Gravitational lensing statistics with extragalactic surveys

III. Joint constraints on λ_0 and Ω_0 from lensing statistics and the m-z relation for type Ia supernovae

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Abstract. I present constraints on cosmological parameters in the λ_0-Ω_0 plane from a joint analysis of gravitational lensing statistics (Helbig et al. 1999b) and the magnitude-redshift relation for Type Ia supernovae (Perlmutter et al. 1999; Riess et al. 1998). I discuss reasons why this particular combination of tests is important and how the constraints can be improved in the future. The lensing statistics and supernova results are not inconsistent, thus it is meaningful to determine joint constraints on λ_0 and Ω_0 by combining the results from both tests. The quantity measured by the lens statistics and the m-z relation for type Ia supernovae discussed here is approximately $\lambda_0 - \Omega_0$. At 95% confidence, the upper limit on $\lambda_0 - \Omega_0$ from lensing statistics alone is 0.45 and from supernovae alone is in the range 0.65–0.81 (depending on the data set). For joint constraints, the upper limit on $\lambda_0 - \Omega_0$ is in the range 0.55–0.60 (again depending on the data set). For a flat universe with $\lambda_0 + \Omega_0 = 1$, this corresponds to upper limits on λ_0, taking the top of the range from different data sets, of 0.72, 0.90 and 0.80 for lensing statistics alone, supernovae alone and the joint analysis, respectively. This is perfectly consistent with the current 'standard cosmological model' with $\lambda_0 \approx 0.7$ and $\Omega_0 \approx 0.3$ and is consistent with a flat universe but, neglecting other cosmological tests, does not require it.

Key words: cosmology: gravitational lensing – cosmology: theory – cosmology: observations – cosmology: miscellaneous

1. Introduction

Recently, several papers (e.g. Ostriker & Steinhardt 1995; Turner 1996; Bagla et al. 1996; Krauss 1998; White 1998; Tegmark et al. 1998a,b; Eisenstein et al. 1998, 1999; Webster et al. 1998; Bridle et al. 1999; Efstathiou et al. 1999) have pointed out the advantages of joint analyses of cosmological parameters, i.e. combining the information from more than one cosmological test. Ideally, such tests would be complementary, i.e. the degeneracy in the λ_0-Ω_0 plane would be in orthogonal directions. However, even if this is not the case, indeed, even if the degeneracy is exactly the same, the combination of tests can tighten the constraints as well as serve as a consistency check. Here, I discuss constraints on cosmological parameters in the λ_0-Ω_0 plane from a joint analysis of gravitational lensing statistics (Helbig et al. 1999b, hereafter Paper II) and the magnitude-redshift relation for Type Ia supernovae, using the results of the Supernova Cosmology Project and the High-Z Supernova Search Team (Perlmutter et al. 1999; Riess et al. 1998, hereafter SCP and HZSST, respectively). Although both tests are preliminary in the sense that they will improve with more and better observational data, the time is already ripe for a joint analysis, to demonstrate both what already can be done and how each test can be improved to lead to tighter joint constraints.

The plan of this paper is as follows. In Sect. 2 I briefly review the basis of each of these two cosmological tests. In Sect. 3 I present and discuss the joint constraints. Sect. 4 provides a summary and conclusions.

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2. Theory review

I here use the notation of Kayser et al. (1997) with regard to cosmology and refer the reader there for the relevant definitions. In particular, Ω_0 refers only to 'ordinary matter' and λ_0 is the normalised cosmological constant, such that $\lambda_0 + \Omega_0 = 1$ for a flat universe.

Both gravitational lensing statistics and the magnitude-redshift relation are 'classical' cosmological tests, i.e. the theoretical dependence of an observable quantity on redshift is compared with observations. This is done straightforwardly in the case of the magnitude-redshift relation, and in a somewhat more roundabout way in the maximum-likelihood analysis of gravitational lens statistics used here.

lation extends at present out to $z \approx 1$. In the case of lensing statistics, the source population extends to quite large redshifts ($z \approx 4$) although the redshift range of significant optical depth is smaller. Thus, the two tests are both 'global' rather than 'local' cosmological tests and probe similar, though not identical, redshift ranges. Otherwise, the tests are completely independent.

The m-z relation is concerned essentially only with the luminosity distance D^{L} whereas lensing statistics deal with several different angular size distances (between observer and lens (D_{d}), observer and source (D_{s}) and lens and source (D_{ds})) (see, e.g., Kayser et al. 1997, for a discussion of the various cosmological distances) and the volume; they also depend on several other 'astrophysical' parameters (e.g. Kochanek 1996; Quast & Helbig 1999, hereafter Paper I).

2.1. The m-z relation for type Ia supernovae

The basic idea of the m-z relation is simple: one has an object of known absolute magnitude M and compares it to the observed magnitude m. The difference or distance modulus is

$$m - M = 5 \log_{10} D^{\mathrm{L}} + K + 42.384 - 5 \log_{10} h, \qquad (1)$$

where D^{L} is in units of the Hubble length, K is the K-correction and h is the Hubble constant in units of 100 km/s/Mpc (see, e.g., Kayser et al. 1997, for a derivation)[1]. This depends on the cosmological model since D^{L} depends on the cosmological parameters λ_0 and Ω_0. Note that, as is often the case in practice, if M is known modulo h, then Eq. (1) does not depend on the Hubble constant at all. On the other hand, if M is known absolutely, this is equivalent to knowing h, assuming one has at least one object at low redshift (where the dependence on λ_0 and Ω_0 is negligible). In any case, our knowledge (or lack of it) about the value of the Hubble constant H_0 does not appreciably affect the ability of this cosmological test to measure the cosmological constant λ_0 and the density parameter Ω_0.

Thus, one has a number of objects with observed magnitudes m_i and a way of calculating the absolute magnitudes M_i (see, e.g., SCP and HZSST for a description of how this is done in practice) and fits for the parameters λ_0 and Ω_0. If all objects are in a narrow redshift range, then confidence contours in the λ_0-Ω_0 plane will only allow one to measure approximately $\lambda_0 - \Omega_0$ whereas having objects at different redshifts breaks this degeneracy (e.g. Goobar & Perlmutter 1995).

[1] Note that the second occurrence of the term 'Hubble length' in Kayser et al. (1997) should actually be 'Hubble length for $h = 1$', although this is obvious from the context.

See Paper I and references therein for a discussion of how constraints on λ_0-Ω_0 are derived from gravitational lensing statistics. Gravitational lensing statistics, at least in the 'interesting' part of parameter space, constrain approximately $\lambda_0 - \Omega_0$ (e.g. Cooray 1999). Thus the degeneracy is approximately the same as that of the m-z test. Thus, rather than reducing the allowed area of parameter space through orthogonal degeneracies, these two cosmological tests provide a consistency check on each other. Also, the m-z relation provides a good *lower* limit on λ_0 while lensings statistics provides an *upper* limit; obviously, the former should be smaller than the latter. If this is the case, then the two cosmological tests are consistent with each other, and it is meaningful to construct joint constraints, which allow a region of parameter space smaller than that allowed by either test alone.

3. Data and results

3.1. Individual results

For the m-z test I used the results presented in SCP and HZSST, which have kindly been made available by the respective collaborations, as well as our own results from the analysis of JVAS, the Jodrell Bank-VLA Astrometric Survey (Paper II and references therein). Fig. 1 shows the likelihood ratio as a grey scale and the 68%, 90%, 95% and 99% confidence contours for the results from SCP and HZSST; Fig. 2 does the same for the results from Paper II. See these references for discussions of these results individually. I use the JVAS results of Paper II, rather than those of Paper I, since the former seem more reliable, despite the remaining uncertainties (see Paper II for a discussion). Also, using only one set of lensing statistics results, rather than a combination, is conservative, since the joint constraints are tighter than individual constraints.[2]

The basic format here is that of a probability density function, i.e. a relative probability as a function of λ_0 and Ω_0. Ideally, this would cover *all* values of λ_0 and Ω_0, or at least all for which there is a non-negligible probability. Alternatively, one can impose a prior constraint on λ_0, Ω_0 or both, such that there is a non-negligible probability only in a comfortably small region of parameter space. The simplest way to do this is to use a top-hat function, such that the a posteriori likelihood is given by the a priori likelihood within some range and is exactly zero outside of this range. This is a conservative approach

[2] Of course, if one is concerned with the consistency of the results, rather than in reducing the parameter space through joint constraints, then one should use as many results as possible. However, it only makes sense in this context to use reliable results, so this is a reason to neglect the results based on optical gravitational lens surveys discussed in Paper I.

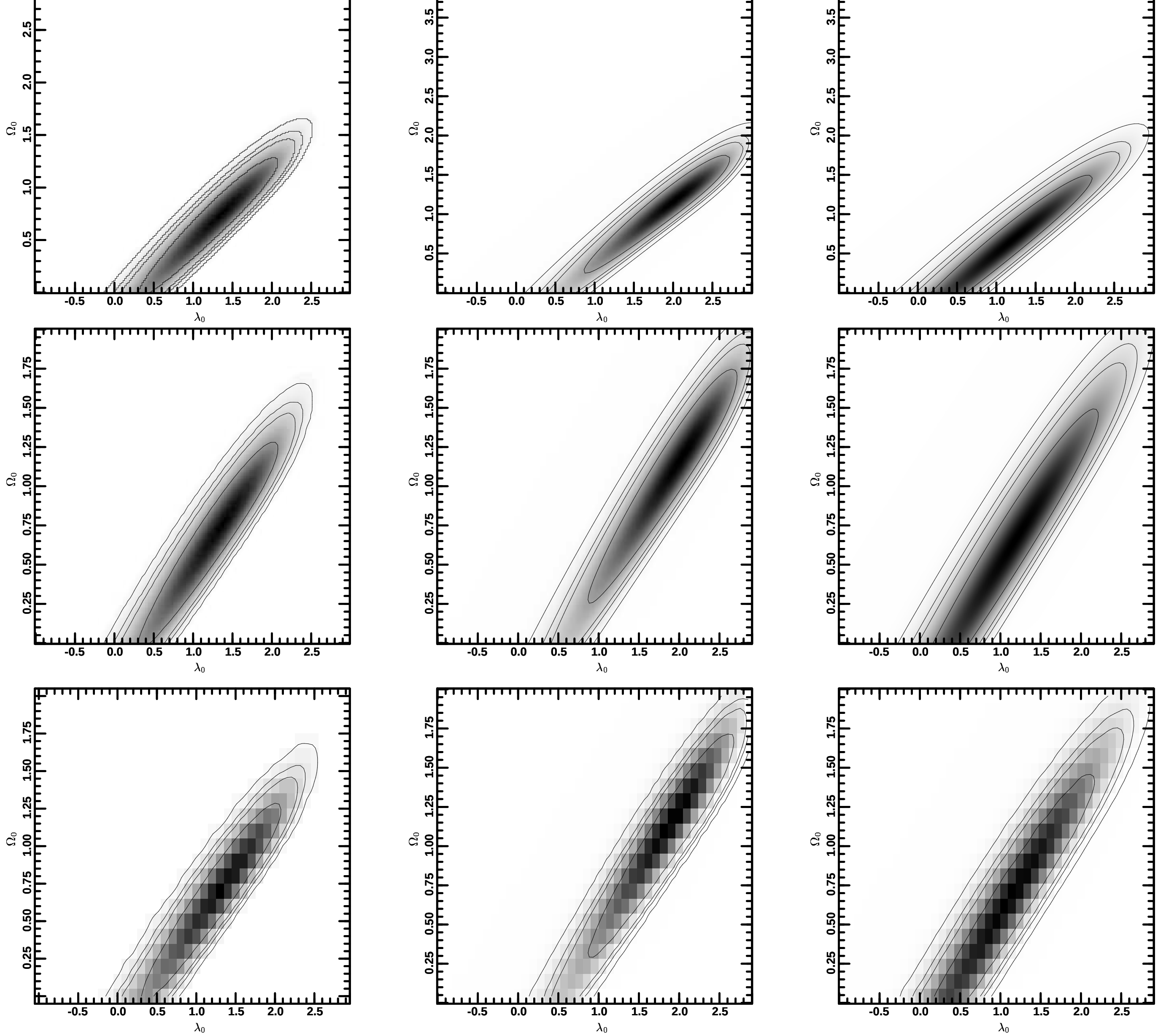

Fig. 1. The likelihood function $p(D|\lambda_0, \Omega_0)$ (cf. Paper I) from Perlmutter et al. (1999) (SCP, equivalent to their Fig. 7; hereafter data set $\mathcal{A}$) (left column), Riess et al. (1998) (HZSST) ($\Delta m_{15}(B)$ method, equivalent to the dotted contours of their Fig. 7; hereafter data set $\mathcal{B}$) (middle column) and Riess et al. (1998) (MLCS method, equivalent to the dotted contours of their Fig. 6; hereafter data set $\mathcal{C}$) (right column) in the original parameter space and resolution (top row), in the parameter space used for the calculations in this paper but in the original resolution (middle row) and in the parameter space used for calculations in this paper in the resolution used for calculations in this paper (bottom row). The pixel grey level is directly proportional to the likelihood ratio, darker pixels reflect higher ratios. The contours mark the boundaries of the minimum 0.68, 0.90, 0.95 and 0.99 confidence regions for the parameters λ_0 and Ω_0

if the allowed range is large enough to include the correct cosmological model in any case and also since, within the allowed range, the likelihood depends only on the cosmological tests considered and not on the priors (which makes for easier interpretation).

The first three rows of Table 1 show the range of parameter space covered by the references which are used to provide input data for this work. I take as the only prior that the likelihood is zero outside the overlap of the various ranges of the various cosmological tests used, as in the last row in Table 1. The lower limit on $\Omega_0 = 0$ is physical and the upper limits $\Omega_0 = 2$ and $\lambda_0 = 2.9$ are certainly large enough (see the discussion in Paper I on these values and on the use of prior information in general). The lower

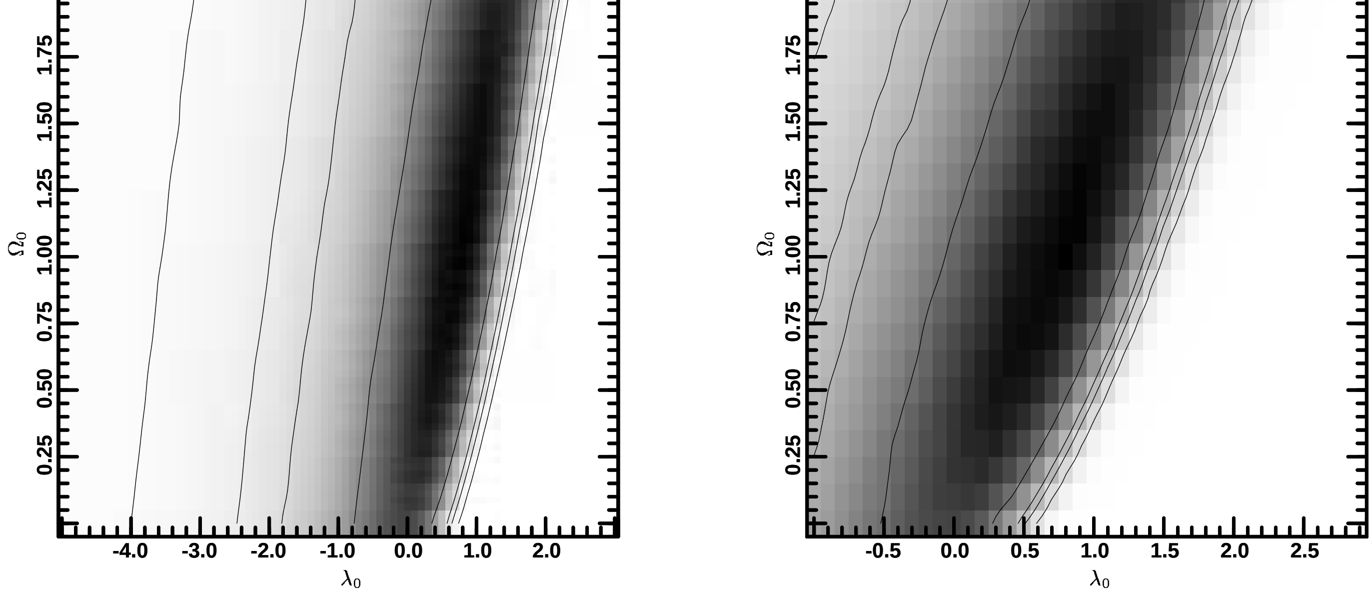

Fig. 2. The likelihood function $p(D|\lambda_0, \Omega_0)$ from Paper II (hereafter data set $\mathcal{D}$) in the original parameter space (left) and in the parameter space used for the calculations in this paper (right). (The resolution in Paper II is (due to the fact that the lens statistics calculations are numerically much more demanding) the worst of all the data sets considered here and is thus the one used for the calculations in this paper.) See Fig. 1 for a description of the plotting scheme

Table 1. The range of λ_0 and Ω_0 explored by the references used here

Reference	λ_0 range		λ_0 resolution	Ω_0 range		Ω_0 resolution
Perlmutter et al. (1999) (SCP)	-1.00	2.98	0.02	0.00	2.99	0.01
Riess et al. (1998) (HZSST)	-1.00	3.00	0.01	0.00	4.00	0.01
Helbig et al. (1999b) (Paper II)	-5.00	3.00	0.10	0.00	2.00	0.10
this work	-1.00	2.90	0.10	0.00	2.00	0.10

limit on λ_0 comes mainly from the fact that $\lambda_0 < -1$ is strongly excluded by the m-z relation itself, although the analysis of current cosmic microwave background observations (e.g. Lineweaver 1998; Macias Perez et al. 1999; Helbig et al. 1999a) also suggests this. In any case, an analysis of joint constraints from other cosmological tests, even excluding the m-z relation, suggests that $\lambda_0 > 0$ is a robust result (e.g. Roos & Harun-or-Rashid 1999).

With an individual test, likelihood contours are found (in all of the cases discussed here; see the discussion in Paper I for possible caveats when comparing the results of various cosmological tests as presented in the literature) by finding the highest contour at constant likelihood such that the corresponding fraction of the total likelihood is enclosed. Note that these contours depend not only on the likelihood ratio but also on the range of parameter space plotted. In this work, the parameter space plotted should be considered to have an a priori likelihood of 1, while the parameter space outside the plot should be considered to have an a priori likelihood of 0. Note that these are 'real' likelihood contours, not approximations based on $\Delta\chi^2$, the assumption that the probability distribution is (a 2-dimensional) Gaussian etc such as one often finds in the literature.

It should also be noted that I take the likelihood as presented in the references in question. In the case of the lens statistics (data set $\mathcal{D}$), all parameters except λ_0 and Ω_0 were held constant. In the case of the m-z relation (data sets $\mathcal{A}$–$\mathcal{C}$), the results are obtained by marginalising over the nuisance parameters (see, e.g., Paper I for a discussion). However, this is of no concern at the level of accuracy I am concerned with here, especially since there are no nuisance parameters common to the m-z test and the lensing statistics test.[3]

[3] The publicly available data from SCP are actually not the likelihood itself, but rather the value for each point in the parameter space is the (normalised) sum of all likelihood values for all points in the parameter space which are not less than the value for the point in question. This format, which allows one to immediately plot a given confidence contour by plotting a contour at that level, I have converted back to the original probability density function.

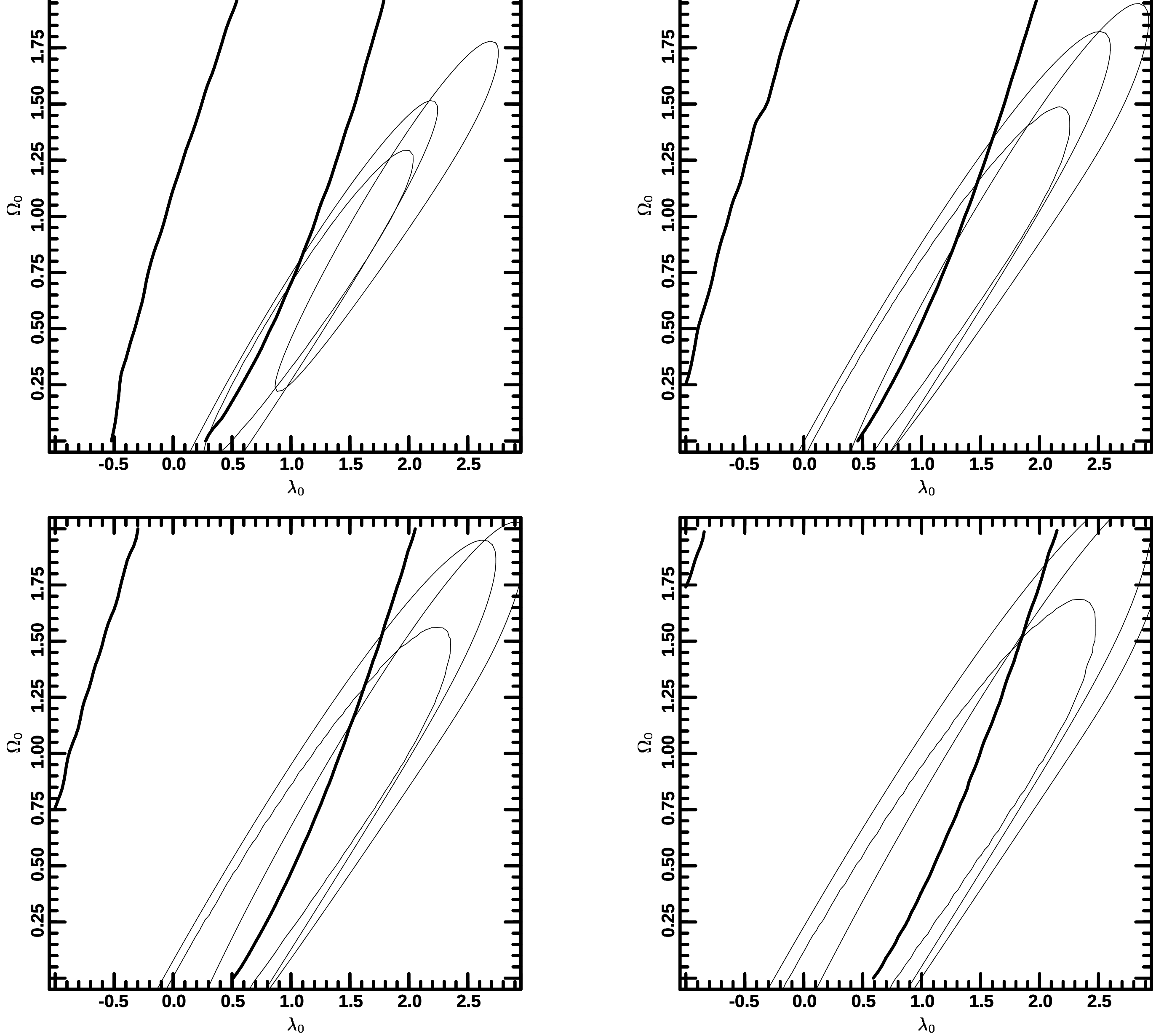

Fig. 3. The 68% (top left), 90% (top right), 95% (bottom left) and 99% (bottom right) confidence contours for each of the data sets. The thick curves are for the lensing statistics results (data set $\mathcal{D}$). In all plots, data set $\mathcal{A}$ has the contour with the lowest value of Ω_0 at its maximum height. Starting from this point and moving left, towards smaller values of λ_0, in all plots one crosses first the contour of data set $\mathcal{B}$ then that of data set $\mathcal{C}$

3.2. Joint constraints

The simplest thing to do when building joint constraints would be to multiply the corresponding probability density functions (PDFs).[4] One can then plot confidence contours in the manner described above. However, it is obvious that this is not meaningful if the PDFs are not consistent with each other, i.e. if the region of confidence for a 'sensible' confidence level from the joint constraints does not overlap with the corresponding confidence level for all component tests. A necessary, though not sufficient, condition for this inconsistency to exist is that the corresponding confidence contours for the individual component tests do not overlap. Fig. 3 shows the 68%, 90%, 95% and 99% confidence contours for the four data sets considered here. As the 90% confidence contours from all supernovae data

[4] Of course, this must be done at the same resolution. Rather than interpolate the low resolution lens statistics results, I have reduced the resolution of the m-z results to that of the lensing statistics results by using only those points in the λ_0-Ω_0 plane which were examined in the lens statistics calculations, all of which were examined by both m-z tests.

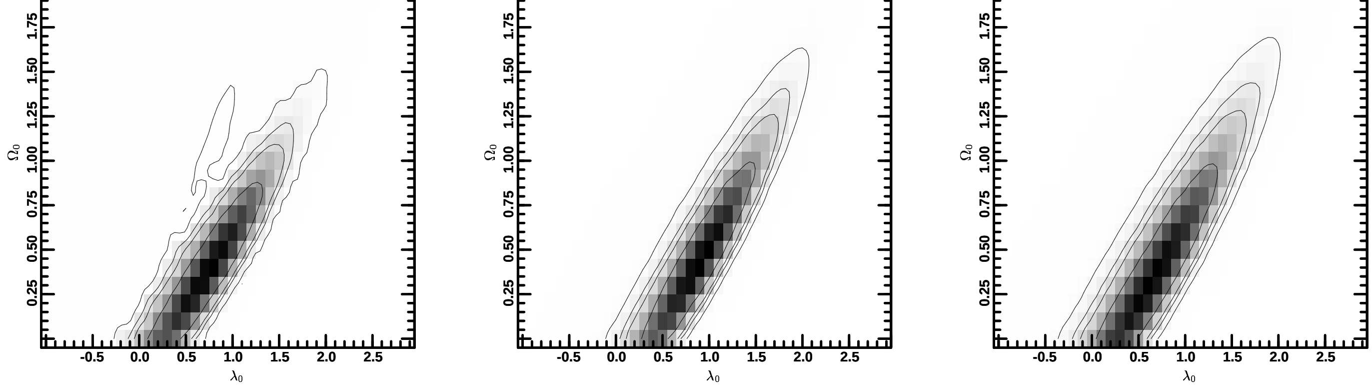

Fig. 4. Joint constraints from the lensing statistics calculations of Helbig et al. (1999b) (Paper II) and the m-z relation for type Ia supernovae as in data set $\mathcal{A}$ (left), data set $\mathcal{B}$ (middle) and data set $\mathcal{C}$ (right). See Fig. 1 for an explanation of the data sets

sets overlap with that of the lensing statistics, and even the 68% confidence contours from two of three supernovae data sets overlap with that of the lensing statistics, the results from the two cosmological tests are consistent and one is justified in calculating joint constraints by multiplying the probability distributions of the individual tests. Interestingly, they are most consistent at small, but not too small, values of Ω_0. The results of this are shown in Fig. 4.

Note that if there is some offset between the allowed regions from each individual test, as is the case here, then certain aspects of the joint constraints, such as in this case the upper limit on λ_0, will not necessarily be tighter than the corresponding aspect from each individual test. The joint constraints are nevertheless better in that the allowed region is smaller and that this allowed region should contain the correct cosmological model, assuming of course that the results of the individual tests are correct as far as they go. The tests are very different in nature and one should not expect the form of the probability density function to be the same in each case. In particular, as lensing statistics is especially sensitive to a large cosmological constant, the gradient in this area of parameter space is quite steep, thus it is not surprising that the lensing statistics upper limit on λ_0 is tighter. The fact that the confidence contours from the individual tests overlap shows that the tests are not inconsistent, and of course the allowed region from the joint constraints, which is consistent with each individual test, is approximated by this overlap.

Since the two m-z results are not completely independent, the question of the consistency of or joint constraints from the two supernovae data sets will not be discussed in this paper. Rather, the question is the consistency of and joint constraints from each of these data sets individually with the lensing statistics constraints.

Fig. 4 is the main conclusion of this paper. Although lensing statistics and the m-z relation individually allow, for appropriate values of λ_0, rather large values of Ω_0, the joint constraints clearly indicate a lower Ω_0, in accordance with observational evidence which measures Ω_0 more 'directly' (see the discussion in Paper I). Compared to the supernovae results, the allowed region of parameter space is shifted somewhat towards lower values of Ω_0 in the joint constraints. Although the actual best-fit value should not be taken too seriously, it is comfortably close to the current 'standard cosmological model' with $\lambda_0 \approx 0.7$ and $\Omega_0 \approx 0.3$

The quantity measured by both the lens statistics and the m-z relation for type Ia supernovae discussed here is approximately $\lambda_0 - \Omega_0$. Table 2 shows the 95% confidence ranges for $\lambda_0 - \Omega_0$ allowed by each of the four data sets individually and by the joint constraints of data set $\mathcal{D}$ with data sets $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$. At 95% confidence, the upper limit on $\lambda_0 - \Omega_0$ from lensing statistics alone is 0.45 and from supernovae alone is in the range 0.65–0.81 (depending on the data set).[5] For joint constraints, the upper limit on $\lambda_0 - \Omega_0$ is in the range 0.55–0.60 (again depending on the data set). For a flat universe with $\lambda_0 + \Omega_0 = 1$, this corresponds to upper limits on λ_0, taking the top of the range from different data sets, of 0.72, 0.90 and 0.80 for lensing statistics alone, supernovae alone and the joint analysis, respectively. Again, this is perfectly consistent with the current 'standard cosmological model' with $\lambda_0 \approx 0.7$ and $\Omega_0 \approx 0.3$ (e.g. Roos & Harun-or-Rashid 1999; Turner 1999) and is consistent with a flat universe but, neglecting other cosmological tests, does not require it.

3.3. Systematic errors

As far as the m-z relation for type Ia supernovae goes, various possible sources of systematic errors have been discussed in detail by SCP and HZSST. See particularly Fig. 5 in SCP. Basically, there is no evidence that the

[5] Note that this is not the same as that quoted in Paper II; this is because, as discussed above, the range of parameter space examined or, equivalently (in this case), the prior is different.

Reference	lower limit	upper limit
Perlmutter et al. (1999) (SCP) (data set $\mathcal{A}$)	-0.05	$+0.65$
Riess et al. (1998) (HZSST, $\Delta m_{15}(B)$) (data set $\mathcal{B}$)	$+0.30$	$+0.81$
Riess et al. (1998) (HZSST, MLCS) (data set $\mathcal{C}$)	-0.12	$+0.79$
Helbig et al. (1999b) (Paper II) (data set $\mathcal{D}$)	-1.90	$+0.45$
$\mathcal{A}+\mathcal{D}$	-0.20	$+0.55$
$\mathcal{B}+\mathcal{D}$	± 0.00	$+0.55$
$\mathcal{C}+\mathcal{D}$	-0.25	0.60

purported effects could significantly bias the results or, as in the case of grey dust, while a modest amount cannot be ruled out, it seems physically rather implausible (but see, however, Aguirre 1999a,b; Aguirre & Haiman 1999). (Note that Falco et al. (1999) find no evidence for grey dust at optical wavelengths based on studies of extinction in gravitational lens galaxies.) It is interesting to note that while one can invoke grey dust to explain the dimming of supernovae relative to the expectation in for example a dust-free $\lambda_0 = 0$ model, instead of invoking for example a low-density model with a positive cosmological constant, this degeneracy can be broken by observing supernovae at higher redshift (e.g. Aguirre 1999a) than has been done up until now: the m-z relation as a function of λ_0 and Ω_0 is exactly known, so the larger the range in redshift for which the m-z relation is observed, the more ad hoc alternative explanations become, provided of course that there is a cosmological model (which is not ruled out on other grounds) which provides an acceptable fit to the data.

Grey dust is also something which can effect gravitational lensing statistics *based on optical samples*, although the effects are not so straightforward. On the one hand, if the grey dust is concentrated in (lensing) galaxies, this could lead to lens systems being missed in the survey. To first order, this would lead to an underestimate of the optical depth and thus of the value of λ_0. On the other hand, again if dust is concentrated in (lensing) galaxies, the sources in the identified lens systems can suffer from extinction, which, depending on the details of the luminosity function, could lead to a wrong estimate of the magnification bias. In the 'normal' case of a flattening of the luminosity function for fainter objects, this will lead to an underestimate of the amplification bias and hence an overestimate of the optical depth and thus the value of λ_0 (Falco et al. 1999). Radio surveys of course are not affected by dust, so in principle one could detect grey dust through a systematic difference in the results from optical and radio surveys. In practice, however, the presence of other systematic effects makes such a detailed comparison impractical.

Radio surveys for gravitational lenses offer many advantages over optical surveys (see the discussion in Paper II). However, at present, the main source of uncertainty, lack of knowledge about the source population, makes them worse than optical surveys in this respect. For the calculation of the amplification bias, one needs to know, at a given redshift, the luminosity function.[6] On the other hand, much information is gained from the sources in a survey which are not lensed (see the discussion in Paper I); to interpret this, one needs to know, at a given flux density, the redshift distribution of the sources. Of course, these two things—the redshift-dependent luminosity function and the flux-density dependent redshift distribution—are different sides of the same coin.

The number counts of the Cosmic Lens All-Sky Survey, of which the Jodrell Bank-VLA Astrometric Survey, the results of which are used here, is a subset, suggest that amplification bias is not a big effect; hence the systematic error from lack of knowledge of the luminosity function is probably small, although it is conceivable that the number counts (integrated over redshift, as in general the CLASS sources have unknown redshifts) of CLASS are not representative of the luminosity function at *all* redshifts. As the lensed sources are generally at higher redshifts, the luminosity function might be different here and thus the true amplification bias different from that which was used in the JVAS analysis of Paper II (where it was assumed that the CLASS number counts are representative of all redshifts).

In Paper II, it was also assumed that the redshift distribution of JVAS is equal to that of CJF, independent of flux-density. There is some preliminary evidence that, as one moves toward lower flux-density levels, the typical redshift of flat-spectrum radio sources decreases. If this is the case, then our JVAS analysis will have underestimated the value of λ_0, as a higher value of λ_0 (all other things being equal) is needed to achieve the same optical depth for a low-redshift source than is needed for a high-redshift source. Although the results from the m-z relation for type Ia supernovae and gravitational lensing statistics are not inconsistent, and although, due to the different dependence on the cosmological parameters, the lower limit on λ_0 will always be stronger from the former and the upper limit from the latter, if it does turn out

[6] Due to the amplification of the gravitational lens effect, lensed sources near the lower flux-density limit of the survey will have an unlensed flux density lower than this, so the luminosity function thus needs to be known down to a flux-density level a factor of several below that of the survey.

ically higher than the true redshift distribution of JVAS, then the results from the m-z relation for type Ia supernovae and gravitational lensing statistics will become even more consistent.

4. Summary and conclusions

I have presented the first detailed analysis of joint constraints between gravitational lensing statistics and the m-z relation for type Ia supernovae, making use of data from Helbig et al. (1999b), Perlmutter et al. (1999) and Riess et al. (1998), presenting the individual results and the new joint constraints in a uniform way. The two tests are not inconsistent, the joint constraints are tighter than those from either test individually and provide additional evidence in favour of the current 'standard cosmological model' with $\lambda_0 \approx 0.7$ and $\Omega_0 \approx 0.3$, although (neglecting constraints from other sources such as the CMB) a reasonable range of other cosmological models is not excluded.

In the near future, gravitational lensing statistics from CLASS, the Cosmic Lens All-Sky Survey (Myers et al. 1999) should reduce both the random and systematic errors. Should the results from lensing statistics and the m-z relation for type Ia supernovae remain consistent, this should reduce the allowed parameter space even further. We are truly entering an era of precision cosmology, where the overlap of the allowed regions of parameter space from many different and independent cosmological tests is very small but not zero.

The data for the figures shown in this paper are available at

`http://multivac.jb.man.ac.uk:8000/ceres/data_from_papers/snlens/snlens.html`

or

`http://gladia.astro.rug.nl:8000/ceres/data_from_papers/snlens/snlens.html`

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