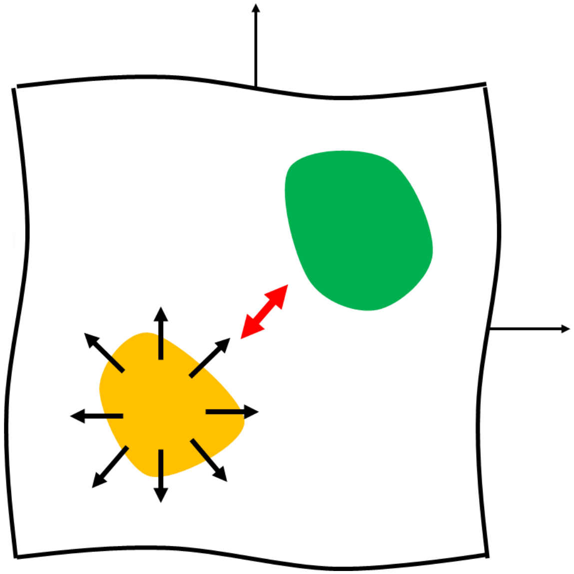
Numerical Evaluation of Interaction Tensors in Heterogeneous Materials



Reduced Order Modeling
for
Composites

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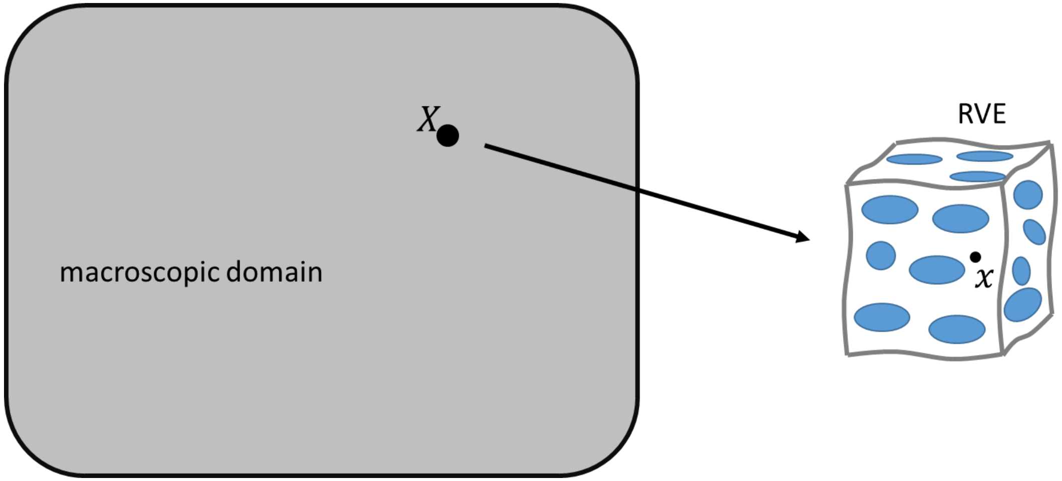
  - Offline and online stage
- Offline stage
  - Spatial decomposition of the RVE
  - Determination of interaction tensors
- First TFA results for UD fibers

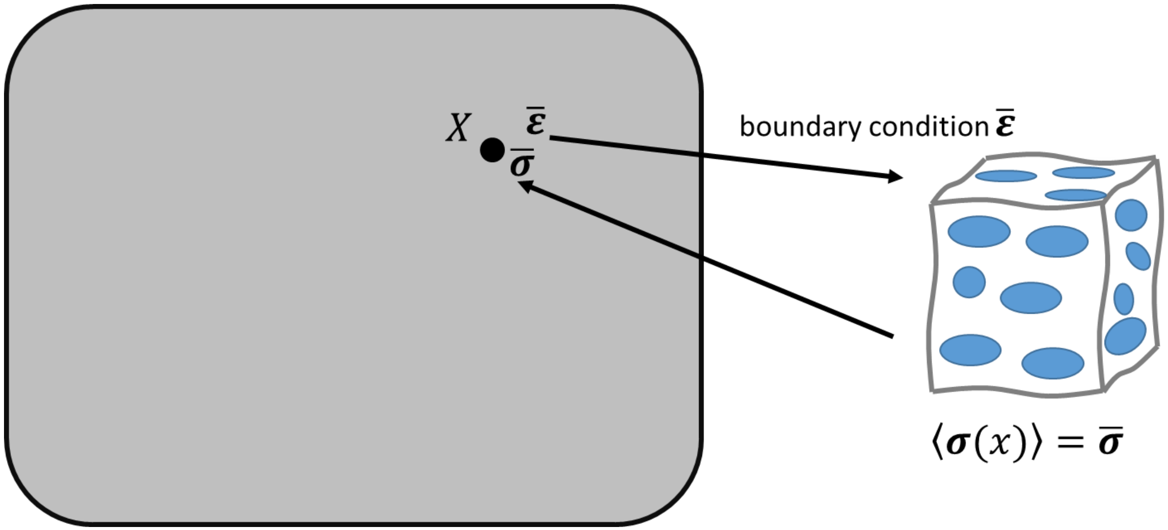
Multiscale Modeling

- Aim: achieving macroscopic material response $\bar{\boldsymbol{\varepsilon}} \rightarrow \bar{\boldsymbol{\sigma}}$
- Linking microscopic and macroscopic scales: $x \leftrightarrow X$
  - Model macroscopic behaviour by respecting microscopic processes
  - Taking into account microscopic structure
- Strategy: finding a representative volume element (RVE)
- RVE represents the micro-structure at a macroscopic point



Multiscale Modeling

- Computing macroscopic response: solving boundary value problems for a RVE
  - FEM: solving equations for all microscopic points



→ computational effort is too high with the FEM

- Finding a less costly way to achieve macroscopic response ?

  → Reduced Order Modeling (ROM) approaches

Transformation Field Analysis

- Subdivision of the full RVE volume V into a number r subvolumes V_r

- Piecewise uniform fields of internal variables in the clusters:

$$\alpha_r(x) = \alpha_r$$

- Local fields

$$\alpha(x) = \sum_{r=1}^{k} \chi_r \alpha_r \,, \qquad \text{where } \chi_r = \begin{cases} 1, & x \in V_r \\ 0, & else \end{cases}$$

- Internal variables:

$$\alpha_r = \boldsymbol{\varepsilon}_r, \boldsymbol{\varepsilon}_r^{in}, \boldsymbol{\sigma}_r, \boldsymbol{\sigma}_r^{eig}$$

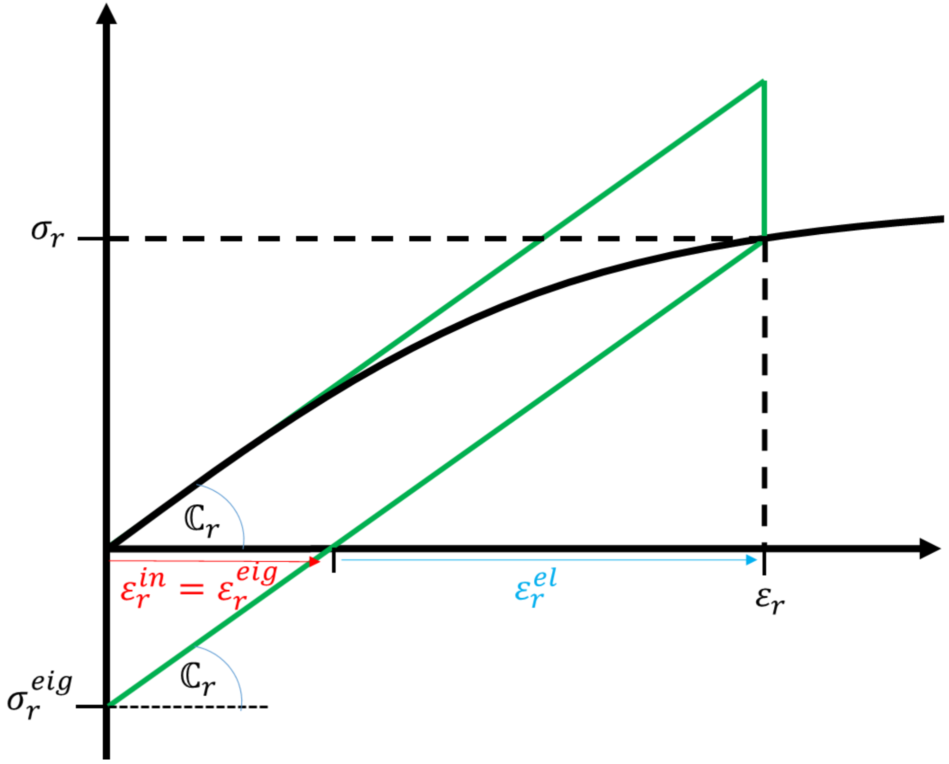
- Overall stress and strain

$$\bar{\boldsymbol{\sigma}} = \sum_{r=1}^{k} \upsilon_r \, \boldsymbol{\sigma}_r, \qquad \bar{\boldsymbol{\varepsilon}} = \sum_{r=1}^{k} \upsilon_r \, \boldsymbol{\varepsilon}_r$$

- Local constitutive relation:

$$\boldsymbol{\sigma}_r = \mathbb{C}_r : (\boldsymbol{\varepsilon}_r - \boldsymbol{\varepsilon}_r^{in})$$

$$\boldsymbol{\sigma}_r^{eig} = -\mathbb{C}_r : \boldsymbol{\varepsilon}_r^{in}$$



Transformation Field Analysis

- Microscopic – macroscopic coupling constitutive relation

$$\boldsymbol{\varepsilon}_r = \mathbb{A}_r : \bar{\boldsymbol{\varepsilon}} + \sum_{s=1}^{k} \mathbb{D}_{rs} : \boldsymbol{\varepsilon}_s^{in}$$

- Elasto-plasticity:

$$\mathbb{A}_r^{ep} = [\,\mathbb{I} + \mathbb{D}_{rs} : (\mathbb{C}_s^{-1} : \mathbb{C}_s^{tan} - \mathbb{I})]^{-1} : \mathbb{A}_r$$

- Solving equations for clusters instead of all microscopic points

- Evaluation of strain concentration tensors $\mathbb{A}$ and interaction tensors $\mathbb{D}$

Transformation Field Analysis

- TFA: reduce computational cost for multiscale simulations by a spatial decomposition of the RVE

- Problem: capturing the highly heterogeneous strain and plastic strain field

  → Cluster decomposition based on strain concentration tensors

- Offline computations
  - Cluster Decomposition
  - Once-for-all computation: strain concentration tensors $\mathbb{A}_r$ and interaction tensors $\mathbb{D}_{rs}$

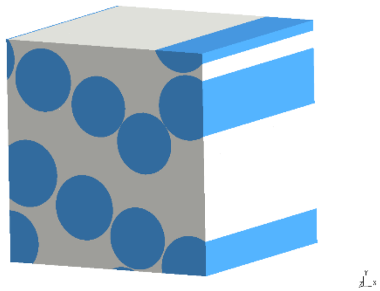
- Online stage
  - Solve TFA equation for the clusters instead for all material points

$$\boldsymbol{\varepsilon}_r = \mathbb{A}_r : \bar{\boldsymbol{\varepsilon}} + \sum_{s=1}^{k} \mathbb{D}_{rs} : \boldsymbol{\varepsilon}_s^{in}$$

TFA Overview

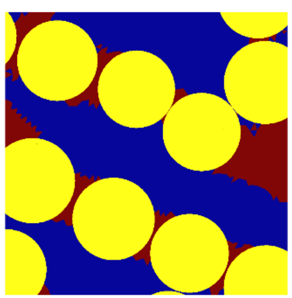
- Offline stage:

1) full FEM simulations



Extract strain concentration tensors $\mathbb{A}(x)$

2) Clustering of the RVE in r clusters and determination of the $\mathbb{A}_r$



3) FEM simulations with eigenstrain to determine the $\mathbb{D}_{rs}$

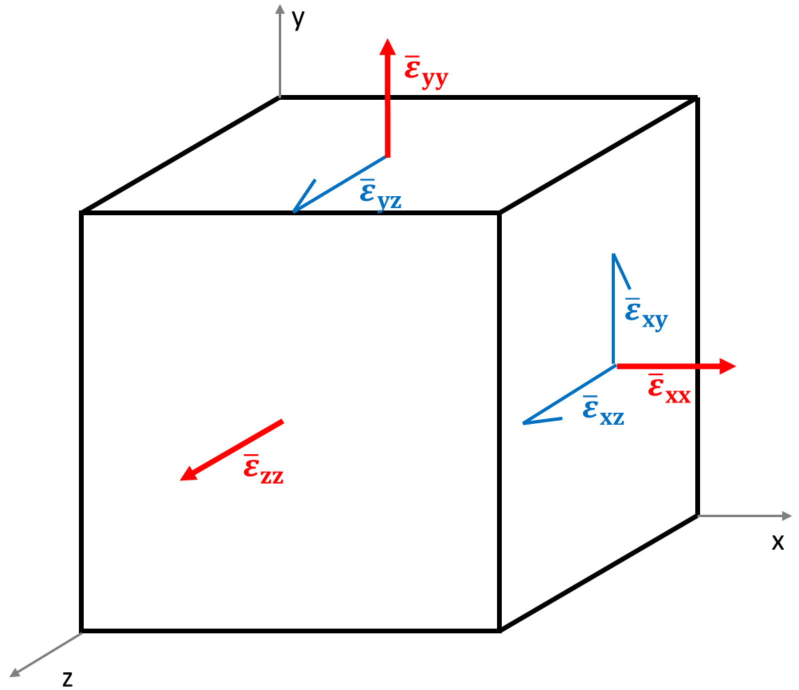
Cluster Decomposition

- Clustering of all microscopic points based on similarity of mechanical behavior
  → Required: strain concentration tensors for all microscopic points

$$\boldsymbol{\varepsilon}(x) = \mathbb{A}(x) : \bar{\boldsymbol{\varepsilon}}$$

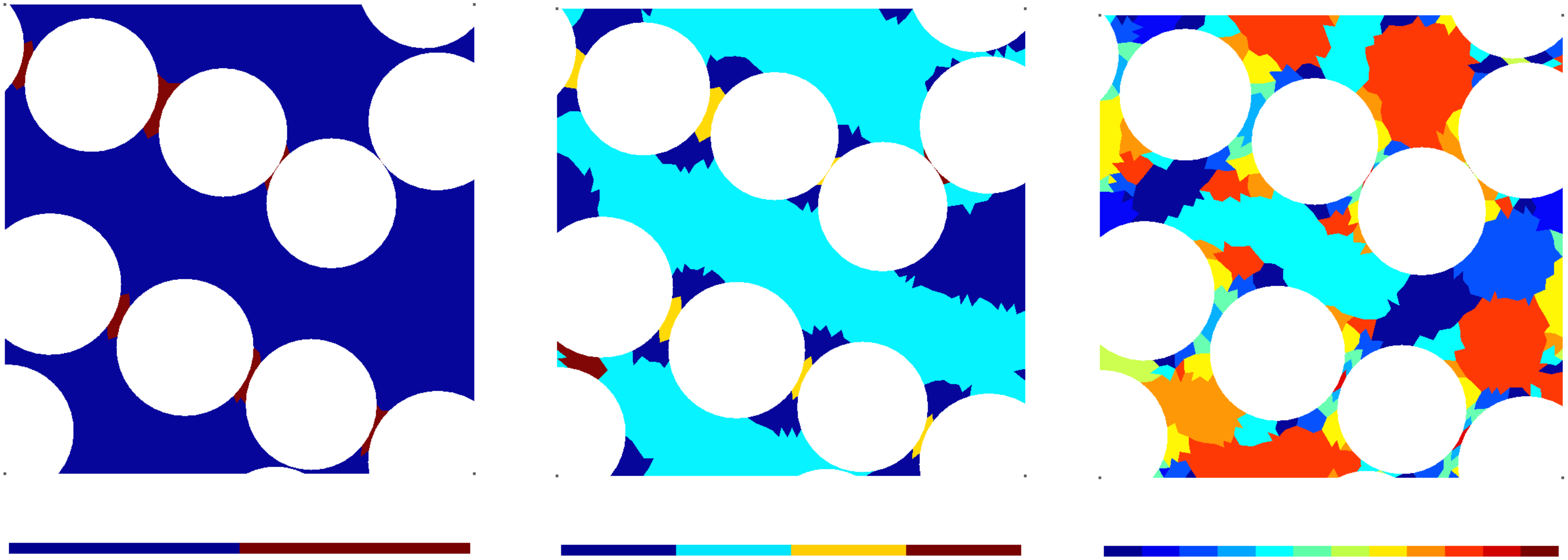
- Determination of 36 independent components of $\mathbb{A}(x)$:
  application of 6 orthogonal loading conditions $\bar{\boldsymbol{\varepsilon}}$ on the RVE

- Simulations in the elastic range



Cluster Decomposition

- UD fibers
- Results for various cluster numbers $k = 2, 4, 12$ for the matrix

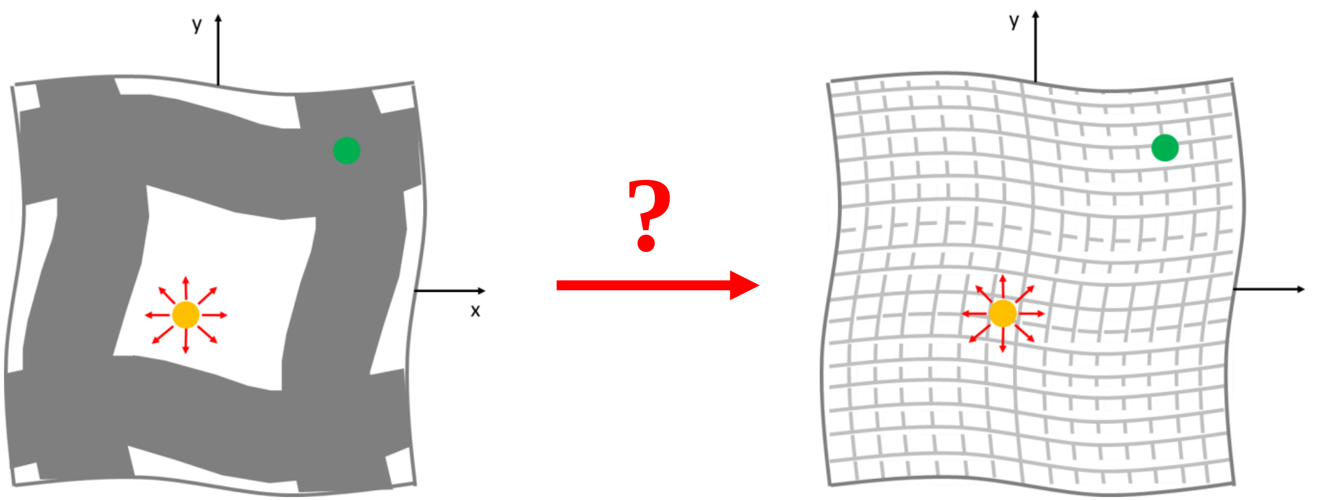


Interaction Tensors

- Analytical evaluation including Green's function
  - Provides influences between material points at different locations
  - Relies on homogenized material properties, isotropic or anisotropic

- Composites: Microscopic strongly heterogeneous properties

  → Homogenized anisotropic stiffness not representative



Woven microstructure *Homogenized woven microstructure*

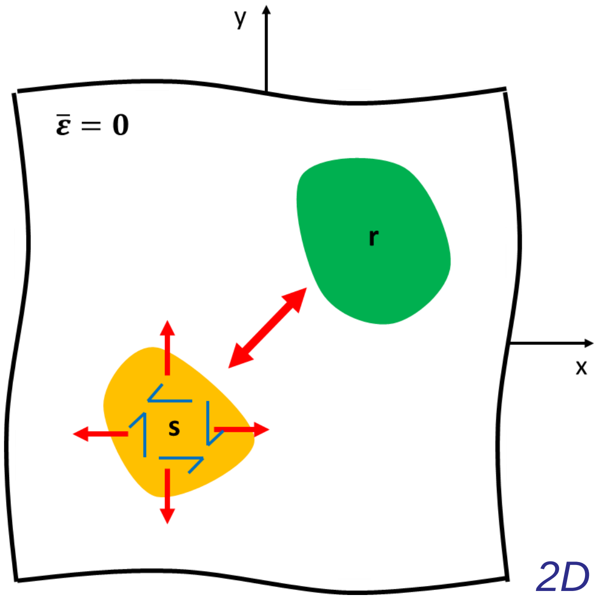
→ Numerical instead of analytical determination of eigenstrain – strain interaction tensors

- Ability to account for complex anisotropic + heterogeneous structures

- BC: periodic, no overall strain: $\bar{\boldsymbol{\varepsilon}} = 0$

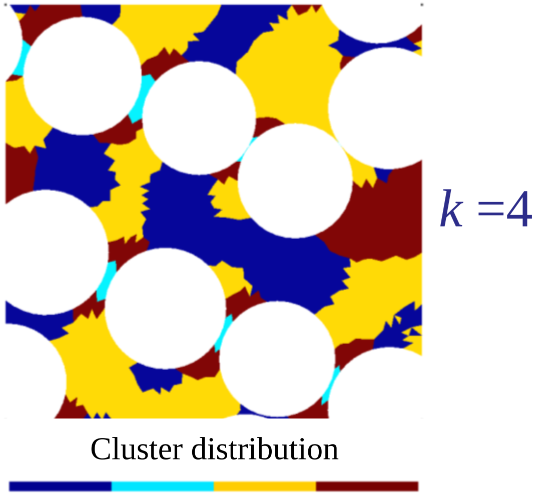
$$\boldsymbol{\varepsilon}_r = \mathbb{A}_r : \bar{\boldsymbol{\varepsilon}} + \sum_{s=1}^{k} \mathbb{D}_{rs} : \boldsymbol{\varepsilon}_s^{eig} \quad \Rightarrow \quad \boldsymbol{\varepsilon}_r = \sum_{s=1}^{k} \mathbb{D}_{rs} : \boldsymbol{\varepsilon}_s^{eig}$$

- 3D: Application of 6 orthogonal eigenstrains in cluster s and computation of resulting strain in cluster r to determine 36 independent components of $\mathbb{D}_{rs}$



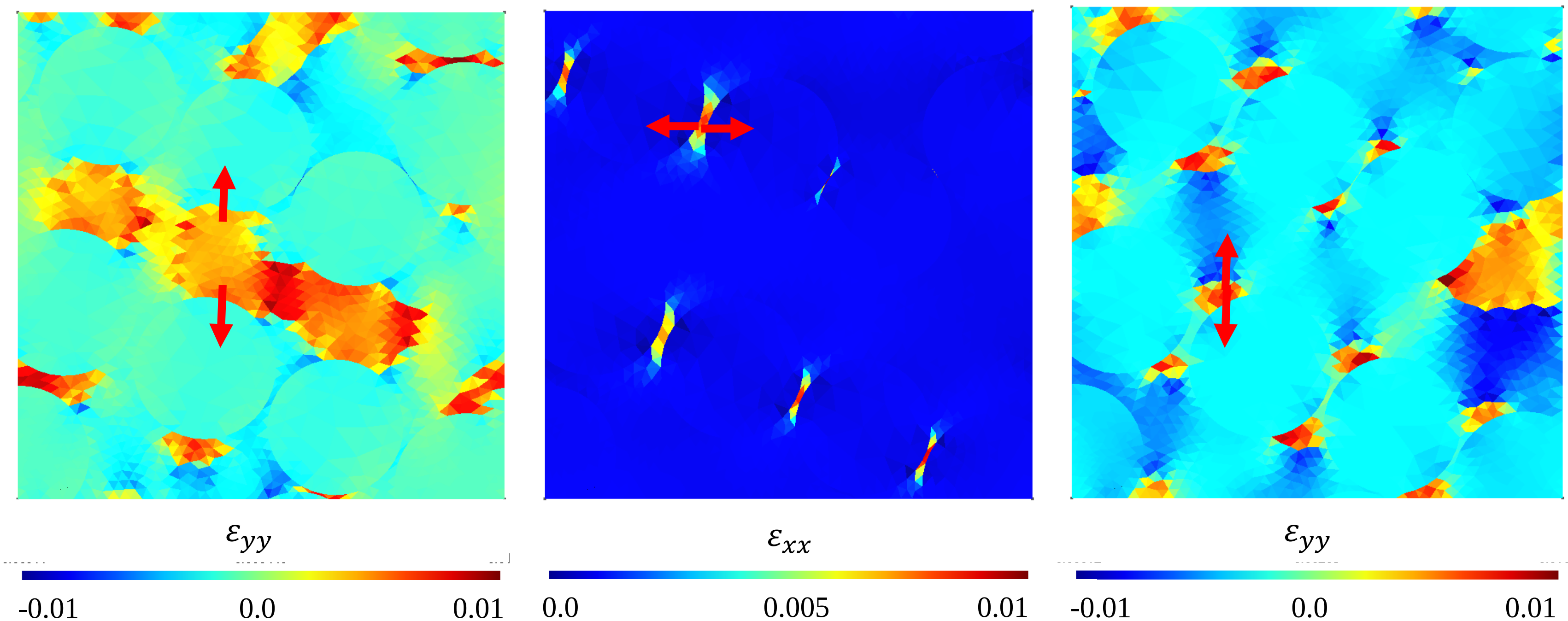
2D

Interaction Tensors

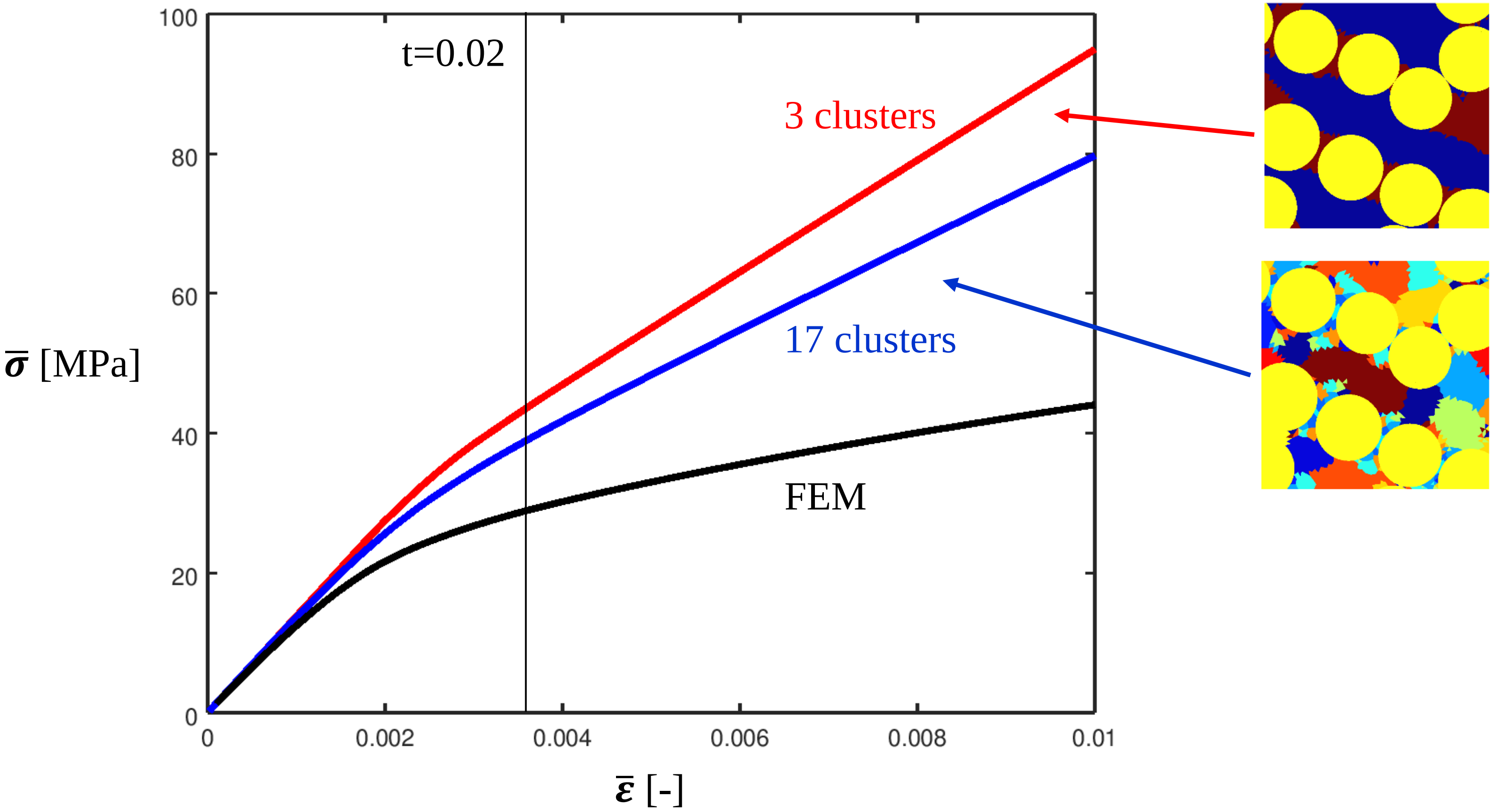


Cluster distribution

- Dilatation of clusters with eigenstrain
- Compression of surroundings

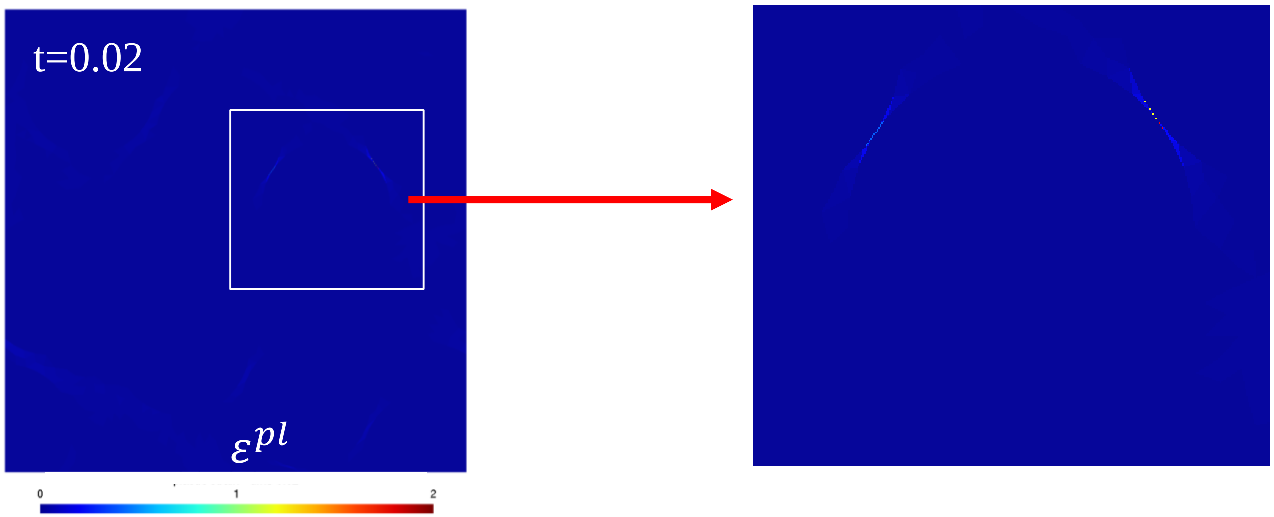


- Results comparison: FEM vs. TFA



Conclusions

- Elasticity: good accuracy
- Plastic deformation starts later
- Highly over-stiff behavior when elastic range is exceeded
- Bad representation of highly localized plastic strain



- Increase number of clusters → increasing computational effort
- Switch to NTFA → extensive pre-simulations and savings required

References

- Transformation Field Analysis:

  George J. Dvorak (1992), Transformation Field Analysis of Inelastic Composite Materials, *Proceedings: Mathematical and Physical Sciences*, 437: 311-327

  Pierre Suquet (1997), Continuum Micromechanics, Springer

- Non-uniform Transformation Field Analysis:

  Jean-Claude Michel and Pierre Suquet (2003), Nonuniform Transformation Field Analysis, *Journal of Solids and Structures*, 40: 6937-6955