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Constraints in the λ_0-Ω_0 plane from gravitational lensing

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Abstract. I review simultaneous constraints on the cosmological parameters λ_0 and Ω_0 from gravitational lensing. The emphasis is on systematic extragalactic surveys for strong gravitational lenses, mainly the largest and best-defined such survey, JVAS/CLASS.

1. Introduction

Since the details of the gravitational-lensing effect depend on the cosmological model, it offers a means of determining the cosmological parameters H_0, λ_0 and Ω_0 by comparing expectations from different cosmological models with observations. Advantages of using gravitational lensing to learn about the cosmological model include the fact that it is based on relatively well understood astrophysics and that it makes use of information from an intermediate redshift range, complementing tests which use information primarily from the low-redshift (e.g. cosmic flows) or high-redshift (e.g. cosmic microwave background fluctuations) regimes.

I review simultaneous constraints on the cosmological parameters λ_0 and Ω_0 from gravitational lensing. (Constraints on H_0 are discussed in other contributions in this volume; in particular, Schechter discusses constraints from lensing.) The emphasis is on systematic extragalactic surveys for strong gravitational lenses, mainly the largest and best-defined such survey, JVAS/CLASS. However, other methods of constraining λ_0 using (mostly strong) gravitational lensing are also discussed. After briefly reviewing the basic theory and history of the subject, I present the currently available constraints and briefly (since this is covered in other contributions in this volume, such as that by Lineweaver) touch on joint constraints with other cosmological tests. Finally, I discuss systematic errors and prospects for the future.

In general, one can derive constraints on λ_0 and Ω_0 from gravitational lensing when there is more than one source plane involved (e.g. Golse, Kneib, & Soucail 2000). This is the case in most examples of weak lensing and cluster lensing. Also, arc statistics (e.g. Bartelmann et al. 1998) can give information about the cosmological model, not only since more than one source plane is involved, but also since this is sensitive to the evolution of structure (the lenses in this context), which also depends on the cosmological model. Another possibility is provided by higher-order effects in measuring H_0 through gravitational-lens time delays; the time delay is inversely proportional to H_0, but there is a weaker, non-linear dependence on λ_0 and Ω_0. Finally, the statistical analysis of

gravitational-lens surveys provides a potentially powerful method of measuring the cosmological parameters.

2. Time Delay

The basic idea behind using the time delay in a gravitational-lens system, i.e. the time between seeing variations in the brightness of one image and seeing similar variations in the brightness of another image, is a simple one: Most observables in a gravitational-lens system (angles, brightness ratios etc) are dimensionless; measuring the time delay provides the scale for the lens system. Cosmological distances depend linearly on (the reciprocal of) H_0 but λ_0 and Ω_0 enter at higher order. Thus, measuring time delays in systems with various values of z_d and z_s might enable one to put constraints on λ_0 and Ω_0. This technique was already mentioned by Refsdal (1966) but at present there are too few lens systems and too many observational and lens-modelling uncertainties for this method to provide useful constraints on λ_0 and Ω_0. For a discussion of future prospects, see Haarsma, Lehár, & Barkana (2000).

The basic equation is

$$H_0 = (\Delta t)^{-1} T f \quad , \tag{1}$$

where f is a quantity which depends only on observables and the lens model and T is the cosmological correction function (Refsdal 1966, Kayser & Refsdal 1983). Since

$$T = \frac{H_0}{c} \frac{D_\mathrm{d} D_\mathrm{s}}{D_\mathrm{ds}} (1 + z_d) \frac{z_\mathrm{s} - z_\mathrm{d}}{z_\mathrm{d} z_\mathrm{s}} \quad , \tag{2}$$

there is the natural behaviour

$$T \to 0 \quad \text{for} \quad z_\mathrm{s} \to 0 \quad .$$

3. Gravitational-Lensing Statistics

3.1. Basic Principles

Gravitational-lensing statistics—the number of gravitational lenses found in a survey and their properties such as image configuration (including multiplicity, image separations and flux ratios), source and lens redshifts, nature of lens galaxies etc—depends on the cosmological model, the properties of the lens galaxy population, the properties of the source population and on selection effects. The influence of the cosmological is quite large, since two effects occur which tend to reinforce each other (see, e.g., Appendix A in Quast & Helbig (1999)):

- The volume element $\frac{\mathrm{dV}}{\mathrm{dz}}$ influences the number of lenses. It is of course the redshift z and related quantities which are observed. The volume element as a function of redshift is strongly dependent on the cosmological model. Thus, if one fixes the space density of galaxies at $z = 0$, varying the cosmological parameters can greatly vary the number of potential lens galaxies per redshift interval at higher redshift. (Of course, if one has *observed* the

number of potential lens galaxies per redshift interval at higher redshift, then this is an additional constraint; here, I assume that the space density of galaxies is fixed at $z = 0$ and free, i.e. determined by the cosmological model through $\frac{\mathrm{dV}}{\mathrm{dz}}$, at higher redshift.)

- The cross section of an individual galaxy depends on a combination of various angular size distances. The dependence of the angular size distances on the cosmological model (e.g. Kayser, Helbig, & Schramm 1997) thus means that the lensing cross section of an individual galaxy has a dependence on the cosmological model.

The total lensing cross section obviously depends on the number of potential lens galaxies and the cross section of an individual galaxy. Both of these depend on the cosmological model and the two effects tend to reinforce each other.

In general, the larger λ_0 the more lenses one expects, especially for $\lambda_0 > 0$. The number of lenses, however, is only one measurable quantity and due to parameter degeneracy, just 'counting lenses' will give weaker constraints. Traditionally, lensing statistics has tended to give comparatively tight upper limits on λ_0, especially in the often studied flat-universe case. It is important to keep in mind that, as with other constraints in the λ_0-Ω_0 plane, the actual values for the best-fit model are less interesting than more robust limits (cf. Helbig 1999).

Following the formalism in Kochanek (1996), for each source in a survey one can calculate the probability p_{lens} that it is a lens system with the observed properties; for non-lens-systems, the probability that they are non-lenses is obviously $1 - p_{\mathrm{lens}}$. For a given cosmological model, the total likelihood is just the product of the likelihoods for the individual objects in the survey, i.e. information from both the lenses and the non-lenses is used (see, e.g., Appendix A in Quast & Helbig (1999)). One can thus assign a relative probability to each cosmological model. Of course, there is a dependence on quantities other than λ_0 and Ω_0 as well. In the new results presented below, we consider all other variables to take fixed, observationally determined values and concentrate on the corresponding constraints in the λ_0-Ω_0 plane (see Helbig et al. (1999) and references therein for more details).

3.2. Some History

Turner, Ostriker, & Gott (1984) presented the first quantitative lensing-statistics analysis, but assumed $\lambda_0 = 0$. Fukugita et al. (1992) generalised the Turner et al. treatment to non-zero λ_0, and concluded that $\lambda_0 < 0.95$. Kochanek (1996) came to the more quantitative conclusions that $\lambda_0 < 0.66$ (95%) for $k = 0$ and $\Omega_0 > 0.15$ (90%) for $\lambda_0 = 0$. This paper is in some sense the definitive analysis, but it is important to remember that it is not based on the best input data. In particular, it is based on optical quasar surveys. Not only is the m-z relation for QSOs not particularly well-known—especially since the m-z relation should apply to QSOs with the same selection criteria as those in the gravitational-lens survey—but selection effects in optical gravitational-lens surveys are more difficult to quantify. It should be noted that Falco, Kochanek, & Muñoz (1998) obtain a higher value of λ_0 based on radio data. This gives some idea of the uncertainties involved. Using only radio data, Falco et al. (1998) obtain $\lambda_0 < 0.73$ (2σ) for $k = 0$; using combined radio and optical data, they get $\lambda_0 < 0.62$

(2σ) for $k = 0$. This paper is also interesting since it quantitatively details how the constraints change depending on various assumptions. Helbig et al. (1999) presented an analysis based on the JVAS gravitational-lens survey, which is reasonably large but also well understood. Since in the interesting part of parameter space, lensing statistics essentially measures $\lambda_0 - \Omega_0$, reducing the results to just a few numbers gives $-2.69 < \lambda_0 - \Omega_0 < 0.68$ (95%); for $k = 0$: $-0.85 < \lambda_0 < 0.84$.

In passing, it should be remembered that the *redshift of lens galaxies alone* is also sensitive to the cosmological model, but the situation here is not clear (see Helbig 2000a for discussion and references).

3.3. Radio Lens Surveys

There are many good reasons to do a gravitational-lens survey in the radio as opposed to optical:

- Using interferometry, the beam size is much smaller than the image separation.
- Flat-spectrum objects are compact (i.e. (almost) point sources), allowing typical lensing morphologies to be recognised easily.
- Somewhat related to the previous point, the lensing probability depends on the cross section determined by the lens population; for extended sources, the source geometry partially determines what is recognised as a lens system.
- Most sources are quasars at high redshift, which leads to a high lensing rate.
- Since flat-spectrum objects are compact, they can be variable on relatively short timescales, which aids in determining time delays.
- There is no bias from lens galaxies due to extinction by the lens or comparable brightness of source and lens, as can be the case with optical surveys.
- High-resolution followup is possible with interferometers such as MERLIN, the VLBA, VLBI....

But there is one disadvantage:

- Additional work is required to get redshifts.

By far the largest gravitational-lens survey is CLASS, the Cosmic Lens All-Sky Survey (e.g. Helbig 2000b). (JVAS is essentially a subset of CLASS, consisting of the stronger sources.) CLASS covers the range in image separation of 0.3–6 arcsec. It was later extended up to 15 arcsec, and a smaller survey using a different strategy extends up to 60 arcsec. (These additions are not reflected in the table.) One wide-separation lens candidate remains (see Phillips et al., these proceedings).

While the actual survey is complete, the followup of the lens system is not. Thus, here I present only preliminary results: $-0.8 < \lambda_0 - \Omega_0 < 0.3$ (95%); for

Table 1. Basic statistics of the JVAS and CLASS gravitational-lens surveys. 'Extra' lens systems are lens systems which were found and followed up but are not part of the statistically complete sample. The '+1' refers to lens candidates which will probably be confirmed.

	JVAS	CLASS	both
sources	2308	6976	9284
lens systems in complete sample	5	10+1	15+1
'extra' lens systems	1	2	3
total lens systems	6	12+1	18+1

$k = 0$: $0.1 < \lambda_0 < 0.65$. Note that the *lower* limit on λ_0 (cf. Quast & Helbig 1999) in a flat universe is >0. It is important to realise the difference between constraints being *compatible with* a certain cosmological model and *favouring* a certain cosmological model. For instance, until recently the constraints in the λ_0-Ω_0 plane from gravitational-lensing statistics were so broad that the Einstein-de Sitter model was not ruled out. However, this was never the *preferred* model; many cosmological models were compatible with the data. Also, since the best-fit model usually occurs in a region of parameter space where the gradient in the probability density is rather flat, the actual best-fit model is very sensitive to noise in the input data; more interesting are the much more stable regions enclosed by a given confidence-level contour.

A more definitive analysis will be done when all lenses are confirmed and enough observational data on lens systems and on the source population are available, taking all errors into account:

- statistical errors (including propagation from errors on input quantities)
- systematic errors $\longrightarrow$ statistical errors, i.e. we hope to eliminate systematic errors, mainly due to our ignorance about details of the source population, by turning them into statistical errors by observationally constraining the corresponding quantities
- sample variance (due to relatively small number of lens systems)

4. Conclusions and Outlook

- Despite vast improvements on both the observational and theoretical sides, the upper limit on λ_0 has been a remarkably stable number in the astronomical literature.
- We can already place a *statistically* strong upper limit on λ_0.
- Gravitational-lensing statistics is already hinting at a positive lower limit on λ_0, at least in a flat universe. While there is abundant evidence for a positive cosmological constant based on various combinations of cosmological tests, at present only the m-z relation for type Ia supernovae (see

e.g. the contributions by Perlmutter and Kirschner in these proceedings) is the only cosmological test which indicates this *by itself*; it would thus be nice to have another test which can do so.

- When the analysis of the CLASS gravitational-lens survey is complete, this will improve in the sense that the statistical errors will become smaller and systematic effects will be reduced; whether or not the upper limit on λ_0 decreases depends, of course, on what the value of λ_0 actually is and to what extent current estimates are biased.

- We will have a *robust* upper limit on λ_0 soon when the S-z plane for CLASS is better understood. The S-z plane enters into the calculation in two respects, since it determines both the flux-density dependent redshift distribution, which is needed to estimate the redshifts of the non-lensed sources in CLASS, and the redshift-dependent luminosity function, needed for the calculation of the amplification bias. (See McKean et al., these proceedings, for some information about current work in this area.)

- We will be able to provide an *independent* check on constraints from m-z relation (SN Ia), CMB, (evolution of) LSS and clusters, weak lensing, cluster lensing etc.

- Looking farther ahead, many more lens systems will be found in the future in both radio and optical surveys. Since the Poisson noise due to the small number of systems is at present a source of appreciable uncertainty, progress can be expected just from finding more lens systems (after doing the corresponding analysis, of course).

- Even when the cosmological parameters are known, perhaps more precisely through (a combination of) other methods, gravitational-lensing statistics will still be interesting: It will be possible to use gravitational-lensing statistics to constrain variables other than λ_0 and Ω_0, and thus study galaxy evolution, the source population etc.

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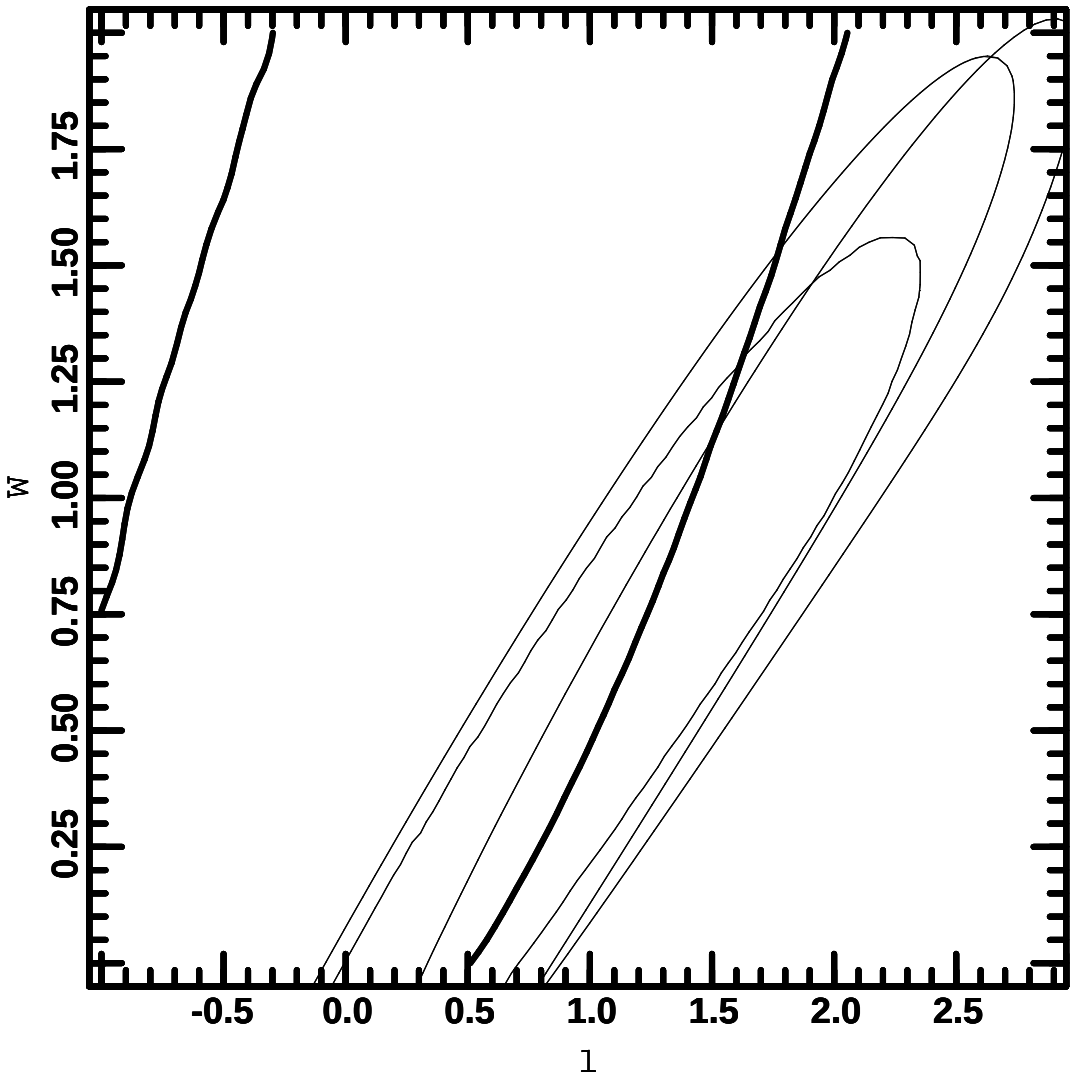

Figure 1. Degeneracy of constraints in the λ_0-Ω_0 plane for gravitational-lensing statistics (thick curves) and the m-z relation for type Ia supernovae (three different results). All curves are 95% confidence contours. See Helbig (1999), from which this figure is taken, for more details.

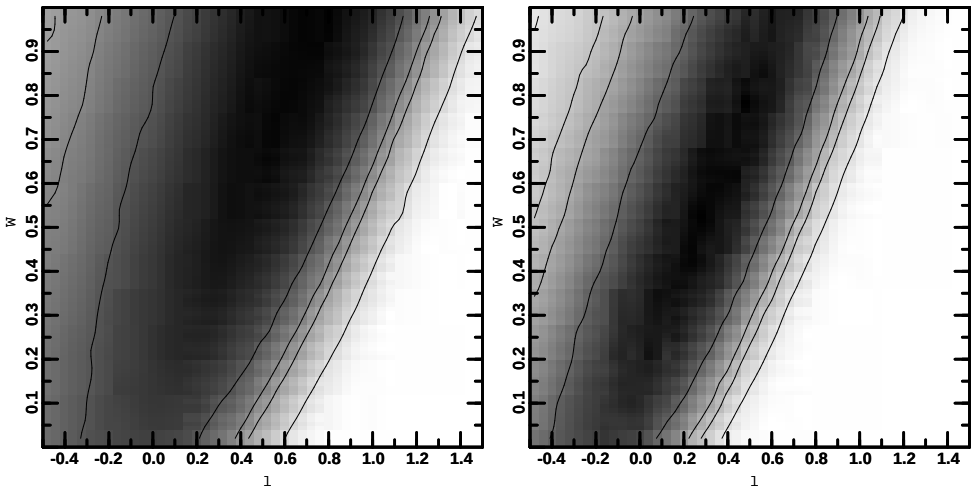

Figure 2. Constraints in the λ_0-Ω_0 plane from JVAS (cf. Helbig et al. 1999) (left) and CLASS (excluding JVAS) (right). Darker means higher likelihood; contours are at 68%, 90%, 95% and 99%.

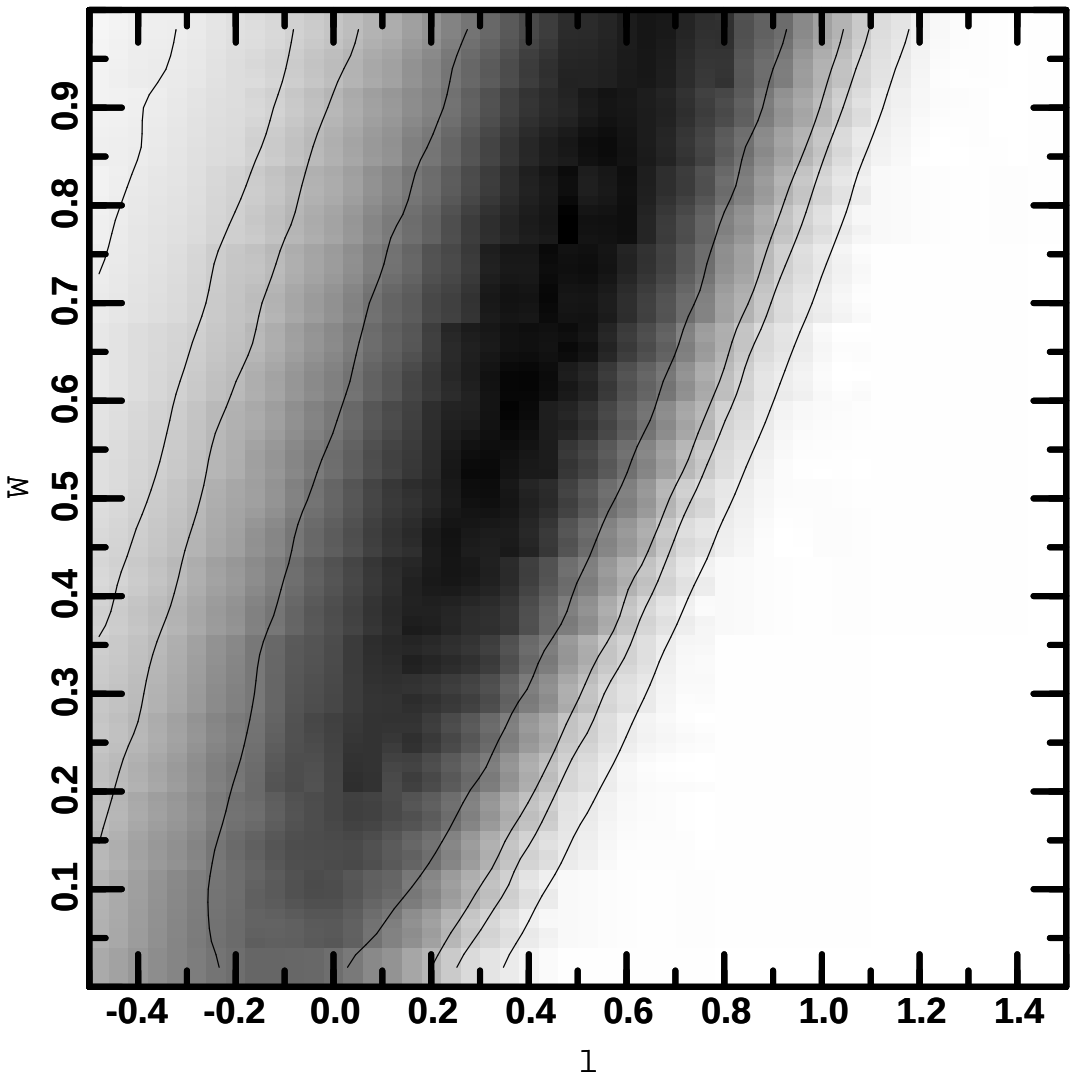

Figure 3. Constraints in the λ_0-Ω_0 plane using the entire CLASS (including JVAS). Darker means higher likelihood; contours are at 68%, 90%, 95% and 99%.