Dynamic directional nonparametric profit efficiency analysis for a single decision making unit: an aggregation approach

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Abstract

We propose a simple and intuitive nonparametric technique to assess the profit performances of a single decision making unit over time. The particularity of our approach lies in recognizing that technological change may be present in the profit evaluation exercise. We partition the periods of time into several time intervals, in such a way that the technology is fixed within intervals but may differ between intervals. Attractively, our approach defines a new Luenberger-type indicator for dynamic profit performance evaluation when a single decision making unit is of interest, and provides a coherent and systematic way to compare the profit performance changes between the periods of time and the time intervals. To define the interval-level concepts, we rely on a flexible weighting linear aggregation scheme. We also show how the new indicator can be decomposed into several dimensions. We illustrate the usefulness of our methodology with the case of the Chinese low-end hotel industry in 2005-2015. Our results highlight a performance regression, which is mainly due to the technical components of the indicator decomposition.

Keywords: profit efficiency analysis; directional distance function; Luenberger indicator; aggregation; Chinese hotel industry.

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1 Introduction

Nonparametric efficiency analysis of production activities is a technique used to evaluate the performance of a Decision Making Unit (DMU; such as a firm, plant, utility, bank) by comparing its input-output performance to that of other DMUs operating in a similar technological environment. One particularity of this type of analysis is that it does not require any functional form for the technology. Instead, the production possibility set is reconstructed using the observed inputs and outputs and by imposing standard technology axioms (such as monotonicity, convexity, returns-to-scale). Efficiency, therefore, is measured in technical terms, i.e. as the distance to the reconstructed production possibility set. In practice, the efficiency scores are obtained by solving linear programs where the peers are the other DMUs (e.g. Charnes et al., 1978).

In many contexts, DMUs have a clear economic objective, e.g. firms seek to maximize their profit, plants seek to minimize their cost given the demand, banks seek to maximize their revenue given their assets. Therefore, much effort has been made to propose nonparametric efficiency analysis models taking the economic objective of the DMUs into account. Recent theoretical advancements include those of Ruiz and Sirvent (2012), Boussemart et al. (2015), Cherchye et al. (2016), and Asharafi and Kaleibar (2017) when profit maximization is considered. At this point, it has to be made clear that while, in many contexts, taking the economic objective into account represents a clear advantage, it also comes with the disadvantage of requiring additional data observation. In these cases, indeed, observing the prices is generally required.[1]

When the DMUs are observed for several periods of time, a well-established approach is to rely on an index or an indicator to measure the performance changes over time. Popular indexes and indicators include the Malmquist index (Caves et al., 1982), the Hicks-Moorsteen index (Bjurek, 1996), the Malmquist-Luenberger index (Chung et al., 1997), the Luenberger indicator (Chambers, 2002), and the Luenberger-Hicks-Moorsteen indicator (Briec and Kerstens, 2004). Two main raisons explain the popularity of these indexes and indicators: only input-output data are required, and they can be decomposed into several components. Initially, these indexes and indicators have been defined from a technical perspective, i.e. when ignoring the economic objective of the DMUs. Later, they have been extended in that direction by e.g. Emrouznejad et al. (2011), Tohidi and Razavyan (2013), Juo et al. (2015), and Walheer and Zhang (2018) for profit maximization settings.

In some situations, the research question involves only a single DMU. Examples include the study of a country, a region, a city, or a sector, e.g. Beijing (Li et al., 2013), the Cali-

[1] Note that solutions have been proposed when the prices are not observed. For example, reconstructing the prices using the input-output data (Färe and Zelenyuk, 2003; Zelenyuk, 2006) and relying on shadow prices (Cherchye et al., 2016; Walheer, 2018b).

fornian water industry (Houtsma, 2003), the electric power generation in the United States (Vaninsky, 2006), the public distribution system in India (Ramaswami and Balakrishnan, 2012), and the Saudi electricity sector (Al-Mahish, 2017). Other examples include the study of a firm, plant, or company, e.g. a vertically integrated electricity utility in Malaysia (See and Coelli, 2013, 2014), British Telecom (Florilo, 2003), Chunghwa Telecom Company (Kang, 2009), Malaysia Airlines (See and Azwan, 2016), Telecom Italia (Mancuso, 2012), the Bell system (Sueyoshi 1991; Parker, 1999; Chang and Mashruwal, 2006), and the Hellenic telecommunications organization (Athanassopoulos and Giokas 1998; Laitsou et al., 2017). Another reason to consider a single DMU, besides the research question itself, is the difficulty to find acceptable comparison peers that are operating in a similar technology environment; a core assumption of the nonparametric efficiency analysis approach. For instance, the DMUs considered in these studies present very particular or unique features, such as a monopoly position, a specific market, a special regulatory system, or a specific law system.

In this paper, we propose a new Luenberger-type indicator to study the profit efficiency behaviour of a single DMU over time. We choose a profit maximization setting for three main reasons. First, profit efficiency evaluations are more stringent than cost efficiency evaluations since cost minimization is, by its initial definition, a necessary condition for profit maximization. As a result, profit efficiency evaluations can signal additional potential performance improvements. Second, profit efficiency analysis takes the overall production process (i.e. the input and output sides of the production process) into account, while other economic behaviour ignores one of the two sides. Third, a profit maximization behaviour fits better with our empirical application to the Chinese low-end hotel sector. Nevertheless, it is worth noting that our approach can fairly easily apply to alternative economic objectives.

Our method presents four main distinguished features. First, we allow for technological changes. The common practice when studying a single DMU over time is to use the periods of time as the peers for the evaluation (as in most of the previous cited works). That is, each individual period of time is considered as a DMU in the linear program. By doing so, it is implicitly assumed that the periods of time are homogeneous in terms of technology.[2] Clearly, when several periods of time are considered, this modeling is rather

[2]Introducing the presence of heterogeneity in nonparametric efficiency analysis is not new (Battese et al., 2004; O'Donnell et al., 2008; Walheer, 2018a). Previous works have considered the presence of heterogeneity between DMUs, while we present a methodology to capture heterogeneity in terms of technology between time intervals for a single DMU. The suggested methodology thus shares the willingness of incorporating technology heterogeneity in performance evaluation methods with these existing techniques. At this point, we highlight that it is important to justify the number of time intervals chosen. The common practice, previous works, important events, descriptive statistics, or statistical methods (e.g. cluster analysis) may help at this stage. See Section 3.1 for an illustration. Window analysis, which suggests a window width of three or four time periods because it tends to yield the best balance of informativeness and stability, may be used as an inspiration for selecting the number of time periods. Finally, one should keep in mind that

restrictive. We suggest partitioning the periods of time into several intervals instead. The intervals are defined so that the technology is similar within intervals, but possibly different between intervals. This partitioning naturally allows us to evaluate technological change between intervals. Second, we propose profit evaluation at two different levels: time period and time interval. In particular, we obtain our profit measurements and indicators for the intervals using a flexible weighting linear aggregation scheme. This allows us to define the interval-level profit measurements and indicators in terms of the period-specific counterparts exclusively. Third, our profit measurements and indicators can be decomposed into a technical and allocative counterpart. The decomposition for the interval-level concepts are also obtained using an economic approach. Last, our new methodology establishes a simple and systematic way to compare profit performances between intervals. When comparing a single DMU over time, we often want to compare the performances before and after specific events (such as new investment, privatization, governmental decision, regulatory change etc.), as yet though no systematic way has been proposed to quantify performance differences.

We use our new indicator to study the profit performance changes of the low-end hotel industry in China. The Chinese hotel industry has grown rapidly in recent years to become a significant factor contributing to the country's high economic growth. This importance is well-recognized by Chinese policy-makers as the hotel industry has benefited from recurrent policy interventions. A particularity of the Chinese star-hotel sector is the important gap between low- and high-end hotels. It turns out that the Chinese low-end hotel industry presents very unique features. The results highlight a profit performance regression which is mainly accountable to the technical components of the indicator decomposition.

The rest of the paper unfolds as follows. In Section 2, we present our methodology. In Section 3, we apply our methodology to the case of the low-end hotel sector in China. In Section 4, we present our conclusions.

2 Methodology

We consider that we observe a single Decision Making Unit (DMU) during T periods of time. The particularity of our method is to consider that the T periods of time can be partitioned into N time intervals. Each time interval contains at least two periods, though the time intervals need not be equally long. The extreme case is when $N = 1$ making the profit performance evaluation for $[0, T]$. The time intervals are defined so that the technology is fixed within every time interval, but could be different between time intervals.

differences between time intervals is not systematically due to technology changes. The method suggested in this paper can be used to test the correctness of such claims.

In other words, we allow for technological change between time intervals, but not within time intervals. For each period t in interval $n \in \{1, \ldots, N\}$, the DMU operates at the netputs $\mathbf{z}_t^n$ and prices $\mathbf{p}_t^n$.[3] Therefore, the actual profit at time t in interval n is given by $\mathbf{p}_t^{n'}\mathbf{z}_t^n$.

We start out by defining our notion of profit efficiency measurement, profit efficiency change, profit technological change, and profit Luenberger indicator. Next, we show how these measurements and indicators can be decomposed into a technical and allocative component.

2.1 Profit efficiency

To define our concept of profit efficiency, we first characterize the technology by interval-level production possibility sets. It is defined for interval n as follows:

$$T^n = \{\mathbf{z} \mid \mathbf{z} \text{ is technically feasible in interval } n\}. \tag{1}$$

We assume that those sets fulfill some general regularity conditions to perform a profit efficiency evaluation.[4] Note that these sets are interval-specific and not period-specific; this exactly captures our approach of allowing for potential technological changes between and not within time intervals. Building on the notion of production possibility sets, we define the concept of maximal attainable profit at time t in interval n as follows:

$$\pi_t^n(\mathbf{p}_t^n) = \max_{\mathbf{z} \in T^n} \mathbf{p}_t^{n'}\mathbf{z}. \tag{2}$$

$\pi_t^n(\mathbf{p}_t^n)$ gives the maximum attainable profit given the prices $\mathbf{p}_t^n$ and the technology in interval n (as such the superscript n on $\pi_t^n(\mathbf{p}_t^n)$ refers to the time interval of the technology; and the subscript t refers the evaluated period of time). By construction, maximal profit can only be greater than actual profit: $\pi_t^n(\mathbf{p}_t^n) \geq \mathbf{p}_t^{n'}\mathbf{z}_t$. Profit is at its maximal level for the DMU when $\pi_t^n(\mathbf{p}_t^n) = \mathbf{p}_t^{n'}\mathbf{z}_t$, and profit improvement is possible when $\pi_t^n(\mathbf{p}_t^n) > \mathbf{p}_t^{n'}\mathbf{z}_t$.

Using the previous definition of maximum attainable profit, we define the notion of

[3] Considering a netput representation instead of the more standard input-output representation allows us to considerably reduce our notation. Note that if we denote the inputs by $\mathbf{x}_t^n$ and the outputs by $\mathbf{y}_t^n$, and their respective price by $\mathbf{p}_{x,t}^n$ and $\mathbf{p}_{y,t}^n$, the netputs and their price are defined by: $\mathbf{z}_t^n = \begin{bmatrix} \mathbf{y}_t^n \\ -\mathbf{x}_t^n \end{bmatrix}$ and $\mathbf{p}_t^n = \begin{bmatrix} \mathbf{p}_{y,t}^n \\ \mathbf{p}_{x,t}^n \end{bmatrix}$.

[4] Note that very weak conditions are needed for profit efficiency evaluation (see, for example, Cherchye et al., 2016 for more discussion).

directional profit efficiency measurement for period t in interval n as follows:

$$PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = \frac{\pi_t^n(\mathbf{p}_t^n) - \mathbf{p}_t^{n'} \mathbf{z}_t^n}{\mathbf{p}_t^{n'} \mathbf{g}_{\mathbf{z}_t^n}}. \tag{3}$$

$PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n})$, introduced by Chambers et al. (1998), indicates profit efficiency at time t in interval n in the direction of $\mathbf{g}_{\mathbf{z}_t^n}$ given the prices $\mathbf{p}_t^n$ and the technology in interval n. In practice, the directional vector $\mathbf{g}_{\mathbf{z}_t^n}$ has to be specified by the practitioners in light of the empirical study. An obvious choice is to set $\mathbf{g}_{\mathbf{z}_t^n} = \mathbf{z}_t^n$, i.e. when looking for profit (in)efficiency behaviour in the netput directions. Note that, in that case, (3) coincides with the percentage profit efficiency measurement suggested by Varian (1990); see our empirical application in Section 3. As pointed out previously maximal profit can only be greater than actual profit, implying that $PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) \geq 0$. When $PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = 0$, it means that $\pi_t^n(\mathbf{p}_t^n) = \mathbf{p}_t^{n'} \mathbf{z}_t^n$ reflecting profit efficiency behaviour at time t in interval n. On the contrary, profit inefficient behaviour is captured by $PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) > 0$. In practice, $PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n})$ is obtained by solving a linear program:

$$\begin{aligned} PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = & \min_{\pi_t^n \in \mathbb{R}} \frac{\pi_t^n - \mathbf{p}_t^{n'} \mathbf{z}_t^n}{\mathbf{p}_t^{n'} \mathbf{g}_{\mathbf{z}_t^n}} \\ & \text{s.t.} \quad \pi_t^n \geq \mathbf{p}_t^{n'} \mathbf{z}_s^n, \text{ for every period } s \in n. \end{aligned} \tag{4}$$

In words, the constraint verifies whether more profit is reachable in period t when comparing to the other periods of time in interval n. This also highlights why pooling all periods of time together is probably not a good idea when technological change is present. Indeed, by doing so, we compare periods of time that are not really comparable, and thus create a sort of bias in the profit performance evaluation exercise. By relying on intervals, we remove (or, at least, reduce) that bias issue. We notice that in (4), we do not put any restriction on the returns-to-scale nature of the technology. It turns out that we allow for variable returns-to-scale.

To define our concepts at the interval level, it is crucial to obtain a profit efficiency measurement at that level. To this aim, we rely on a flexible linear weighting aggregation scheme. In particular, let $\omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})$ be the weights associated with period t in interval n. In words, $\omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})$ gives us the relative importance of period t in interval n. We only impose that these weights are between zero and one and sum to unity:

$$\forall t \in n : 0 < \omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) < 1; \text{ and } \sum_{t \in n} \omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) = 1. \tag{5}$$

In that definition, $\mathbf{z}^n$, $\mathbf{p}^n$, and $\mathbf{g}_{\mathbf{z}^n}$ stands for the netputs, prices, and directional vectors for interval n, respectively. The weights can be defined by following a mathematical approach (Liang et al., 2006; Bichou, 2011; Tohidi et al., 2012; Tohidi and Razavyan, 2013) or an economic approach (Färe and Zelenyuk, 2003; Zelenyuk, 2006; Mayer and Zelenyuk, 2014; Walheer, 2018b; Walheer and Zhang, 2018). The main advantage of the mathematical approach is that it does not require any additional assumption about the technology or the behaviour of the DMU, while its disadvantage is its lack of economic intuition. Clearly, the opposite holds true for the economic approach.

Both approaches are popular for empirical works. This is why we choose to define our method in general terms, i.e. without specifying the weights. We leave that choice to the practitioners instead. Refer, for example, to our application for an illustration of our method with the economic approach. This is also why our weights depend on all the netputs, prices, and directional vectors of interval n. This clearly represents the most general case. Note that, in the following, we assume that the weights are observable, i.e. they can be computed independently of the other measurements. For most empirical applications, this is not a strong assumption. Solutions have been proposed when it is not the case by, for example, Färe and Zelenyuk (2003), Cherchye et al. (2016), and Walheer (2018b).

We obtain our profit efficiency measurement for interval n as follows:

$$PE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) = \sum_{t \in n} \omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) \times PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) \tag{6}$$

In words, directional profit efficiency at the interval level is defined as a weighted sum of the period-specific directional profit efficiency measurements, where the weights capture the relative importance of each period of time in the interval. When profit efficiency is observed for every period of time in interval n, we obtain that $PE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) = 0$, since $PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = 0$ for every $t \in n$ in that case. It suffices that one period of time in interval n presents a profit inefficient situation to obtain $PE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) > 0$, i.e. profit inefficiency is observed at the interval level.

Finally, it is worth noticing that other weights can be used as an alternative to those proposed in (6). For example, we may wish to rely on a non-linear aggregation scheme (e.g. Zha and Liang, 2010; Li et al., 2012). It will not impact the definitions of the profit-based concepts. The weights we present offer the advantage to be based on a simple and intuitive approach, and to give the option to decompose the profit concepts into several dimensions (see Section 2.5).

2.2 Profit efficiency change

We can now define the profit efficiency change between two consecutive periods of time (say t and $t+1$) in interval n. It is simply given by taking the difference between two consecutive profit efficiency measurements:

$$PEC^n_{t,t+1}(\mathbf{z}^n_t, \mathbf{z}^n_{t+1}, \mathbf{p}^n_t, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_t}, \mathbf{g}_{\mathbf{z}^n_{t+1}}) = PE^n_t(\mathbf{z}^n_t, \mathbf{p}^n_t, \mathbf{g}_{\mathbf{z}^n_t}) - PE^n_{t+1}(\mathbf{z}^n_{t+1}, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_{t+1}}). \quad (7)$$

We obtain that $PEC^n_{t,t+1}(\mathbf{z}^n_t, \mathbf{z}^n_{t+1}, \mathbf{p}^n_t, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_t}, \mathbf{g}_{\mathbf{z}^n_{t+1}}) > 0$ when there is a profit efficiency improvement between t and $t+1$ in interval n since, in that case, $PE^n_t(\mathbf{z}^n_t, \mathbf{p}^n_t, \mathbf{g}_{\mathbf{z}^n_t}) > PE^n_{t+1}(\mathbf{z}^n_{t+1}, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_{t+1}})$. On the contrary, $PEC^n_{t,t+1}(\mathbf{z}^n_t, \mathbf{z}^n_{t+1}, \mathbf{p}^n_t, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_t}, \mathbf{g}_{\mathbf{z}^n_{t+1}}) < 0$ implies a profit performance regression since, in that case, $PE^n_t(\mathbf{z}^n_t, \mathbf{p}^n_t, \mathbf{g}_{\mathbf{z}^n_t}) < PE^n_{t+1}(\mathbf{z}^n_{t+1}, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_{t+1}})$. Finally, $PEC^n_{t,t+1}(\mathbf{z}^n_t, \mathbf{z}^n_{t+1}, \mathbf{p}^n_t, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_t}, \mathbf{g}_{\mathbf{z}^n_{t+1}}) = 0$ implies that profit efficiency behaviour is constant between t and $t+1$ in interval n. To practically evaluate $PEC^n_{t,t+1}(\mathbf{z}^n_t, \mathbf{z}^n_{t+1}, \mathbf{p}^n_t, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_t}, \mathbf{g}_{\mathbf{z}^n_{t+1}})$, it is needed to compute profit efficiency at time t and $t+1$. To obtain the former, it suffices to use the linear program given (4). For the latter, we can also use the same linear program when replacing t by $t+1$.

Besides profit efficiency change within every interval, it is also important to propose profit efficiency change measurement between consecutive intervals. The profit efficiency change between two consecutive intervals (say n and $n+1$) is given as follows:

$$PEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) = PE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - PE^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}). \quad (8)$$

$PEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ has to be interpreted in a similar manner as $PEC^n_{t,t+1}(\mathbf{z}^n_t, \mathbf{z}^n_{t+1}, \mathbf{p}^n_t, \mathbf{p}^n_{t+1}, \mathbf{g}_{\mathbf{z}^n_t}, \mathbf{g}_{\mathbf{z}^n_{t+1}})$, but for intervals n and $n+1$ instead of periods t and $t+1$ in interval n. A value greater (smaller) than zero implies profit improvement (regression). Again a value of zero implies the status quo.

In practice, to compute the profit efficiency change effects at the interval level, we have to compute two interval-specific profit efficiency measurements: $PE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})$ and $PE^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})$. Given the definition of these measurements in (6), it suffices to obtain all profit efficiency measurements in the intervals n and $n+1$ using the linear program in (4). In practice, n has to be replaced by $n+1$ when computing $PE^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})$. We recall that the weights are exogenous as given by the data.

2.3 Profit technological change

Not only is profit efficiency change important, it also matters to know how the technology changes. In practice, technological change is found when evaluating counterfactual profit efficiency measurements, i.e. profit efficiency measurements when the netputs are fixed for the specific interval and the technology for another interval. Two ways can be considered to define the profit technological change effect, when interval n or $n+1$ is considered for the netputs of the profit evaluation:

$$PTC^{n,n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}^n}) = PE^{n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}^n}) - PE^{n}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}^n}). \tag{9}$$

$$PTC^{n,n+1}(\mathbf{z}^{n+1},\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^{n+1}}) = PE^{n+1}(\mathbf{z}^{n+1},\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^{n+1}}) - PE^{n}(\mathbf{z}^{n+1},\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^{n+1}}). \tag{10}$$

The only difference between $PTC^{n,n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}_t^n})$ and $PTC^{n,n+1}(\mathbf{z}^{n+1},\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^{n+1}})$ is the interval chosen for the profit evaluation. When $PTC^{n,n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}_t^n}) > 0$, it implies that more profit inefficient behaviour for the netputs at interval n is detected when the technology is fixed at $n+1$ than when it is fixed at n. It implies that we observe a technological progress in profit terms. Conversely, when less profit inefficiency behaviour is observed when the technology is fixed at $n+1$ than for n, it implies that we have a technological regression. In that case $PTC^{n,n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}_t^n}) < 0$. Finally, the technological status quo in profit terms is captured by $PTC^{n,n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}_t^n}) = 0$. $PTC^{n,n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}_t^n}) > 0$ has to be interpreted in an analogous manner except that the netputs are those of interval $n+1$.

To avoid choosing between the two intervals, a commonly agreed procedure is to define the profit technological change as the arithmetic averages of the two previous (netput-dependent) components (see our discussion of (13) for more detail on the arithmetic average):

$$PTC^{n,n+1}(\mathbf{z}^n,\mathbf{z}^{n+1},\mathbf{p}^n,\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^n},\mathbf{g}_{\mathbf{z}^{n+1}}) = \frac{1}{2}\left[PTC^{n,n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}^n}) + PTC^{n,n+1}(\mathbf{z}^{n+1},\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^{n+1}})\right]. \tag{11}$$

It turns out that $PTC^{n,n+1}(\mathbf{z}^n,\mathbf{z}^{n+1},\mathbf{p}^n,\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^n},\mathbf{g}_{\mathbf{z}^{n+1}})$ measures the profit technological change irrespective of the interval chosen for the netputs. A value greater (smaller) than zero implies a technological progression (regression) in profit terms. A value of zero captures the benchmark value for the indicator.

In practice, to compute the profit technological change effects, we have to compute four interval-specific profit efficiency measurements. $PE^{n}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}^n})$ and $PE^{n+1}(\mathbf{z}^{n+1},\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^{n+1}})$ are obtained, as explained before, by solving the linear program (4) for all periods in the interval n and $n+1$, respectively. Clearly, the definition in (6) also holds true for the two counterfactual interval-level profit efficiency measurements $PE^{n+1}(\mathbf{z}^n,\mathbf{p}^n,\mathbf{g}_{\mathbf{z}^n})$ and $PE^{n}(\mathbf{z}^{n+1},\mathbf{p}^{n+1},\mathbf{g}_{\mathbf{z}^{n+1}})$. It turns out that it suffices to compute the counterfactual profit efficiency measurements for every period in the intervals to obtain the counterfactual interval-level profit efficiency measurements (the weights are again given by the data). Attractively, we can adapt the linear program in (4) to compute these measurements. In particular, we obtain the following linear program for $PE^{n+1}(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n})$:

$$PE_t^{n+1}(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = \min_{\pi_t^{n+1}\in\mathbb{R}} \frac{\pi_t^{n+1} - \mathbf{p}_t^{n\prime}\mathbf{z}_t^n}{\mathbf{p}_t^{n\prime}\mathbf{g}_{\mathbf{z}_t^n}}$$
$$\text{s.t.} \quad \pi_t^{n+1} \geq \mathbf{p}_t^{n\prime}\mathbf{z}_s^n, \text{ for every period } s \in n+1. \tag{12}$$

This linear program is very similar to (4). In fact, the only difference is that the comparison peers are the periods of time of interval $n+1$. This naturally captures the idea of counterfactual profit efficiency measurements. $PE^n(\mathbf{z}_t^{n+1}, \mathbf{p}_t^{n+1}, \mathbf{g}_{\mathbf{z}_t^{n+1}})$ can be obtained, for all t in interval $n+1$, by the same linear program. It suffices to interchange n and $n+1$ in (12).

2.4 Profit Luenberger indicator

Attractively, when summing our notions of profit efficiency change $PEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ and profit technological change $PTC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$, we obtain a profit Luenberger-type indicator:

$$\begin{aligned}
&PEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) + PTC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) = \\
&= \frac{1}{2}\left\{\left[PE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - PE^n(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})\right] + \left[PE^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - PE^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})\right]\right\}, \\
&= \frac{1}{2}\left\{\left[PLI_n^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})\right] + \left[PLI_{n+1}^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})\right]\right\}, \\
&= PLI^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}).
\end{aligned} \tag{13}$$

The Luenberger indicator, introduced by Chambers (2002) in the production context after Luenberger (1992), is defined as the difference of profit efficiency measurements. To avoid a dependence of the chosen technology, it is commonly agreed to rely on a simple arithmetic average. In our context, it means that the Luenberger indicator is defined as the arithmetic average of two indicators when n and $n+1$ are chosen for the technology, denoted by $PLI_n^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ and $PLI_{n+1}^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$, respectively. A value greater (smaller) than zero implies an improvement (decline) of the profit performances. A value of zero implies no profit performance change, and thus captures the benchmark situation. Contrary to the two technology-dependent Luenberger indicators, $PLI^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ provides a profit performance indicator irrespective of the chosen interval for the technology.

2.5 Decomposition

Since the classical paper of Farrell (1957), it is well-known that structural efficiency, generally, can be decomposed into technical and allocative counterparts. The technical counterpart ignores the economic objective of the DMU, but rather considers the input-output combinations (and is thus independent of the prices). The allocative counterpart is interpreted as the residual inefficiency behaviour, i.e. when removing the technical inefficiency behaviour from the structural inefficiency behaviour (and is thus dependent of the prices). In other words, allocative inefficiency is defined as inefficiency due to non-optimal allocation of the netputs given their price.

2.5.1 Profit efficiency

In our directional profit efficiency context, we can decompose $PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n})$ into two parts as follows:

$$PE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = \vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) + AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}), \tag{14}$$

where $\vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = \max\left\{\beta \mid \left(\mathbf{z}_t^n + \beta \cdot \mathbf{g}_{\mathbf{z}_t^n}\right) \in T^n\right\}$ is the directional distance function, introduced by Chambers et al. (1998), adapted to our context. Attractively, as for our profit efficiency measurement, the directional distance function can be obtained by solving a linear program:

$$\begin{aligned}
\vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = & \max_{\lambda_s (s\in n)} \beta \\
\text{s.t.} \quad & \sum_{s\in n} \lambda_s \mathbf{z}_s^n \geq \mathbf{z}_t^n + \beta \cdot \mathbf{g}_{\mathbf{z}_t^n} \\
& \sum_{s\in n} \lambda_s = 1 \\
& \forall s \in n : \lambda_s \geq 0 \\
& \beta \geq 0.
\end{aligned} \tag{15}$$

The first constraint verifies whether the netputs can be simultaneously reduced in the direction of $\mathbf{g}_{\mathbf{z}_t^n}$. The second constraint is necessary to ensure that variable returns-to-scale is satisfied. The last two constraints are needed to be sure that the variables have the correct signs when solving the linear program. The directional distance function has an analogous interpretation to our profit efficiency measurement (this is also intuitive when looking at their connection in (14)). That is $\vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = 0$ stands for a technical efficient behavior, while greater values imply more technical inefficiency behaviour. The allocative efficiency measurement is therefore obtained a posteriori once both the profit efficiency measurement and the directional distance function have been computed (using (4) and (15), respectively). It highlights the residual nature of the allocative efficiency measurement in (14). It turns out that $AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = 0$ implies that no allocative inefficiency behaviour is present, while $AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) > 0$ reflects the presence of allocative inefficiency behaviour.

We can obtain a similar decomposition at the interval-level. Combining our definition of profit efficiency measurement in (6) and the decomposition in (14), we obtain:

$$\begin{aligned} PE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) &= \sum_{t \in n} \omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) \times \left(\vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) + AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) \right), \\ &= \left(\sum_{t \in n} \omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) \times \vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) \right) + \left(\sum_{t \in n} \omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) \times AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) \right), \\ &= \vec{D}^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + AE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}). \end{aligned} \tag{16}$$

Profit efficiency measurement for interval n can be decomposed into two parts, where each component is exclusively defined in terms of period-specific measurements. Interestingly, the weights for the two components are the same as the weights found previously for the profit efficiency measurement (see our discussion of (6)). The first component $\vec{D}^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})$ is equal to zero only when the directional distance functions are also equal to zero for every time period in interval n. We notice that $\vec{D}^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})$, contrary to $\vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n})$ depends on the prices. This comes from the profit maximization behaviour assumed for the DMU (see Färe and Zelenyuk, 2003 and Walheer 2018a for related discussion). When more technical inefficiency behaviour is observed for the time periods, i.e. when $\vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n})$ is further from zero for at least one t, $\vec{D}^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) > 0$. It turns out that a greater value implies more technical inefficiency behaviour for the interval. A similar reasoning holds true for $AE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})$: $AE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) = 0$ when allocation efficiency is observed for all periods of time in interval n (i.e. $AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = 0$, $\forall t \in n$), while $AE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) > 0$ when allocation inefficiency is observed for at least one period of time in interval n (i.e. $AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) > 0$, for at least one $t \in n$).

2.5.2 Profit efficiency change

Using the decomposition of the profit efficiency measurement for period t in (14), we can obtain a similar decomposition for the profit efficiency change between t and $t+1$ in interval

n as follows:

$$
\begin{aligned}
&PEC_{t,t+1}^{n}(\mathbf{z}_t^n, \mathbf{z}_{t+1}^n, \mathbf{p}_t^n, \mathbf{p}_{t+1}^n, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}_{t+1}^n}) = \\
&= \left(\vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) + AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n})\right) - \left(\vec{D}_{t+1}^n(\mathbf{z}_{t+1}^n, \mathbf{g}_{\mathbf{z}_{t+1}^n}) + AE_{t+1}^n(\mathbf{z}_{t+1}^n, \mathbf{p}_{t+1}^n, \mathbf{g}_{\mathbf{z}_{t+1}^n})\right), \\
&= \left(\vec{D}_t^n(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) - \vec{D}_{t+1}^n(\mathbf{z}_{t+1}^n, \mathbf{g}_{\mathbf{z}_{t+1}^n})\right) + \left(AE_t^n(\mathbf{z}_t^n, \mathbf{p}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) - AE_{t+1}^n(\mathbf{z}_{t+1}^n, \mathbf{p}_{t+1}^n, \mathbf{g}_{\mathbf{z}_{t+1}^n})\right), \\
&= TEC_{t,t+1}^n(\mathbf{z}_{t+1}^n, \mathbf{z}_{t+1}^n, \mathbf{g}_{\mathbf{z}_{t+1}^n}, \mathbf{g}_{\mathbf{z}_{t+1}^n}) + AEC_{t,t+1}^n(\mathbf{z}_t^n, \mathbf{z}_{t+1}^n, \mathbf{p}_t^n, \mathbf{p}_{t+1}^n, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}_{t+1}^n}). \qquad (17)
\end{aligned}
$$

$TEC_{t,t+1}^n(\mathbf{z}_{t+1}^n, \mathbf{z}_{t+1}^n, \mathbf{g}_{\mathbf{z}_{t+1}^n}, \mathbf{g}_{\mathbf{z}_{t+1}^n})$ and $AEC_{t,t+1}^n(\mathbf{z}_t^n, \mathbf{z}_{t+1}^n, \mathbf{p}_t^n, \mathbf{p}_{t+1}^n, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}_{t+1}^n})$ have to be interpreted as $PEC_{t,t+1}^n(\mathbf{z}_t^n, \mathbf{z}_{t+1}^n, \mathbf{p}_t^n, \mathbf{p}_{t+1}^n, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}_{t+1}^n})$ but in technical and allocative terms, respectively. Thus, a value smaller (greater) than one implies a efficiency regression (progression), while zero is the benchmark value. Note that to compute the technical component, it suffices to replace t by $t+1$ in (15). The allocative component is again obtained a posteriori by its definition in (17).

The decomposition of the profit efficiency measurement at the interval-level in (16) allows us to obtain a similar decomposition for the profit efficiency change between intervals n and $n+1$:

$$
\begin{aligned}
&PEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) = \\
&= \left(\vec{D}^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + AE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})\right) - \left(\vec{D}^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) + AE^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})\right), \\
&= \left(\vec{D}^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - \vec{D}^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})\right) + \left(AE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - AE^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})\right), \\
&= TEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) + AEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}). \qquad (18)
\end{aligned}
$$

Again, $TEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ and $AEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ have to be interpreted as $PEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ but in technical and allocative terms, respectively. As for the profit efficiency change, it suffices to obtain all directional distance functions in the intervals n and $n+1$ using the linear program in (15) to obtain the interval-level technical efficiency change component. The allocative counterpart is obtained a posteriori using (18).

2.5.3 Profit technological change

Building on the decomposition for the profit efficiency change, we can apply a similar procedure for the profit technological change. In particular, when the netputs are those of interval n, we obtain the following decomposition:

$$\begin{aligned}
&PTC^{n,n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) = \\
&= \left(\vec{D}^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + AE^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})\right) - \left(\vec{D}^{n}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + AE^{n}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})\right), \\
&= \left(\vec{D}^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - \vec{D}^{n}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})\right) + \left(AE^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - AE^{n}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n})\right), \\
&= TTC^{n,n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + ATC^{n,n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}).
\end{aligned} \tag{19}$$

$TTC^{n,n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}_t^n})$ and $ATC^{n,n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}_t^n})$ are defined in an analogous manner to $PTC^{n,n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}_t^n})$: a value of zero represents the technological stagnation between n and $n+1$, while greater (smaller) values imply a technological progress (regress). The main difference between these three components is that they are defined in profit, technical, and allocative terms. Note that these components are dependent on the netputs chosen for the profit evaluation.

In a similar vein, when the netputs are those of interval $n+1$, we can obtain the following decomposition:

$$PTC^{n,n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) = TTC^{n,n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) + ATC^{n,n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}). \tag{20}$$

Let us define the technological changes in technical and allocative terms as follows:

$$TTC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) = \frac{1}{2}\left[TTC^{n,n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + TTC^{n,n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})\right]. \tag{21}$$

$$ATC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) = \frac{1}{2}\left[ATC^{n,n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + ATC^{n,n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}})\right]. \tag{22}$$

As $PTC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$, these two components capture the technological change irrespective of the netputs chosen for the profit efficiency evaluation. Combining equation (11) with equations (19) to (22), we obtain the desired decomposition of the profit technological change between intervals n and $n+1$:

$$\begin{aligned}
PTC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) =& TTC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) \\
&+ ATC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}).
\end{aligned} \tag{23}$$

We observe that to practically measure the technical technological changes, it is necessary to compute counterfactual direction distance functions (see also our discussion of (11)).

As done previously for the profit component, we can adapt the linear program in (15) to obtain the counterfactual concepts. In particular, $\vec{D}_t^{n+1}(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n})$ is obtained as follows:

$$\begin{aligned}
\vec{D}_t^{n+1}(\mathbf{z}_t^n, \mathbf{g}_{\mathbf{z}_t^n}) = & \max_{\lambda_s (s \in n+1)} \beta \\
\text{s.t.} \quad & \sum_{s \in n+1} \lambda_s \mathbf{z}_s^{n+1} \geq \mathbf{z}_t^n + \beta \cdot \mathbf{g}_{\mathbf{z}_t^n} \\
& \sum_{s \in n+1} \lambda_s = 1 \\
& \forall s \in n+1 : \lambda_s \geq 0 \\
& \beta \geq 0.
\end{aligned} \tag{24}$$

It suffices to interchange n and $n+1$ in the linear program to obtain the other counterfactual directional distance function $\vec{D}_t^n(\mathbf{z}_t^{n+1}, \mathbf{g}_{\mathbf{z}_t^{n+1}})$. The interval-level concepts are obtained after computing the weights (using only the data) as explained in (6). Finally, all allocative components are found a posteriori using their respective definition in (19) to (23).

2.5.4 Profit Luenberger indicator

Finally, the profit Luenberger indicator can also be decomposed into two parts by replacing the decomposition of the profit efficiency measurements at the interval level in the definition in (13):

$$\begin{aligned}
& PLI^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) = \\
& = \frac{1}{2} \Big\{ \Big[\Big(\vec{D}^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + AE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) \Big) - \Big(\vec{D}^n(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) + AE^n(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) \Big) \Big] \\
& + \Big[\Big(\vec{D}^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) + AE^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) \Big) - \Big(\vec{D}^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) + AE^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) \Big) \Big] \Big\}, \\
& = \frac{1}{2} \Big\{ \Big[\vec{D}^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - \vec{D}^n(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) \Big] + \Big[\vec{D}^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - \vec{D}^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) \Big] \Big\} \\
& + \frac{1}{2} \left\{ \left[AE^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - AE^n(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) \right] + \left[AE^{n+1}(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) - AE^{n+1}(\mathbf{z}^{n+1}, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^{n+1}}) \right] \right\}, \\
& = \frac{1}{2} \Big\{ \left[TLI_n^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) \right] + \Big[TLI_{n+1}^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) \Big] \Big\} \\
& + \frac{1}{2} \Big\{ \left[ALI_n^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) \right] + \Big[ALI_{n+1}^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) \Big] \Big\}, \\
& = TLI^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) + ALI^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}).
\end{aligned} \tag{25}$$

In that decomposition, $TLI^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ is the technical Luenberger indicator irrespective of the chosen interval for the technology. It is greater than zero when there is a technical performance improvement, smaller than zero when there is a technical performance regression, and equal to zero when it is the technical status quo. $TLI_n^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ and $TLI_{n+1}^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}})$ measure exactly the same thing, but when interval n and $n+1$ are chosen for the technology. A similar interpretation holds true for the allocative Luenberger indicators.

As a final remark, we highlight that, by construction, the technical and allocative Luenberger indicators can be decomposed into efficiency and technological change parts as follows:

$$\begin{aligned} &TLI^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}_t^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) = \\ &= TEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) + TTC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}). \end{aligned} \tag{26}$$

$$\begin{aligned} &ALI^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) = \\ &= AEC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}) + ATC^{n,n+1}(\mathbf{z}^n, \mathbf{z}^{n+1}, \mathbf{p}^n, \mathbf{p}^{n+1}, \mathbf{g}_{\mathbf{z}^n}, \mathbf{g}_{\mathbf{z}^{n+1}}). \end{aligned} \tag{27}$$

These last two equations reveal the consistency of our approach and its practical usefulness, as also demonstrated in the next Section with the case of the Chinese low-end industry.

3 Application

In China, the hotel industry has grown rapidly in recent years to become a significant factor contributing to the country's high economic growth. To give some key figures: in 2016, the total income of the hotel industry reached 893.81 billion RMB, while the income of the tourist attractions and the travel agencies were about 39.15 and 463.34 billion RMB respectively (National Bureau of Statistics of China); the contribution of the hotel industry to GDP has continually increased over the last 10 years (Ministry of Culture and Tourism of China); and China is ranked in the fourth position globally regarding both international tourism arrivals and receipts in 2016 (World Tourism Organization, 2017). This growing importance is also well-recognized by scholars in light of the increasing interest to study the performance of that industry (Huang et al., 2012; Zhang and Cheng, 2014; Sun et al., 2015; Yang and Cai, 2016; Su and Sun, 2017; Yang et al., 2017; Walheer and Zhang, 2018; see Gross et al., 2013 and Law et al., 2014 for a recent literature review). While these papers use different methodologies to the one suggested here, they all highlight the benefits of using nonparametric efficiency analysis to study performances of the Chinese hotel sector.

This importance is also well-recognized by Chinese policy-makers as the hotel sector has benefited from recurrent policy interventions. They recognize that a strong hotel industry is crucial to developing a strong tourism industry. In particular, Chinese policy-makers have emphasized the importance of transforming and upgrading the tourism industry, while focusing on the quality and efficiency improvement of the tourism components. Recently, for instance, the concept of 'all-for-one tourism' has been suggested.[5] This concept refers to both the promotion of China as a tourism destination and to tourism as an advantageous industry to develop economic growth in China. In particular, 'all-for-one tourism' contains the modernization, intensification, quality improvement and internationalization of the Chinese tourism industry.

A particularity of the Chinese star-hotel sectors is the important gap between low- (below three stars) and high-end (four and five star) hotels. For instance, there are important differences in terms of revenue and investment between high-end and low-end hotels. For example, the average revenue-total asset ratio was 41.92% for high-end hotels, while it was 50.17% for low-end hotels during 2007-2016; and the number of high-end hotels increased from 1,424 to 3,164, while it decreased from 10,404 to 7,386 for low-end hotels during the same period. Also, high-end hotels have attracted the major part of foreign investment (Su and Sun, 2017). For example, in 2015, there were 261 foreign investments in high-end hotels, while this number falls to 122 for low-end hotels (China Star-Rated Hotel Report). This gap, in a sense, is encouraged by policy-makers since most of the policy implementations have occurred for high-end hotels (Qu et al., 2014; Liu et al., 2015; Zhang and Gao, 2017).

It turns out that the Chinese low-end hotel industry presents very unique features. That is, comparing their performance to the Chinese high-end hotel industry or to the low-end hotel industry in other countries does not seem to be an accurate decision (the technology homogeneity assumption may not hold). We instead propose to use our new methodology to assess the profit performance changes of the low-end hotel industry in 2005-2015. We are particularly interested to capture the performance change between two intervals: 2005-2009 and 2010-2015. As discussed below, we suspect potential technological change between these two intervals. We first present our data and some key descriptive statistics, and discuss the results next.

3.1 Variable selection, data and descriptive statistics

We select three inputs: the number of employees, the number of rooms, and the total fixed assets (Barros et al., 2011; Assaf and Agbola 2011; Yang et al., 2017; Walheer and Zhang, 2018). An important aspect of the hotels is the quality of the services provided (Arbelo-

[5] Document "Guiding Opinions on Promoting Development of all-for-one tourism" by the General Office of the State Council, March 2018.

Perez et al., 2017). As such, we make use of the total revenue as our single output.[6] Indeed, greater quantity results in higher revenue, while greater quality implies higher price and thus higher revenue. We thus use an indirect measure of quality since no direct measure is available. Note that several recent studies have highlighted the importance of taking both the quantity and the quality into consideration when conducting an efficiency analysis (Fukuyama and Weber, 2008; Sahoo et al., 2014; Cherchye et al., 2016; Walheer and Zhang, 2018). We make use of two databases to obtain our data for the inputs, outputs, and their respective price: the Wind Database and the Supplement of China Tourism Statistic Year Books (note that the wages are taken from the China Statistic Bureau). We finish with a sample of 31 provinces and a period spanning from 2005 to 2015.

To contextualize our study and highlight the relative importance of the low-end hotels in China, we first present the total and the low-end hotel share for our inputs and outputs in Table 1. We also present the total number of hotels in that Table. Note that the total fixed assets and the total revenue are given in 1,000 RMB.

Several main lessons arise from that Table. Let us start with the figures for the total inputs and outputs. First, while the number of hotels are decreasing over the period, the number of rooms are increasing, showing that larger hotels have been built. Next, the total fixed assets have significantly increased over the period, showing the new investments made in the Chinese star-hotel sector, while the number of employees has decreased, showing a rationalization of the labour input. Finally, the total revenue has gone up over the period, but not in 2013-2015. For the shares of the low-end hotels, an initial observation is the decreasing importance of this type of hotel over the period. Indeed, all shares have gone down between 2005 and 2015. In particular, this type of hotel represents less than 50% of the number of rooms in 2015, while this share in 2005 was almost 75%. Nevertheless, 70% of the hotels are low-end hotels in 2015. The decreasing importance of this type of hotels is also clear when looking at the share for the total assets (i.e. investment) and total revenue (i.e. profitability). These shares move from around 45% to 30% and 47.5% to 30% respectively. We notice that low-end hotels employ slightly less than 45 % of the employees as compared with almost 65% in 2005.

The Table also shows the critical disappearance of 2,500 hotels in 2010, i.e. a decrease of around 180,000 rooms in turn implying a significant fall in labour input of around 80,000 persons. The total fixed asset and the total revenue are not impacted on a contemporaneous basis, probably because of their long-run nature. The main reason for this drop has to

[6] Let us illustrate the concept of netput using our application. Let the number of employees, the number of rooms, and total fixed assets be denoted by x_1, x_2, and x_3, respectively; the total revenue by y. The netput vector is thus giving by $\mathbf{z} = \begin{bmatrix} y \\ -x_1 \\ -x_2 \\ -x_3 \end{bmatrix}$.

Table 1: Descriptive statistics

Year	Number of hotels	Inputs			Output
		Number of rooms	Total Fixed Assets	Employed Persons	Total Revenue
Total					
2005	11,828	1,276,210	36,789,500	1,461,887	13,188,456
2006	12,751	1,400,230	38,752,158	1,520,108	14,546,974
2007	13,583	1,510,786	42,136,136	1,613,687	16,152,097
2008	14,099	1,530,396	42,677,570	1,620,445	17,302,135
2009	14,237	1,607,357	43,448,832	1,622,251	17,783,605
2010	11,779	1,426,736	44,629,053	1,541,941	20,896,719
2011	11,676	1,428,769	44,992,723	1,503,840	22,769,678
2012	11,367	1,447,505	46,498,930	1,547,259	23,861,068
2013	11,687	1,488,236	49,105,707	1,464,794	22,487,660
2014	11,180	1,447,849	48,972,479	1,327,943	21,111,604
2015	10,550	1,413,469	53,294,444	1,307,787	20,660,241
Low-end hotel share (%)					
2005	87.94	73.10	45.49	64.41	47.62
2006	86.90	71.38	43.23	63.96	46.14
2007	85.54	68.98	41.00	58.78	44.37
2008	84.02	66.07	38.47	55.65	42.53
2009	82.51	64.65	35.95	54.92	43.52
2010	78.38	58.33	32.70	48.92	35.85
2011	76.34	55.91	29.93	46.51	34.06
2012	75.14	53.40	28.61	46.46	33.92
2013	73.47	52.55	27.27	42.75	32.68
2014	72.11	50.88	25.82	42.46	30.10
2015	70.01	49.18	29.22	42.71	29.22

be found in the low-end hotel industry. Indeed, in 2010, most of the shares decreased importantly for this hotel type. In particular, the total revenue decreased by almost 8 percentage points, the number of hotels by more than 4 percentage points, the number of rooms by more than 6 percentage points, and the number of employees by 6 percentage points. Various reasons may explain this turning point: the sensibility to the economic crisis (Yang and Cai, 2016; Walheer and Zhang, 2018), the lack of returns of new investments (Yang et al., 2017; Walheer and Zhang, 2018), and the new policy implementations in favour of high-end hotels (Qu et al., 2014; Zhang and Gao, 2017). Interestingly, the shares for the low-end hotels continued to decrease after 2010, but more consistently. In a sense, we may conclude that the 2005-2015 period can be partitioned into two intervals: 2005-2009 and 2010-2015. At this point, we observe that we test for other partitions of the time periods. In light of the obtained results (see, in particular, the technological changes in Tables 3, 5 and 7), this partitioning seems reasonable. One must bear in mind that the intervals are defined so that the technology is constant within but not between intervals.

3.2 Results

We present in this Section our results when considering two intervals 2005-2009 and 2010-2015. Our method also requires choosing directional distance vectors and weights. It seems reasonable to take both the inputs and the outputs into consideration in the case of the Chinese hotel sector. Indeed, we may argue that hotels have the power to choose both the input and output levels.[7] That is, we are investigating for the presence of profit (in)efficiency behaviour in the netput direction. Next, we follow an economic approach by defining the weights as the actual profit shares.[8] This gives a natural way to take the relative importance of the periods of time when defining the interval-level concepts. Note that these weights are also coherent with our choice for the directional vectors.

We start by presenting our results when considering the profit maximization behaviour of the hotels. Next, we decompose the profit efficiency measurements and indicators into a technical and allocative counterpart.

3.2.1 Profit efficiency

We start by presenting our results for the profit efficiency changes at both the period- and interval-levels. As explained in detail in Section 2.1, the first step is to compute the profit efficiency measurements for every period of time (second column in Table 2). These

[7]Formally, we obtain that $\mathbf{g}_{\mathbf{z}_t^n} = \mathbf{z}_t^n$ for period t in interval n. Clearly, other exogenous and endogenous ways are possible to define the directional vectors (Hampf and Kruger, 2014; Atkinson and Tsionas, 2016; and Färe et al., 2017).

[8]Formally, the weights are defined, in our case, as follows: $\omega_t^n(\mathbf{z}^n, \mathbf{p}^n, \mathbf{g}_{\mathbf{z}^n}) = \frac{\mathbf{p}_t^{n\prime}\mathbf{z}_t^n}{\sum_{t\in n}\mathbf{p}_t^{n\prime}\mathbf{z}_t^n}$.

measurements equal zero when profit efficiency is detected, while greater values imply more profit inefficiency. For the first interval 2005-2009, profit efficiency is detected in 2005, 2006, and 2007. The profit inefficiency found for the period 2008-2009 may be attributed to contemporaneous consequence of the economic crisis (Yang and Cai, 2016; Walheer and Zhang, 2018). For the second interval 2010-2005, we find profit inefficiency for all periods of time, probably due to lack of returns of new investments (Yang et al., 2017; Walheer and Zhang, 2018), and new policy implementations in favour of high-end hotels (Qu et al., 2014; Zhang and Gao, 2017).

The second step is to compute the interval-level profit efficiency measurements (see Section 2.1). They are obtained as weighted sums of the period-specific profit efficiency measurements. In light of our empirical application, we make used of the actual profit shares for the weights. The weights are all included in the intervals $14\% - 22\%$, meaning that all periods have comparable relative importance when computing the interval-specific profit efficiency measurements. The interval-level measurements tell us that more profit inefficiency behaviour is detected for the second interval (see Section 2.2). There is a regression of 0.0206.

Table 2: Profit efficiency change

Year	Profit efficiency	Profit efficiency change	Weight (%)
Analysis per year			
2005	0.0000	-	17.81
2006	0.0000	0.0000	19.04
2007	0.0000	0.0000	20.33
2008	0.0307	-0.0307	20.87
2009	0.0029	0.0278	21.95
2010	0.0000	-	17.39
2011	0.0418	-0.0418	18.00
2012	0.0492	-0.0074	18.79
2013	0.0641	-0.0149	17.06
2014	0.0000	0.0641	14.75
2015	0.0000	0.0000	14.01
Analysis per interval			
2005-2009	0.0070	-	-
2010-2015	0.0277	-0.0206	-

Next, we present our results for the profit technological change in Table 3. These indicators are only computed for the intervals since no technological change is allowed within intervals. To do so the first step is to compute counterfactual profit efficiency measurements, i.e. when the netputs and the technology are not for the same intervals (see Section 2.3). These counterfactual profit efficiency measurements are provided in the first column of

Table 3. Note that they are always zero for the first interval and positive for the second intervals. This also shows a technology consistency for our interval choice. Next, they are aggregated using new weights to compute the interval-level counterfactual profit efficiency measurements. Finally, we can compute the profit technical change indicators. When the netputs are fixed for the first interval, this indicator is -0.0070 revealing a technological regression. When the netputs are fixed for the second interval, the value is also negative -0.2170. As explained in Section 2.3, to avoid the dependence of the netputs chosen, the profit technological change indicator is computed as the simple average of the netput dependent indicators. It has a value of -0.1120 revealing a technological regression. This regression may be due, as explained, before, to the consequences of the economic crisis, lack of returns of new investments, or political decisions.

Table 3: Profit technological change

Year	Profit efficiency	Profit technological change	Weight (%)
Analysis per year			
2005	0.0000	-	16.75
2006	0.0000	-	22.14
2007	0.0000	-	16.25
2008	0.0000	-	22.17
2009	0.0000	-	22.69
2010	0.1793	-	16.78
2011	0.2748	-	21.24
2012	0.2834	-	16.45
2013	0.2747	-	16.85
2014	0.3501	-	15.25
2015	0.0740	-	13.43
Analysis per interval			
2005-2009	0.0000	-0.0070	-
2010-2015	0.2447	-0.2170	-
Average	-	-0.1120	-

As explained in Section 2.4, we can obtain a profit Luenberger-type indicator when summing the profit efficiency change and the profit technological change. In this case, it is -0.1327. It indicates a profit performance regressions between intervals 2005-2009 and 2010-2015. This is explained by both a profit efficiency regression (-0.0206) and a profit technological regression (-0.1120). The technological regression is the main explanatory component in our case. This finding gives credits to our choice for the intervals. As a final note, we remark that we redo the computations using other specifications for the intervals, but we never found a higher profit technological change effect. It therefore confirms our simple observation using only the descriptive statistics from Table 1.

3.2.2 Decomposition

As explained in Section 2.5, an attractive feature of our profit measurements and indicators is that they can be decomposed into two components: a technical component, defined as a directional distance function, and an allocative component, defined as a residual part. The main advantage of the decomposition is that it allows us to better understand the reasons for the profit performance changes. We recall that the directional distance functions are computed, as the profit efficiency measurements, by means of linear programs; while the allocative concepts are obtained a posteriori using their respective definition. The technical and allocative efficiency changes are given in Tables 4 and 5, respectively.

Table 4: Technical efficiency change

Year	Technical efficiency	Technical efficiency change	Weight (%)
Analysis per year			
2005	0.0000	-	17.81
2006	0.0000	0.0000	19.04
2007	0.0000	0.0000	20.33
2008	0.0000	0.0000	20.87
2009	0.0000	0.0000	21.95
2010	0.0000	-	17.39
2011	0.0399	-0.0399	18.00
2012	0.0387	0.0012	18.79
2013	0.0505	-0.0118	17.06
2014	0.0000	0.0505	14.75
2015	0.0000	0.0000	14.01
Analysis per interval			
2005-2009	0.0000	-	-
2010-2015	0.0231	-0.0231	-

These Tables reveal that the profit inefficiency behaviour in the first interval is only due to the allocative efficiency change. That is, the problem of non-optimal allocation of the netputs given the prices. In fact, the first interval presents a technical efficiency situation, i.e. the interval-level efficiency score is zero. We recall that this can only occur when all periods in the interval present a technical efficiency situation. Allocative inefficiency is only observed for the periods 2008 and 2009. We can attribute this result to the impact of the economic crisis on the low-end hotel industry. This means that the netputs have been non-optimally used during the economic crisis. Both technical and allocative inefficiencies explain the profit inefficiency behaviour found for the second interval. We find a regression in both technical and allocative terms (-0.0213 and -0.0024). This reveals that technical is the main explanatory component of the profit efficiency regression (-0.0206). Finally, we note that the weights are the same for the technical and allocative components as those for

Table 5: Allocative efficiency change

Year	Allocative efficiency	Allocative efficiency change	Weight (%)
Analysis per year			
2005	0.0000	-	17.81
2006	0.0000	0.0000	19.04
2007	0.0000	0.0000	20.33
2008	0.0307	-0.0307	20.87
2009	0.0029	0.0279	21.95
2010	0.0000	-	17.39
2011	0.0019	-0.0019	18.00
2012	0.0105	-0.0086	18.79
2013	0.0136	-0.0031	17.06
2014	0.0000	0.0136	14.75
2015	0.0000	0.0000	14.01
Analysis per interval			
2005-2009	0.0070	-	-
2010-2015	0.0046	0.0024	-

the profit component in Table 2. This comes from our methodology, and not from the data (see (16)).

We can also decompose the profit technological change into two components: technical technological change and allocative technological change. As explained before, to compute these components, we rely on counterfactual measurements. The technical and allocative efficiency measurements are provided in Tables 6 and 7. These results again indicate that the technical component is the main explanatory component of the decomposition. As for the profit technological change in Table 3, the first interval presents values of zero for the technical and allocative counterparts; and the second interval presents positive values for the two components for all periods (note that the allocative component has a negative value in 2015). It turns out that there is both a technical and allocative technological regression between the two intervals (-0.1037 and -0.0083). Note that, as for the profit technological change, the technical and allocative technological change components are computed as the simple arithmetic averages of the netput-dependent components. Finally, note also that the weights for both components are the same as those used for the profit component in Table 3.

Finally, as explained at the end of Section 2.5.4, by summing the technical (allocative) efficiency and technological changes, we obtain a technical (allocative) Luenberger-type indicator. They can therefore be used to investigate the cause of the profit performance change. In our case, we find that the technical Luenberger indicator is -0.1268 and the allocative Luenberger indicator is -0.0059 (We found previously that the profit Luenberger

Table 6: Technical technological change

Year	Technical efficiency	Technical technological change	Weight (%)
Analysis per year			
2005	0.0000	-	16.75
2006	0.0000	-	22.14
2007	0.0000	-	16.25
2008	0.0000	-	22.17
2009	0.0000	-	22.69
2010	0.1692	-	16.78
2011	0.2578	-	21.24
2012	0.2748	-	16.45
2013	0.2568	-	16.85
2014	0.3314	-	15.25
2015	0.0624	-	13.43
Analysis per interval			
2005-2009	0.0000	0.0000	-
2010-2015	0.2305	-0.2075	-
Average	-	-0.1037	-

Table 7: Allocative technological change

Year	Allocative efficiency	Allocative technological change	Weight (%)
Analysis per year			
2005	0.0000	-	16.75
2006	0.0000	-	22.14
2007	0.0000	-	16.25
2008	0.0000	-	22.17
2009	0.0000	-	22.69
2010	0.0101	-	16.78
2011	0.0170	-	21.24
2012	0.0086	-	16.45
2013	0.0179	-	16.85
2014	0.0187	-	15.25
2015	-0.0624	-	13.43
Analysis per interval			
2005-2009	0.0000	-0.0070	-
2010-2015	0.0141	-0.0095	-
Average	-	-0.0083	-

is -0.1327). This reveals that the decomposition of the profit Luenberger indicator is true empirically: $-0.1327 = (-0.1268) + (-0.0059)$. This decomposition once more supports our two main results. First, there is a performance regression between the two intervals for all dimensions. Second, the main cause of the profit performance regression has to be found in the technical (efficiency and technological) components.

4 Conclusion

Nonparametric efficiency analysis of production activities is a technique used to evaluate the performance of a Decision Making Unit (DMU). While this type of analysis has grown in popularity for both static and dynamic settings (for which, in the latter case, indexes and indicators are generally used), most theoretical advancements have been proposed when other DMUs are used as peers in the performance evaluation exercise. In some situations, the research question involves but a single DMU. In this paper, we propose a new Luenberger-type indicator to assess the profit performance change of a single DMU over time. The particularity of our approach is to recognize that technological change may be present in the profit evaluation exercise. Attractively, the new indicator can be decomposed into several dimensions, and provides a coherent and systematic means of comparing the profit performance changes.

We apply our Luenberger-type indicator to the case of the Chinese low-end hotel industry. The Chinese hotel industry has grown fast in recent years, and become a significant factor of the country's high economic growth. Its importance is well-recognized by Chinese policy-makers, as the hotel industry has benefited from recurrent policy interventions. Particular to the Chinese star-hotel sectors is the sensible gap between low- and high-end hotels. The Chinese low-end hotel industry thus presents unique features. Our results highlight a performance regression which can largely be accounted for by the technical components of the indicator decomposition, and holds for efficiency as well as technological changes.

Our results have important managerial and policy implications. First, the profit efficiency regression should be taken seriously. While the number of low-end hotels has gone down significantly, there is no evidence of performance progression. This fact questions the relevance of closing so many low-end hotels. Moreover, the performance regression is largely due to the technical components whereas allocative efficiency appears to have played a minor positive role. These findings reveal that prices are not to blame, but instead the inputs and outputs. Managers and policy-makers therefore should direct their effort at the quantity of inputs and outputs. Next, technological regression implies that assuming a homogeneous technology for the periods considered is not credible. Instead, we reveal that low-end hotels have faced a contraction of their technology over time. In other words, they are less able to achieve higher performances. One possible explanation is the lack of policy support. Overall, while high-end hotels have benefited from recent policy interventions, it is not the case for low-end ones; this probably explains a major part of our findings.

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