The m-z relation for Type Ia supernovae, locally inhomogeneous cosmological models, and the nature of dark matter

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Abstract

The magnitude-distance relation for Type Ia supernovae is one of the key pieces of evidence supporting the cosmological concordance model. The resulting constraints on the cosmological parameters are often derived under the idealized assumption that the universe is perfectly homogeneous on small scales (at least as far as light propagation is concerned). However, we know that the universe is not homogeneous on small scales, and we know that such local inhomogeneities affect light propagation and hence distances which depend on angles, such as the luminosity distance. What does this mean for constraints on cosmological parameters derived from the magnitude-distance relation for Type Ia supernovae? And, conversely, what does the fact that these constraints, when small-scale homogeneity is assumed, agree with other constraints mean for the nature of dark matter?

1 Introduction

In the last 20 years or so, cosmological observations have improved greatly and it also appears that the measured values are converging on the true values. Among the most important of these observations are those by the Supernova Cosmology Project [9, 26, 25, 24, 1, 30] and the High-z Supernova Search Team [7, 28, 29] which provide joint constraints on λ_0 and Ω_0. Combined with other observations [21, 27], these lead to quite well constrained values for the cosmological parameters [30]. Although the supernova data alone allow a relatively wide range of significantly different other models, it is interesting that the best-fitting values obtained from these observations using the current data are quite close to the much better constrained values using combinations of several observations without the supernova data, at least under the assumptions with which the former were calculated. However, since the m-z relation depends not only on λ_0 and Ω_0 (with H_0 as a scale factor) but also on the distribution of matter along and near the line of sight, the dependence of conclusions drawn from the m-z relation for Type Ia supernovae on this matter distribution should be investigated. Alternatively, these observations can perhaps tell us something about this distribution.

2 Background

Kayser, Helbig, & Schramm (herafter KHS) [20] developed a general and practical method for calculating cosmological distances in the case of a locally inhomogeneous universe. See KHS for details (and for a description of the notation, which is followed here). If the universe is homogeneous, then the fact that light propagates along null geodesics provides sufficient information to calculate distances from redshift. If the universe is locally inhomogeneous, then distances which depend on angular observables related to the propagation of radiation will differ from the homogeneous case because more or less convergence will change the angle involved (the angle at the observer in the case of the angular-size distance, that at the source in the case of the luminosity distance). The basic idea is that one considers a universe which is homogeneous and isotropic on large scales, this determining the global dynamics via the Friedmann-Lemaître equation. Local inhomogeneities are modelled as clumps, where the extra matter in the clumps is taken from the surrounding matter. Thus, a beam which propagates between clumps will have only this thinned-out matter inside the beam, while outside the beam the average density (taking both the thinned-out background matter and the clumps into account) is approximately equal to the global density (precisely so in the limit of an infinitesimal beam).

KHS described the inhomogeneity via the parameter $0 \leq \eta \leq 1$, where η is ratio of the density inside the beam to the global density or, alternatively, the fraction of matter which is homogeneously distributed, as opposed to being clumped.[1] This leads to a second-order differential equation for the angular-size distance (equation (33) in KHS) which can be efficiently integrated numerically. In the locally inhomogeneous case as well the luminosity distance, which is needed here, is larger than the angular-size distance by a factor of $(1+z)^2$. η of course depends on the beam size. Since the beams of supernovae at cosmological distances are extremely thin objects (the thinnest objects ever studied by science), evidence for $\eta < 1$ should be most obvious in the m-z relation for Type Ia supernovae.

Note that η does not have to be constant as a function of redshift. Also, it could be different for different lines of sight. It was pointed out by Weinberg [31] that η must be 1 when averaged over all lines of sight, which follows from flux conservation. However, in practice lines of sight will probably avoid concentrations of matter, due to selection effects or design. If these selection effects do not exist, and if the sample is large enough, then the 'safety in numbers' effect [13] allows one to effectively assume $\eta = 1$ on average, even if it varies from one line of sight to another, with inhomogeneity merely increasing the dispersion, roughly linearly with redshift; see section 3.

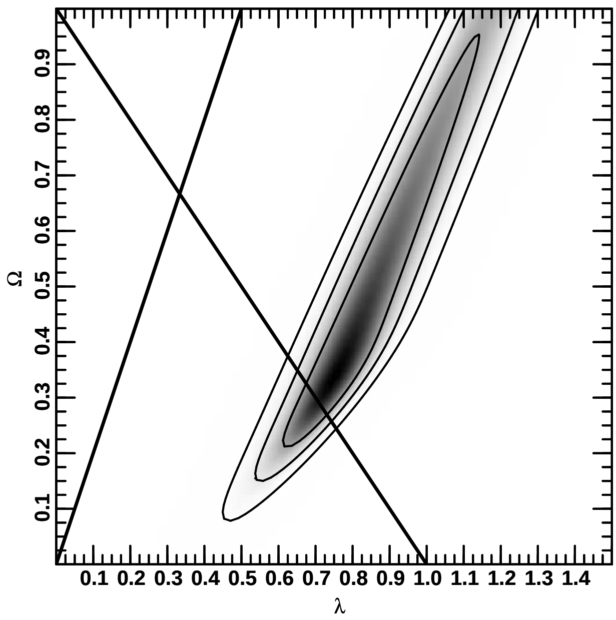
The effects of a locally inhomogeneous universe on quantities important for observational cosmology were first investigated in a series of papers by Zel'dovich and collaborators [32, 4, 3]. Dyer & Roeder [5, 6] discussed the special case of $\lambda_0 = 0$ but with Ω_0 as a free parameter for $\eta = 0$ (where there is an analytic solution) and for general η values. As a result, the distance for $\eta \neq 1$, in particular for $\eta = 0$, is sometimes referred to as the Dyer-Roeder distance. KHS presented a second-order differential equation and numerical implementation valid for the general case ($-\infty < \lambda_0 < \infty$, $0 \leq \Omega_0 < \infty$, $0 \leq \eta \leq 1$). Kantowski and collaborators [14, 19, 15, 17, 18, 16] have stressed the importance of η for the interpretation of the m-z relation for Type Ia supernovae and have provided numerical implementations using elliptic integrals for the special values of η of 0, $\frac{2}{3}$, and 1. The Supernova Cosmology Project [24] considered the effect of $\eta \neq 1$ on their results (see their fig. 8) and concluded that, at least in the 'interesting' region of the λ_0-Ω_0 parameter space, it had a negligible effect. The reason for this work is that, with the larger number of supernovae now available, this is no longer the case. Further investigation has often been motivated by the m-z relation for Type Ia supernovae [8, 23]. It has also been investigated, via comparison with explicit ray-tracing through mass distributions derived from simulations or observations, whether η is a useful parametrization for local inhomogeneity [2, 22] (and the conclusion is that it is a useful approximation, at least for cosmological models which are otherwise realistic).

3 Calculations, results and discussion

I have used the publicly available 'Union2.1' sample of supernova data [30] and calculated χ^2 and the associated probability following Amanullah et al. [1] on a regularly-spaced grid in the λ_0-Ω_0-η parameter space. (More details on these calculations can be found in one of the papers related to this talk [11]; many more plots are available there and in the slides of my talk available at the conference website.) I have used the standard contour values 0.683, 0.954 and 0.997.

The 'standard procedure' for reducing the number of parameters shown in a plot is to *marginalize* over the less interesting or 'nuisance' parameters. This is shown in Fig. 1 (left). In all figures, the grey-scale corresponds to the probability. Fig. 1 (left) also contains two straight lines corresponding

[1] This is sometimes denoted by α. I, and some others, use η because locally inhomogeneous cosmological models are often used in gravitational lensing (which per se implies local inhomogeneities) where α is almost always used to denote the deflection angle.



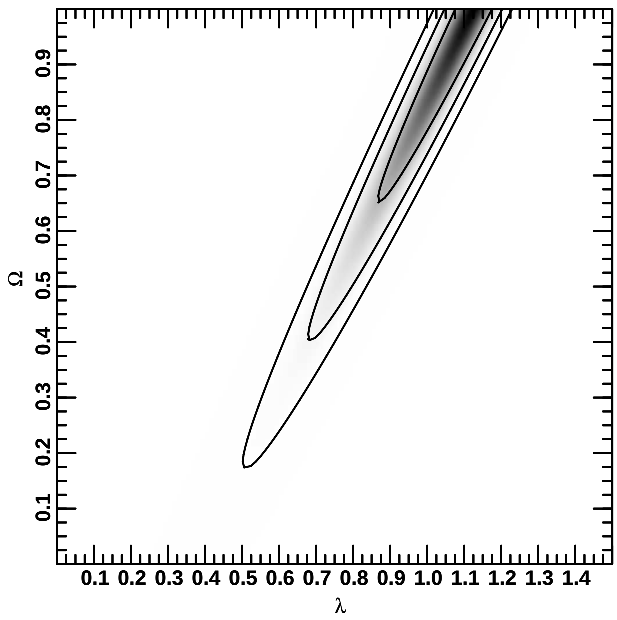


Figure 1: *Left:* Two-dimensional probability distribution obtained by marginalizing over η. *Right:* Two-dimensional probability distribution for $\eta = 0$.

to a flat universe with $\lambda_0 + \Omega_0 = 1$ (negative slope) and zero acceleration ($q_0 = \frac{\Omega_0}{2} - \lambda_0 = 0$) (positive slope).

Most discussion of the m-z relation for Type Ia supernovae has concentrated not on contours of more than two dimensions, nor on some reduction (projection, cut, marginalization, maximization) of these higher-dimensional contours to two dimensions, but rather on two-dimensional contours, i.e. with a δ-function prior on the nuisance parameters. Almost always, of course, the (often implicitly assumed) prior is $\eta = 1$. For comparison, in Fig. 1 (right) and Fig. 2 I show constraints in the λ_0-Ω_0 plane for fixed values of η, namely 0, 0.455 (the value at the maximum of the three-dimensional probability distribution), and 1.

Of course, little significance should be placed on variations in the probability within the innermost contour, since the probability that the point representing the true values of λ_0 and Ω_0 is only about twice as likely to lie inside this contour than outside it. Nevertheless, it is remarkable that the maximum of the probability in Fig. 1 (right) is at $\lambda_0 \approx 0.72$ and $\Omega_0 \approx 0.28$, i.e. at the values of the concordance model (within the small uncertainties; these are much smaller than even the 68.3 per cent contour in Fig. 1 (right)). (Note that when fewer supernova data were available, the best-fitting value was at much higher values of λ_0 and Ω_0 [10].) However, the best-fitting values for the (current) supernova data correspond to the concordance model only if one assumes $\eta \approx 1$. For $\eta = 0.455$, the concordance model lies very near the 95.4 per cent contour, and for $\eta = 0$ it is even outside the 99.7 per cent contour. The plots above illustrate that it is not possible to appreciably constrain η from the supernova data alone. However, the fact that the supernova data suggest the concordance model only for high values of η could be seen as evidence that $\eta \approx 1$.

A similar result is shown in Fig. 3 (left), where a flat universe ($\lambda_0 + \Omega_0 = 1$) has been assumed. As in the other plots, λ_0 is reasonably well constrained, while η is quite weakly constrained. Note that the best-fitting value is for $\eta = 1$ and $\lambda_0 \approx 0.72$; in other words, again the best fit is for the concordance model with $\eta = 1$.

While the supernova data cannot usefully constrain η, as has been shown above, the fact that they result in the concordance model if one assumes $\eta \approx 1$ suggests that $\eta \approx 1$. Since there are many cosmological tests completely independent of the supernova data, and also independent of the value of η, which suggest the concordance model (this is of course why it is called the concordance model), one can assume the concordance values for λ_0 and Ω_0 and calculate the probability of η from the supernova data with these additional constraints; this is shown in Fig. 3 (right).

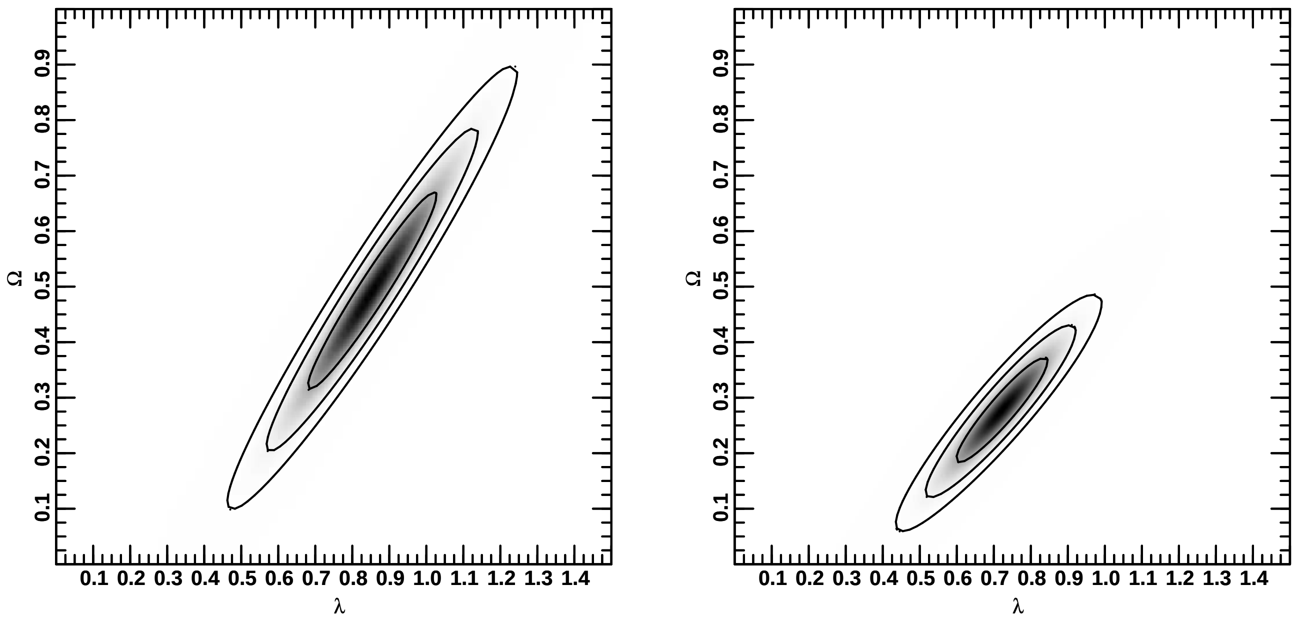


Figure 2: *Left:* Two-dimensional probability distribution for $\eta = 0.455$. *Right:* Two-dimensional probability distribution for $\eta = 1$.

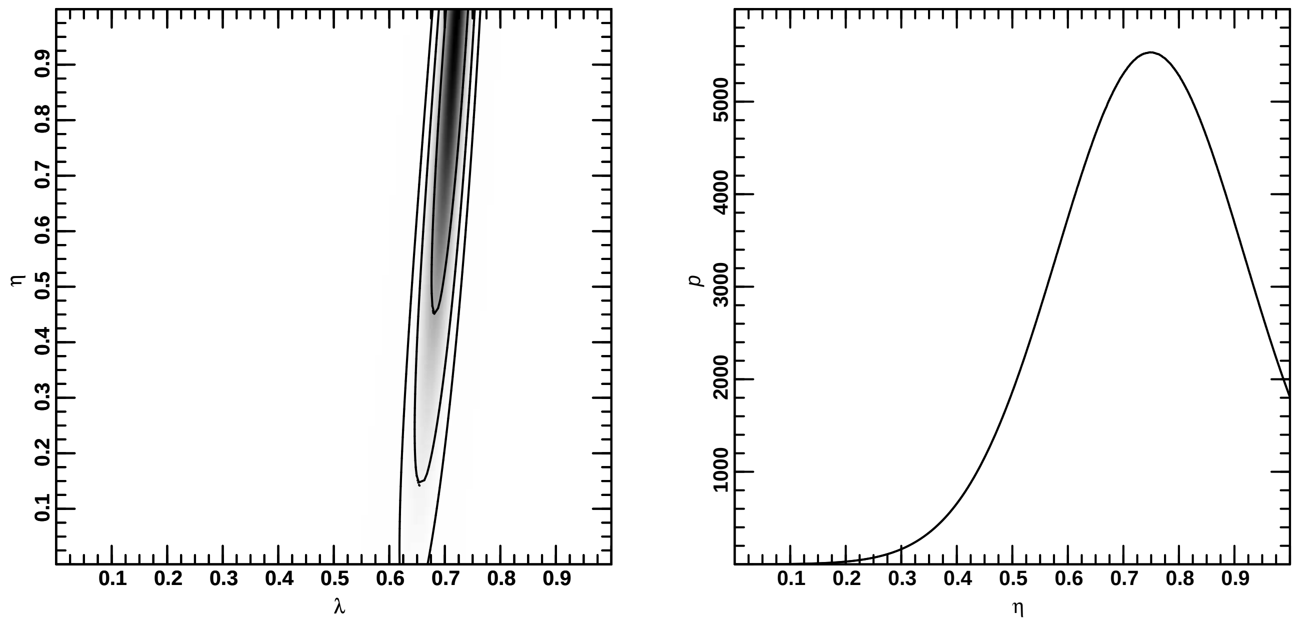


Figure 3: *Left:* Two-dimensional probability distribution for $k = 0$. *Right:* One-dimensional probability distribution for the concordance model.

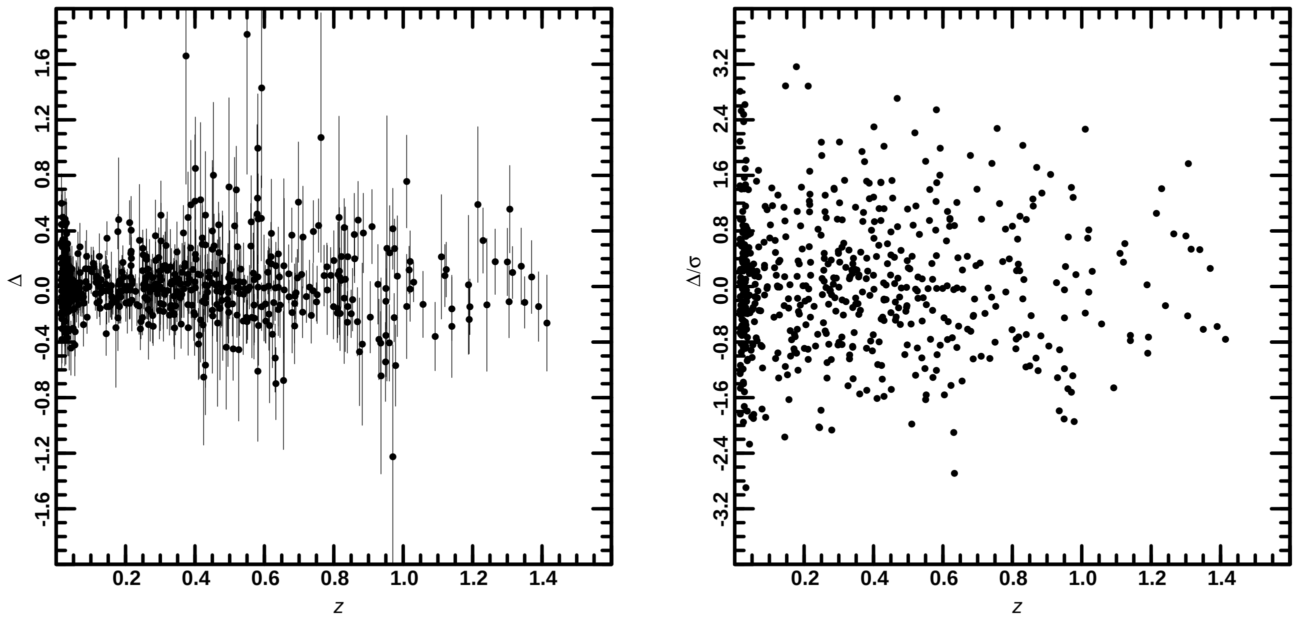


Figure 4: *Left:* Residuals (points) and observational uncertainties (error bars). *Right:* Quotients of residuals and observational uncertainties.

What does it mean that $\eta \approx 1$? This could be due to the density along each line of sight being equal to the overall cosmological density, appropriately averaged along *each* line of sight, or to 'safety in numbers', with variation in the density along *all* lines of sight averaging out if the sample is large enough. In the former case, residuals (differences between observed distance moduli and those calculated for the best-fitting model) and observational uncertainties should be proportional; in the latter case, the residuals should become larger, compared to the uncertainties, as redshift increases. Fig. 4 demonstrates that the quotient of residual and observational uncertainty does not show a trend with redshift. This indicates that the former scenario is more likely [12].

4 Conclusions

The following conclusions were more or less expected: **1:** Constraints on λ_0 and Ω_0 are weaker if η is not constrained. **2:** The concordance model is reasonably probable. **3:** There is a degeneracy between η and the amount of spatial curvature ($\lambda_0 + \Omega_0$). **4:** λ_0 is constrained best, then Ω_0, then η. The following conclusion was neither expected nor surprising: **5:** Even when η is allowed to be a free parameter, the m-z relation for Type Ia supernovae is not compatible with $q_0 = \frac{\Omega_0}{2} - \lambda_0 \geq 0$, and thus implies that the universe is currently accelerating. (Even though the m-z relation for Type Ia supernovae is one of the key pieces of evidence supporting the cosmological concordance model with $\lambda_0 \approx 0.7$ and $\Omega_0 \approx 0.3$, it is not an essential piece in the sense that combinations of other tests still result in the same concordance model. Nevertheless, it is still an important piece of evidence in favour of the concordance model since it is the only single test which, without additional assumptions, implies $q_0 < 0$, i.e. a universe which is currently accelerating.) The following conclusions are somewhat surprising: **6:** The overall (in the three-dimensional parameter space) best-fitting values for λ_0 and Ω_0 are ruled out by other cosmological tests. Probably, this best-fitting point is the result of overfitting: its probability is not significantly higher than elsewhere and the allowed region is quite large. **7:** If one *assumes* $k = 0$ then the best fit is very close to the concordance model *and* has $\eta = 1$. **8:** If one *assumes* $\eta = 1$, then the best fit is very close to the concordance model. **9:** If one *assumes* the concordance model, then one can probably rule out low values of η, even though the relevant scale is extremely small, which implies that dark matter is much less clustered than galaxies are. **10:** We cannot rule out $\eta = 1$, and there is some tentative evidence for it. **11** It appears that each line of sight has, on average, $\eta = 1$, not just that this holds when averaging over many lines of sight.

One might have thought that the increase in the number of data points since the first results of the Supernova Cosmology Project [24] would allow some sort of useful constraint to be placed on η from the supernova data without further assumptions. This is not the case. Even worse, if η is allowed to vary, then the conclusions about the cosmological model derived from the m-z relation for Type Ia supernovae are not as robust. However, as discussed in section 1, current constraints from combinations of cosmological tests without using the supernova data determine the concordance model with $\lambda_0 \approx 0.7$ and $\Omega_0 \approx 0.3$ to rather high precision. It is thus perhaps more interesting to assume the concordance model and use the supernova data to constrain η, especially since η is otherwise difficult to measure. It is also extremely interesting that the supernova data have the best-fitting values for λ_0 and Ω_0 corresponding to those of the concordance model if and only if $\eta \approx 1$ is assumed. If this is not a statistical fluke, it could indicate that $\eta \approx 1$, which is somewhat surprising since the value of η as 'felt' by the supernova might be expected to be somewhat less, because the corresponding beams are extremely thin. The fact that even the supernova data 'want' $\eta \approx 1$ could indicate that dark matter is distributed extremely homogeneously.

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