

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