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The $\Delta\theta$-$z_{\rm s}$ relation for gravitational lenses as a cosmological test

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ABSTRACT

Recently, Park & Gott (1997) claimed that there is a statistically significant, strong, negative correlation between the image separation $\Delta\theta$ and source redshift $z_{\rm s}$ for gravitational lenses. This is somewhat puzzling if one believes in a flat ($k=0$) universe, since in this case the typical image separation is expected to be independent of the source redshift, while one expects a negative correlation in a $k=-1$ universe and a positive one in a $k=+1$ universe. Park & Gott explored several effects which could cause the observed correlation, but no combination of these can explain the observations with a realistic scenario. Here, I explore this test further in three ways. First, I show that in an inhomogeneous universe a negative correlation is expected regardless of the value of k. Second, I test whether the $\Delta\theta$-$z_{\rm s}$ relation can be used as a test to determine λ_0 and Ω_0, rather than just the sign of k. Third, I compare the results of the test from the Park & Gott sample to those using other samples of gravitational lenses, which can illuminate (unknown) selection effects and probe the usefulness of the $\Delta\theta$-$z_{\rm s}$ relation as a cosmological test.

Key words: gravitational lensing – cosmology: theory – cosmology: observations.

1 INTRODUCTION

Historically, there has been little interest in the $\Delta\theta$-z_s relation compared to other cosmological tests based on gravitational lensing statistics, perhaps because the inflationary paradigm (e.g. Guth 1981), which began about the same time as the discovery of the first gravitational lens (Walsh, Carswell & Weymann 1979), has become so influential. Since a flat ($k=0$) universe is a robust prediction of inflation, many researchers assume this and consider only flat universes (or, at most, $k=-1$ cosmological models with $\lambda_0=0$). Due to the fact that for the popular singular isothermal sphere model for a single-galaxy lens the average image separation $\Delta\theta$, integrated over the lens redshift $z_{\rm d}$ from $z_{\rm d}=0$ to $z_{\rm d}=z_s$, is *completely independent* of the source redshift z_s in a flat universe, there is little point in pursuing the $\Delta\theta$-z_s relation if one is interested primarily in flat cosmological models. If one is not committed to a flat universe, then of course one should not assume $k=0$, but even if one believes that the universe must be flat, it is still important to test this belief observationally. The situation is somewhat worsened by the fact that most 'standard' cosmological tests such as the m-z (magnitude-redshift or 'standard candle') and θ-z (angular size-redshift or 'standard rod') relations, 'conventional' gravitational lensing statistics, age of the universe) are relatively insensitive to the radius of curvature of the universe ($R_0 \sim (|\Omega_0+\lambda_0-1|)^{-\frac{1}{2}}$), being degenerate in combinations of λ_0 and Ω_0 in directions roughly perpendicular to lines of constant R_0 in the λ_0-Ω_0 plane. A notable exception are constraints derived from CMB anisotropies (e.g. Scott, White & Silk 1995; Hu, Sugiyama & Silk 1997).

2 THEORY

For a singular isothermal sphere lens, the angular image separation is given by (e.g. Turner, Ostriker & Gott 1984)

$$\Delta\theta = 8\pi\left(\frac{v}{c}\right)^2 \frac{D_{\rm ds}}{D_{\rm s}} \quad , \tag{1}$$

where v is the velocity dispersion and D is the angular size distance (see below). Even if the singular isothermal sphere is not a perfect model for the gravitational lens systems considered, it is still a good approximation when one is concerned only with the image separation. For a given v, by combining Eqs. (5) and (6) in Gott, Park & Lee (1989) and using the more appropriate and more general angular size distances, one obtains an expression for the average image separation $\Delta\theta$, by integrating over the lens redshift $z_{\rm d}$ from

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$z_{\rm d}=0$ to $z_{\rm d}=z_s$,

$$\frac{\Delta\theta(z_s)}{\Delta\theta(0)} = \frac{\left(\int\limits_0^{z_s} {\rm d}z_{\rm d}\,\frac{D_{{\rm d}s}^3 D_{\rm d}^2\,(1+z_{\rm d})^2}{D_s^3 Q}\right)}{\left(\int\limits_0^{z_s} {\rm d}z_{\rm d}\,\frac{D_{{\rm d}s}^2 D_{\rm d}^2\,(1+z_{\rm d})^2}{D_s^2 Q}\right)}\,, \qquad (2)$$

where

$$Q=\sqrt{\Omega_0\,(1+z_{\rm d})^3-(\Omega_0+\lambda_0-1)\,(1+z_{\rm d})^2+\lambda_0}\quad . \qquad (3)$$

The D_{ij} (with $D_k := D_{0k}$) in Eqs. (1) and (2) are angular size distances, which are functions of the lens and source redshifts $z_{\rm d}$ and z_s, the cosmological parameters λ_0 and Ω_0 as well as the 'homogeneity parameter' η, which gives the fraction of smoothly, as opposed to clumpily, distributed matter along the line of sight. Note that Eq. (2) is valid for all combinations of λ_0, Ω_0 and η. The angular size distances can be computed for arbitrary combinations of these parameters by the method outlined in Kayser, Helbig & Schramm (1997).

Figures 1 and 2 show $\Delta\theta$ as a function of z_s for various cosmological models, for $\eta=1$ (the traditional case assuming a completely homogeneous universe) and $\eta=0$ as extreme cases. Note in Fig. 1 that the curve is a horizontal line for $k=0$, has positive slope for $k=+1$ and negative slope for $k=-1$, where $k:=\mathrm{sign}(\Omega_0+\lambda_0-1)$. In Fig. 2, for $\eta=0$, the slope is negative regardless of the value of k. Thus, at first sight it appears that an inhomogeneous universe, a possibility not investigated by Park & Gott (1997, hereafter PG), might be able to explain the puzzling negative correlation between $\Delta\theta$ and z_s. However, it is shown in Sect. 5 that even the extreme $\eta=0$ scenario produces an anticorrelation which is much weaker than that found by PG. This effect can be qualitatively understood by realizing how Eq. (2) is affected by decreasing η: inspection shows that this might be estimated by examining $D_{{\rm d}s}/D_s$. All other things being equal, the angular size distance increases with decreasing η. Also, the effect of η is more noticeable at large redshift differences. Since $z_s \geq z_s - z_{\rm d}$, the denominator is the more important term, and so decreasing η increases D_s and so decreases $D_{{\rm d}s}/D_s$ and thus $\Delta\theta(z_s)/\Delta\theta(0)$.

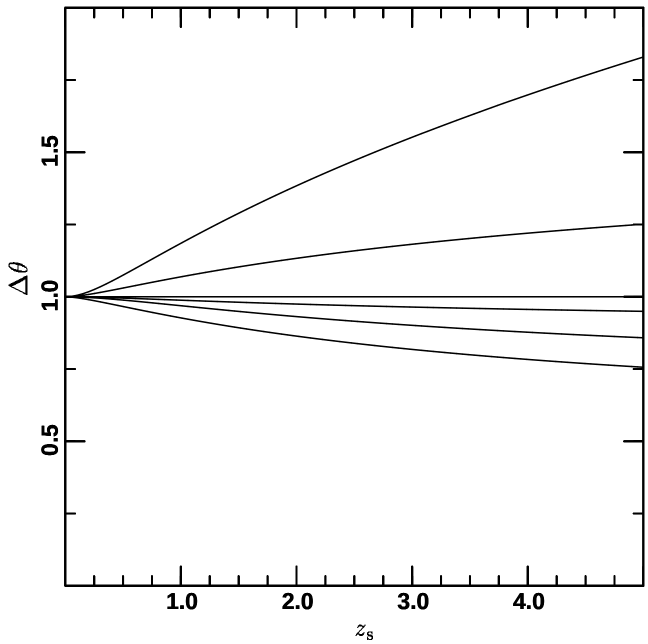

Figure 1. Normalized image separation as a function of source redshift. From the top, the (λ_0,Ω_0) values are (2,4), (0,4), $k=0$, (0,0.7), (0,0.3) and (-5,1). For $k=0$ the result is valid for all (λ_0,Ω_0) values whose sum is 1. $\eta=1$.

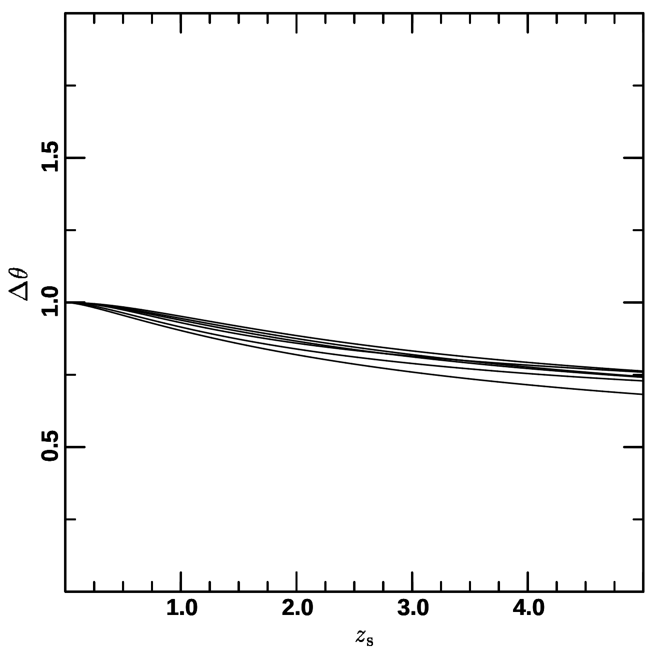

Figure 2. The same as Fig. 1 except that here $\eta=0$.

3 DATA

PG used an inhomogeneous sample of gravitational lenses from the literature. While this seems problematic at first sight, PG noted that there is no reason to believe that this should influence the analysis. Nevertheless, it is worth comparing the PG results to those obtained from a better defined sample.

The observational data provided by the JVAS and CLASS surveys offer an independent sample of gravitational lenses. JVAS is the Jodrell Bank VLA Astrometric Survey (Patnaik et al. 1992); CLASS is the Cosmic Lens All-Sky Survey (Myers et al. 1998). Even though the observational tasks are not yet complete, the JVAS and CLASS surveys which constitute the database have already yielded sufficient gravitational lenses to enable one to make an independent analysis. Table 1 shows the current state of knowledge about the JVAS/CLASS gravitational lenses. Note that the questionable source redshift for $2114+022$ is probably the redshift of an additional lensing galaxy (this interpretation is supported by several independent lines of evidence).

Although not all source redshifts in the JVAS/CLASS sample are known, 8 out of 11 are, and based on our sur-

Table 1. The JVAS/CLASS gravitational lenses.

Name	# images	$\Delta\theta$ [arcsec]	lens galaxy type	z_d	z_s
0218+357	ring + 2	0.33	spiral	0.6847	0.96
0414+0534	4	2.0	elliptical	?	2.62
0712+472	4	1.2	?	0.406	1.339
1030+074	2	1.6	peculiar	0.599	1.535
1422+231	4	1.2	?	0.65	3.62
1600+434	2	1.4	spiral	0.4144	1.589
1608+656	4	2.2	spiral?	0.64	1.39
1933+503	4+4+2	0.9	?	0.755	?
1938+666	4+2	0.9	?	?	?
2045+265	4+1?	2.0	?	0.87	1.28
2114+022	2+2?	2.4	?	0.316	0.588?

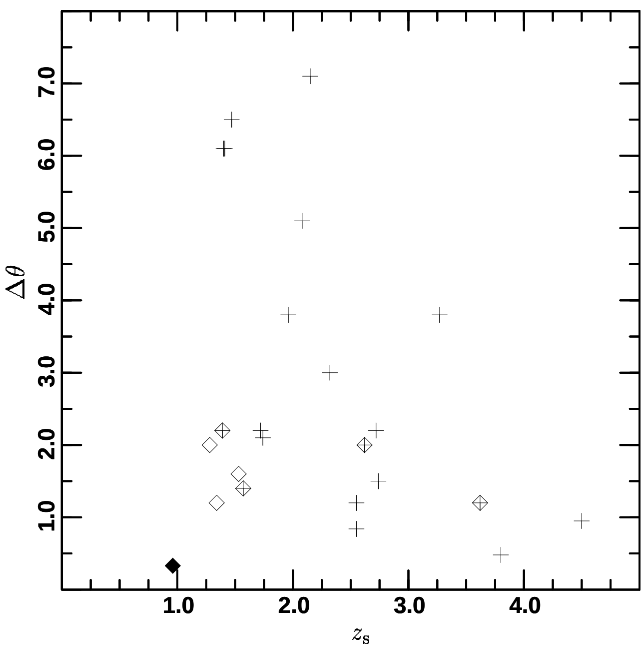

Figure 3. Source redshifts z_s and image separations $\Delta\theta$ (in arcsec) for the gravitational lens systems studied in this paper. Crosses represent the PG sample (20 systems; note that two data points with $\Delta\theta \approx 6$ arcsec almost coincide); diamonds represent the JVAS/CLASS sample (8 systems; of course only those with known source redshifts are included). Note that there is an overlap of four data points. The filled diamond represents the system 0218+357, which was not used by PG although its source redshift had been published before the PG analysis was done (Lawrence 1996).

vey, discovery and followup strategies there is no reason to suspect the unknown source redshifts to be statistically different from those already known. Figure 3 shows the source redshifts and image separations of the gravitational lens systems used in this paper: the PG sample and the JVAS/CLASS sample.

4 CALCULATIONS

All calculations here implement the method of PG, which uses the Spearman rank correlation test to generate a relative probability for a given cosmological model. PG noted the fact that they always obtained a low probability with their sample, even when allowing for non-flat cosmological models (albeit in a limited area of parameter space), galaxy evolution or departure from the singular isothermal sphere model. As PG noted, allowing for these effects increases the probability, since they all tend to create a negative correlation in a flat universe, but the magnitude of the effect is not large enough to explain the observations. Again as noted by PG, if the lenses are parts of clusters, then this will work in the opposite direction, making the observed negative correlation even more puzzling.

Calculations were done for four samples:

the PG sample
the PG sample with the addition of the system $0218+357$
the JVAS/CLASS sample
the union of all samples

Note that the source redshift for $0218+357$ had been published before the PG analysis was done (Lawrence 1996). Since $0218+357$ lies below and to the left of all other data points, it is clear that including it will weaken the puzzling negative correlation found by PG; this is discussed more quantitatively in Sect. 5.

5 RESULTS AND DISCUSSION

Since the PG test assigns a low probability to a $k = 0$ universe, the question arises as to whether it can be used as a general cosmological test to determine the values of λ_0 and Ω_0. This is not the case. For all four samples I have calculated the Spearman rank correlation probability as a function of λ_0 and Ω_0 in a range of parameter space ($-8 < \lambda_0 < 2$ and $0 < \Omega_0 < 10$) much larger than that allowed even by a generous interpretation of observations. This was done with a resolution of 0.1 in both λ_0 and Ω_0 for both $\eta = 1$ and $\eta = 0$. The Spearman rank correlation probability is essentially constant over a wide range of parameter space; basically, either all cosmological models are probable, or all are improbable, depending on the sample used.

The probability is a weak function of the cosmological model, with the sharpest transition occuring when crossing the $k = 0$ line in the λ_0-Ω_0 plane. For all samples except the PG sample, the probability is >5% in almost the entire parameter space;† those cosmological models with a lower probability are among those ruled out by current observations. Thus, the Spearman rank correlation probability does not allow one to reject any otherwise viable cosmological models, which shows both that there is no reason to expect unknown effects in the gravitational lens samples and that it is not very useful as a cosmological test. For the PG sample, the 1% contour corresponds almost exactly to the $k = 0$ line, with higher values for a negatively curved universe. Thus, the PG sample is *marginally* compatible with a $k = -1$ cosmological model, although the probability values are low throughout the λ_0-Ω_0 plane, with values near the maximum of 0.025 being attained only for small (but realistic) Ω_0 values and large (in absolute value) negative values of λ_0. Since there are no known selection effects which can account for the differences between the PG sample and other samples, either the test is not very useful and/or it is pointing to unknown selection effects in the literature sample used by PG. The fact that the PG result changes dramatically (probability $\approx$ 10–20% in most of the λ_0-Ω_0 plane) by the inclusion of just 1 additional data point, which could have been included in their analysis, argues in favour of the former possibility.

The above discussion was for $\eta = 1$. For $\eta = 0$ the situation is qualitatively the same and quantitatively involves only slightly different values of probabilities derived from the Spearman rank correlation test.

It is interesting to compare the probabilities from the Spearman rank correlation test for the PG sample using the actual values of z_s and $\Delta\theta$ as used by PG to those obtained using more up-to-date data for the *same* lens systems. If two values are very near each other, rounding them off to the same values produces a different result for the rank correlation test than if they differ by even a small amount. Using more up-to-date data, an even lower probability is obtained for the PG sample, for $\eta = 1$ and $\eta = 0$, for a wide variety of cosmological models.

6 CONCLUSIONS

Park & Gott (1997) pointed out that the image separations in gravitational lens systems show a strong significant negative correlation with the source redshift, while in a flat universe one would expect no correlation (while a negative correlation would be expected in a universe with negative curvature and a positive one in a universe of positive curvature). None of the possibilities they examined were strong enough to explain the effect. A possibility not examined by them, namely an inhomogeneous universe, produces a negative correlation regardless of the sign of the curvature, but it too is not strong enough to account for the effect. As a general test for the values of λ_0 and Ω_0 the test is of no use, all cosmological models being assigned roughly the same probability, but *which* value they are assigned depends on the sample used.

The strong dependence of the result on the sample used seems to indicate that the result of Park & Gott (1997) is due not to some physical cause but rather to unidentified selection effects in the sample of gravitational lenses taken from the literature. The large number of JVAS and CLASS lenses gives us an independent comparison sample, thus demonstrating the need for discovering a large number of lenses in a well-defined sample. As Park & Gott (1997) point out, since many conclusions based on 'conventional' gravitational lensing statistics are based on essentially the same lenses as in their literature sample, if this sample is for some unknown reason atypical, then conclusions drawn from statistical analyses of it must be examined with care. It will thus be interesting to see what conclusions can be drawn from a statistical analysis of the JVAS/CLASS sample after the observational tasks have been completed. (We expect to find more lenses, but have no qualms about using the present incomplete sample in this analysis since there is no reason to believe that a larger sample would show a different $\Delta\theta$-z_s relation.)

7 NOTE

Since this work was completed, two other responses to Park & Gott (1997) (apart from Helbig (1998)) have appeared. The first (Williams 1997) is complementary to this work in that it assumes the effect is real and explores the astrophysical consequences while the second (Cooray 1998) is more similar to this analysis, arriving at essentially the same conclusions though using different observational data (and exploring neither the question of usefulness as a general test for λ_0 and Ω_0 nor the effects of a locally inhomogeneous universe).

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† For the JVAS/CLASS sample, the maximal probability is 0.955 and is realized in almost the entire $k = +1$ area of the parameter space.

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