THE $\Delta\theta$-$z_{\rm s}$ RELATION AS A COSMOLOGICAL TEST

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Recently, it was noted by Park & Gott[1] that there is a statistically significant, strong, negative correlation between the image separation $\Delta\theta$ and source redshift z_s for gravitational lenses. Park & Gott explored several effects which could cause the observed correlation, but no combination of these can explain the observations with a realistic scenario. Here, I point out that in an inhomogeneous universe a negative correlation is expected regardless of the value of k. I also compare the results of the test from the Park & Gott sample to those using other samples of gravitational lenses and explore whether the $\Delta\theta$-z_s relation can be used as a test to determine λ_0 and Ω_0, rather than just the sign of k.

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`http://multivac.jb.man.ac.uk:8000/helbig/research/publications/info/pocowo97.html`

For a singular isothermal sphere, the image separation is given by[2]

$$\Delta\theta = 4\pi \left(\frac{v}{c}\right)^2 \frac{D_{\rm ds}}{D_{\rm s}} \tag{1}$$

where v is the velocity dispersion. For a given v, one can show that

$$\frac{\Delta\theta(z)}{\Delta\theta(0)} = \left(\int_0^{z_s} \frac{D_{\rm ds}^3 D_{\rm d}^2 \left(1+z_{\rm d}\right)^2}{D_{\rm s}^3 Q(z_{\rm d})}\right) \left(\int_0^{z_s} \frac{D_{\rm ds}^2 D_{\rm d}^2 \left(1+z_{\rm d}\right)^2}{D_{\rm s}^2 Q(z_{\rm d})}\right)^{-1} \tag{2}$$

where

$$Q(z_{\rm d}) = \sqrt{\Omega_0 \left(1+z_{\rm d}\right)^3 - \left(\Omega_0 + \lambda_0 - 1\right)\left(1+z_{\rm d}\right)^2 + \lambda_0} \tag{3}$$

The D_{ij} (with $D_k := D_{0k}$) are angular size distances, which depend on $z_{\rm d}$, $z_{\rm s}$, λ_0, Ω_0 and η.[3] Park & Gott's analysis implicitly assumed an ideally homogeneous universe; generally, the more inhomogeneous the universe, the stronger the tendency for a negative correlation between $\Delta\theta$ and $z_{\rm s}$ regardless of the cosmological model.

I examined four gravitational lens samples: that used by Park & Gott (PG); PG with the addition of $0218+357$, for which $\Delta\theta$ and $z_{\rm s}$ were known when Park & Gott made their results known; the JVAS/CLASS sample; and the union of PG and the JVAS/CLASS samples. I used the method of Park & Gott, which uses the Spearman rank correlation test to generate a relative probability

Table 1: The JVAS/CLASS gravitational lenses

Name	#images	$\Delta\theta$ [$''$]	lens galaxy type	$z_{\rm d}$	$z_{\rm s}$
0218+357	ring + 2	0.33	spiral	0.6847	0.96
0414+0534	4	2.0	elliptical	?	2.62
0712+472	4	1.2	?	0.406	1.34
1030+074	2	1.6	peculiar	0.599	1.53
1422+231	4	1.2	?	0.65	3.62
1600+434	2	1.4	spiral	0.415	1.57
1608+656	4	2.2	spiral?	0.64	1.39
1933+503	4+4+2	0.9	?	0.755	?
1938+666	4+2	0.9	?	?	?
2045+265	4+1?	2.0	?	0.87	1.28
2114+022	2+2?	2.4	?	0.316	0.588?

for a given cosmological model. I did calculations for many cosmological models in a large region of the λ_0-Ω_0 plane, for all four samples. Park & Gott noted the fact that they always obtained a low probability with their sample, even when allowing for possible effects which would tend to result in a more negative correlation between $\Delta\theta$ and $z_{\rm s}$, including $k \neq 0$ cosmological models.

I find that the probability is a weak function of the cosmological model and a strong function of the sample used. Thus, the test is not very useful and/or it is pointing to unknown selection effects in the literature sample used by Park & Gott. A possibility not examined by Park & Gott, namely an inhomogeneous universe, can produce a negative correlation regardless of the sign of the curvature, but it too is not strong enough to account for the effect. As a general test for the values of λ_0 and Ω_0 the test is of no use, all cosmological models being assigned roughly the same probability, but *which* value they are assigned depends on the sample used.

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