Monitoring Process Variation Using Modified EWMA

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ABSTRACT

A new control chart, namely modified EWMA control chart, for monitoring the process variance is introduced in this work by following the recommendations of Khan et *al.*[15]. The proposed control chart deduces the existing charts to be its special cases. The necessary coefficients, which are required for the construction of modified EWMA chart, are determined for various choices of sample sizes and smoothing constants. The performance of the proposed modified EWMA is evaluated in terms of its run length (RL) characteristics such as average RL (ARL) and standard deviation of RL (SDRL). The efficiency of the modified EWMA chart is investigated and compared study with some existing control charts. The comparison reveals the superiority of proposal as compared to others control charts in terms of early detection of shift in process variation. The application of the proposal is also demonstrated using a real life dataset.

Keywords: Control Charts, Modified statistic, EWMA, run length, Variation, Markov Chain.

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1. Introduction

Control chart is an important tool of statistical process control (SPC) for monitoring the process deviation and improving the quality of the target variable (see, Montgomery[1]). Two types of control charts i.e., variable and attribute are commonly used in the literature of the control chart. Variable control charts are used for the numeric characteristics of the interested quality as the height, weight or temperature while the attribute control charts are developed for the non-numeric characteristics of the interested quality as the good/defective items, satisfied/not-satisfied or the small/medium/large items. The variable quality characteristics provide more information about the process as compared to the attribute[2]. Location charts are developed for monitoring of the process average while the dispersion charts are applied for monitoring the process variation. Dispersion charts are considered the speedy chart for the process variation monitoring. The study of the dispersion chart is also important as it is common assumption of the Shewhart average chart to assume that the process standard deviation rests constant (see, Acosta-Mejia *et al.*[3]). The dispersion charts have been explored by many quality control researchers including Lowery *et al.*[4].

Exponentially weighted moving average (EWMA) charts was introduced by Roberts[5] and originally, it was developed for normally distributed data, but today the method has found widespread usage for online monitoring of analytical processes (Abbasi[6]), industrial production processes (Lucas and Saccucci[7]), public health surveillance (Woodall[8]), and other procedures, where the outcomes of the event are obtained sequentially, but not necessarily coming from a normal distribution. An EWMA chart dominates the Shewhart chart in detecting small to moderate shifts in terms of statistical performance. The general form for the two-sided EWMA chart is given in many textbooks, including Montgomery[1]. The EWMA statistic is the exponentially weighted average of the previous as well as the current observations. The typical weight parameter λ (ranging from0 to 1), also known as a smoothing parameter, performs well for small values (between 0.05 and 0.25) for monitoring the production processes[1]. A lot of work has been done in the literature on designing and studying the performance of EWMA control charts including[9-11].

The researchers are continuously endeavoring to develop a robust technique for the early detection of the shifted process so that the losses incurred by the manufactured process be avoided. Gan[12] developed the modified EWMA control chart for the monitoring of the binomial counts.

Later, Gan[13] introduced three modified EWMA statistics for detecting smaller shifts in the process for which the Shewhart chart is less effective. Patel and Divecha[14] proposed a modified EWMA chart for monitoring the process mean when small and abrupt shifts occur in the process. Recently, in order to increase the fast detecting ability of the EWMA statistic, a modified EWMA statistic was introduced by Khan *et al.*[15]. Unlike the conventional EWMA statistic a second parameter k is introduced in it for quick detection of the small process shifts. It has been observed that the modified EWMA statistic is capable of detecting overall mean shift as compared to the traditional EWMA statistic. Latterly, Schmid[16] proposed modified EWMA control chart for the monitoring the time series data. Other work on the development of EWMA control charts based on modified EWMA statistics includes Aslam *et al.*[17], Khan *et al.*[18] , Herdiani *et al.*[19], Zhang *et al.*[20], Lampereia *et al.*[21], etc. All these articles concluded that the modified EWMA control chart is more capable of detecting early shift in process parameter under study as compared to the traditional EWMA control chart. Therefore, this article proposes a EWMA control chart using modified EWMA statistic for efficient monitoring of process variation.

The rest of the paper is organized as follow: design of the proposed chart is developed in Section 2. In Section 3, the performance of the proposed chart is given. In Section 4, the designing for the estimation of parameters of the modified EWMA chart is explained. In Section 5, the designing of the out-of-control performance of the proposed chart and its comparison is elaborated. The application of the proposed chart is demonstrated in Section 6. Finally, the conclusions and the future recommendations are given in the Section 7.

2. Design of the Proposed Chart

It is assumed that the quality of interest $X_t\,(t \geq 1)$ follows the normal distribution with mean μ and variance σ^2. Our interest is only to detect any change in the process dispersion $\delta_t^2 = \sigma_t^2/\sigma_0^2$ where σ_t^2 is shifted variance and σ_0^2 in-control variance. Without loss of generality, it is assumed that $\mu = 0$. Define a random variable $Y_t = \ln(S_t^2/\sigma_0^2)$, where S_t^2/σ_0^2 is a gamma distributed random variable with parameters $(n-1)/2$ and $2\delta_t^2/(n-1)$. The resulting distribution of Y_t is log-gamma distribution and Lawless[22] showed that it can be approximated by a normal distribution, i.e., $Y_t \approx N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \ln(\delta_t^2) - \frac{1}{n-1} - \frac{1}{3(n-1)^2} + \frac{2}{15(n-1)^4} \quad \text{and} \quad \sigma_Y^2 = \frac{2}{n-1} + \frac{2}{(n-1)^2} + \frac{4}{3(n-1)^3} - \frac{16}{15(n-1)^5}.$$

Based on this assumption, we propose a control chart using Modified EWMA statistic for monitoring the process variation.

Let a random sample $X_{t1}, X_{t2}, \dots, X_{tn}$ of size n is randomly drawn at time (or subgroup) t and measure Y_t. The modified EWMA statistic, denoted by M_t, having a smoothing constant λ at time t can be defined as:

$$M_t = (1-\lambda)M_{t-1} + \lambda Y_t + k\,(Y_t - Y_{t-1}) \tag{1}$$

Here, $M_0 = 0$ and $Y_t = \ln(S_t^2/\sigma_0^2)$ is log-transformed random variable which is ratio of sample variance to population variance. The smoothing constant in the range of $0.05 < \lambda \leq 0.25$ is usually recommended by Montgomery[1]. The constant $-1 \leq k \leq 1$ may be chosen independently of λ, but in this study the optimal choice for k is $k = -\lambda/2$, which is derived by Khan *et al.*[15]. Further note that, when k=0, the modified EWMA statistics reduces to Crowder and Hamilton[23] EWMA statistic and at k =1, it reduces to Patel and Divecha[14] first order auto-correlated EWNMA statistic. After successive substitution, statistic $M_t$defined in equation (1) can be rewritten as:

$$M_t = (1-\lambda)^t M_0 + \sum_{i=0}^{t-1}(1-\lambda)^i[(\lambda+k)Y_{t-i} - kY_{t-i-1}] \tag{2}$$

The mean and variance of modified EWMA statistic are given as follows

$$E(M_t) = \mu_Y \text{ and } V(M_t) = \frac{(\lambda+2\lambda k+2k^2)}{(2-\lambda)}\sigma_Y^2 \tag{3}$$

To detect an increase in process variance, the EWMA statistic Q_t= max $(M_t, 0)$ signals an out of control if Q_t is greater than

$$UCL = L_Q\sqrt{\frac{(\lambda+2\lambda k+2k^2)}{(2-\lambda)}\sigma_Y^2} \tag{4}$$

where L_Q can be determined for fixed value of n and to achieve a desired ARL_0.

Similarly, a decrease in process variance signals an out of control if $\acute{Q}_t$= min $(M_t, 0)$ is greater than

$$LCL = -L_{\acute{Q}}\sqrt{\frac{(\lambda+2\lambda k+2k^2)}{(2-\lambda)}\sigma_Y^2} \tag{5}$$

where $L_{\acute{Q}}$ is control charting constant and can be determined for fixed value of n to achieve a desired ARL_0.

The limits defined in equations (3) and (4) are called one sided control limits of the modified EWMA statistic. However, the two sided control limits for modified EWMA chart can be determine using both statistics $\acute{Q}_t$ and Q_t. In the rest of this article, we will consider a two sided modified EWMA control chart.

The two sided control limits for the modified EWMA are defined as

$$\left.\begin{aligned} LCL &= -L\sqrt{\frac{(\lambda+2\lambda k+2k^2)}{(2-\lambda)}\sigma_Y^2} \\ UCL &= L\sqrt{\frac{(\lambda+2\lambda k+2k^2)}{(2-\lambda)}\sigma_Y^2} \end{aligned}\right\}, \tag{6}$$

where $L = L_Q = L_{\acute{Q}}$ is control chart coefficient need to be determined for a fixed value of n, λ, k and desire in-control ARL_0.The proposed modified EWMA chart is based on two constants λ and k. The proposed chart is the extension of the existing control charts. The proposed control chart reduces to control chart proposed by Crowder and Hamilton[23] when k = 0.

The proposed modified EWMA control chart will signal if $\mathrm{UCL} < \mathrm{M}_t < LCL$, otherwise the process is declared to be work under in-control scenario. The probability of signal can be find as:

$$\text{Prob. of signal} = \Pr(UCL < M_t < LCL|\delta_t^2) \tag{7}$$

3. The Performance Evaluation

The average run length (ARL), which is the mean of RL distribution, is the most important and widely used measure to evaluate the performance of control charts. The ARL is defined as *the average number of subgroups until an out-of-control signal is raised*. In case, the process is in control, the ARL should be sufficiently large to avoid too many false alarms while for the out-of-control process; the ARL should be sufficiently small for rapid detection of shift.

$$\mathrm{ARL}_0 = 1/\Pr(LCL < M_t < UCL|\delta_t^2 = 1) \tag{8}$$

and

$$\mathrm{ARL}_1 = \frac{1}{\text{Prob.of Signal}} \tag{9}$$

The accurate measure of ARL has been thoroughly discussed by Hussain *et al.*[24]. ARL for the EWMA chart have been discussed by Chananet *et al.*[25] and Li *et al.*[26]

The equation (8) and (9) can be solved easily if the probability density function of modified EWMA statistic M_t is known. The exact distribution of modified EWMA statistic defined in equation (1) is more explicit and up to our best knowledge is not available in the literature. Therefore, the run length characteristic of the proposed control chart can be investigated through Monte Carlo simulation method. For this purpose, the following algorithm in *R* package is developed and run length characteristic are obtained for various choices of parameters.

Algorithms

The following algorithms has been used in *R* to find the results of this study.

Algorithm 1: Determination of control charting constant (*L*)

Step 1. Sample of size *n* is drawn from Normal distribution (0, 1), without loss of generality, and $Y_i = \ln(S_i^2/\sigma_0^2)$ is calculated.

Step 2. Using the value of Y_i obtained in step 1, modified EWMA statistic M_t is calculated for a specified value of λ and *k*. This process is repeated up to 10,000 times and at the end of this step we have 10,000 modified EWMA statistic's.

Step 3. The control charting constant *L* is determined to satisfying the in-control condition of $\Pr(LCL < M_t < UCL) = \alpha$.

Step 4. The above steps 1 to 3 are repeated 1,000 times and at the end of this step we have 1000 *L* values for specific choice of *n*, λ and *k*.

Step 5. Finally, the average value of *L* is chosen for further analysis.

Algorithm 2: Determination of run length characteristics

Step 1. Control limits of the modified EWMA chart are constructed using the charting constant determined in above algorithm for specific choice of *n*, λ and *k*.

Step2. Sample of size *n* is drawn from Normal distribution (0, σ_1^2), and $Y_i = \ln(S_i^2/\sigma_1^2)$ is calculated.

Step 3. Using the value of Y_i obtained in step 2, modified EWMA statistic M_t is calculated.

Step 4. The probability of signal using the equation (7) and run length using the equation (9) are calculated.

Step 4. The above steps 1 to 4 are repeated 10,000 times and at the end of this step we have 10,000 average run length values.

Step 5. Finally, the average and standard deviation of 10,000 average run length values. These ARL and SDRL are used

4. Designing for the estimation of parameters of the modified EWMA chart

The traditional design procedure for selecting the parameters of EWMA charts based on only the average run length (ARL) may lead to high probability of false out-of-control signal for some types of EWMA charts as discussed by Chan and Zhang[27]. They also recommended the procedure for selecting the parameters based on the average run length and standard deviation of the run length (SDRL) with the constraint that $SDRL \leq ARL$ when the process is in-control and out-of-control. Using the algorithm 1, a Monte Carlo simulation approach with 10,000 iterations is used to approximate the RL distribution of the proposed modified EWMA chart. Note that Kim[28] and Schaffer and Kim[29] indicated that 5,000 replications are sufficient to estimate the ARL to an acceptable level of precision in many control chart settings. The zero-state or initial-state ARL's are computed in this study. The control charting constant $L$is determined in such a way that the two one-sided charts produces equal individual ARL_0 so that the overall ARL_0 of the two-sided chart is approximately equal to 200. The values of L, for a fixed value of n, λ, k and ARL_0= 200, are given in Table 1.

Table 1: The control chart multiplier L for various values of n, λ and k to achieve the in-control $ARL_0 = 200$.

k	n	λ				
		0.05	0.10	0.15	0.20	0.50
0	5	1.194	1.570	1.761	1.971	2.009
	8	1.285	1.616	1.835	2.010	2.025
	10	1.360	1.672	1.870	2.027	2.052
	15	1.473	1.758	1.933	2.057	2.088
$-\lambda/2$	5	0.975	1.269	1.469	1.635	2.107
	8	1.121	1.386	1.550	1.681	2.164
	10	1.207	1.467	1.620	1.735	2.274
	15	1.340	1.590	1.725	1.821	2.367
1	5	4.122	3.824	3.640	3.515	3.296
	8	4.082	3.803	3.624	3.481	3.164
	10	4.045	3.785	3.601	3.465	3.105
	15	4.001	3.762	3.585	3.440	3.055

From Table 1, it can be seen that:

a. the control charting constant depend on k, n and λ for a fixed ARL_0.
b. the values of L decreases with increase in sample size when others parameters are fixed.
c. increase in smoothing weight increase the value of L for a fixed k and n except the value of k = 1.

5. Out-of-control Performance

Suppose that the process goes out-of-control with respect to variance and $\sigma_1^2 = \rho.\sigma_0^2$ be the new process variance with ρ time change in σ_0^2. Under the assumption that the data follows a Normal distribution with mean 0 (without loss of generality) and variance σ_1^2, the probability of signal given in (7) can be redefined as;

$$P_{out} = Pr(\mathrm{M_t} > UCL|\sigma_1^2)\ orP(\mathrm{M_t} < LCL|\sigma_1^2)$$
$$P_{out} = Pr(\mathrm{M_t} > UCL|\sigma_1^2) + P(\mathrm{M_t} < LCL|\sigma_1^2)$$
$$P_{out} = 1 - F_{\mathrm{M_t}}(UCL|\rho\sigma_0^2) + F_{\mathrm{M_t}}(LCL|\rho\sigma_0^2) \tag{10}$$

where $F_{\mathrm{M_t}}(.)$ be the cumulative density function of $\mathrm{M_t}$.

The exact probability density function of modified $\mathrm{M_t}$ is explicitly complicated and is not available in close form. Therefore, again we have used simulation procedure to determine the empirical CDF of $\mathrm{M_t}$. The above algorithm 2 is used and P_{out} determined for various choices of n, k, λ and ρ.

Table 2 through 5 the values of ARL and SDRL have been estimated using the above mentioned equations. The zero state run length performance of the modified EWMA (MEWMA) control chart for the sample of size 5 and 10, the smoothing constant λ=0.05 and 0.10 and control constant k = 0, $-\lambda/2$ and 1 are given for different shift levels 0.50 to 2.00. It can be observed from these tables that SDRL is always smaller than ARL for all the process settings. A control chart scheme is considered as a better one if it has smaller ARL values, so the proposed chart is studied at different choice of k to identify the suitable value of k .

Table 2: Zero state run length performance of the MEWMA chart at n=5 and λ=0.05

ρ	k=0		k=$-\lambda/2$		k=1	
	ARL	SDRL	ARL	SDRL	ARL	SDRL

0.50	4.567	2.345	2.287	1.154	7.314	5.670
0.60	8.995	7.956	4.933	3.005	16.161	14.223
0.70	19.104	14.234	15.424	13.256	29.717	26.745
0.80	40.097	31.658	29.671	23.228	55.216	52.325
0.90	85.707	65.323	77.093	68.098	104.284	93.2715
1.00	200.747	188.776	201.071	189.751	200.890	192.356
1.10	84.345	75.563	71.508	60.005	96.345	85.456
1.20	33.756	25.097	26.504	20.534	61.345	55.445
1.30	18.677	11.876	15.841	10.054	41.223	34.449
1.40	13.087	7.082	11.170	9.113	25.334	19.777
1.50	10.542	5.667	9.083	4.264	18.544	14.332
2.00	6.793	3.045	5.005	1.813	11.223	7.556

Table 3: Zero state run length performance of the MEWMA chart at n = 5 and λ = 0.10

ρ	k=0		$k=-\lambda/2$		k=1	
	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.50	3.987	1.567	2.106	0.986	5.445	3.099
0.60	9.177	5.470	4.086	2.896	17.189	11.234
0.70	21.123	17.209	11.037	9.556	38.334	26.223
0.80	60.223	51.890	35.888	30.022	90.990	80.224
0.90	110.445	98.990	102.924	91.990	130.776	122.113
1.00	201.005	192.113	200.628	192.556	200.888	189.445
1.10	85.456	76.003	76.998	69.977	101.12	94.332
1.20	42.220	35.344	32.755	27.564	72.339	63.224
1.30	25.539	19.223	18.706	12.867	48.228	39.002

1.40	16.435	10.066	12.532	7.725	31.097	21.091
1.50	13.112	7.199	9.976	5.378	25.489	13.223
2.00	8.509	3.244	5.020	2.114	14.559	8.223

Table 4: Zero state run length performance of the MEWMA chart at n = 10 and λ = 0.05

ρ	k=0		k=$-\lambda/2$		k=1	
	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.50	4.223	2.005	1.850	0.653	7.889	4.889
0.60	8.778	5.334	3.672	2.417	15.224	9.225
0.70	29.334	21.234	16.871	10.077	54.334	43.532
0.80	156.445	143.223	144.05	130.091	165.223	155.229
0.90	177.445	170.089	159.009	141.223	186.056	156.228
1.00	200.998	192.345	200.934	196.058	201.279	191.287
1.10	35.445	29.770	29.116	22.572	71.890	60.011
1.20	21.224	17.556	12.535	6.758	43.245	37.220
1.30	14.283	10.990	8.333	3.508	28.346	20.078
1.40	8.724	5.332	6.609	2.255	15.489	9.881
1.50	7.665	3.020	5.661	1.709	12.114	6.546
2.00	6.670	1.776	4.243	0.8042	10.134	3.078

Table 5: Zero state run length performance of the MEWMA chart at n = 10 and λ = 0.10.

ρ	k=0		k=$-\lambda/2$		k=1	
	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.50	2.980	1.107	1.676	0.524	4.309	3.002

0.60	4.329	2.128	2.766	1.455	7.998	4.978
0.70	15.224	9.298	8.433	6.766	33.234	21.223
0.80	78.334	64.297	52.211	48.550	108.334	98.768
0.90	142.223	136.556	105.671	99.224	167.889	151.244
1.00	200.005	192.334	200.211	191.456	199.098	189.223
1.10	42.099	36.445	31.366	25.446	68.345	59.456
1.20	20.334	14.234	13.033	7.953	42.998	36.890
1.30	13.229	9.078	8.232	4.005	28.334	21.003
1.40	11.870	7.345	6.356	2.493	20.889	14.334
1.50	8.889	4.329	5.366	1.809	15.754	9.329
2.00	5.768	2.248	3.912	0.825	9.356	4.982

Table 2 through 5 shows that:

1. The proposed chart tends to become Crowder and Hamilton[23] chart when the control k=0.
2. The proposed control chart performs better for the choice of $k=-\lambda/2$ than any other value of k (0 or 1 considered here). For example when a sample of n=5 and λ=0.05 is used then a shift of 0.70 is detected by an average samples of 19.104 for the existing chart (k=0) while the proposed control chart detects the same shift in average of samples of 15.424 ($k=-\lambda/2$). This shows that the proposed chart is 23% better in detecting this shift. The same performance can be seen for other process settings.
3. The modified EWMA control chart performs better than the case when k =1 (as considered by Patel and Divecha[14]), for example when a sample of n=5 and λ=0.05 is used then a shift of 0.70 is detected by an average samples of 29.717 for the existing chart while the proposed control chart detects the same shift in average of samples of 15.424. This shows that the proposed chart is 93% better in detecting this shift. The same performance can be seen for other process settings.

The comparative efficiency of the proposed chart has also been shown on the graph. It can be seen in the Figures 1 through 3. The proposed chart shows better efficiency in early detecting the out-of-control process for all process settings at different shift levels.

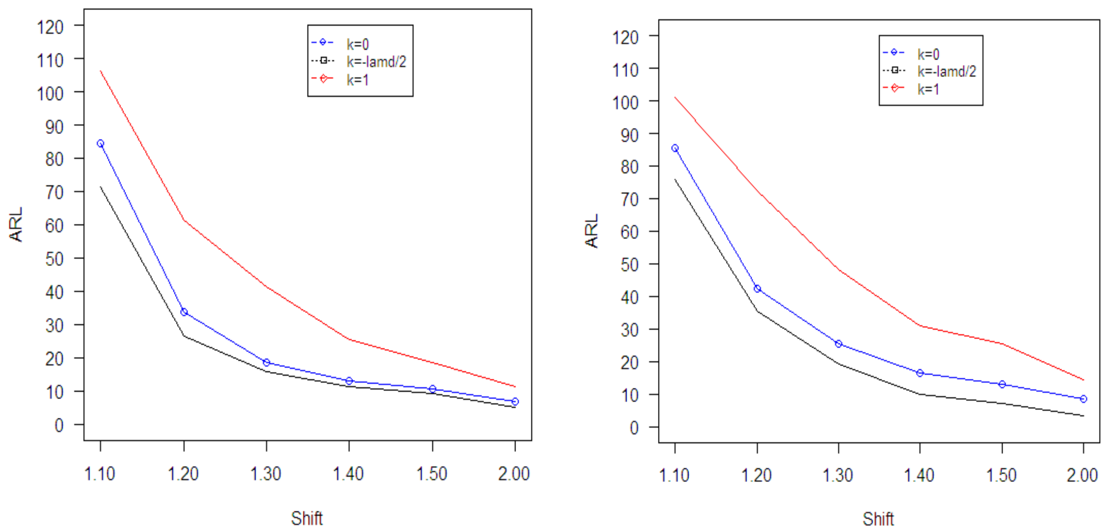
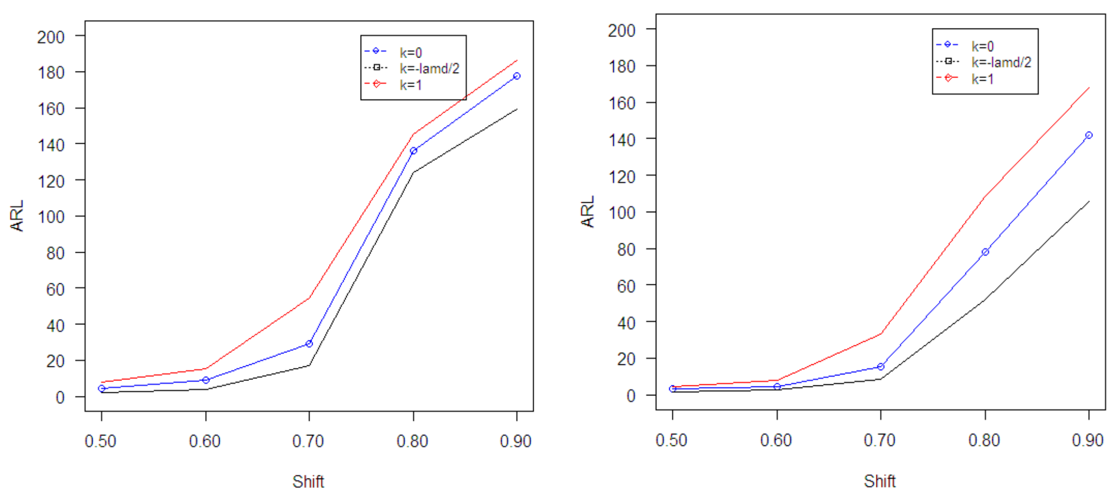


Figure 1. ARL curves of the modified EWMA chart at (a) $\lambda = 0.05$ (b) $\lambda = 0.10$ for n =5 when process variance increases.



(a) (b)

Figure 2. ARL curves of the modified EWMA chart at (a) $\lambda = 0.05$ (b) $\lambda = 0.10$ for n =10 when process variance decreases.

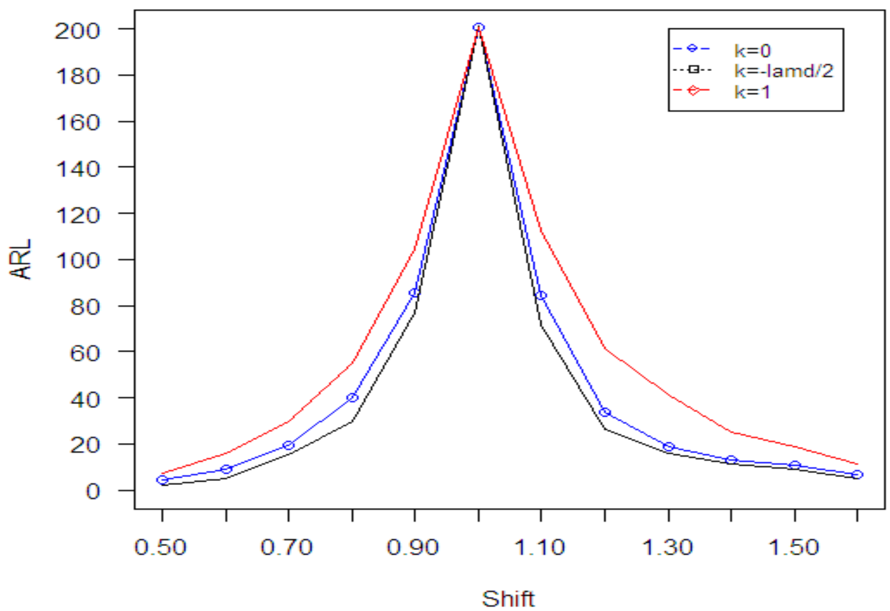


Figure 3. ARL curves of the modified EWMA chart for *n* =5 and $\lambda = 0.05$ for up and down shifts in variance.

5.1 Comparison with other EWMA charts

Further, the efficiency of the proposed modified EWMA variance chart is compared with the recently proposed charts by Huwang *et al.*[30] and Zou and Tsung[31]. Fixing the in-control average run length of two charts, the out-of-control average run length is measured for both charts at various values of ρ as efficiency measure. The results of out-of-control ARL are provided here in table 6 and table 7 for ARL_0. However, the results can be determined for any other value.

Table 6 and Table 7 reveals that the proposed modified EWMA variance chart perform better than the charts of Huwang *et al.*[30] and Zou and Tsung[31]. The efficiency of the proposed chart in terms of producing smaller values of out-of-control ARL increases with the increase in sample size. The proposed chart is better than Huwang *et al.*[30] control chart for small shift in the process which is desirable in the industry.

Table 6: Zero state run length performance of the proposed EWMA with Zou and Tsung[31] chart at $ARL_0 = 370.00$.

ρ	λ=0.05					
	Modified EWMA chart at $k=-\lambda/2$				Zou and Tsung[31] chart	
	n=5		n=10			
	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.00	369.092	350.220	368.995	347.400	370.067	355.505
1.10	123.346	114.45	115.334	99.122	123.055	115.098
1.20	57.045	38.980	48.766	23.334	58.098	39.955
1.30	32.322	21.908	27.698	15.332	33.889	25.660
1.40	21.900	12.898	17.443	9.089	23.657	14.230
1.50	17.334	6.890	14.190	4.556	18.909	7.905
2.00	6.113	4.234	4.909	3.245	7.129	5.220

Table 7: Zero state run length performance of the proposed EWMA with Huwang *et al.*[30] chart at $ARL_0 = 200.00$.

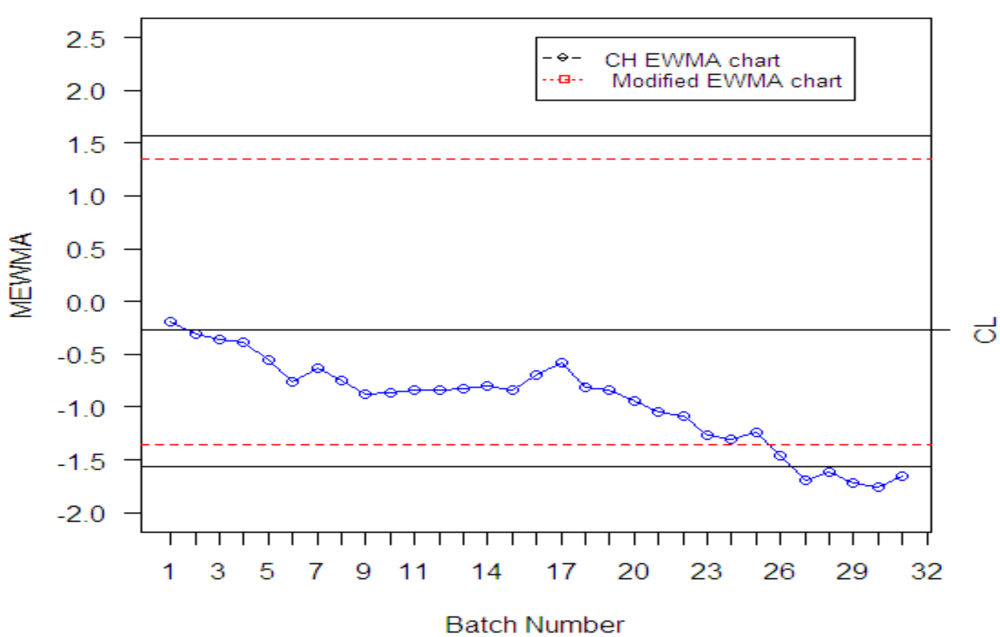
ρ	λ=0.05					
	Modified EWMA chart at $k = -\lambda/2$				Huwang *et al.*[30] chart	
	n =5		n =10			
	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.50	2.287	1.154	1.850	0.653	6.531	4.898
0.60	4.933	3.005	3.672	2.417	9.940	7.045
0.70	15.424	13.256	16.871	10.077	16.925	12.233
0.80	29.671	23.228	144.05	130.091	34.042	27.520
0.90	77.093	68.098	159.009	141.223	91.410	78.586
1.00	201.071	189.751	200.934	196.058	199.000	189.660
1.10	71.508	60.005	29.116	22.572	103.011	90.105
1.20	26.504	20.534	12.535	6.758	49.082	38.552

1.30	15.841	10.054	8.333	3.508	29.062	21.253
1.40	11.170	9.113	6.609	2.255	20.360	14.558
1.50	9.083	4.264	5.661	1.709	15.402	10.998
2.00	5.005	1.813	4.243	0.8042	6.931	4.045

6. Illustrative Example

The application of the proposed modified EWMA chart is investigated in this section using a real dataset given by Jones-Farmer *et al.*[32]. The data consist of patients waiting time (in minutes) for a colonoscopy procedure. Jones-Farmer *et al.*[32] shown that the process is in-control with respect to dispersion parameter. In order to highlight the ability of the modified EWMA structure to detect changes in the process dispersion parameter, we have introduced contaminations in the original data by multiplying the last 6 observations by 3 similar to Abbasi and Miller[33]. Then, the Crowder and Hamilton[23] chart, named as CH chart, and the proposed modified EWMA chart are constructed using $\lambda = 0.10$ and $k = -\frac{\lambda}{2}$ in figure 4 below.

Figure 4:Modified EWMA Control chart for patients data at lambda=0.10



It is obvious from Figures 4 that the CH chart detects out-of-control signals at five sample points, whereas the proposed modified EWMA chart detects all the six problem points. Hence, it is concluded that the modified EWMA chart is more efficient than the CH EWMA chart to detect shift in dispersion more quickly.

7. Conclusions and Recommendations

In this article, a modified EWMA control chart is proposed for efficient monitoring of the process dispersion following the work of Khan *et al.*[15]. The control charting constant L, which is necessarily required for the construction of the proposed control chart, is determined using Monte Carlo simulation. The performance of the proposed modified EWMA control chart is measured in terms of run length characteristics such as ARL and SDRL for several process dispersion level settings using simulation. Tables of the ARL and the SDRL are generated for different shifts levels, different smoothing constant values and for different control constant values. The proposed control chart has been compared with two existing charts. It has been observed that the proposed chart is efficient in quick detection of the out-of-control process as compared to existing control charts for monitoring process dispersion. These results are very helpful for the practitioners and researchers for monitoring the process dispersion. Further work along the direction to develop modified EWMA control charts for the monitoring multivariate process dispersion and attribute process can be done and authors are currently working on this.

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