Determining Cosmological Parameters through Statistics of Redshifts of Lens Galaxies

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Abstract: For known gravitational lens systems the redshift distribution of the lenses is compared with theoretical expectations for 10^4 Friedmann-Lemaître cosmological models, which more than cover the range of possible cases. The comparison is used for assigning a relative probability to each of the models. The entire procedure is repeated for different values of the inhomogeneity parameter η as well as H_0 and the limiting spectroscopic magnitude, which are important for selection effects. The dependence on these three parameters is examined in more detail for $\lambda = 0$ and $k = 0$.

The previous result of other authors that this method is a good probe for λ_0 is confirmed, but it appears that the low probability of models with large λ_0 values reported by these authors may be due to a selection effect. The power of this method to discriminate between cosmological models can be improved dramatically if more gravitational lens systems are found.

1 Introduction

It has recently been suggested by many authors (see, for example, Fukugita et al. (1992) and references therein) that gravitational lensing statistics can provide a means of distinguishing between different cosmological models, most effectively concerning the value of the cosmological constant. This is fortunate, since most of the classical methods for determining cosmological parameters are more sensitive to other quantities such as the density or deceleration parameter. It has even been suggested (Carroll et al., 1992) that gravitational lens statistics based on *current* observations *already* give the best upper limits on λ_0 for world models with $k > 0$, and are the most promising method of doing so for $k = 0$.

Kochanek (1992) has suggested a method based not on the total number of lens systems but rather on the redshift distribution of known lens systems characterised by observables such as redshift and image separation. Looking at a few different models, he concludes that flat, λ-dominated models are five to ten times less probable than more 'standard' models. It was my aim to extend this formalism[1] to arbitrary Friedmann-Lemaître cosmological models as well as to look at the influence of observational biases.

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[1] A formalism which has the advantages of being (almost) independent of the Hubble constant and not being plagued by normalisation difficulties as are most schemes involving the total number of lenses.

2 Theory

I make the 'standard assumptions' that the Universe can be described by the Robertson-Walker metric and that lens galaxies can be modelled as non-evolving singular isothermal spheres (SIS). If one drops the first assumption, the cosmological parameters λ_0, Ω_0 and H_0 lose their significance; the second assumption makes for easy calculation but, more importantly, is probably justified within the attainable accuracy (see Krauss & White (1992) for a discussion). In order to have a well-defined statistical quantity, which is based on the optical depth $d\tau$ for 'strong' lensing events,[2] this discussion is limited to gravitational lens systems with sources which are multiply imaged ($\rightarrow$ image separation) by isolated ($\rightarrow$ negligible cluster influence) single galaxies and with known source and lens redshifts. An additional requirement is that the system must have been found without any biases concerning the redshift of the lens. (See Kochanek (1992) for a discussion of these selection criteria.)

Making use of the fact that the SIS produces a constant deflection angle, i.e., independent of the position of the source with respect to the optical axis (defined as passing through observer and lens), one can define the angular cross section *of a single lens* for 'strong' lensing events (Turner et al., 1984):

$$\pi a^2 = 16\pi^3 \left(\frac{v}{c}\right)^4 \left(\frac{D_{ds}}{D_s}\right)^2 \tag{1}$$

where v is the one-dimensional velocity dispersion of the lens galaxy, D_d and D_{ds} the angular size distances between observer and lens and, respectively, lens and source. Following Kochanek (1992), one can arrive at an expression for the optical depth as follows:
For a *fixed mass* and mass distribution ($\rightarrow v$), world model and z_s, the *differential* optical depth due to all lenses of a given mass as a function of z_d is of course proportional to the number of lenses per z_d-interval. In order to arrive at an expression for $d\tau$ for a *fixed image separation*, one needs to know the relative number of lenses which, under the given circumstances, can produce this image separation. This can be done by using the Schechter luminosity function (Schechter, 1976) as well as the Faber-Jackson and Tully-Fisher relations (Faber & Jackson, 1976, Tully & Fisher, 1977), which give the dependence of the velocity dispersion on the luminosity for elliptical and spiral galaxies, respectively. Bringing in the familiar parameters and neglecting all terms which are concerned only with normalisation, one arrives, after some tedious but trivial calculations, at the expression

$$\frac{d\tau}{dz_d} = (1+z_d)^2 \frac{a}{a*}\frac{\gamma}{2}\left(\frac{a}{a*}\frac{D_s}{D_{ds}}\right)^{\frac{\gamma}{2}(1+\alpha)} D_d^2 \frac{1}{\sqrt{P(z_d)}} \exp\left(-\left(\frac{a}{a*}\frac{D_s}{D_{ds}}\right)^{\frac{\gamma}{2}}\right) \tag{2}$$

where $a* := 4\pi\left(\frac{v*}{c}\right)$ ($v* := v$ of an $L*$ galaxy), γ is the Faber-Jackson/Tully-Fisher exponent, α the Schechter exponent, D_d the angular size distance between the observer and the lens and $P(z_d) := (1+z_d)^2(\Omega_0 z_d + 1 - \lambda_0) + \lambda_0$. The optical depth depends on the cosmological model through $P(z_d)$ as well as through the angular size distances, because of the fact that $D_{ij} = D_{ij}(z_i, z_j; \text{cosmological model})$. In general, there is no analytic expression for the D_{ij}, which also depend on η; they can be obtained by the solution of a second-order differential equation. (See Kayser (1985); for an equivalent derivation for $\lambda = 0$ see Schneider et al. (1992).)

If one has an efficient method of calculating the angular size distances, it is trivial to evaluate Eq. 2 for various world models described by the parameters λ_0, Ω_0 and η. (The influence of η, which gives the fraction of homogeneously distributed, as opposed to compact, matter is felt only in the calculation of the angular size distances, whereas the cosmological

[2]See Schneider et al. (1992) for a clarification of the concept of optical depth in lensing.

model in the narrower sense makes its influence felt here as well as through $P(z_d)$.) Worthy of note is the *independence* of Eq. 2 on the source luminosity function (which of course will generally itself depend on z_s as well), the relative numbers of galaxy types (the galaxy type for a particular lens is assumed to be known) and the fraction of galaxies in clusters (the method looks only at field galaxies); these factors have to be taken into account when doing statistics based on the total number of lenses. Also, Eq. 2 is insensitive to finer points of the mass model such as core radius and ellipticity (Krauss & White, 1992, Narayan & Wallington, 1992). The main idea is to compare the observed distribution of lens redshifts with theoretical expectations for various world models; the method is described in the next section.

Although there is an exponential cutoff towards $z_d = 0$ and $z_d = z_s$ caused by the fact that the required mass of the lensing galaxy for producing a given image separation diverges at these points and the number of such galaxies declines exponentially in the Schechter function, $d\tau$ can nevertheless take on appreciable values at intermediate redshifts *even though the lens galaxy would be too faint to be seen at the redshift in question.* In order to correct for this effect, I have calculated the redshift at which the lens galaxy would become too faint to have its redshift measured for the investigated cosmological model and truncated $d\tau$ at this point. (Details in the next section.) It is immediately obvious that failure to correct for the faintness of the lens galaxies will artificially exclude cosmological models with a high median redshift in Eq. 2, which might otherwise not be excluded.

3 Calculations

The following gravitational lens systems[3] meet the selection criteria: UM 673, 0218+357, 1115+080 (Triple Quasar), 1131+0456, 1654+1346 and 3C324. I considered the following ranges of values[4] for the cosmological parameters: $-10 < \lambda_0 < +10$, $0 < \Omega_0 < 10$, $0 < \eta < 1$ and $30 < H_0 < 110$ (H_0 is in the usual units of $\frac{\mathrm{km}}{\mathrm{s \cdot Mpc}}$). Keeping the other two parameters constant at their default values, I looked at 100×100 models in the λ_0-Ω_0 plane for $\eta = 0.0, 0.3, 0.5, 0.7, 1.0$, $H_0 = 40, 50, 70, 90$ and $m_{\mathrm{lim}} = 23.5, 24.5, \infty$ (Johnson R magnitudes). I used the following default values: $\eta = 0.5$, $H_0 = 70$ and $m_{\mathrm{lim}} = 23.5^{\mathrm{m}}$. In addition, I looked at 100×100 models in the η-Ω_0, H_0-Ω_0 and m_{lim}-Ω_0 planes for the special cases of $\lambda = 0$ and $k = 0$.

To measure the relative probability of a given cosmological model, I defined the quantity f as follows:

$$0 < f := \frac{\int_0^{z_l} d\tau}{\int_0^{z_s} d\tau} < 1$$

where z_l is the *observed* lens redshift for a particular system. The distribution of the different f values (one for each lens system in the sample) in b bins in the interval]0,1[gives the relative probability p of a given cosmological model, with $p = \prod_{i=1}^{b} \frac{1}{n_i!}$ (normally; if $m_{\mathrm{gal}} > m_{\mathrm{lim}}$ or $z_{s,\mathrm{max}} > z_{max}$ then $p := 0$) where n_i is the number of systems in the i-th bin. The variable b

[3]For observational data on these systems, see Surdej (1993).

[4]Of course, these are much larger than contemporary wisdom demands. However, I think that there are at least two reasons for using such large ranges:

- It would be an additional, though by no means necessary, point in favour of the validity of the method if it assigns the highest probability to a cosmological model within the presently accepted canonical parameter space.
- The history of cosmology shows that the prejudices of today are often out of fashion tomorrow.

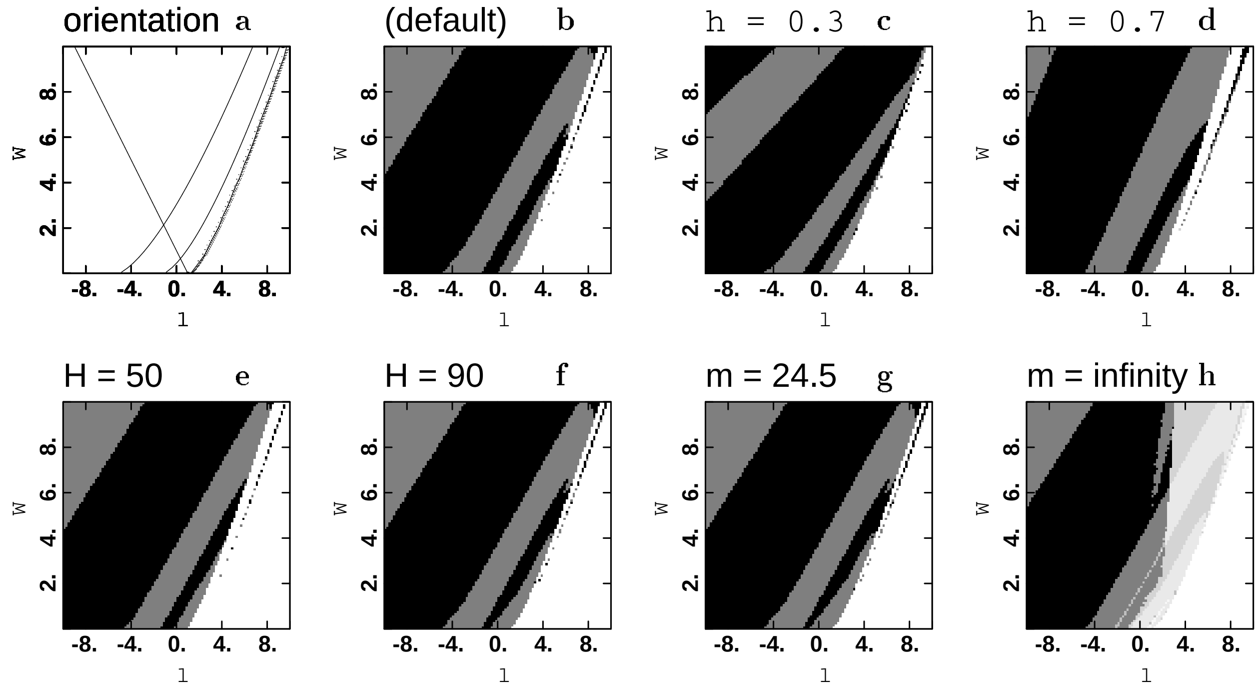

Figure 1: Plots of the relative probability in the λ_0-Ω_0 plane. Deviations from the default values are indicated.

is a free parameter, but it is easily seen that the most information is obtained when b is equal to the number of systems. The relative probability is thus 0 if the lens galaxy is too faint to have its redshift measured and/or if $z_{\rm max}$, the maximum redshift possible in the cosmological model in question, is smaller than the largest source redshift $z_{s,\rm max}$ in the sample. The apparent luminosity of the lens galaxy was calculated for the Johnson R-band using the K-corrections of Coleman, Wu & Weedman (1980). (These are based on displacement of standard spectra at $z = 0$ which extend into the UV-band and are given only up to $z = 2.0$, where evolutionary effects would in any case have to be considered. However, in most cases the galaxy becomes too faint at modest redshifts, so the assumption of no evolution is probably justified.)

4 Results and Discussion

Plot **a** gives some orientation in the λ_0-Ω_0-plane. The vertical line shows $\lambda_0 = 0$, the slanted line $k = 0$, the curve at the right $z_{\rm max} = 10$, the other two curves correspond to $t_0 h_0 = 5, 7.5$ in units of 10^9a (left to right, $h_0 := \frac{H_0 \cdot \rm s \cdot Mpc}{100 \cdot km}$); the area between the world age curve and the z_{max} curve is the 'allowed area' based on present knowledge. Taking **b** as an example plot, one notices first of all the region of relative probability 0 in the parameter space containing the 'bounce models'[5] and the fact that most of the 'structure' occurs in the 'allowed area'. Since the gradient runs more nearly parallel to the λ_0-axis than to the Ω_0-axis, one can learn more about λ_0 than about Ω_0.

Different values of η (plots **c** and **d**) produce a continuous transformation of the plot structure, but interestingly very little changes in the 'allowed area'. The fact that η plays a relatively unimportant rôle in the optical depth has previously been demonstrated in another manner by Fukugita et al. (1992).

[5]See Bondi (1952), Stabell & Refsdal (1966) or Feige (1992) for a discussion of the different models and their relation to the cosmological parameters.

Qualitatively, the same thing holds for different values of H_0 (**e** and **f**). For small values, a region of probability 0 covers part of the 'interesting' parameter space, but this should not be taken too seriously, resulting as it does from the sudden 'disappearance' of a lens galaxy—the calculated apparent magnitudes are probably accurate to only a magnitude or so (Kochanek, 1992). The fact that the structure in the plots does not severely and/or discontinuously change hints at the fact that the influence of the Hubble constant, which makes itself felt only in the calculation of the apparent magnitudes of the lens galaxies, is not too severe.

A comparison between **b** or **g** and **h** shows the effect of neglecting $m_{\rm lim}$; on the other hand, there is relatively little difference between **b** and **g**, so that the exact determination of $m_{\rm lim}$ is not crucial and one sees that using the same $m_{\rm lim}$ for all systems is good enough for a first approximation.

It can be qualitatively understood why the value of H_0 or $m_{\rm lim}$ doesn't exert a larger influence: around the redshift at which the lens galaxy becomes too faint, the graph of m as a function of z_d is very steep,[6] so that even a relatively large change in H_0 (moving the entire curve parallel to the m-axis) or $m_{\rm lim}$ (changing the cutoff value) corresponds to just a slight change in z_d. Since the graph of $d\tau$ as a function of z_d is typically not very steep at this value of z_d, a small change in the value of z_d at which the probability distribution is truncated makes little difference as far as the integral up to this point is concerned.

Returning to the example plot, one notices that, at least in the 'allowed area', roughly oblong areas of constant relative probability run approximately parallel to curves of constant world age.

5 Concluding Remarks

A comparison of the plots shows that neglecting the limiting magnitude produces more 'structure'. This arises from the fact that the current sample contains mostly lenses of relatively low redshift; one obtains a relatively low probability of world models with a higher median redshift. Correcting for this effect means looking only at the difference in distribution at low redshift, which makes the method less able to discriminate between different cosmological models. Nevertheless, plot **g** *gives an idea of* what could be done, if one were able to measure the redshifts of the faintest lens galaxies. For a given image separation, the brightness of the lens galaxy has a minimum at some intermediate redshift; this is typically at about $30^{\rm m}$ in R, so that larger telescopes and advances in image processing will probably be able to make substantial progress on this front in the next few years. If one were able to measure the lens redshift at the minimum brightness, this would have the side-effect of eradicating the dependence on H_0. On the other hand, probably more would be gained than lost, because it would no longer be possible to neglect evolutionary effects. For this reason, the most progress in the immediate future (barring a revolution in the understanding of evolutionary effects) will probably come from increasing the number of usable systems rather than from pushing $m_{\rm lim}$ to fainter values.

The dramatic difference caused by not neglecting $m_{\rm lim}$ casts doubt on the degree to which *present* observations, based only on the redshift distribution, are able to rule out certain cosmological models; in particular, flat models with a large cosmological constant, having a high median expected lens redshift, become more probable through introducing $m_{\rm lim}$.[7] It is reassur-

[6] The apparent brightness decreases much faster as a function of z_d than in the 'normal' case, because in this z_d-interval the required galaxy mass ($\rightarrow$ absolute brightness) for a given image separation typically decreases with increasing z_d; see Kochanek (1992).

[7] More information is available in theory by looking at not only the redshift distribution, i.e., the shape of the curve, but also the number of lenses, i.e., the area under the curve. This, however, introduces additional

ing that one can nevertheless see 'structure' in the 'allowed area', so that a larger sample might be able to put limits on cosmological parameters comparable to other methods.

At present, it is difficult to quantify the conclusions, since it is difficult to define, for example, a confidence region in the λ_0-Ω_0 plane. The values of the relative probabilities are known, of course, but it is too simple to conclude that a quotient of, say, 5 between two areas means that the one area is 'five times more likely' than the other; the fact that there are only (a few) *discrete* values of the relative probability makes the situation a bit more complicated. The best method seems to be to carry out simulations using artificial samples and then to use these to define a useful confidence region. This will also allow the statistical fluctuations (arising because of the small number of systems in the sample) to be taken into account.[8]

To conclude, I find that the method outlined here can probably be used to set useful limits on λ_0 and perhaps Ω_0, although at present there are too few useful systems to allow one to make firm confidence estimates, if one takes $m_{\rm lim}$ into account, as seems to be essential. A method of usefully quantifying the estimates probably requires information derived from simulations. As the number of systems increases and the interpretation of the results becomes clearer, it might prove useful to estimate $m_{\rm lim}$ for each system individually, taking into account all observational factors.

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uncertainties due to normalisation. It may be possible to obtain more information by considering the completeness of the sample, i.e., the fraction of systems with measured lens redshifts from the sample of systems which are otherwise suitable, as suggested by Kochanek (see the Discussion).

[8]Such a programme is currently under investigation.