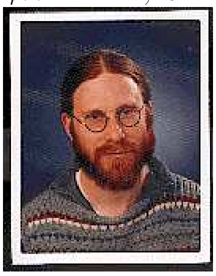
THE MAGNITUDE-REDSHIFT RELATION FOR TYPE IA SUPERNOVAE, LOCALLY INHOMOGENEOUS COSMOLOGICAL MODELS, AND THE NATURE OF DARK MATTER

PHILLIP HELBIG
Thomas-Mann-Str. 9,
D-63477 Maintal, Germany



The magnitude-distance relation for Type Ia supernovae is one of the key pieces of evidence supporting the cosmological concordance model. The resulting constraints on the cosmological parameters are often derived under the idealized assumption that the universe is perfectly homogeneous on small scales. Since we know that this is not the case, what does this mean for constraints on cosmological parameters derived from the magnitude-distance relation for Type Ia supernovae? And, conversely, what does the fact that these constraints, when small-scale homogeneity is assumed, agree with other constraints mean for the nature of dark matter?

1 Introduction and Background

In the last 20 years or so, cosmological observations have improved greatly and it also appears that the measured values are converging on the true values. Among the most important of these observations are those by the Supernova Cosmology Project [3,12,11,10,1,16] and the High-z Supernova Search Team [2,14,15] which provide joint constraints on λ_0 and Ω_0. Combined with other observations [9,13], these lead to quite well constrained values for the cosmological parameters [16]. Although the supernova data alone allow a relatively wide range of significantly different other models, the best-fitting values obtained from these observations using the current data are quite close to the better-constrained values using combinations of several observations without the supernova data, at least under the assumptions with which the former were calculated. However, since the m-z relation depends not only on λ_0 and Ω_0 (with H_0 as a scale factor) but also on the distribution of matter along and near the line of sight, conclusions drawn from the m-z relation for Type Ia supernovae depend on this matter distribution. Alternatively, such observations can perhaps tell us something about this distribution. Early on, the Supernova Cosmology Project [10] considered the effect of $\eta \neq 1$ on their results (see their fig. 8) and concluded that, at least in the 'interesting' region of the λ_0-Ω_0 parameter space, it had a negligible effect. With the larger number of supernovae now available, this is no longer the case.

Kayser, Helbig, & Schramm (KHS) [8] developed a general and practical method for calculat-

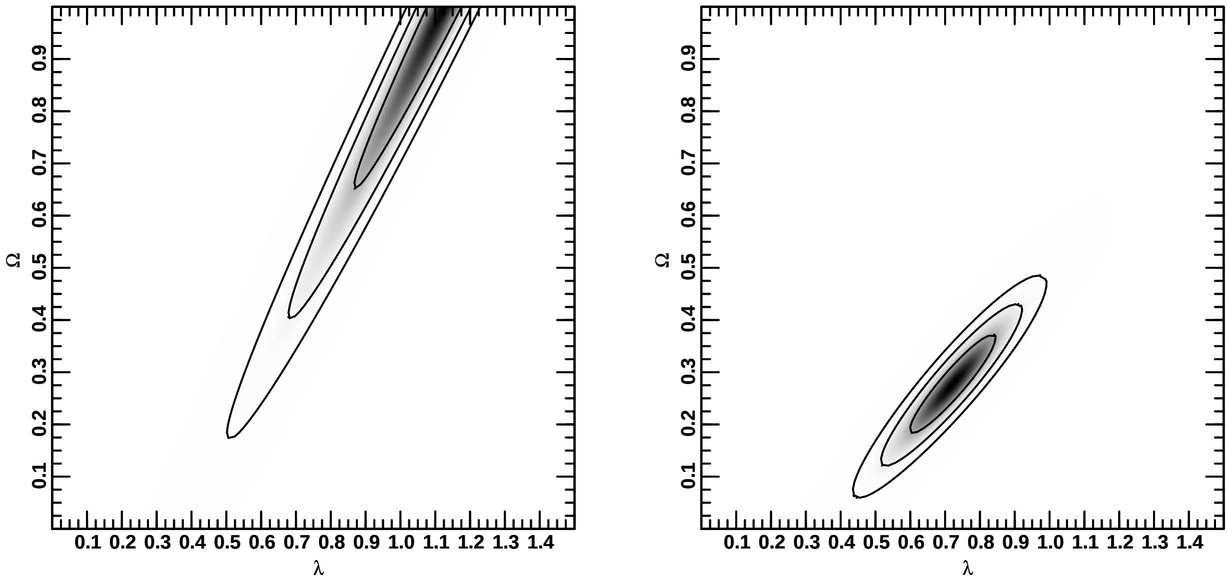


Figure 1 – *Left:* Two-dimensional probability distribution (PD) for $\eta = 0$. *Right:* The same for $\eta = 1$.

ing cosmological distances in the case of a locally inhomogeneous universe. In this case, distances which depend on angular observables related to the propagation of radiation will differ from the homogeneous case because more or less convergence will change the angle involved. The basic idea is that one considers a universe which is homogeneous and isotropic on large scales, this determining the global dynamics via the Friedmann-Lemaître equation. Local inhomogeneities are modelled as clumps, where the extra matter in the clumps is taken from the surrounding matter; the assumption is that light propagates between the clumps.

KHS described the inhomogeneity *via* the parameter $0 \leq \eta \leq 1$, where η is ratio of the density inside the beam to the global density or, alternatively, the fraction of matter which is homogeneously distributed. η of course depends on the beam size. Since the beams of supernovae at cosmological distances are extremely thin objects, evidence for $\eta < 1$ should be most obvious in the m-z relation for Type Ia supernovae.

It was pointed out by Weinberg [17] that, roughly speaking, η must be 1 when averaged over all lines of sight. However, in practice lines of sight will probably avoid concentrations of matter, due to selection effects or design. If these selection effects do not exist, and if the sample is large enough, then the 'safety in numbers' effect [7] allows one to effectively assume $\eta = 1$ on average, even if it varies from one line of sight to another, with inhomogeneity merely increasing the dispersion, roughly linearly with redshift; see section 2.

2 Calculations, results and discussion

I have used the publicly available 'Union2.1' sample of supernova data [16] and calculated χ^2 and the associated probability following Amanullah *et al* [1] on a regularly-spaced grid in the λ_0-Ω_0-η parameter space. (More details on these calculations can be found in one of the papers related to this talk [5]; many more plots are available there and in the slides of my talk available at the conference website.) I have used the standard contour values 0.683, 0.954 and 0.997. In all figures, the grey-scale corresponds to the probability.

Most discussion of the m-z relation for Type Ia supernovae has concentrated on two-dimensional contours, often *i.e.* with a δ-function prior on the nuisance parameters. Almost always, $\eta = 1$ is assumed. For comparison, in Fig. 1 I show constraints in the λ_0-Ω_0 plane for fixed values of η, namely 0 and 1.

Though little significance should be placed on variations in the probability within the in-

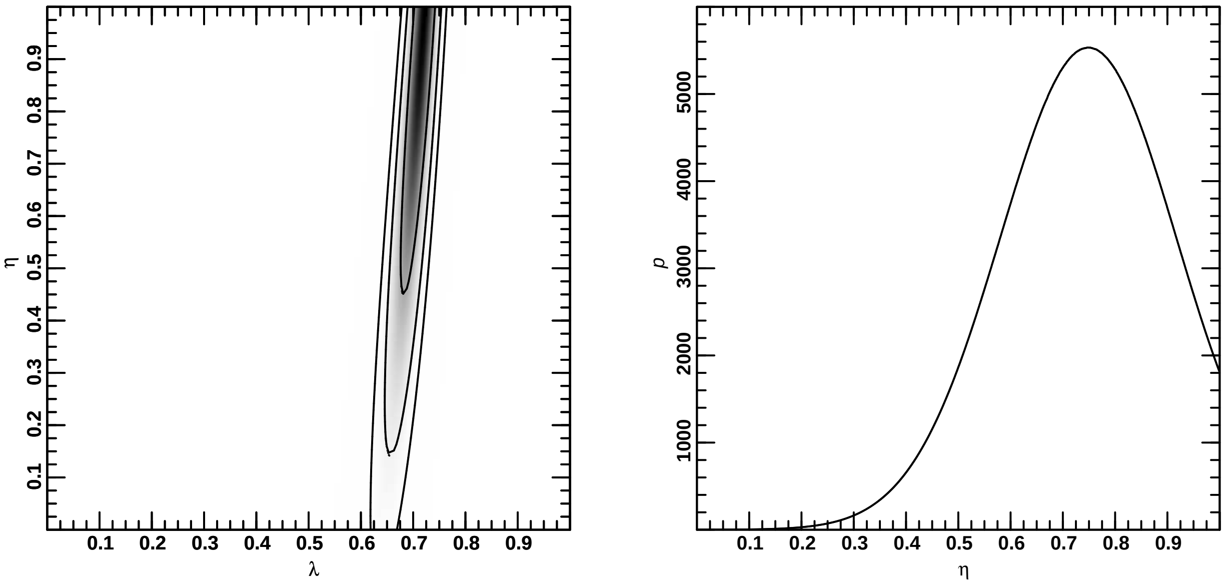


Figure 2 – *Left:* Two-dimensional PD for $k = 0$. *Right:* One-dimensional PD for the concordance model.

nermost contour, it is remarkable that the maximum of the probability in Fig. 1 (right) is at $\lambda_0 \approx 0.72$ and $\Omega_0 \approx 0.28$, *i.e.* at the values of the concordance model. (Note that when fewer supernova data were available, the best-fitting value was at much higher values of λ_0 and Ω_0 [4].) However, the best-fitting values for the (current) supernova data correspond to the concordance model only if one assumes $\eta \approx 1$. For $\eta = 0.455$ (not shown), the concordance model lies very near the 95.4 per cent contour, and for $\eta = 0$ it is even outside the 99.7 per cent contour. Thus, it is not possible to appreciably constrain η from the supernova data alone. However, the fact that the supernova data suggest the concordance model only for high values of η could be seen as evidence that $\eta \approx 1$.

A similar result is shown in Fig. 2 (left), where a flat universe ($\lambda_0 + \Omega_0 = 1$) has been assumed. As in the other plots, λ_0 is reasonably well constrained, while η is quite weakly constrained. Note that the best-fitting value is for $\eta = 1$ and $\lambda_0 \approx 0.72$; in other words, again the best fit is for the concordance model with $\eta = 1$.

While the supernova data cannot usefully constrain η, the fact that they result in the concordance model if one assumes $\eta \approx 1$ suggests that $\eta \approx 1$. Since there are many cosmological tests completely independent of the supernova data, and also independent of the value of η, which suggest the concordance model, one can assume the concordance values for λ_0 and Ω_0 and calculate the probability of η from the supernova data with these additional constraints; this is shown in Fig. 2 (right).

What does it mean that $\eta \approx 1$? This could be due to the density along each line of sight being equal to the overall cosmological density, appropriately averaged along *each* line of sight, or to 'safety in numbers', with variation in the density along *all* lines of sight averaging out if the sample is large enough. In the former case, residuals and observational uncertainties should be proportional; in the latter case, the residuals should become larger, compared to the uncertainties, as redshift increases. Fig. 3 demonstrates that the quotient of residual and observational uncertainty does not show a trend with redshift. This indicates that the former scenario is more likely [6].

3 Conclusions

One might have thought that the increase in the number of data points since the first results of the Supernova Cosmology Project [10] would allow some sort of useful constraint to be placed on

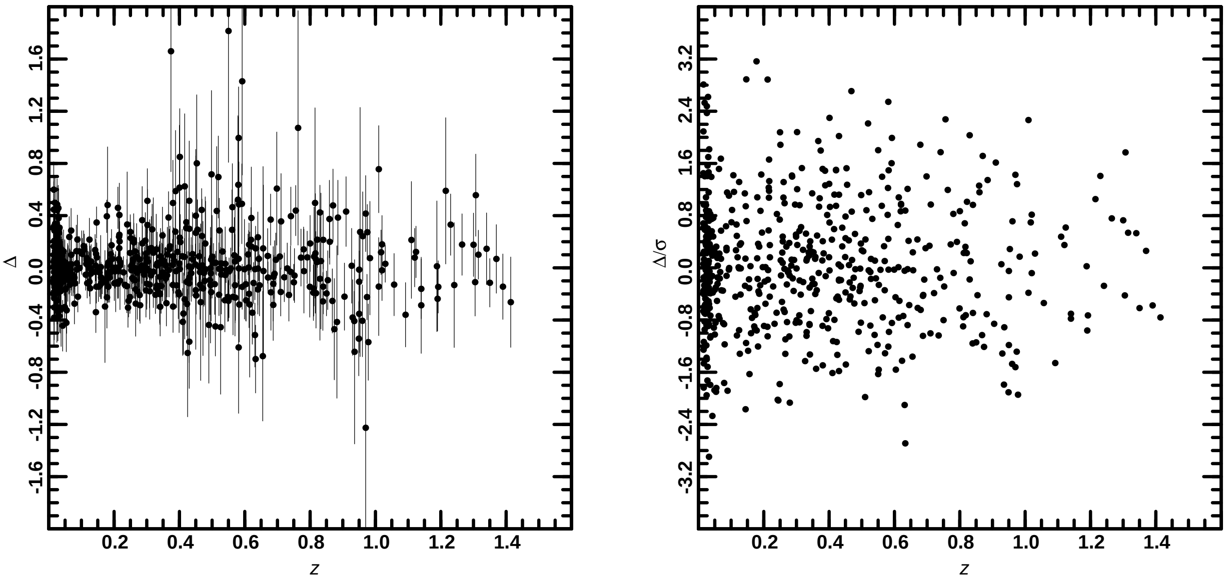


Figure 3 – *Left:* Residuals (points) and observational uncertainties (error bars). *Right:* Quotients of residuals and observational uncertainties.

η from the supernova data without further assumptions. This is not the case. Even worse, if η is allowed to vary, then the conclusions about the cosmological model derived from the m-z relation for Type Ia supernovae are not as robust. However, current constraints from combinations of cosmological tests without using the supernova data determine the concordance model with $\lambda_0 \approx 0.7$ and $\Omega_0 \approx 0.3$ to rather high precision. It is thus perhaps more interesting to assume the concordance model and use the supernova data to constrain η, especially since η is otherwise difficult to measure. It is also extremely interesting that the supernova data have the best-fitting values for λ_0 and Ω_0 corresponding to those of the concordance model if and only if $\eta \approx 1$ is assumed. This could indicate that $\eta \approx 1$, which is somewhat surprising since the value of η as 'felt' by the supernova might be expected to be somewhat less, because the corresponding beams are extremely thin. The fact that even the supernova data 'want' $\eta \approx 1$ could indicate that dark matter is distributed extremely homogeneously.

References

1. R. Amanullah *et al*, *ApJ* 716, 712 (2010).
2. P. M. Garnavich *et al*, *ApJ* **493**, L53 (1998).
3. A. Goobar and S. Perlmutter, *ApJ* **450**, 14 (1995).
4. P. Helbig, *A&A* **350**, 1 (1999).
5. P. Helbig, *MNRAS* **451**, 2097 (2015).
6. P. Helbig, *MNRAS* **453**, 3975 (2015).
7. D. E. Holz and E. V. Linder, *ApJ* **631**, 678 (2005).
8. R. Kayser, P. Helbig, and T. Schramm, *A&A* **318**, 680 (1997).
9. E. Komatsu *et al*, *ApJS* **192**, 18 (2011).
10. S. Perlmutter *et al*, *ApJ* **517**, 565 (1999).
11. S. Perlmutter *et al*, *Nature* **391**, 51 (1998).
12. S. Perlmutter *et al*, *ApJ* **440**, L41 (1995).
13. *Planck* Collaboration, *A&A* **571**, A16 (2014).
14. A. G. Riess *et al*, *AJ* **116**, 1009 (1998).
15. A. G. Riess *et al*, *ApJ* **536**, 62 (2000).
16. N. Suzuki *et al*, *ApJ* **746**, 85 (2012).
17. S. Weinberg, *ApJ* **208**, L1 (1976).