

Active Nonlinear Energy Sink Using Integral Force Feedback

Guoying Zhao¹, Ghislain Raze², Ahmad Paknejad¹, Arnaud Deraemaeker³, Gaetan Kerschen² and
Christophe Collette^{1,2}

¹ Department of Bio-, Electro- and Mechanical Systems
Université Libre de Bruxelles, 50, F.D.Roosevelt Av, B-1050, Brussels, Belgium

² Department of Aerospace and Mechanical Engineering
University of Liège, Allée de la Découverte 9, B-4000, Liège, Belgium

³ Department of Building Architecture and Town Planning
Université Libre de Bruxelles, 50, F.D.Roosevelt Av, B-1050, Brussels, Belgium

ABSTRACT

Excessive vibrations in mechanical structures can cause many problems such as reducing structural integrity and compromising the commissioning of precise instruments. Tuned mass dampers (TMDs) are often employed to suppress undesirable vibrations. However, they are known to be effective only in a frequency band limited around one vibration mode and to be sensitive to the variation of primary structures. Alternatively, nonlinear energy sinks (NESs) can be used as they do not have a preferential resonance frequency making them more robust and capable of damping multiple resonances. In this paper, the performance of an active nonlinear energy sink (ANES) is investigated, which is realized using a novel integral force feedback controller. Unlike the traditional NES, which is realized by a cubic spring, a dashpot and an inertial mass, the proposed ANES is equivalent to a mechanical system which consists of a cubic root inerter, a dashpot and a linear spring. Because of the full analogy with a mechanical network, the stability of the proposed active system is guaranteed. Although the form of the proposed ANES is different from that of traditional NESs, it is found that the targeted energy transfer phenomenon also occurs with the ANES and the control effectiveness is similar to traditional NESs.

Keywords: inerter, integral force feedback, nonlinear energy sink

INTRODUCTION

Tuned mass dampers [1] have been widely used to suppress undesirable vibrations of various types of mechanical structures such as machinery, helicopters, bridges, buildings, etc. The most generic form of a TMD is an auxiliary system which consists of a proof mass and a spring-dashpot pair. It has been shown that they are only effective around one particular vibration mode. Deviation from the desired settings can to some extent degrade their performance [2]. Instead, nonlinear strategies in vibration suppression have been proposed aiming to overcome this limitation. Nonlinear energy sinks have been extensively investigated for such a purpose [3,4]. The most popular form of a NES consists of a proof mass, a dashpot and a cubic nonlinear spring. The introduction of the essential nonlinearity allows NESs to absorb energy from the primary structure in an irreversible way and dissipate it locally. NESs do not have a preferential resonance which thus enables NESs to impede primary structure vibrations in a larger frequency band compared to TMDs. Substantial work has been done to better understand the underlying dynamics of NESs when coupled to single-degree-of-freedom (SDOF) systems and also multi-degree-of-freedom (MDOF) systems [3,5,6]. Nguyen and Pernot [7] established some design guidance for choosing the nonlinear stiffness and the damping coefficient of a NES coupled with a SDOF system under transient regime. Gendelman et al. studied the dynamics of a NES when it is mounted onto a forced SDOF system [8,9]. Similar to TMDs, the weight of the proof mass of NESs is important and better control performance comes with a heavier proof mass. However, the added mass may be penalizing in light weight applications, e.g. automotive and aerospace structures. Zhang et al. [10] and Javidialesaadi and Wierschem [11] have proposed to integrate inerters into NESs aiming to boost the performance as the inertance of inerters can be significantly greater than their actual mass [12].

The inerter was initially proposed to complete the analogy between the mechanical and electrical networks, where the effect of inerters on the dynamic behaviour of mechanical systems is designed to be similar to that of electrical capacitors in electrical systems [13]. It is defined as a one-port mechanical element which impedes the relative acceleration across its terminals. Several mechanical forms have been proposed to realize inerters in practice such as rack and pinion based inerters [12], ball and screw based inerters [13] and hydraulic inerters [14]. However, some imperfections due to the mechanical construction will be inevitably present preventing them to act as idealized inerters. For instance, the performance of rack and pinion and ball-screw inerters may degrade because of the friction and backlash or elastic effect of gears or screws [15], and hydraulic inerters may exhibit some nonlinear damping in addition to the inertance-like behavior.

In order to address the aforementioned problems associated with passive inerters, the potential of using active means has been investigated. Zhao et al. [16] proposed to realize an active inerter using reactive actuators and force sensors. An active inerter can be realized by feeding back the output of the force sensor through a double integrator. Following up this work, an active nonlinear energy sink is proposed in this paper as an alternative approach to traditional passive NESs. The ANES is equivalent to a pure mechanical system consisting of a cubic root inerter, a dashpot and a linear spring. The aim of this paper is to understand the energy transfer mechanisms of the proposed ANES and to validate the feasibility of targeted energy transfer from the primary structure to the ANES.

MODELLING

The system under investigation is shown in Figure. 1, which represents a linear oscillator. It is defined through a lumped mass m_1 , a linear spring k_1 and a dashpot d_1 , a reactive actuator with its stiffness denoted by k_a and a collocated force sensor which measures the actual force, represented by F_s , transmitted to the structure. The active control loop is implemented by feeding the output of the force sensor F_s through a nonlinear controller $C(F_s)$ to drive the actuator.

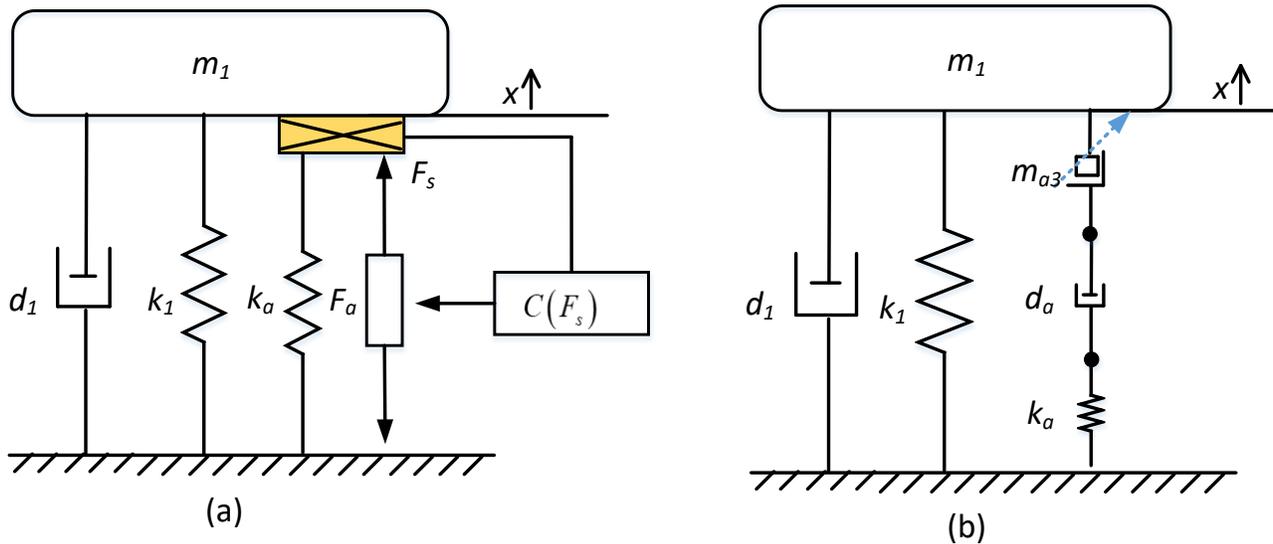


Figure 1 (a) The scheme of the system under investigation and (b) its equivalent mechanical model

The analysis hereafter will be conducted under transient regime. The governing equations of the system read:

$$m_1 \ddot{x} + d_1 \dot{x} + k_1 x = F_s \quad (1)$$

$$F_s = -C(F_s) - k_a x \quad (2)$$

The nonlinear controller $C(F_s)$ is built upon the linear controller proposed in [16], but modified to have a dynamic behavior similar to that of nonlinear energy sinks. The controller $C(F_s)$ reads:

$$C(F_s) = g_s \int_0^t F_s dt + g_{d3} \int_0^t \int_0^t (F_s)^3 dt dt \quad (3)$$

In fact, the proposed system can be alternatively realized by a pure mechanical network composed of a spring, a dashpot and a cubic root inerter connected in series. This equivalent mechanical scheme is shown in Fig. 1 (b). The equivalent damping coefficient and the inertance can be expressed by:

$$d_a = \frac{k_a}{g_s}, \text{ and } m_{a3} = \sqrt[3]{k_a/g_{d3}} \quad (4)$$

Substituting the proposed form of $C(F_s)$ into Eqs. (1) and (2), and normalizing the resulting equations, yields:

$$y_1'' + \xi y_1' + y_1 - \mu y_2 = 0 \quad (5)$$

$$y_2'' + g_1 y_2' + g_2 y_2^3 + y_1'' = 0 \quad (6)$$

where $y_1 = x$, $y_2 = F_s/k_2$, $\omega_0 = \sqrt{k_1/m_1}$, $\tau = \omega_0 t$, $\mu = k_a/k_1$, $g_1 = g_s/\omega_0$, $\xi = c/\sqrt{k_1 m_1}$, $g_2 = (g_{d3} k_2^2)/\omega_0^2$

ANALYSIS

The energy dissipation within the ANES is used as the performance index to evaluate the capacity of the ANES to absorb the impulsive energy from the primary structure. It is defined as in [3]:

$$E_{ANES}(t) = \frac{2g_1 \int_0^\tau y_2'^2 d\tau}{\mu (y_{1_initial})^2} \quad (7)$$

where $y_{1_initial}$ denotes the normalized displacement of the primary structure.

$E_{ANES}(t)$ in fact represents the percentage of the impulsive energy that is dissipated by the ANES up to the normalized time τ . The dependence of the performance index E_{ANES} upon the initial displacement of the primary structure is studied. The following parameters are used: $\mu = 0.1$, $g_1 = 0.02$, $g_2 = 1$ and $\xi = 0.002$. The results are depicted in Figure 2. It is shown that there exists a threshold of the initial displacement below which the nonlinear dynamics of the coupled system is not triggered and it behaves linearly. Above the threshold, the dissipation efficiency is suddenly increased which indicates the occurrence of targeted energy transfer. The damping effectiveness does not hold when the impulse energy is further increased which might be due to another energy exchange mechanism.

Figure 3-5 plot the time history of the response when the initial displacement is chosen at A, B, and C as given in Figure 2. At point A, the coupled system behaves linearly and it takes a quite long time to damp the initial energy. At point B as shown in Figure 4, the primary mass exhibits a nonlinear beating phenomenon and it vibrates in phase with the ANES which indicates the occurrence of the targeted energy transfer as also observed with traditional NESs [3]. In the regime around point C, the ANES vibrates three times faster than the primary system. The transient dynamics is captured on a 1:3 resonant manifold of the dynamics.

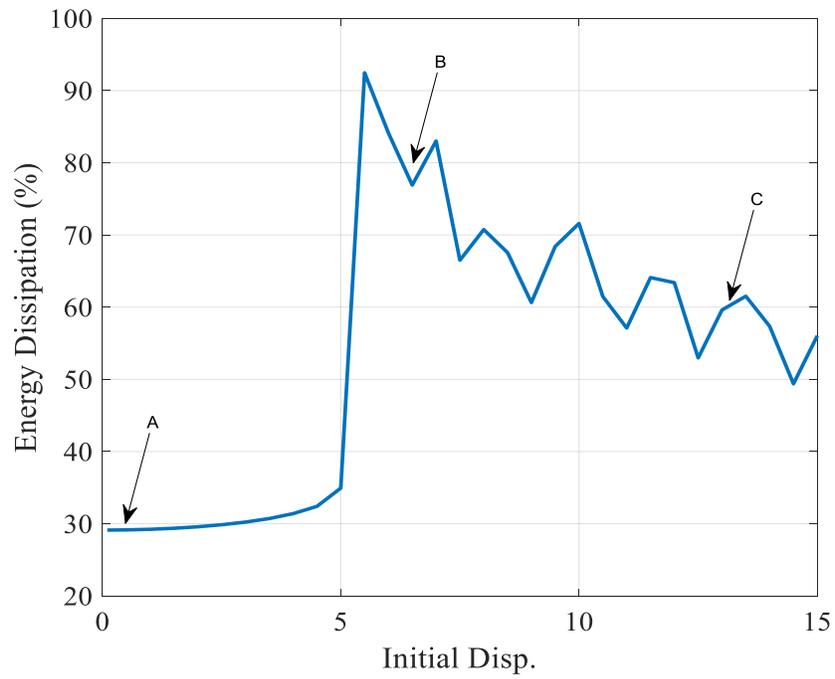


Figure 2 Energy dissipation ratio by the ANES with respect to different initial displacements

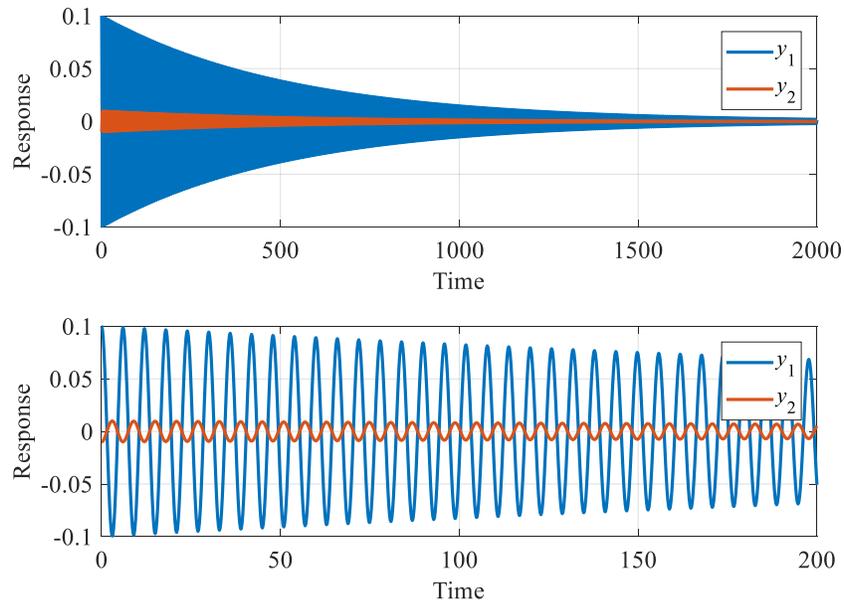


Figure 3 System response when the initial displacement is set to point A

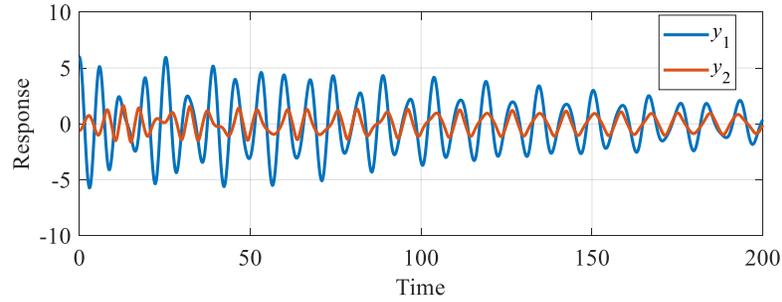
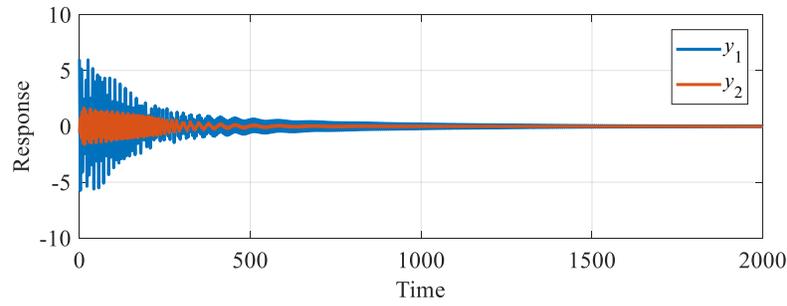


Figure 4 System response when the initial displacement is set to point B

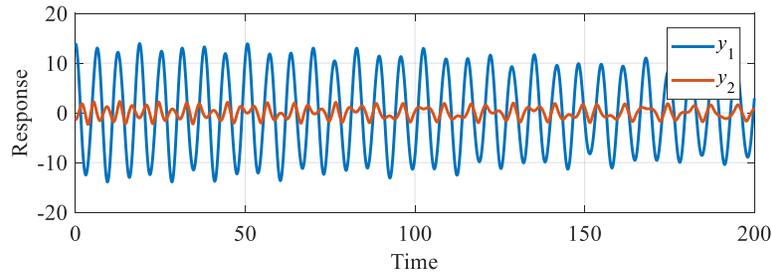
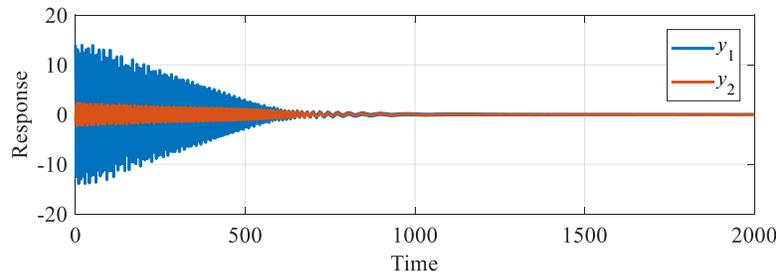


Figure 5 System response when the initial displacement is set to point C

CONCLUSION

This paper discusses a novel control concept for realizing an active nonlinear energy sink by using a pair of collocated reactive actuator and force sensor. The equivalent mechanical models of the controller's components are derived in order to better illustrate the coupling of the electrical controller with the mechanical system. A cubic root inerter is introduced in the ANES. Numerical studies have been performed to explore the feasibility of using ANES for vibration mitigation. It is found that the proposed ANES behaves similarly to the traditional NESs. The targeted energy transfer phenomenon and the 1:3 resonance capture phenomenon in the transient regime are also observed with the proposed ANES. Analytical analysis on the underlying dynamics will be continued in the future work.

ACKNOWLEDGEMENTS

The financial supports from Wal'innov (MAVERIC project 1610122) and F.R.S.-FNRS (IGOR project F453617F) are gratefully acknowledged.

REFERENCES

- [1] J.P. Den Hartog, *Mechanical Vibrations*, Dover Publications, 1985.
- [2] A.H. von Flotow, A. Beard, D. Bailey, Adaptive tuned vibration absorbers: Tuning laws, tracking agility, sizing, and physical implementations, in: *Adapt. Tuned Vib. Absorbers Tuning Laws, Track. Agil. Sizing, Phys. Implementations*, In: Noise-con 94, Fort Lauderdale, New York, 1994: pp. 437–454.
- [3] A.F. Vakakis, O. V. Gendelman, L.A. Bergman, D.M. McFarland, G. Kerschen, L.Y. Lee, nonlinear targeted energy transfer in mechanical and structural systems, 2008. doi:10.1007/978-1-4020-9130-8.
- [4] Y. Starosvetsky, O. V. Gendelman, Vibration absorption in systems with a nonlinear energy sink: Nonlinear damping, *J. Sound Vib.* 324 (2009) 916–939. doi:10.1016/j.jsv.2009.02.052.
- [5] E. Gourc, G. Michon, S. Seguy, A. Berlioz, Experimental investigation and design optimization of targeted energy transfer under periodic forcing, *J. Vib. Acoust. Trans. ASME.* 136 (2014). doi:10.1115/1.4026432.
- [6] K. Dekemele, R. De Keyser, M. Loccufier, Performance measures for targeted energy transfer and resonance capture cascading in nonlinear energy sinks, *Nonlinear Dyn.* 93 (2018) 259–284. doi:10.1007/s11071-018-4190-5.
- [7] T.A. Nguyen, S. Pernot, Design criteria for optimally tuned nonlinear energy sinks-part 1: Transient regime, *Nonlinear Dyn.* 69 (2012) 1–19. doi:10.1007/s11071-011-0242-9.
- [8] Y. Starosvetsky, O. V. Gendelman, Attractors of harmonically forced linear oscillator with attached nonlinear energy sink. II: Optimization of a nonlinear vibration absorber, *Nonlinear Dyn.* 51 (2008) 47–57. doi:10.1007/s11071-006-9168-z.
- [9] O. V. Gendelman, Y. Starosvetsky, M. Feldman, Attractors of harmonically forced linear oscillator with attached nonlinear energy sink I: Description of response regimes, *Nonlinear Dyn.* 51 (2008) 31–46. doi:10.1007/s11071-006-9167-0.
- [10] Z. Zhang, Z.Q. Lu, H. Ding, L.Q. Chen, An inertial nonlinear energy sink, *J. Sound Vib.* 450 (2019) 199–213. doi:10.1016/j.jsv.2019.03.014.
- [11] A. Javidialesaadi, N.E. Wierschem, An inerter-enhanced nonlinear energy sink, *Mech. Syst. Signal Process.* 129 (2019) 449–454. doi:10.1016/j.ymsp.2019.04.047.
- [12] M.C. Smith, Synthesis of mechanical networks: The inerter, *IEEE Trans. Automat. Contr.* 47 (2002) 1648–1662. doi:10.1109/TAC.2002.803532.
- [13] M.Z.Q. Chen, C. Papageorgiou, F. Scheibe, F.C. Wang, M. Smith, The missing mechanical circuit element, *IEEE Circuits Syst. Mag.* 9 (2009) 10–26. doi:10.1109/MCAS.2008.931738.
- [14] X. Liu, J.Z. Jiang, B. Titurus, A. Harrison, Model identification methodology for fluid-based inerters, *Mech. Syst. Signal Process.* 106 (2018) 479–494. doi:10.1016/j.ymsp.2018.01.018.
- [15] C. Papageorgiou, N.E. Houghton, M.C. Smith, Experimental Testing and Analysis of Inerter Devices, *J. Dyn. Syst. Meas. Control.* 131 (2009) 011001. doi:10.1115/1.3023120.
- [16] G.. Zhao, G.. Raze, A.. Paknejad, A.. Deraemaeker, G.. Kerschen, C.. Collette, Active tuned inerter-damper for smart structures and its \mathcal{H}_∞ optimisation, *Mech. Syst. Signal Process.* 129 (2019) 470–478. doi:10.1016/j.ymsp.2019.04.044.