Fault Detection Algorithm based on Null-Space Analysis for On-Line Structural Health Monitoring

Aimin Yan, Jean-Claude Golinval, Frédéric Marin
University of Liege, Department of Aerospace, Mechanics and Materials
Vibrations & Identification of Structures,
1, Chemin des Chevreuils, B-4000 Liege, Belgium
Email: am.yan@ulg.ac.be, jc.golinval@ulg.ac.be, f.marin@ulg.ac.be

ABSTRACT

Early diagnosis of structural damages or machinery malfunctions allows to reduce the maintenance cost of systems and to increase their reliability and safety. This paper addresses the damage detection problem by statistical analysis on output-only measurements of structures. The developed method is based on subspace analysis of the Hankel matrices constructed by vibration measurement data. The column active subspace of the Hankel matrix defined by the first principal components is orthonormal to the column null-subspace defined by the remaining principal components. The residue in the orthonormality relation obtained from different data sets may be used to detect structural damages. It is illustrated that this null-space-based method constitutes an enhancement of the classical damage detection method based on principal component analysis (PCA). Several damage indicators are proposed to characterize the resulting residue matrices. The method is first illustrated on a a numerical example and then, it is applied to vibration fatigue testing of a street-lighting device. Because of its simplicity and efficiency, the proposed algorithm is expected to be suitable for continuous on-line health monitoring of structures in practical situations.

1 Introduction

In recent years, early diagnosis of structural damages or machinery malfunctions has been paid more and more attention because it allows to increase reliability and safety of systems. Visual and systematic inspections performed in practice are generally simple, but sometimes they are not effective so that the shutdown (and disassembly) of the system may be required. Therefore, it is of interest to use condition-based inspections by continuous or periodical monitoring of the system. The combined application of these two types of inspections allows to take an optimal maintenance decision. In developing on-line monitoring tool, both efficiency and simplicity are pursued.

Structural damage detection techniques based on multi-variate statistics and neural networks have been largely developed in the past few years. Due to these developments, some simplest measurement data (acceleration, stress, etc.) may be directly used for structural health monitoring and sensitivity to noise could be reduced to maximum extent. Generally, neither an identification of modal parameters nor a construction of a finite element model is needed for damage detection. For instance, Basseville *et al*. [1] proposed a subspace identification-based fault detection algorithm using a χ^2-type test; Worden *et al.* [2] developed a novelty analysis for damage detection, which was also used by Sohn & Farrar [3] with a two-stage (AR-ARX) prediction model; Friswell & Inman [4] applied PCA for the sensor validation. PCA has also been used in [5-6] for the detection and localization of structural damages, and the method has been further enhanced in [7] to improve its efficiency. Zang *et al.* [8] introduced independent component analysis in damage detection. A novelty analysis based on the Kalman model was recently proposed in [9]. The basic assumption is that there is a significant increase in the residual errors when a Kalman model identified from the undamaged system is used to predict the response of a damaged system. Besides, special effort has been paid to eliminate the effect of varying environmental conditions (e.g. temperature variations) on damage detection during a long-term monitoring [10].

The present work provides an alternative algorithm to the above developments, which pursues both efficiency and simplicity. The proposed method relies on the concept of subspace identification and null-space analysis on the data structure. The output data, which are supposed to be periodically collected on the monitored structure, are used to construct the Hankel matrix, which is further partitioned into subspaces. If no structural damage occurs, the orthornomality between subspaces of the Hankel matrices made from different data sets remains approximately valid according to small residues. Those residues may be used in turn as damage-sensitive features: they increase obviously only when structural damages (or other changes in the system) occur. The threshold level of the residues has to be determined on the basis of the healthy structure, possibly under various excitations of different levels. Several damage indicators are proposed to assess the residues for damage detection.

The efficiency of the proposed method is first discussed on a numerical application, where it is clearly shown that changes due to damage on any modal parameters (especially on the natural frequencies and mode shapes) may be identified. The results are compared to the PCA-based damage detection technique for which only changes in the mode shapes may be detected. Finally, the efficiency of the method is illustrated on an experimental application.

2. Damage Detection by Null-Space Analysis of the Hankel Matrix

Let us consider structural vibration responses that are periodically measured by *m* accelerometers (or strain gauges), leading to a series of output records of *N* data points. In the case of continuous monitoring, one may separate the records into a series of data sets of a chosen time interval. From any data set, we construct the Hankel matrix $\mathbf{H}_{p,q}$ filled up with *p* block rows and *q* block columns of the output covariance matrix Λ_i :

$$\mathbf{H}_{p,q} = \begin{bmatrix} \Lambda_0 & \Lambda_1 & \dots & \dots & \Lambda_{q-1} \\ \Lambda_1 & \Lambda_2 & \dots & \dots & \Lambda_q \\ \dots & \dots & \dots & \dots & \dots \\ \Lambda_{p-1} & \Lambda_p & \dots & \dots & \Lambda_{p+q-2} \end{bmatrix} ; \quad \mathrm{q} \geq \mathrm{p} \tag{1}$$

where *p* and *q* are user-defined parameters (in this work, *p*=*q*); the output covariance matrices Λ_i are approximately estimated in the following way:

$$\Lambda_i \approx \frac{1}{N-i} \sum_{k=1}^{N-i} \mathbf{y}_{k+i} \mathbf{y}_k^{\mathrm{T}} \tag{2}$$

As known from the subspace identification theory [11-12], the Hankel matrix defined by eq. (1) contains all the modal information of the structure, from which the state matrices may be extracted and modal parameters (natural frequencies, damping ratios and mode shapes) may be identified. From the point of view of damage detection, we are not concerned with the precise values of modal parameters or other structural features. Instead, only relative changes of the features are necessary to provide information on structural damage.

The proposed damage detection method is based on the null-space concept [13] of the Hankel matrices. Performing the singular value decomposition (SVD) of the weighted Hankel matrix $\bar{\mathbf{H}}$ leads to:

$$\bar{\mathbf{H}} = \mathbf{W}_1 \mathbf{H}_{p,q} \mathbf{W}_2 \approx \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_2 \end{bmatrix}^{\mathrm{T}} = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1^{\mathrm{T}} \tag{3}$$

where $\mathbf{W}_1$ and $\mathbf{W}_2$ are invertible weighting matrices (both are chosen as equal to the identity matrix in this work); the diagonal matrix $\mathbf{S}_1$ contains $n \approx 2N_m$ (N_m is the number of vibrating modes) non-zero singular values sorted in decreasing order.

On the one hand, eq. (3) allows to write the following relations:

$$\mathbf{U}_1^{\mathrm{T}}\overline{\mathbf{H}}\mathbf{V}_1 \approx \mathbf{S}_1 \tag{4}$$

$$\overline{\mathbf{H}}\mathbf{V}_2 \approx \mathbf{0} \text{ or } \overline{\mathbf{H}}\mathbf{V}_2 = \zeta_v \tag{5}$$

$$\mathbf{U}_2^{\mathrm{T}}\overline{\mathbf{H}} \approx \mathbf{0} \text{ or } \mathbf{U}_2^{\mathrm{T}}\overline{\mathbf{H}} = \zeta_u \tag{6}$$

It is shown that performing SVD on the weighted Hankel matrix $\overline{\mathbf{H}} \in \Re^{r\times c}$, where $r = m\times p$ and $c = m\times q$, leads to four fundamental subspaces: $\mathbf{U}_1$ contains the maximum number (i.e. *n*) of independent column vectors that span the column space of $\overline{\mathbf{H}}$; $\mathbf{V}_1^{\mathrm{T}}$ contains the maximum number (*n*) of independent row vectors that span the row space of $\overline{\mathbf{H}}$; $\mathbf{U}_2$ contains the maximum number (*c-n*) of independent column vectors that span the column null-space of $\overline{\mathbf{H}}$; $\mathbf{V}_2^{\mathrm{T}}$ contains the maximum number (*r-n*) of independent row vectors that span the row null-space of $\overline{\mathbf{H}}$. In almost all practical applications, eq. (3) only holds approximately; so do eqs. (4-6) because the exact order *n* of the system is difficult to determine. Generally, the residue matrices ζ_v and ζ_u are mainly due to noise effects and to weakly-excited high modes that are neglected by cutting off with a chosen order *n*. Sometimes, the variation of the residue matrices from different data sets may be large enough to mask the variation due to small structural damages. Therefore it is difficult to directly use these residue matrices for damage detection.

On the other hand, due to orthonormality property of matrices $\mathbf{U}$ and $\mathbf{V}$, the following equations are always exactly verified in the mathematical sense for any data set:

$$\mathbf{U}_2^{\mathrm{T}}\mathbf{U}_1 = \mathbf{0} \tag{7}$$

or

$$\mathbf{U}_2^{\mathrm{T}}\left(\mathbf{U}_1\mathbf{S}\mathbf{V}_1^{\mathrm{T}}\right) = \mathbf{0} \tag{8}$$

$$\mathbf{V}_1^{\mathrm{T}}\mathbf{V}_2 = \mathbf{0} \tag{9}$$

or

$$\left(\mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^{\mathrm{T}}\right)\mathbf{V}_2 = \mathbf{0} \tag{10}$$

From the numerical point of view, the equations above show that the active part (with the chosen order *n*) of column or row spaces is used instead of the full Hankel matrix. Unlike (5-6), equalities (7-10) do not depend on the precise determination of system order *n*.

In the following, we will consider only the left kernel relations (7-8) of the Hankel matrix for the damage detection problem**.** It may be shown that $\mathbf{U}_1$, containing the first *n* active principal components, represents a generalized subspace (also called hyperplane), around which the response data locate [7]. Without damage or environmental variation, this subspace of the Hankel matrix should remain unchanged (i.e. no rotation of subspace occurs) and the orthonormality relations (7-8) apply between different data sets. Therefore, observing the rotation of this subspace (i.e. changes in orthonormality between different data sets) provides information on structural damage. For convenience of description, we will call $\mathbf{U}_1$ and $\mathbf{U}_2$, respectively, column active subspace and column null-subspace of the weighted Hankel matrix $\overline{\mathbf{H}}$.

Note that the same methodology may also be considered when the Hankel matrix is constructed using response data blocks instead of data covariance matrices:

$$\mathbf{H}_d = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_j \\ \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_{j+1} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_d & \mathbf{y}_{d+1} & & \mathbf{y}_N \end{bmatrix} \tag{11}$$

where d is the user-defined number of row blocks, and j the number of columns (in practice, $j = N\text{-}d\text{+}1$). It may be demonstrated [7] that equation (11) defines a dynamic model that contains richer information than the classical model based on PCA. In the present context, dynamic modeling means using time-shift output records in the analysis i.e. not only looking at the vibration signal at the same instant but also at other time instants. Using eq. (1) consists of another type of dynamic modeling.

As only the left kernel of the Hankel matrix is considered, SVD is performed on the covariance of the Hankel matrix:

$$\bar{\mathbf{H}}_d \bar{\mathbf{H}}_d^{\mathrm{T}} \approx \begin{bmatrix} \bar{\mathbf{U}}_1 & \bar{\mathbf{U}}_2 \end{bmatrix} \begin{bmatrix} \bar{\mathbf{S}}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{U}}_1 & \bar{\mathbf{U}}_2 \end{bmatrix}^{\mathrm{T}} = \bar{\mathbf{U}}_1 \bar{\mathbf{S}}_1 \bar{\mathbf{U}}_1^{\mathrm{T}} \tag{12}$$

where $\bar{\mathbf{H}}_d = \mathbf{W}_1 \mathbf{H}_d \mathbf{W}_2$; (both weight matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ are chosen as identity matrix in this work). As the dimension of $\bar{\mathbf{H}}_d \bar{\mathbf{H}}_d^T$ is much smaller than the dimension of the original matrix $\bar{\mathbf{H}}_d$, the SVD computational cost is reduced drastically. By a similar analysis as with the COV-driven Hankel matrix, we obtain the following expressions that are similar to (7-8):

$$\bar{\mathbf{U}}_2^{\mathrm{T}} \bar{\mathbf{U}}_1 = \mathbf{0} \tag{13}$$

$$\bar{\mathbf{U}}_2^{\mathrm{T}} \left(\bar{\mathbf{U}}_1 \bar{\mathbf{S}}_1 \bar{\mathbf{U}}_1^{\mathrm{T}} \right) = \mathbf{0} \tag{14}$$

Again, equalities (13-14) do not depend on the chosen system order n. As (7-8) and (13-14) are of the same form, the following discussion is suitable for both definitions of the Hankel matrix.

When two different reference data sets are examined: one for column active subspace $\mathbf{U}_1$ or active Hankel matrix ($\mathbf{U}_1 \mathbf{S}_1 \mathbf{U}_1^{\mathrm{T}}$), and the other one for column null-subspace $\mathbf{U}_2$, equalities (7-8) or (13-14) no longer strictly apply due to noise effects and other sources of errors (e.g. variation of the excitation). Therefore a series of tests of the system in structural healthy state should be performed to provide a reliable reference residue. It is expected that tests may be performed at different excitation levels because in most practical situations, the natural exciting forces may change from time to time although they may be assumed to remain stationary during a short acquisition period. Assuming that column null-subspace $\mathbf{U}_{2,0}$ is determined using reference data (indicated by index 0), and column active subspace $\mathbf{U}_{1,r}$ (and $\mathbf{S}_{1,r}, \mathbf{V}_{1,r}$) using the r-th current data set, the residue matrices corresponding to (7-8) or (13-14) are

$$\delta_r = \mathbf{U}_{2,0}^{\mathrm{T}} \mathbf{U}_{1,r} \tag{15}$$

$$\Delta_r = \mathbf{U}_{2,0}^{\mathrm{T}} \left(\mathbf{U}_{1,r} \mathbf{S}_{1,r} \mathbf{V}_{1,r}^{\mathrm{T}} \right) \tag{16}$$

When using the data-driven Hankel matrix, $\mathbf{V}_{1,r}$ is replaced by $\mathbf{U}_{1,r}$. The residue matrix δ_r represents the orthonormality change between the subspaces of responses due to noise effects and/or mainly due to structural damages if any. In contrast to (15), the residue matrix Δ_r represents a similar orthonormality change but weighted by the active singular matrix $\mathbf{S}_{1,r}$ and the right kernel matrix $\mathbf{V}_{1,r}^{\mathrm{T}}$. Both residue matrices (15-16) are good candidates as damage-sensitive features. From the numerical point of view however, the former seems better than the latter due to the fact that, although the effect of $\mathbf{V}_{1,r}^{\mathrm{T}}$ is not clear, $\mathbf{S}_{1,r}$ decreases with damage; this may reduce the sensitivity of the residue indicator to damages (if the measured data are not well normalized). Nevertheless, it may be shown that when (16) is adopted for damage detection, the present method can be related to the method proposed in [1].

From the orthonormality of subspaces (15), we propose to take the complementary angle between subspaces $\mathbf{U}_{2,0}$ and $\mathbf{U}_{1,r}$, defined as below, as damage indicator:

$$\alpha_r = \sin^{-1}\left[\mathrm{norm}(\delta_r)\right] \tag{17}$$

where norm(.) is an operator giving the maximal singular value of a matrix. Obviously, the value of α remains in the range (0-90°). In the same manner, a damage indicator may be defined on the basis of eq. (16):

$$\sigma_i = \mathrm{norm}\left(\Delta_i\right) \tag{18}$$

Additionally, another damage indicator may be built from the residue matrix, following an idea proposed by Basseville *et al.* [1] and Fritzen *et al*. [14]. It consists first of transforming the residue matrix $\mathbf{\Delta}_i$ into a vector $\left\{\Delta\right\}_i$ and then, to define a normalized scalar by an inner-product operation of the vector

$$\overline{\Delta}_i = \frac{\left\{\Delta\right\}_i^{\mathrm{T}}\left\{\Delta\right\}_i}{\Sigma_\Delta} \tag{19a}$$

with

$$\left\{\Delta\right\}_i = \mathrm{vec}\left(\Delta_i\right) \tag{19b}$$

$$\Sigma_\Delta = \frac{1}{N_{ref}}\sum_{j=1}^{N_{ref}}\left\{\Delta\right\}_j^{\mathrm{T}}\left\{\Delta\right\}_j \tag{19c}$$

where vec(.) denotes the column stacking operator; Σ_Δ is a scalar used for normalization (different from the matrix defined in [1] and [14]). Similarly, we have

$$\overline{\delta}_i = \frac{\left\{\delta\right\}_i^{\mathrm{T}}\left\{\delta\right\}_i}{\Sigma_\delta} \tag{20a}$$

with

$$\left\{\delta\right\}_i = \mathrm{vec}\left(\delta_i\right) \tag{20b}$$

$$\Sigma_\delta = \frac{1}{N_{ref}}\sum_{j=1}^{N_{ref}}\left\{\delta\right\}_j^{\mathrm{T}}\left\{\delta\right\}_j \tag{20c}$$

In the simplest case, N_{ref} =1 may be applied. The rotation angle (17) indicates global change of the subspace, while the normalized indicators (19-20) are more affected by local changes of vibration modes. If the structure is undamaged, the rotating angle remains small, and the normalized indicators (19) and (20) are close to unity. Conversely, it is expected that structural damages will cause obvious increase of the proposed indicators.

3. Examples of Application

3.1. Numerical example: a 4-DOF system under random excitation

The first example is designed to illustrate the theoretical predictions and verify the efficiency of the proposed NSA-based damage detection method. The proposed method is also compared to the PCA-based damage detection method described in [6-7]. Consider first a simple 4-DOF system under random force along the *x*-direction: the responses at all the 4 DOF are recorded and 1% noise is added to the data.

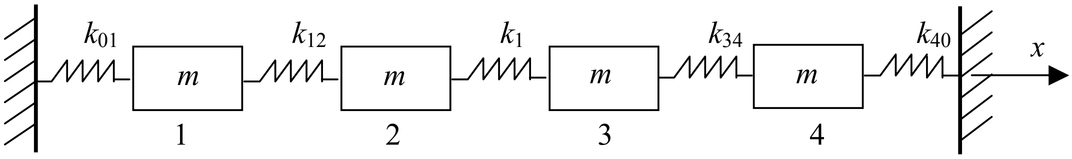

Fig. 1. A 4-DOF system under random excitation

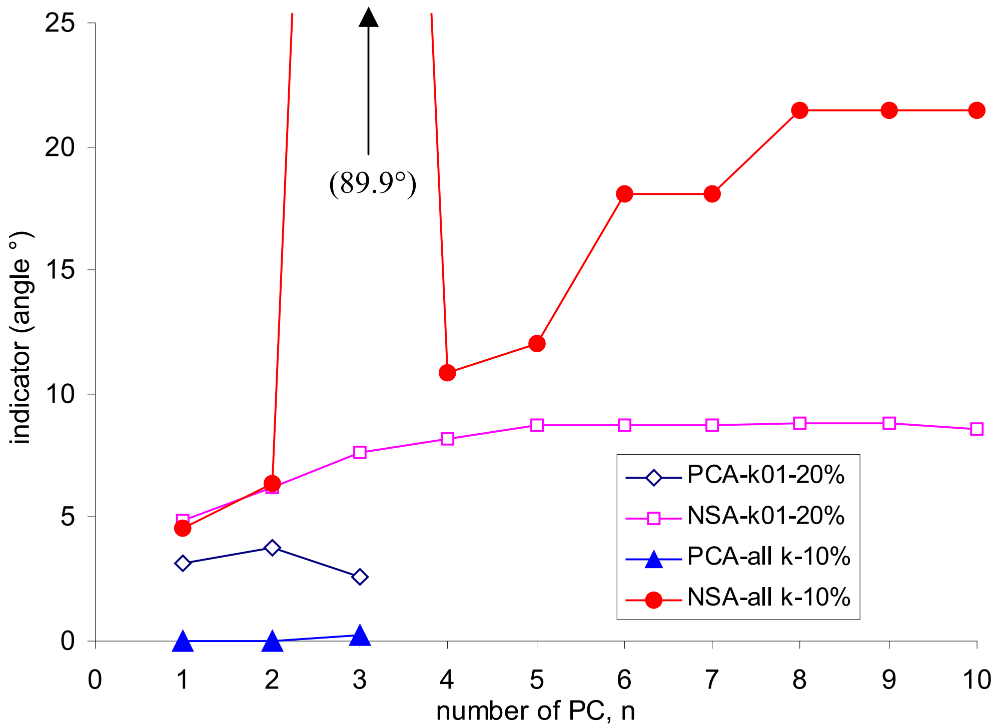

Fig. 2 Damage detection by PCA and NSA

Damage case n°1 is simulated by reducing the stiffness coefficient k_{01} by 20%. This damage leads to a reduction of the four natural frequencies of the order of 4.15% to 0.343% and to changes in the mode-shapes. For this type of damage, both damage detection methods based on PCA and NSA allow to detect the damage; however, the NSA-based method provides a better detection as shown in Fig. 2.

Damage case n°2 corresponds to a uniform degradation of the system: the stiffness coefficients of all the four springs are reduced by 10%. This damage leads to a reduction of all the four natural frequencies of 5.137%, but the mode-shapes remain unchanged. As shown in Fig. 2, the PCA-based damage detection method is unable to detect such a damage because no change occurs in the mode-shapes of the system, while the NSA-based method does. Obviously, in this case, NSA allows to detect the damage thanks to changes in the natural frequencies of the system. Let us recall that, when using PCA, the system order n (i.e. the number of principal components) cannot overpass the number of measured DOF (4 in this example). This limitation is removed when performing NSA. Referring to Fig. 2, it can be also noticed that, when taking n = 3, the NSA-based damage indicator (the rotation angle) increases drastically. This may be explained by observing the singular value diagram given in Fig. 3: as the singular values corresponding to n = 3 and n = 4 are very close, their ordering changes when passing from the reference to the damaged state. Therefore, the rotation angle is calculated between two hyperplanes constructed by swapped sets of principal components.

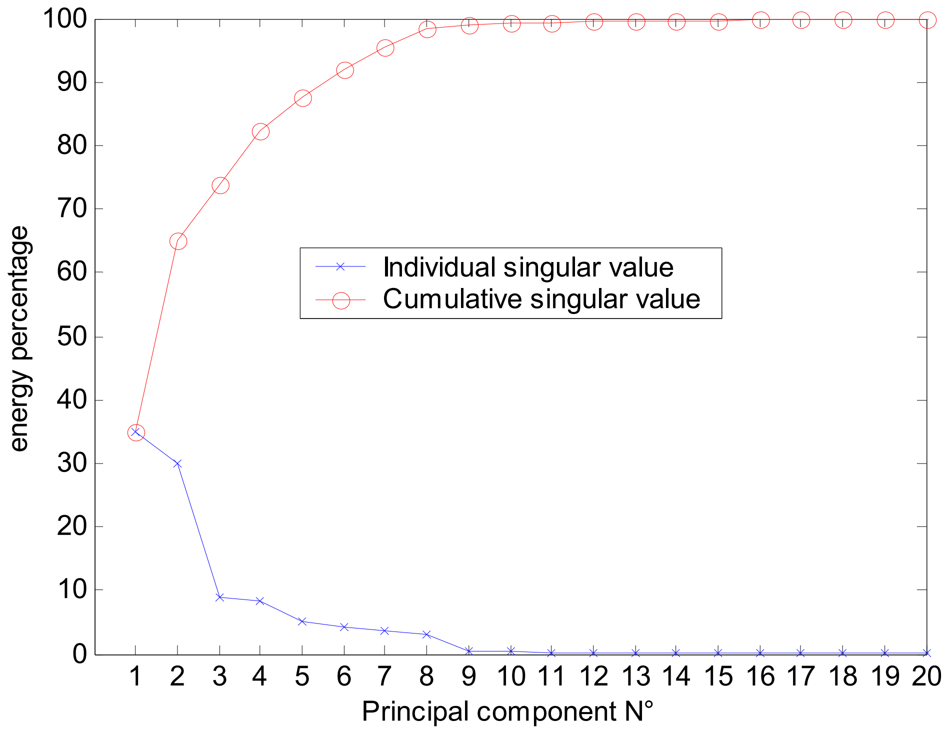

Fig. 3 Singular value diagram

3.2 Fatigue testing of a street-lighting device

Fatigue vibration testing is currently performed on street-lighting devices to simulate the effect of wind-induced vibrations and to assess its mechanical integrity. Various types of excitation (such as random or sinusoidal forces, sine sweep) can be used. The NSA-based detection method was successfully applied to monitor the damage evolution during fatigue testing of the device [15]. In this paper, a vibration test consisting of repeated impulsions is reported. The street-lighting device is placed on a vibration shake table that is driven by a cam type mechanism as shown in Fig. 4. The excitation corresponds to a shock applied every 2 seconds. Twelve accelerometers and four strain gauges are installed at different locations to monitor the structure with a sampling frequency of 512 Hz. The total duration of the test is 2 hours and 38 minutes. Fig. 5 shows the crack induced near the fixation at the end of the test.

The sensor responses were recorded every five minutes. They are collected in a series of windows of 8447 points as shown in Fig. 6 for the first data set. The NSA-based detection method is then performed on each window. The difference between the reference data set (i.e. the first set) and each current set of observations is calculated by evaluating the characteristic angle (19), or the normalized indicators given by (21) noted x^2H or (22) noted x^2. The results obtained by both NSA (*d* or *p* equal to 10) and PCA (*d* or *p* equal to 1) are shown in Fig. 7-9. During the first hour of the test, the damage indicator (i.e. the angle) increases slightly and linearly: no damage is observed during this period of time. The PCA-based damage detection method indicates damage at the beginning of the second hour while the NSA-based method detects damage earlier after 1 hour and 45 minutes. This difference shows that the NSA-based method is more sensitive to slight damage than the PCA-based method. Once damage is detected, the crack growth is very rapid.

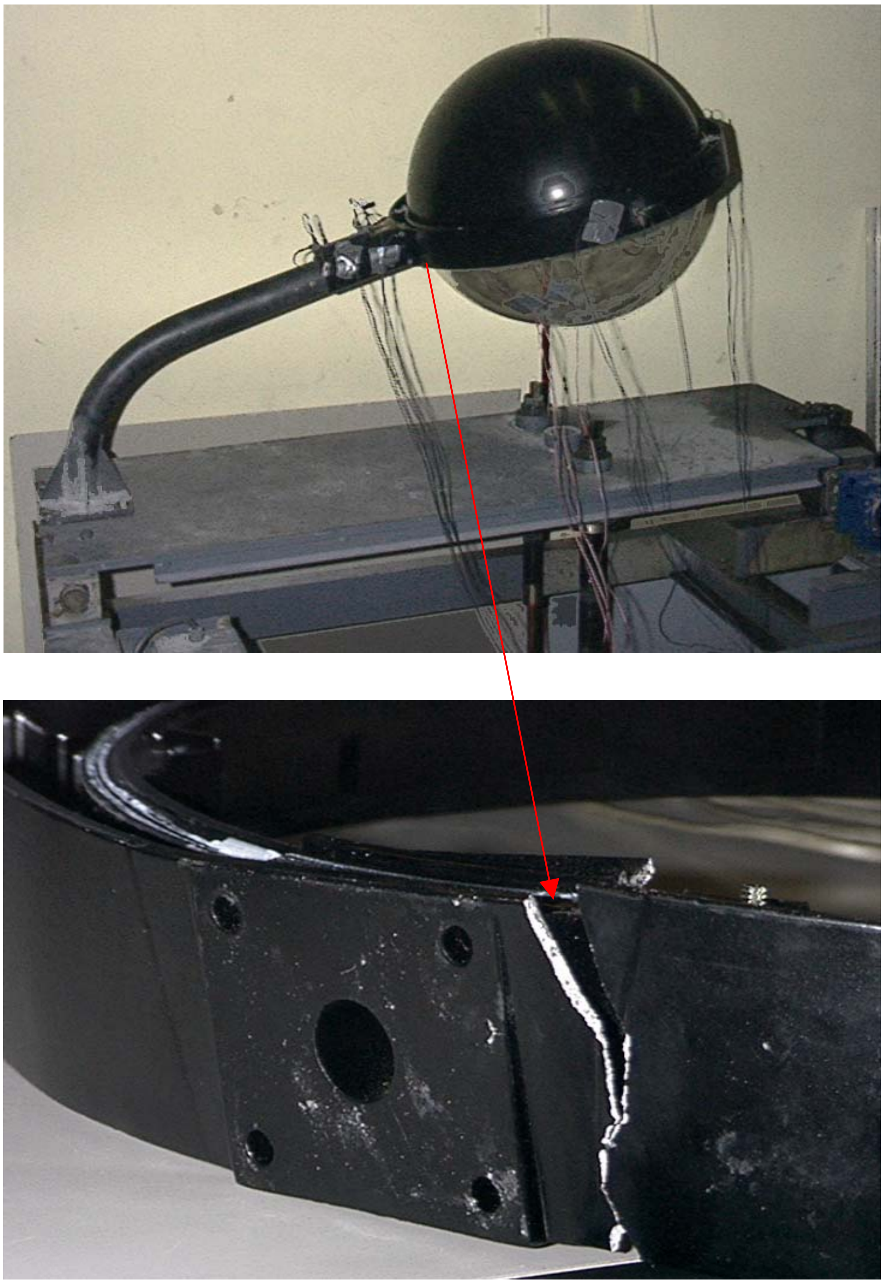

Fig. 4 Fatigue testing of a street-lighting device (upper picture), Fig. 5 View of the crack (lower picture)

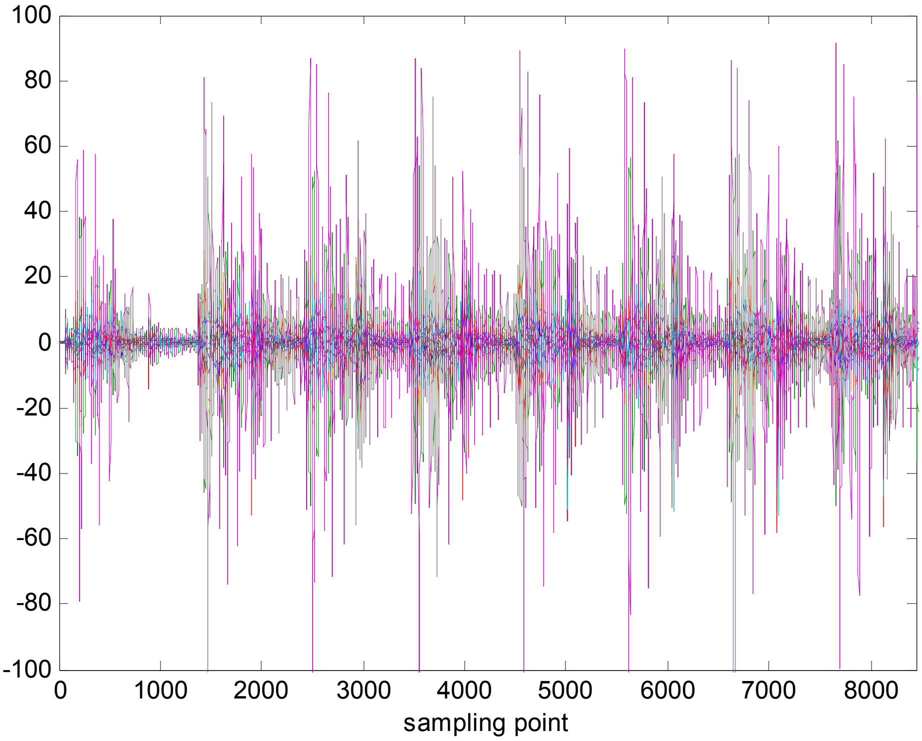

Fig. 6. Response of the system under repeated impulsions

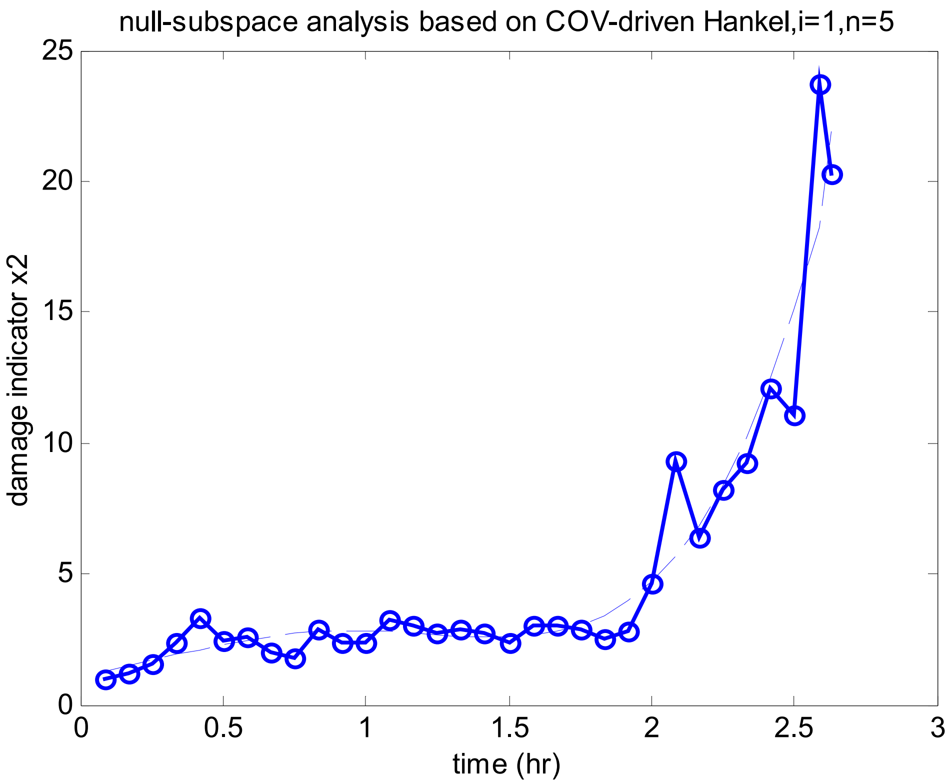

Fig. 7. Time evolution of the damage indicator (angle), PCA-based detection method

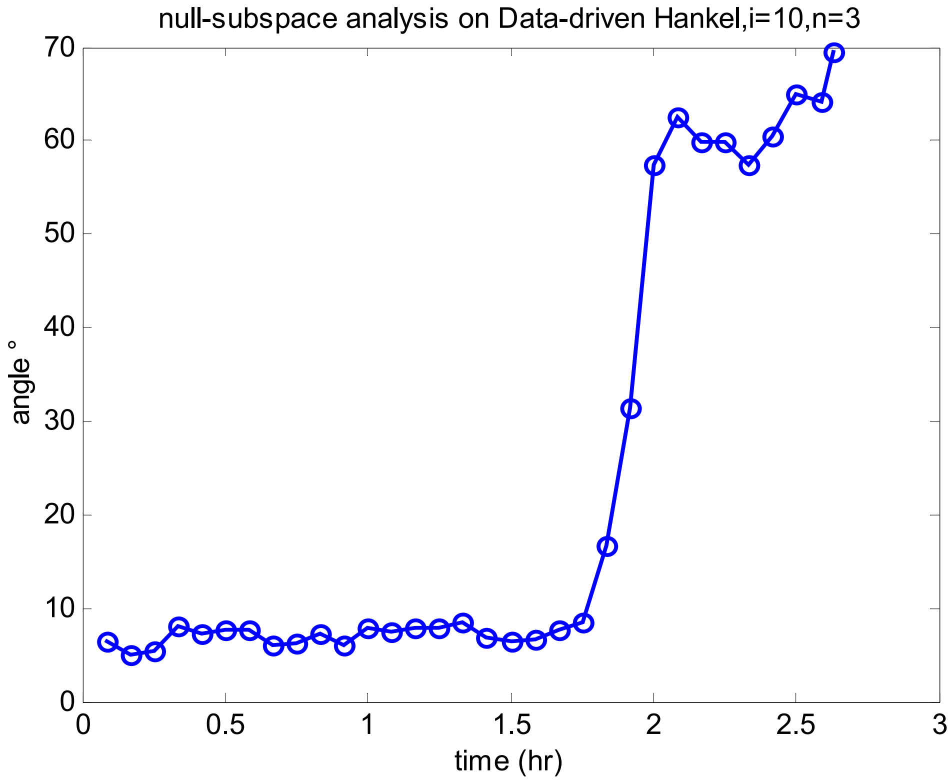

Fig. 8. Time evolution of the damage indicator (angle), NSA-based method

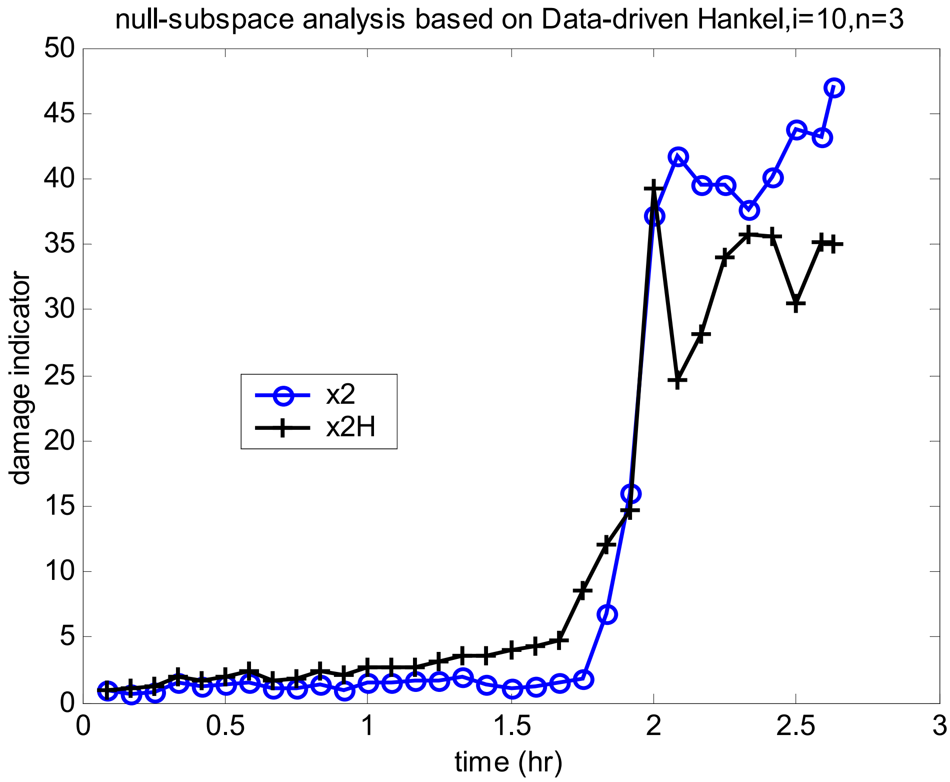

Fig. 9. Time evolution of the normalized damage indicators (NSA-based method)

4. Conclusion

In this paper, a simple and effective damage detection algorithm has been presented, which is based on null-space analysis of output-only measurement data. Residues on orthonormality relations between subspaces of the Hankel matrices are used to assess changes of the dynamic behavior due to structural damage. It has been demonstrated that the proposed method may be considered as an enhancement of the PCA-based detection method. Several damage indicators have been proposed to characterize the residue matrices. Numerical and experimental applications with various kinds of excitation illustrate the efficiency of the proposed method. Because of its simplicity, the NSA-based detection method is suitable for continuous on-line monitoring of structures.

Acknowledgements

This work is supported by the Walloon Region government of Belgium (convention RW - ULg n° 9613419), which is gratefully acknowledged.

References

1. Basseville, M., Abdelghani, M. and Benveniste, A. (2000): Subspace-based fault detection algorithms for vibration monitoring. *Automatica* **36**: 101-109
2. Worden, K., Manson, G. and Fieller, N.R.J. (2000): Damage detection using outlier analysis. *Journal of Sound and Vibration,* **229**(3): 647-667.
3. Sohn, H. and Farrar, C.F. (2001): Damage diagnosis using time series analysis of vibration signals. *Smart Mater. Struct.* **10:** 1-6.
4. Friswell, M.I. and Inman, D.J. (2000) : Sensor validation for SMART structures. *IMAC XVIII (18th International modal analysis conference)*, San Antonio, 483-489.
5. De Boe, P. and Golinval, J.C. (2003): Principal component analysis of piezo-sensor array for damage localization, *Structural Health Monitoring- A International Journal*, **2(2):** 137-144.
6. De Boe, P., Yan A.M., Golinval J.C. (2003): Substructure damage detection by principal component analysis: application to environmental vibration testing. *Structural Health Monitoring 2003*, edited by Fu-Kuo Chang. 642-649. Stanford, CA, USA, September 15-17, 2003.
7. Yan A.M., De Boe P., Golinval J.C. (2003): Structural integral monitoring by vibration measurements. *FM2003- Structural Integrity and Materials Aging*. G.C. Sih, S.T. Tu & Z.D. Wang (Eds.), 363-370.
8. Zang C., Friswell M.I., Imregun M. (2004), Structural damage detection using independent component analysis. *Structural Healthy Monitoring*, **3**(1), 69-83.
9. Yan A.M., De Boe P., Golinval J.C. (2004): Structural diagnosis by Kalman model based on stochastic subspace identification. *Structural Healthy Monitoring*, **3**(2), 103-119.
10. Yan A.M., Kerschen G., De Boe P., Golinval J.C. (2004), Structural damage diagnosis under changing environmental conditions - Part I: linear analysis & Part II: local PCA for nonlinear analysis, submitted to *Mechanical systems & Signal Processing.*
11. Van Overschee, P. and De Moor, B. (1996): *Subspace Identification for Linear Systems: Theory Implementation – Applications.* Dordrecht, Netherlands: Kluwer Academic Publishers.
12. Hermans L., Van Den Auweraer H. (1999): Modal Testing and Analysis of Structures under Operational Conditions : Industrial Applications. *Mechanical Systems and Signal Processing* **13**(2), 193-216.
13. Juang J.N., Phan M.Q. (2001), *Identification and control of mechanical systems.* p29, Cambridge University Press.
14. Fritzen C.P., Mengelkamp G., Güemes A. (2003), Elimination of temperature effects on the damage detection within a smart structure concept. *Structural Health Monitoring 2003*, edited by Fu-Kuo Chang. 1530-1538. Stanford, CA, USA, September 15-17, 2003
15. Yan A.M., Golinval J.C., F. Marin (2004). Null-Subspace analysis for structural damage monitoring in fatigue test of luminaries. *ISMA 2004 - international conference on Noise and Vibration Engineering.* Paper N° 107. September 20-22, 2004. Leuven