The $\Delta\theta$-z_{s} relation as a cosmological test

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Abstract

Recently, it was noted by Park & Gott (1997) that there is a statistically significant, strong, negative correlation between the image separation $\Delta\theta$ and source redshift z_{s} for gravitational lenses. This is somewhat puzzling if one believes in a flat ($k = 0$) universe, since in this case the typical image separation is expected to be independent of the source redshift, while one expects a negative correlation in a $k = -1$ universe and a positive one in a $k = +1$ universe. Park & Gott explored several effects which could cause the observed correlation, but no combination of these can explain the observations with a realistic scenario. Here, I explore this test further in three ways. First, I show that in an inhomogeneous universe a negative correlation is expected regardless of the value of k. Second, I test whether the $\Delta\theta$-z_{s} relation can be used as a test to determine λ_0 and Ω_0, rather than just the sign of k. Third, I compare the results of the test from the Park & Gott sample to those using other samples of gravitational lenses, which can illuminate (unknown) selection effects and probe the usefulness of the $\Delta\theta$-z_{s} relation as a cosmological test.

Introduction

Historically, there has been little interest in the $\Delta\theta$-z_{s} relation compared to other cosmological tests based on gravitational lensing statistics, perhaps because the inflationary paradigm (*e. g.* Guth (1981)), which began about the same time as the discovery of the first gravitational lens (Walsh *et al.* 1979), has become so influential. Since a flat ($k = 0$) universe is a robust prediction of inflation, many researchers assume this and consider only flat universes (or, at most, $k = -1$ cosmological models with $\lambda_0 = 0$). Due to the fact that for the popular single isothermal sphere model for a single-galaxy lens the image separation $\Delta\theta$ is *completely independent* of the source redshift z_{s} in a flat universe, there is little point in pursuing the $\Delta\theta$-z_{s} relation if one is interested primarily in flat cosmological models. If one is not committed to a flat universe, then of course one should not assume $k = 0$, but even if one believes that the universe must be flat, it is still important to test this belief observationally. The situation is somewhat worsened by the fact that most 'standard' cosmological tests (m-z and θ-z relations, 'conventional' gravitational lensing statistics, age of the universe) are relatively insensitive to the radius of curvature of the universe ($R_0 \sim 1/\sqrt{|\Omega_0 + \lambda_0 - 1|}$), being degenerate in combinations of Ω_0 and λ_0 in directions roughly perpendicular to lines of constant R_0 in the λ_0-Ω_0 plane. (A notable exception are constraints derived from CMB anisotropies.)

One of the goals of the CERES project is the determination of cosmological parameters from gravitational lensing statistics. Even though the observational tasks are not yet complete, the JVAS and CLASS surveys which constitute the data base have already yielded enough gravitational lenses to enable one to make an independent analysis, as in this poster. See the companion poster by Marlow *et al.* for another example of the uses to which this observational data base can be put.

Theory

For a singular isothermal sphere, the image separation is given by (Turner *et al.* 1984)

$$\Delta\theta = 4\pi\left(\frac{v}{c}\right)^2\frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \tag{1}$$

where v is the velocity dispersion. Even if the singular isothermal sphere is not a perfect model for the gravitational lens systems considered, it is still a good approximation when one is only concerned with the image separation. For a given v, one can show that

$$\frac{\Delta\theta(z)}{\Delta\theta(0)} = \left(\int_0^{z_{\mathrm{s}}}\frac{D_{\mathrm{ds}}^3 D_{\mathrm{d}}^2\left(1+z_{\mathrm{d}}\right)^2}{D_{\mathrm{s}}^3 Q(z_{\mathrm{d}})}\right)\left(\int_0^{z_{\mathrm{s}}}\frac{D_{\mathrm{ds}}^2 D_{\mathrm{d}}^2\left(1+z_{\mathrm{d}}\right)^2}{D_{\mathrm{s}}^2 Q(z_{\mathrm{d}})}\right)^{-1} \tag{2}$$

where

$$Q(z_{\mathrm{d}}) = \sqrt{\Omega_0\left(1+z_{\mathrm{d}}\right)^3 - \left(\Omega_0+\lambda_0-1\right)\left(1+z_{\mathrm{d}}\right)^2+\lambda_0} \tag{3}$$

The D_{ij} (with $D_k := D_{0k}$) are angular size distances, which are functions of the lens and source redshifts z_{d} and z_{s}, the cosmological parameters λ_0 and Ω_0 as well as the 'homogeneity parameter' η, which gives the fraction of smoothly, as opposed to clumpily, distributed matter along the line of sight. Note that Eq. (1) is valid for all combinations of λ_0, Ω_0 and η. The angular size distances can be computed for arbitrary combinations of these parameters by the method outlined in Kayser *et al.* (1997). A well-tested, portable Fortran code for the caculation of angular size distances for arbitrary values of λ_0, Ω_0 and η is publicly available from

`http://multivac.jb.man.ac.uk:8000/phillip/angsiz_prog/`

Figures 1 and 2 show $\Delta\theta$ as a function of z_{s} for various cosmological models, for $\eta = 1$ (the traditional case assuming a completely homogeneous universe) and $\eta = 0$ as extreme cases. Note in Fig. 1 that the curve is a horizontal line for $k = 0$, has positive slope for $k = +1$ and negative slope for $k = -1$. In Fig. 2, for $\eta = 0$, the slope is negative regardless of the value of k. Thus, it appears that an inhomogeneous universe, a possibility not investigated by Park & Gott, might be able to explain the puzzling negative correlation between $\Delta\theta$ and z_{s}.

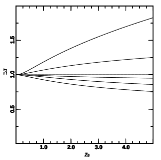

Figure 1: Normalised image separation as a function of source redshift. From the top, the (λ_0,Ω_0) values are (2,4), (0,4), $k = 0$, (0,0.7), (0,0.3) and (-5,1). For $k = 0$ the result is valid for all (λ_0,Ω_0) values whose sum is 1. $\eta = 1$.

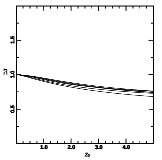

Figure 2: The same as Fig. 1 except that here $\eta = 0$.

Data

Figure 3 shows the sample of gravitational lenses, taken from the literature, which were used by Park & Gott in their analysis. Figure 4 includes the gravitational lens system $0218 + 357$, whose source redshift had been published at the time of the appearance of the Park & Gott paper (Lawrence 1996) and which lies at the extreme lower left of the group of points, weakening the puzzling negative correlation as is shown below. Although the source redshift in Lawrence (1996) was somewhat uncertain at the time of publication, it has since been confirmed.

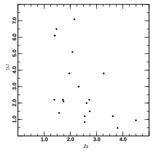

Figure 3: The sample used by Park & Gott.

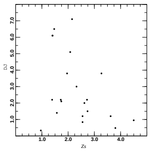

Figure 4: The sample used by Park & Gott with the addition of the gravitational lens system $0218 + 357$.

The table shows the current state of knowledge about the JVAS/CLASS gravitational lenses. (JVAS is the Jodrell Bank VLA Astrometric Survey (Patnaik *et al.* 1992); CLASS is the Cosmic Lens All-Sky Survey (de Bruyn *et al.* 1998).) Note that the questionable source redshift for $2114 + 022$ is probably the redshift of an additional lensing galaxy (this interpretation is supported by several independent lines of evidence).

Name	#images	$\Delta\theta$ [″]	lens galaxy type	z_{d}	z_{s}
0218+357	ring + 2	0.33	spiral	0.6847	0.96
0414+0534	4	2.0	elliptical	?	2.62
0712+472	4	1.2	?	0.406	1.34
1030+074	2	1.6	peculiar	0.599	1.53
1422+231	4	1.2	?	0.65	3.62
1600+434	2	1.4	spiral	0.415	1.57
1608+656	4	2.2	spiral?	0.64	1.39
1933+503	4+4+2	0.9	?	0.755	?
1938+666	4+2	0.9	?	?	?
2045+265	4+1?	2.0	?	0.87	1.28
2114+022	2+2?	2.4	?	0.316	0.588?

Figure 5 shows the CLASS sample while Figure 6 shows a union of the Park & Gott and CLASS samples.

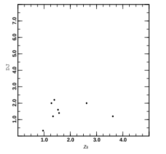

Figure 5: The CLASS sample of gravitational lenses.

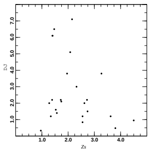

Figure 6: Combined sample: union of Park & Gott and CLASS samples.

Calculations

All calculations here implement the method of Park & Gott, which uses the Spearman rank correlation test to generate a relative probability for a given cosmological model. Park & Gott noted the fact that they always obtained a low probability with their sample, even when allowing for non-flat cosmological models (albeit in a limited area of parameter space), galaxy evolution or departure from the singular isothermal sphere model. As Park & Gott noted, allowing for these effects increases the probability, since they all tend to create a negative correlation in a flat universe, but the magnitude of the effect is not large enough to explain the observations. (Again as noted by Park & Gott, if the lenses are part of clusters, then this will work in the opposite direction, making the observed negative correlation even more puzzling.)

Results and discussion

Since the Park & Gott test assigns a low probability to a $k = 0$ universe, the question arises as to whether it can be used as a general cosmological test to determine the values of λ_0 and Ω_0. This is not the case, as is demonstrated in Figs. 7–10 for each of the four samples. The Spearman rank correlation probability is essentially constant over a wide range of parameter space—basically, either all cosmological models are probable, or all are improbable, depending on the sample used. Since there are no known selection effects which can account for the differences, either the test is not very useful and/or it is pointing to unknown selection effects in the literature sample used by Park & Gott.

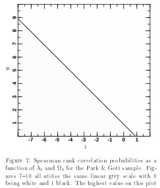

Figure 7: Spearman rank correlation probabilities as a function of λ_0 and Ω_0 for the Park & Gott sample. Figures 7–10 all utilise the same linear grey scale with 0 being white and 1 black. The highest value on this plot is 0.025, barely visible in the lower left corner. The diagonal line is the 1% contour level. The area above contains smaller values; larger ones are below.

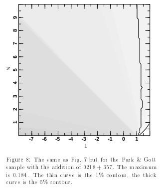

Figure 8: The same as Fig. 7 but for the Park & Gott sample with the addition of $0218 + 357$. The maximum is 0.184. The thin curve is the 1% contour, the thick curve is the 5% contour.

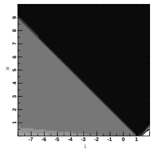

Figure 9: The same as Fig. 8 but for the CLASS sample. The maximum is 0.955.

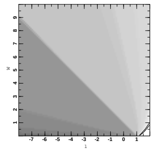

Figure 10: The same as Fig. 8 but for the combined sample. The maximum is 0.583.

It is interesting to compare the probabilities from the Spearman rank correlation test for the Park & Gott sample using the actual values of z_{s} and $\Delta\theta$ as used by Park & Gott to those obtained using more up-to-date data for the *same* lens systems. If two values are very near each other, rounding them off to the same values produces a different result for the rank correlation test than if they differ by even a small amount. Using more up-to-date data, an even lower probability is obtained for the Park & Gott sample, for $\eta = 1$ and $\eta = 0$, for a wide variety of cosmological models.

Another aspect of round-off error is seen in computing the Spearman rank probability for $k = 0$ models. Park & Gott give a probability of 0.012 for a flat universe. I can reproduce this value by using $\lambda_0 = 0.5$ and $\Omega_0 = 0.5$. Other values of λ_0 and Ω_0 (with the sum of 1, corresponding to $k = 0$) result in values between 0.008 and 0.017, while inserting $k = 0$ 'by hand' instead of doing the computations for explicit λ_0 and Ω_0 values (that is, using the fact that the $\Delta\theta$-z_{s} relation is flat in this case rather than computing it) results in a value of 0.011. This, and the problem with inexact observations mentioned above, suggest that the probabilities computed using the Spearman rank correlation test should be taken with a grain of salt.

Conclusions

Park & Gott pointed out that the image separations in gravitational lens systems are strongly significantly negatively correlated with the source redshift, while in a flat universe one would expect no correlation (while a negative correlation would be expected in a universe with negative curvature and a positive on in a universe of positive curvature). None of the possibilities they examined were strong enough to explain the effect. A possibility not examined by them, namely an inhomogeneous universe, produces a negative correlation regardless of the sign of the curvature, but it too is not strong enough to account for the effect. As a general test for the values of λ_0 and Ω_0 the test is of no use, all cosmological models being assigned roughly the same probability, but *which* value they are assigned depends on the sample used.

The strong dependence of the result on the sample used seems to indicate that the result of Park & Gott is due not to some physical cause but rather to ununderstood selection effects in the sample of gravitational lenses taken from the literature. The large number of CLASS lenses gives us an independent comparison sample, thus demonstrating the need for discovering a large number of lenses in a well-defined sample. As Park & Gott point out, since many conclusions based on 'conventional' gravitational lensing statistics are based on essentially the same lenses as in their literature sample, if this sample is for some unknown reason atypical, then conclusions drawn from statistical analyses of it must be examined with care. It will thus be interesting to see what conclusions can be drawn from a statistical analysis of the CLASS sample after the observational tasks have been completed. (We expect to find more lenses, but have no qualms about using the present incomplete sample in this analysis since there is no reason to believe that a larger sample would show a different $\Delta\theta$-z_{s} relation.)

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