Finite Element Model Updating Based On Holographic and Speckle Interferometry Measurements

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ABSTRACT

Optical measurement techniques look very promising for finite element (F.E.) model updating or error localisation of plate-like structures in the field of structural dynamics. The purpose of this paper is to investigate a way to better exploit the high spatial resolution inherent to these techniques in order to correct FE mesh discretisation errors and/or model parameter errors. An important assumption in F.E. model error detection is first to consider the initial mesh as sufficiently fine to well represent the measured (displacement or stress) field. In the case of model updating, the adjustment of the model is performed by minimising the difference between the outputs of the model and the exact solution with respect to design parameters. In the case of FE mesh adaptation, the exact solution has to be estimated whereas in the case of model parameter errors, it is directly measured. The idea developed in this paper is to take advantage of the high spatial resolution offered by optical techniques to calculate successively two error estimators using only measurements. The experimental field is first used for the detection of singular regions corresponding to high gradients. This estimator indicates the regions where a mesh refinement is required. Thus a second estimator is calculated and used for parameter error detection.

INTRODUCTION

Although the capabilities of numerical structural modelling are increasing, industrials are more and more looking for more accurate predictions of the dynamic behaviour of structures. This implies more and more effective procedures to correct the structural models.

When dealing with finite element (FE) models, two types of errors may arise :

- discretisation errors that are related to the number of elements, to their type and their spatial distribution;
- parameter errors which include the uncertainty on the boundary conditions and the inaccuracy on model parameters.

The minimisation of model errors may be performed either by adaptive mesh refinement or by model parameter correction which is also called model updating. Mesh refinement and model updating are based on the comparison of an appropriate indicator of the discrepancy between the analytical solution and another solution assumed to be the correct one. For model updating, the measurements are used as the reference solution. For mesh adaptation, a better numerical solution has to be sought using the mathematical model.

Model Updating in Structural Dynamics

In order to compare experimental results with FE model predictions with the aim of model error localisation, the measured mode-shape vectors have to be expanded to the full size of degrees of freedom (DOF) of the model. Different mode-shape expansion methods are proposed in the literature [2, 6]. They can be classified as mathematically-based or physically-based methods in the sense that they make use or not of the physical behaviour of the structure. It can be shown that the reliability of the error localisation method is closely related to the quality of the eigenvector expansion process. As classical expansion techniques, let us cite the dynamic expansion method or the expansion by minimisation of errors on constitutive equations (MECE) [2]. The main drawback of mode-shape vector expansion techniques is that mathematical errors due to the expansion process are spread all over the structure. A large number of measured co-ordinates helps to enhance the expansion quality and thus the model error localisation results.

Mesh Refinement

Mesh refinement methods used in linear static analysis aim at minimising errors due to the discretisation of the displacement field. It is well known that the use of a cinematically acceptable FE model leads to residual errors in the equilibrium equation and to stress field discontinuities along the interfaces between the elements. Mesh refinement methods [1, 3] look to transform these errors into a discretisation error measurement indicator. The principle of the methods consists in approaching the exact field by a field which satisfies at least one of the two missing properties of the FE discretised field. The exact solution is approximated by polynomials of a higher degree than those of the FE solution. The calculation of the difference between the two fields gives an approximation of the discretisation error. This error is locally defined as the difference between the exact field of displacements (or stresses) φ and the FE field φ_h :

$$e_h = \varphi - \varphi_h \qquad (1.)$$

The error can be calculated element by element or on the whole structure. It can also be estimated using different norms. The FE method being based on the minimisation of the potential energy, it is usual to evaluate the error in the form of an energetic norm.

Experimental Modal Analysis

The use of distributed sensors (e.g. accelerometers) in dynamic structural testing is a widespread measurement technique. However, it usually leads to a very poor spatial resolution. To be able to compare efficiently experimental data with calculated results from a fine FE mesh or to use them as a starting base for structural acoustic calculations, it is recommended and even required to measure a sufficient number of co-ordinates. Because of this limitation, it is often difficult to identify the measured frequency response functions in the medium to high frequency range where the wavelength of the structural motion is in the range of the distance between two neighbouring sensors or even of their diameter. Moreover, the quality of the error localisation strongly depends on the expansion reliability which is itself related to the number of well identified experimental modes.

Another drawback to the use of distributed sensors comes from their interaction with the tested structure. Additional mass and spurious stiffness effects due to the presence of sensors and excitation devices lead to shifts of the resonance frequencies and to a change in the mode shapes. This is especially true for lightweight and stiff structures like turbomachine blades for example.

Plate-like Structures

Plate-like structures offer the advantage of presenting the majority of their nodes used for the modelling on the external surface. In this case, field measurement techniques allow to gather a greater amount of measurement points than the number of DOF used to model the plate surface. This feature can be exploited to generate successively two error estimators using only measurements. The experimental field is first used to build an estimator which indicates the regions where a mesh refinement is required. Thus a second estimator is defined to detect parameter errors. In order to better exploit the richness of the experimental data, the whole measured field is compared with the FE field. The problem reduces then to the comparison between two fields which has been largely studied in the literature for mesh refinement purposes.

MEASUREMENTS

Optical measurement techniques can provide a whole-field measurement of a vibrating object and make possible the visualisation of deformation shapes. Furthermore they have also the advantages to have a high sensitivity, to be non-contacting techniques and to run (sometimes) in real-time. However, they are not convenient for measuring Frequency Response Functions (FRF) over a limited frequency interval. One major assumption when using these techniques is that the structure vibrates along a single mode-shape. We must also add the constraint that this single mode can only be excited by an harmonic excitation signal at the resonance frequency. Only under these assumptions will the deformation shape due to a single point harmonic excitation be equal to a mode-shape. Otherwise, the response of the structure to an harmonic excitation at a frequency close to a resonance frequency will be close to the corresponding mode-shape if :

- the modal responses of all the other modes are low compared to the modal response of the excited mode;
- the resonance frequencies of the other modes are far from the natural frequency of the excited mode;
- damping is low.

The application of the excitation force (especially in the case of multiple excitation when the forces are tuned to force pure normal mode response) induces technical problems such as mass loading and spurious stiffness effects. For this reason, mechanical excitation has to be applied at the mounting base of the structure. Thus the excitation force can be measured by a load cell and an accelerometer can be mounted at the same time in order to have the driving point FRF and to be able to scale the resulting modes. This FRF can also help to locate resonance frequencies.

In the case of highly coupled modes, the solution is to measure Operating Deflection Shapes (ODS) i.e. the stationary dynamic responses rather than mode-shapes. Data acquisition of a set of FRF can be done at appropriated locations using a laser Doppler vibrometer so that a multiple degree of freedom parameter identification may be performed. It allows to identify the damping factors.

Interferometry

Laser-based optical techniques (holography, electronic speckle pattern interferometry (ESPI), scanning laser Doppler vibrometer) have gain a wide acceptance in the field of experimental mechanics for vibration measurement of plate-like structures.

By using phase shifting technique for quantitative deformation measurement, and by doing stroboscopic laser illumination at the vibration frequency, it is possible to extract amplitude and phase information for every point (pixel) of the fringe pattern at a single frequency. With a 3D-ESPI where the position of the camera or the illumination direction can be changed, and thus the sensitivity vectors, a 3D displacement can be calculated.

The measurement and averaging of interference fringes between a rest position and a recording of several periods (called averaging technique) can be used as a quick but rough modal analysis method.

GEOMETRICAL CORRELATION

Before using the measured field in the updating process, it is necessary to establish the geometric correspondence between the experimental structure and the finite element model. This implies to determine with precision the transformation between the 2-D system of the measurement co-ordinates and the 3-D system of the model. The measured field is obtained on a rectangular grid of pixels. Each pixel may be associated to a specific point on the structure surface. So a geometrical correlation between the tested structure and the model geometry has to be performed.

Once this correspondence has been established, the measurements can be "projected" on the model. In reference [4], it is proposed to find the dilatation and rotation factors by minimising the distance between the image of the structure contour and the projection of the model contour in the measurement plane. The minimisation process requires the introduction of the co-ordinates of a reference point and the evaluation of the horizontal and vertical dilations. So a pixel is associated to a co-ordinate *(x,y)*. The third co-ordinate (depth) is obtained by a projection of the grid of measurement on the field representation of the surface geometry.

DISPLACEMENT OR STRESS FIELD RECOVERY

In this paper, the exact field φ to be used in the error localisation equation (1) is replaced by the measured field φ_{exp}. At this point, one would like to be able to compare the measured value at each pixel with the corresponding value of the discretized solution. However, the transformation of the experimental data (defined in the absolute reference frame) in the intrinsic co-ordinates system of a single finite element is a difficult inverse problem. Moreover, the stress field is not continuous in the cinematically acceptable models. For these reasons, it is preferred to fit the (displacement or stress) discretized field using the same polynomial expressions as the ones used in the finite element formulation. The advantages are that :

- the polynomial functions of the FE solution are already known;
- the experimental fields of displacements and of stresses are already naturally continuous;
- it allows to use directly the experimental data so that a possible deterioration of the measurements by a smoothing technique is avoided ;
- it allows to build a continuous FE stress field.

The reconstructed field may either be a naturally continuous displacement or stress field of the same degree as in the FE formulation. It may be expressed element by element according to :

$$\tilde{\mathbf{u}}_i = \sum_{i=1}^{n} N_i \tilde{\mathbf{q}}_\mathbf{i} \qquad (2.)$$

or

$$\tilde{\sigma}_i = \sum_{i=1}^{n} N_i \tilde{\mathbf{s}}_\mathbf{i} \qquad (3.)$$

where
n = the number of nodes in the element
$\mathbf{N}_i$ = the shape function at node *i*
$\mathbf{q}_i$, $\mathbf{s}_i$ = nodal FE displacements or stresses.

The construction of the chosen displacement or stress continuous field may be performed at a local level (element by element) but also at a more global level (region by region). This method is known as the "patch recovery" method [1, 3] : it is based on the recovery of the continuous field by interpolation of the FE field on a given zone called a « patch ». A patch may be defined as a set of elements connected to a node of the mesh. The reconstructed field on the different patches is then used for the evaluation, element by element, of different types of errors.

A method which has given good results is the Super-convergence recovery procedure in element patch. It can be shown that the FE stress field crosses the exact stress field at particular points of a given element : these points are called super-convergence points and their co-ordinates may be found theoretically for the different types of finite elements. The principle of the super-convergence recovery procedure is then to force the smoothed field to pass through those particular points. As a consequence, this method overestimates the quality of the FE solution at these nodes in the case of the stress field.

Definition of the patch area

The patches may comprise several finite elements and in general, they superpose partly each other. These areas must be chosen to have enough data to allow the interpolation of the finite element field. Usually, the patch area associated to a node *j* is made up of the set of elements that are connected to this node.

$$\Omega_{rj} = \bigcup_{i=1}^{m_j} \Omega_i \qquad (5.)$$

where Ω_i represents the element *i* and m_i the number of elements connected to the node *j*.

Representation of the field

Whether speaking about the displacement or the stress field, the notation $\varphi_\mathbf{r}$ will be used to represent the recovered field. Each component of this field is represented by the polynomial function:

$$\varphi_\mathbf{r} = P_n(\chi)a \qquad (4.)$$

with

$$P_n(\chi)a = \sum_{i=1}^{T_n} \left(x^j \ y^k \ z^l\right)_i a_i \qquad (6.)$$

$$j + k + l \leq n$$

where the number of terms is equal to :

$$T_n = \frac{(n+1)(n+2)(n+3)}{6}$$

It is shown in [7] that the results of the recovery of a field represented by a bilinear polynomial depend on the co-ordinate system of the area, on its position, on its size and its orientation. In order to obtain the most general possible procedure, it is necessary to have a recovery method that is independent of the system of axes chosen to evaluate the polynomial coefficients. It has been shown in reference [3] that it is advisable to use a complete polynomial and a normalised system of co-ordinates.

Normalisation of the co-ordinates

The normalisation used for two-dimensional problems is defined in the space *[-1,1] X [-1,1*] by:

$$\chi_n = -1 + 2\,\frac{\chi - \chi_{min}}{\chi_{max} - \chi_{min}}$$

where χ represents the co-ordinates *(x, y).*

This normalisation guarantees the definition of an area which touches all the lines defining the limits of the normalised space [3] and which is weakly dependent on a particular direction.

Conditions on the field φ_r.

The unknown parameters ***a*** in equation (6) are obtained by minimisation of the difference between the FE results and the smoothed values at the points of super-convergence in the "patch". This results in minimising the function :

$$R_i(a) = \omega(x_i, y_i, z_i)\left(\tilde{\varphi}_r(x_i, y_i, z_i) - \varphi_{EF}(x_i, y_i, z_i)\right) \quad (7.)$$

$$R_i(a) = \omega(x_i, y_i, z_i)\left(P_n(x_i, y_i, z_i)\, a - \varphi_{EF}(x_i, y_i, z_i)\right) \quad (8.)$$

where $\omega(x_i,\ y_i,\ z_i)$ is a weighting function that can be used to balance the influence of the points $(x_i,\ y_i,\ z_i)$ according to their distance to the central node which defines the area. It is chosen here to give the same weight to all the points used.

The problem defined by equation (8.) is in general over-determined and its solution can be estimated using a least square method. For this purpose, let us consider the following function :

$$F_i(a) = \sum_{i=1}^{m_i} \left(R_i(x_i, y_i, z_i)\right)^T \left(R_i(x_i, y_i, z_i)\right) \quad (9.)$$

where $(x_i,\ y_i,\ z_i)$ are the co-ordinates of the selected group of points; $m_i = n_e\ K$ is the total number of these points (K = the number of points per element of the area).

The minimisation procedure $\dfrac{\partial\, F(a)}{\partial\, a} = 0$ leads to the system of equations :

$$\sum_{i=1}^{m_i} \omega^2(x_i, y_i, z_i) P_n^T(x_i, y_i, z_i) P_n(x_i, y_i, z_i)\; a = \sum_{i=1}^{m_i} \omega^2(x_i, y_i, z_i) P_n^T(x_i, y_i, z_i)\, \varphi_{EF}(x_i, y_i, z_i) \quad (10.)$$

which can be put in the matrix form:

$$\{a\} = [A]^{-1}\{b\} \quad (11.)$$

Note that the number of equations to be solved for each component in each area is low. This makes the method less expensive than a method that would use a global projection. It should also be noted that the use of the stress field super-convergence property leads to consider smoothed stress values that are better than those obtained by the FE method. However, the difference between this improved field and the FE one is usually quantified at a previous stage when the FE mesh quality is checked. If the FE model discretisation of the structure is well adapted, the difference between the two fields is weak.

CALCULATION OF ERRORS

Once the field φ_r has been recovered on the patch areas, it becomes possible to evaluate the value of the continuous field φ at any point of the continuous structure in the global co-ordinates. Consider a point located on the border of the structure : one keeps first the values of φ_r on all the areas to which the point belongs. Thus one calculates the weighted average using the following equation :

$$\tilde{\varphi}(x_j, y_j, z_j) = \sum_{r=1}^{m_j} \frac{\varphi^r(x_j, y_j, z_j)/C_j^r}{\sum_{r=1}^{m_j} \dfrac{1}{C_j^r}} \quad (12.)$$

where m_j is the number of areas including the point *j ;* C_j^r is a weighting factor corresponding to the distance between the point and the node associated to area *r*.

The value of the displacement or stress field is then available at any measurement points and more particularly, at any points inside an element like the points of Gauss of the external surface for instance.

Correlation between experimental and FE results

The most common technique used to assess the correlation between measured and FE mode-shape vectors is the Modal Assurance Criterion (MAC) defined as follows :

$$MAC(\varphi_{\exp}, \varphi_{FE}) = \left(\frac{\varphi_{\exp}^T\ \varphi_{FE}}{\|\varphi_{\exp}\|\ \|\varphi_{FE}\|}\right)^2 \quad (13.)$$

MAC values give a good idea of the closeness between two different mode-shapes φ_{exp} and φ_{FE}. They oscillate between *0* and *1*. An unitary value means a perfect correlation.

FE mesh discretisation errors

The field recovery technique presented previously may also be used to express the measurements in the form of a smoothed field. If degree *N* of the measurement field polynomial expression is such that :

$$\sum_e \int_{\Omega_e} \tilde{\varphi}_{\exp}^N - \varphi_{\exp} \leq tolérance$$

the elementary error introduced by the discretisation of the measured field and the use of a polynomial function of degree N is written:

$$\int_{\Omega_e}\left(\sum_{j=1}^{n_e} N_j^N(\chi_s)\tilde{s}_{j_{\exp}} - \sum_{j=1}^{n_e} N_j^n(\chi_s)\tilde{s}_{j_{\exp}}\right)d\Omega_e \qquad (14.)$$

The total error is:

$$\sum_e\left(\int_{\Omega_e}\left(\sum_{j=1}^{n_e} N_j^N(\chi_s)\tilde{s}_{j_{\exp}} - \sum_{j=1}^{n_e} N_j^n(\chi_s)\tilde{s}_{j_{\exp}}\right)d\Omega_e\right) \qquad (15.)$$

N_j^N and N_j^n are the polynomial interpolation functions, respectively of degree *N* and *n* at the node *j*.
n is the degree of the FE study.
χ_s are Gauss points of integration in the local co-ordinates on the element surface.
$\tilde{s}_{j_{\exp}}$ is the value of the smoothed experimental field at node *j*.
n is the total number of elements.

Errors on the parameters

In this case, the experimental field is used for the detection of singular regions corresponding to high gradients of errors defined in equation (1). The elementary error is obtained by integration using the Gauss points located on the surface of the element.

CASE STUDY

The error detection procedure has been validated on the example of a clamped plate structure (100 x 50 x 2 mm) (***figure 1***) using simulated optical measurements. The plate is made of Titanium (Young modulus = 11 E+4 N/mm^2). For the purpose of this demonstration, no noise was added to the data. The optical measurements were simulated using a very fine FE mesh (!!! dof) while the FE calculation results are obtained using a coarse mesh (!!! dof).

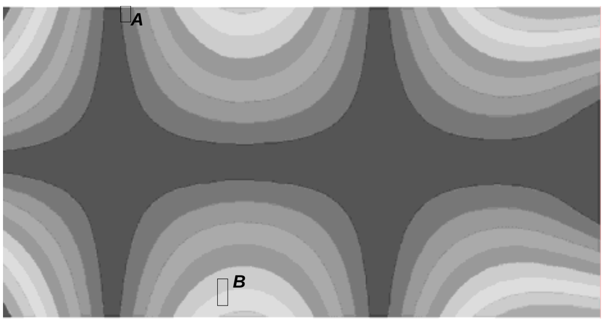

Figure 1 : *Simulated optical measurements (Mode n° 7 at 116.25Hz). (Fixations are on the right side.)*

Errors on the parameters

The FE model and the dynamic analysis were performed using the finite element programme Samcef (***figure 2***).

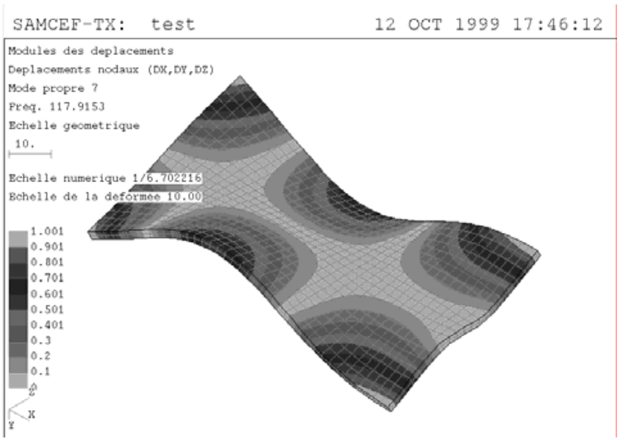

Figure 2: *FE results (Mode 7 at 117.9 Hz)*

The simulated defects are stiffness losses located at points A and B (**figure 1**). The first defect (*A*) is a 1.66 x 3.33 x 2 mm area where the Young modulus has been reduced to 3.25 E+4 and the second defect (*B*) is a 1.66 x 5 x 1 mm area with a Young modulus equal to 1.6 E+4.

Mode 7 was used for error detection because it is the last mode which gives a good correlation between the measurements and the FE results.

The correlation between the FE smoothed field and the measured field for the 7th mode and for the three measurement directions *x*, *y* and *z* gives MAC values equal to respectively: 0.9051, 0.8983, 0.9526.

The results of the error localisation technique are shown in ***figure 3***. The two zones where the stiffness was reduced are clearly identified. Moreover, as shown in ***figure 3***, an error at the clamping is also detected. This error comes from the FE mesh discretisation error.

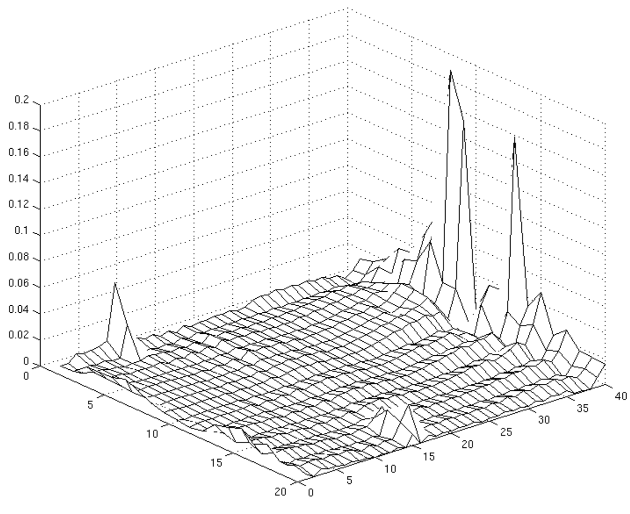

Figure 3: *elementary error localisation (mode 7)*
(The fixations are at line 40 on the x axis.)

FE mesh discretisation errors

The simulated experimental structure is a clamped plate with a very thin square area at the middle as shown in ***figure 4***. The third mode was investigated.

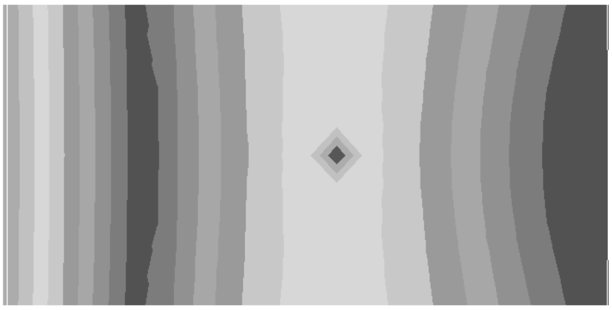

Figure 4:*clamped (on the right side) plate with a very thin square zone, (mode 3)*

Results of the discretisation error localisation procedure are shown in ***figures 5*** and **6**. It can be observed **in figure 5** that a discretised model using only 1026 DOF and 363 nodes is not convenient to well represent the 3^{th} mode. The error is more or less 10% of the experimental displacement field.

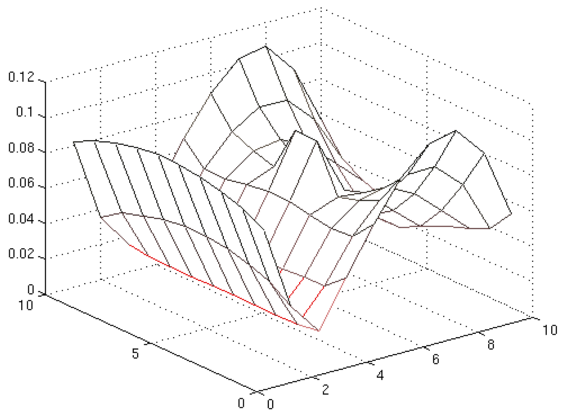

Figure 5: *elementary structural error localisation (1026 DOF and 363 nodes)*

In ***figure 6,*** the error localisation procedure is applied with a model containing 2016 DOF and 693 nodes. It results that :

- the global mode is well represented;
- the mesh is not sufficiently fine near the defect area to well represent the measured displacements;
- an error at the clamping remains.

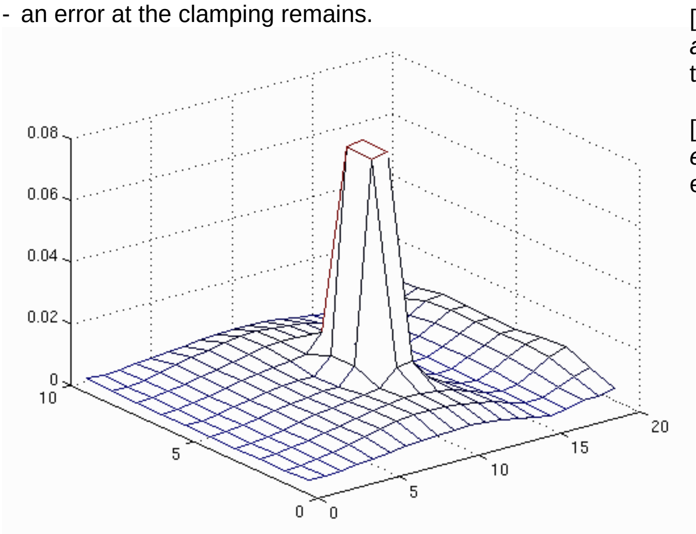

Figure 6: *elementary structural error localisation (2016 DOF and 693 nodes)*

CONCLUSION

The measurement results of optical techniques were exploited for the detection and the localisation of errors in the FE model of the experimental structure. The proposed method has been tested using simulated data and has shown its ability and its performance. The next step in the future will be to validate the method on a compressor blade using true optical measurement data.

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