Active nonlinear inerter damper for vibration mitigation of Duffing oscillators

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Abstract

In this paper, a nonlinear active damping strategy based on force feedback is proposed. The proposed device is composed of a pair of collocated actuator and force sensor. The control law is formed by feeding back the output of the force sensor, through one single, one double integrator and another double integrator of its cube. An equivalent mechanical network which consists of a dashpot, an inerter and a cube root inerter is developed to enable a straightforward interpretation of the physics behind. Closed-form expressions for the optimal feedback gains are derived. Numerical validations are performed to demonstrate the proposed control strategy.

Keywords: Inerter, force feedback, nonlinear vibration control

1. Introduction

Flexible structures such as beams, cables and rods are commonly seen in engineering applications. The associated vibration problems have drawn the attention of many researchers as they often exhibit low damping characteristics. In some cases, especially when the flexible structures are excited around their resonant frequencies, nonlinear vibrations may occur due to, for example, nonlinear properties of materials, geometric nonlinearities, and nonlinear external forces [1]. Traditional linear solutions based on tuned absorbers via passive means [2] or active means [3] are no longer effective. This is because the resonant frequency of the nonlinear oscillations depends intrinsically on the motion amplitudes [4]. In order to recover their control effectiveness, mechanisms that can deliver nonlinear reacting forces should be added in these linear approaches. Habib et al. [5] and Sun et al. [6] proposed to use nonlinear tuned vibration absorbers (NLTVAs) for vibration mitigation of nonlinear resonances. Although this concept is promising, it may become cumbersome and expensive to realise NLTVAs in practice using passive means for complex nonlinear primary systems.

On the other hand, active approaches might be appealing which might lead to a less complex solution. Zhao et al. [7] applied the same tuning strategy as proposed in [5] on a nonlinear positive position feedback (NPPF) controller aiming to optimally damp a Duffing oscillator based on the $\mathscr{H}_\infty$ criterion. However, the stability of this active control system is not unconditionally guaranteed, which might be problematic in practical engineering applications. Zhao et al. [8] proposed an unconditionally stable controller using force feedback for linear applications. The stability is presumed given idealised force sensors and actuators are employed. In fact, the realised active system is equivalent to a pure mechanical system consisting of an inerter and a damper.

This work is built upon the previous developments [7,8] to investigate the potential of using an active nonlinear inerter damper (ANLID) for vibration mitigation of a Duffing oscillator (primary structure). More specifically, a cubic nonlinear term is added in the original linear controller [8] in order to counteract the nonlinear dynamics of the primary structure. The added nonlinear term plays the same role as a cube root inerter in the equivalent mechanical network. The optimal nonlinear control gain is derived such that the control effectiveness achieved with the linear controller-linear primary structure is maintained for a nonlinearly coupled system. It is shown that the ANLID further outperforms the previously proposed NPPF controller for suppressing the occurrence of the detached resonance curves (DRC) which may limit the practical application of the NPPF approach. As the ANLID may be alternatively realized with passive means, the optimal tunings derived in the paper can be also used to guide the design of its equivalent passive counterpart.

The paper is organised as follows. In the next section, the mathematical model of the system under consideration is developed, based on which the optimal nonlinear control gain is derived. In Section 3, numerical analysis is performed in order to validate the derived formulae and to examine the control effectiveness of the ANLID. Conclusions are drawn in Section 4.

2. Mathematical modelling and parameter optimisation

A Duffing oscillator is considered as the primary structure which is shown in Fig. 1(a). It is defined through a lumped mass m_1, a linear spring k_1 and a cubic spring k_3. A harmonic excitation force $F = F_d \cos(\omega t)$ is applied. An actuator is placed in parallel to the suspension of the Duffing oscillator, whose stiffness is denoted by k_a. A collocated force sensor which measures the transmission force represented by F_s is installed. The active control loop is implemented by feeding the output of the force sensor F_s through a nonlinear controller $C(F_s)$ to drive the actuator.

The governing equations of the coupled system read:

$$m_1\ddot{x}+k_1x+k_3x^3=F_d\cos(\omega t)+F_s \tag{1}$$

$$F_s=C(F_s)-k_ax \tag{2}$$

The nonlinear controller $C(F_s)$ is modified from the linear controller proposed in [8] by including a cubic term in order to counteract the nonlinear dynamics of the primary structure. The controller $C(F_s)$ reads:

$$C(F_s)=-g_s\int_0^t F_s\mathrm{d}t-g_d\int_0^t\int_0^t F_s\mathrm{d}t\mathrm{d}t-g_{d3}\int_0^t\int_0^t F_s^{\,3}\mathrm{d}t\mathrm{d}t \tag{3}$$

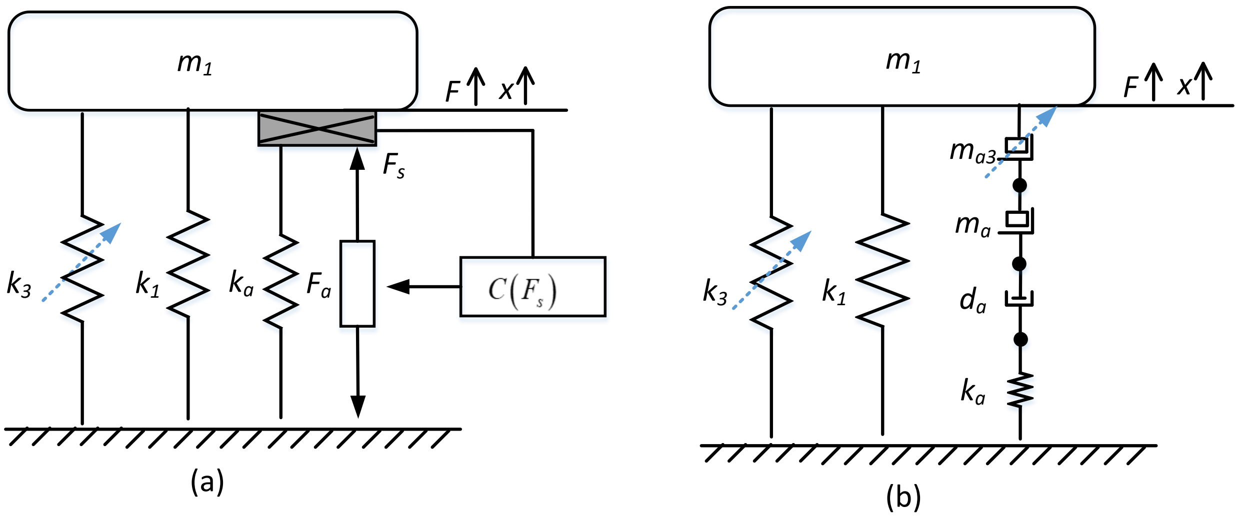

Fig. 1 (a) The sketch of the coupled system under investigation and (b) its equivalent mechanical model.

The following parameters are introduced to normalise the system governing equations:

$$\begin{gathered}\tau=\omega_1 t,\quad y_1=x/x_d,\quad y_2=F_s/(k_1x_d),\quad x_d=F_d/k_1,\quad \omega_1=\sqrt{k_1/m_1},\\ \mu=k_a/k_1,\quad g^{sn}=g_s/\omega_1,\quad g^{dn}=g_d/\omega_1^{\,2},\quad \delta=k_3x_d^{\,2}/k_1,\quad \beta=g_{d3}k_1^{\,3}/k_3\end{gathered} \tag{4}$$

The governing equations with normalised parameters are written as:

$$y_1''+y_1+\delta y_1^{\,3}-y_2=\cos(\Omega\tau) \tag{5}$$

$$y_2''+g^{sn}y_2'+g^{dn}y_2+\delta\beta y_2^{\,3}+\mu y_1''=0 \tag{6}$$

where Ω is the normalised frequency defined as $\Omega=\omega/\omega_1$. As suggested by Eqs. (5) and (6), the proposed system can be alternatively realised by a pure mechanical network composed by a spring, a dashpot, an inerter and a cube root inerter connected in series. This equivalent mechanical scheme is shown in Fig. 1(b).

It has been shown that there exists two fixed points for the linearly coupled system ($\delta = 0$ and $\beta = 0$) [8]. The optimal control parameters g^{sn} and g^{dn} have been derived based on the equal peak method. For the nonlinearly coupled system, the optimal nonlinear control gain is sought to maintain the equal peak property with the presence of primary structure's nonlinearity. However, it is difficult to derive the explicit expression of the performance index i.e. $|y_1|$ from Eqs. (5) and (6). Instead, a pair of one-term harmonic balance approximation is assumed as the solutions as in [7]:

$$y_1 = \left(A_{11} + \delta A_{12}\right)\cos\left(\Omega\tau\right) + \left(B_{11} + \delta B_{12}\right)\sin\left(\Omega\tau\right) \tag{7}$$

$$y_2 = \left(A_{21} + \delta A_{22}\right)\cos\left(\Omega\tau\right) + \left(B_{21} + \delta B_{22}\right)\sin\left(\Omega\tau\right) \tag{8}$$

where the coefficients of $\cos\left(\Omega\tau\right)$ and $\sin\left(\Omega\tau\right)$ are expanded into series with respect to the primary nonlinear coefficient δ.

Substituting Eqs. (7) and (8) into Eqs. (5) and (6), and applying the approximations $\cos^3\left(\Omega\tau\right) \approx 3/4\cos\left(\Omega\tau\right)$ and $\sin^3\left(\Omega\tau\right) \approx 3/4\sin\left(\Omega\tau\right)$, a set of polynomial equations can be obtained by balancing cosine and sine terms and collecting the resulting expressions with respect to the order of the parameter δ. After omitting the expressions whose orders are higher than δ^1, one obtains:

$$-A_{11}\Omega^2 + A_{11} - A_{21} = 1 \tag{9}$$

$$-A_{12}\Omega^2 + A_{12} - A_{22} + 3/4 A_{11}{}^3 + 3/4 A_{11} B_{11}{}^2 = 0 \tag{10}$$

$$-B_{11}\Omega^2 + B_{11} - B_{21} = 0 \tag{11}$$

$$-B_{12}\Omega^2 + B_{12} - B_{22} + 3/4 A_{11}{}^2 B_{11} + 3/4 B_{11}{}^3 = 0 \tag{12}$$

$$\left(-A_{11}\mu - A_{21}\right)\Omega^2 + g^{sn} B_{21}\Omega + g^{dn} A_{21} = 0 \tag{13}$$

$$\left(-A_{12}\mu - A_{22}\right)\Omega^2 + g^{sn} B_{22}\Omega + 1/4\left(3A_{21}{}^3 + 3A_{21}B_{21}{}^2\right)\beta + g^{dn} A_{22} = 0 \tag{14}$$

$$\left(-B_{11}\mu - B_{21}\right)\Omega^2 - g^{sn} A_{21}\Omega + g^{dn} B_{21} = 0 \tag{15}$$

$$\left(-B_{12}\mu - B_{22}\right)\Omega^2 - g^{sn} A_{22}\Omega + 1/4\left(3A_{21}{}^2 B_{21} + 3B_{21}{}^3\right)\beta + g^{dn} B_{22} = 0 \tag{16}$$

Solving for A_{ij} and B_{ij} ($i = 1,2$, $j = 1,2$) from Eqs. (9)-(16), the resulting solutions are found to be in terms of g^{sn} , g^{dn}, μ and Ω. Due to the complexity, these expressions are not given here. The modulus of the normalised receptance $\left|y_1(\Omega)\right|$ can be expressed as:

$$\left|Q(\Omega)\right| = \sqrt{A_1{}^2 + B_1{}^2} = \sqrt{A_{11}{}^2 + B_{11}{}^2 + 2\delta\left(A_{11}A_{12} + B_{11}B_{12}\right) + \mathrm{O}\left(\delta^2\right)} \tag{17}$$

An additional condition is imposed in order to derive the optimal coefficient of the nonlinear compensator β, which is sought to ensure the equal peak property at the fixed points:

$$\left|Q\left(\Omega_{f1}\right)\right| = \left|Q\left(\Omega_{f2}\right)\right| \tag{18}$$

where Ω_{f1} and Ω_{f2} denote the location of the fixed points of the linearly coupled system.

The expressions for Ω_{f1} and Ω_{f2} as well as the optimal values for g^{sn} and g^{dn} are given as [8]:

$$g_{opt}^{sn} = \sqrt{3\mu/2}, \quad g_{opt}^{dn} = 1-\mu/2, \quad \Omega_{f1} = \sqrt{1+\sqrt{\mu/2}}, \quad \Omega_{f2} = \sqrt{1-\sqrt{\mu/2}} \tag{19}$$

Substituting Eq. (19) as well as the solutions of Eqs. (9)-(16) for A_{ij} and B_{ij} ($i=1,2$, $j=1,2$) into Eq. (18), one obtains

$$\beta_{opt} = \frac{\mu^2 - 16\mu - 512}{56\mu^2 - 256\mu} \tag{20}$$

3. Discussion

Numerical studies are performed to validate and examine the control effectiveness of the ANLID controller. The governing equations are computed using a path-following algorithm combining harmonic balance and pseudo-arclength continuation [9]. A modal damping of 1% is added to the primary structure.

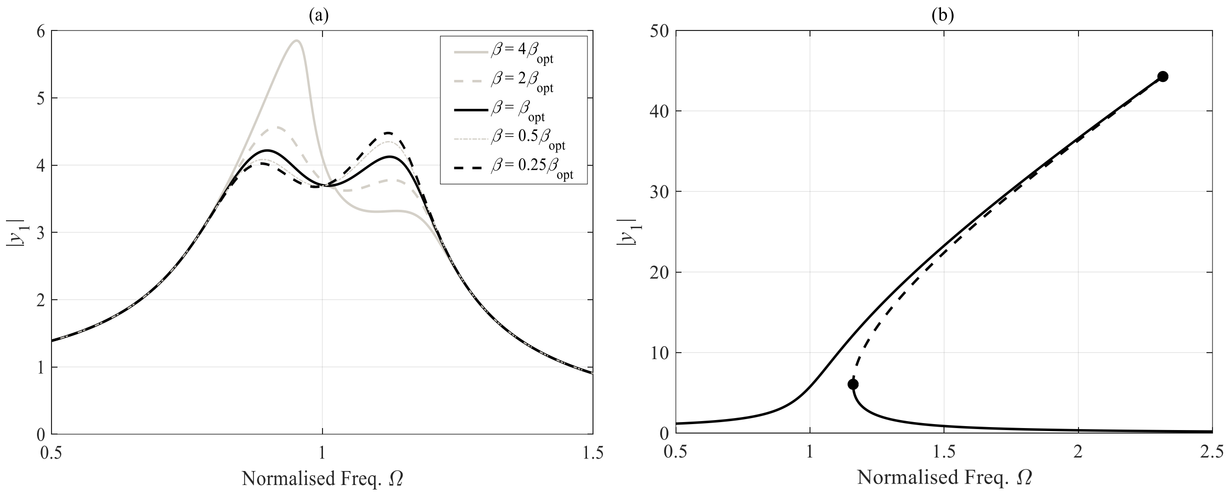

Fig. 2 The performance index $|y_1|$ for (a): the optimal ANLID with the nonlinear gain β varying with respect to its optimal value and (b) the ANLID when control is off; The system parameters $\mu = 0.1$ and $\delta = 0.003$ (•: fold bifurcation)

The first study is focused on the validity of Eq. (20) which describes the optimal coefficient of the nonlinear compensator β. Fig. 2 (a) plots the frequency response of the performance index $|y_1|$, where the nonlinear coefficient δ is set to 0.003, the stiffness ratio μ is set to 0.1, the linear control gains g^{sn} and g^{dn} are calculated as in Eq. (19), and the parameter β varies with respect to its optimal value as β/β_{opt} : 1/4, 1/2, 1, 2 and 4. It is seen that the response at the first resonance frequency increases with an increase of the parameter β and an opposite trend is observed for the second resonance peak. Peaks of equal amplitudes are obtained with the optimal setting of the parameter β as given in Eq. (20). Fig. 2 (b) shows the performance index when the control is set off, which clearly indicates the system is in the regime of nonlinear motions. With the proposed ANLID, the system response can be substantially suppressed as shown in Fig. 2 (a).

For the second study, the control effectiveness of ANLID is investigated and compared with that of the linear inerter-damper [8] (ANLID for $\beta = 0$). Fig. 3 (a) compares the resonance peaks associated with the two controllers. The loci of resonant peaks are computed using the method proposed in [10]. As can be seen, the linear inerter-damper gets rapidly detuned and a clear nonlinear dependence with respect to the nonlinear forcing coefficient δ is observed i.e. non-unique peak values are captured when $0.0088 < \delta < 0.01$. The underlying dynamic mechanism is as follows: there is a DRC, also termed an isola, coexisting with the main frequency response function curve and it merges with the main curve at the second resonance when δ approaches 0.01. On the contrary, the equal peak property is still maintained when the nonlinear controller is applied for the value of δ up to 0.02. Fig. 3 (b) continues to show the evolution of the resonance peaks for the case of the ANLID by extending δ to unity. It can be seen that the difference between the two peaks monotonously increases with an increase of δ up to 0.2. When the nonlinearity is more pronounced i.e. in the extremely strong nonlinear regime (δ >0.2), the distance of the two peaks seems to keep constant. Unlike the linear controller or the NPPF controller, there is no occurrence of isolas when the ANLID is used for damping the Duffing oscillator. Although the equal peak property does not hold for large values of δ, ANLID remains more effective compared to its linear counterpart in terms of the resonance peak difference. Interestingly, it has been noticed that the nonlinearly coupled system i.e. the Duffing oscillator and the active nonlinear inerter damper exhibit a dynamic behaviour similar to that of its linearly coupled counterpart in a large range of forcing amplitudes. In order to further improve the control performance, a stiffer actuator can be used as illustrated in [8] for the linear active inerter damper.

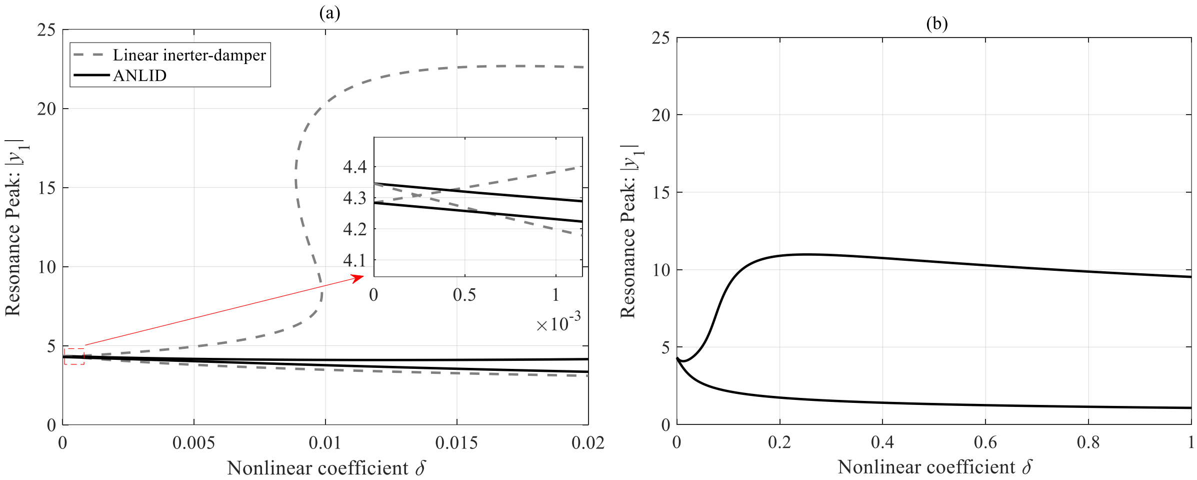

Fig. 3 (a) comparison of the resonance peaks between the linear inerter-damper and ANLID; (b) the resonance peaks for the ANLID when δ is extended to unity

4. Conclusion

A nonlinear active damping strategy based on force feedback has been proposed. The equivalent mechanical representative, i.e. ANLID, has been derived to better understand the working principle of the active control system. Closed-form expressions of the control parameters have been derived. A Duffing oscillator has been considered to illustrate the proposed tuning of the optimal control gains. It was shown that the control effectiveness of ANLID can be maintained for a relatively large range of forcing amplitudes. More interestingly, the optimally configured ANLID can also suppress the presence of isolas which occurs for the linear inerter-damper or the NPPF controller. Experimental validation and further theoretical investigations are left for a future work.

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