A Digital Absorber for Nonlinear Vibration Mitigation

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ABSTRACT

In this study, a digital impedance is used to realize both a linear and a nonlinear piezoelectric tuned vibration absorber in order to mitigate the vibrations of a nonlinear structure. The digital processing unit enables the synthesis of impedances with arbitrary functional forms, thereby easing the implementation of nonlinear absorbers. The superior performance of the nonlinear absorber over its linear counterpart is demonstrated experimentally. Various nonlinear functional forms are also tested in the absorber and illustrate the relevance of the principle of similarity (i.e. the same nonlinear functional form as that in the host structure should be used in the absorber).

Keywords: piezoelectric vibration absorber, synthetic impedance, nonlinear vibrations, nonlinear vibration absorber, principle of similarity

INTRODUCTION

Resonant piezoelectric shunt damping as proposed by Hagood and von Flotow [1] is a popular approach for passive vibration control. A piezoelectric transducer bonded to a vibrating structure converts part of its mechanical energy into electrical energy. This energy may in turn be dissipated in a so-called shunt circuit. Efficient vibration mitigation by resonant shunt circuits is conditioned upon a precise tuning of the shunt circuit to the frequency of the host structure.

Real-world structures exhibit a nonlinear behavior to some extent. This can be detrimental to tuned absorbers, since the frequency of the host structure may vary with the forcing amplitude and the absorber may get detuned, leading to high-amplitude vibrations. Agnes and Inman [2] observed that including nonlinearities in the absorber as well may have a beneficial effect on performance. Habib and Kerschen [3] showed how these nonlinearities may be tuned so as to increase the range of forcing amplitudes over which the absorber is efficient. A central idea in their approach is the principle of similarity, i.e., the nonlinear absorber should possess the same nonlinearity type as that of the host structure. Soltani and Kerschen [4] extended this principle to piezoelectric vibration absorbers using a capacitor with a nonlinear capacitance.

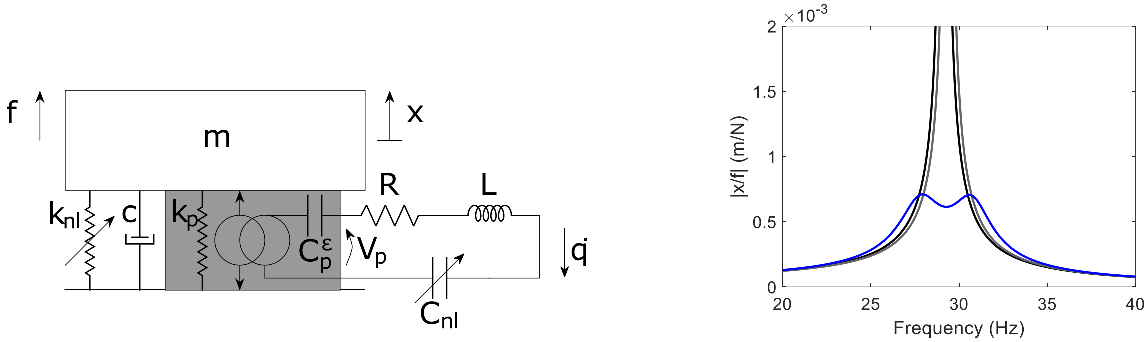
The theoretical developments in [4] were experimentally validated by Lossouarn et al [5] using saturable inductors instead of a nonlinear capacitance. By contrast, digital piezoelectric absorbers, as proposed by Fleming et al [6], are used in this work. A digital absorber comprises a digital unit that can be programmed to follow arbitrary input-output relations. This unit is coupled to an analog board containing a current source and electronic circuitry to make the interface between the digital unit and the piezoelectric transducer. The absorber is connected to the transducer and acts as an impedance from its point of view. The advantage of using a digital absorber comes from the flexibility provided to synthesize virtually any impedance; this however comes at the expense of energy consumption.

PIEZOELECTRIC SHUNT DAMPING

The structure to be controlled is modelled as a single-degree-of-freedom (SDOF) Duffing oscillator coupled to a piezoelectric stack connected to a shunt circuit, as shown in Fig. 1. The equations relating the displacement x of the SDOF oscillator, the external forcing f and the piezoelectric charge q read

$$\begin{cases} m\ddot{x} + c\dot{x} + k_{oc}x + k_3x^3 - \theta q = f \\ L\ddot{q} + R\dot{q} + \dfrac{1}{C_p^{\varepsilon}}q + C_n|q|^{n-1}q - \theta x = 0' \end{cases} \tag{1}$$

where m, c, k and k_3 represent the mass, damping coefficient, open-circuit stiffness and nonlinear stiffness coefficient of the SDOF oscillator, respectively, θ is a piezoelectric coupling coefficient, L, R and C_n are the inductance, resistance and nonlinear capacitance coefficient ($C_n = 0$ for a linear shunt circuit) of the shunt circuit, respectively, and C_p^{ε} is the piezoelectric capacitance at constant strain. The values of L and R may be tuned such that the frequency response of the underlying linear system exhibits two peaks of equal amplitude in place of the original high-amplitude resonance [7], as depicted in Fig. 1. When the structure behaves nonlinearly, detuning of the linear absorber may occur. The purposeful use of nonlinearities in the absorber can mitigate this undesirable phenomenon. In line with [3], the value of n should be chosen according to the principle of similarity, i.e. $n = 3$. The value of the coefficient C_n may then be found from [4].

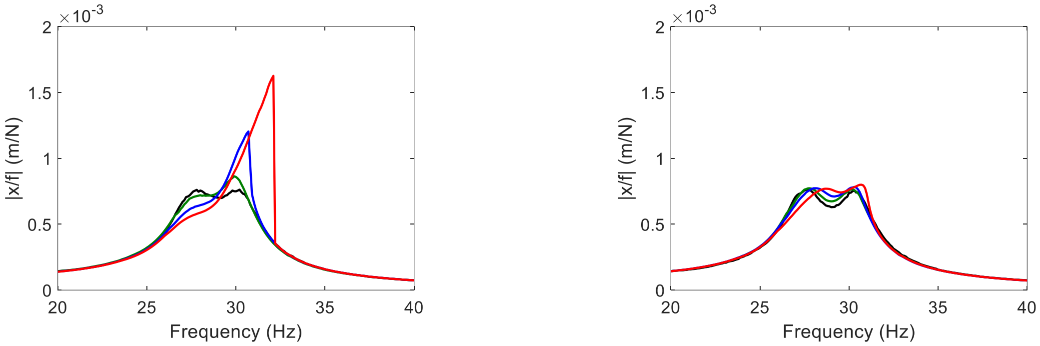


(a) (b)

Fig. 1 Schematics of the system (a) and frequency response function of the underlying linear system (b) when the piezoelectric transducer is short-circuited (▬), open-circuited (▬) and connected to an optimally tuned RL shunt circuit (▬).

EXPERIMENTAL RESULTS

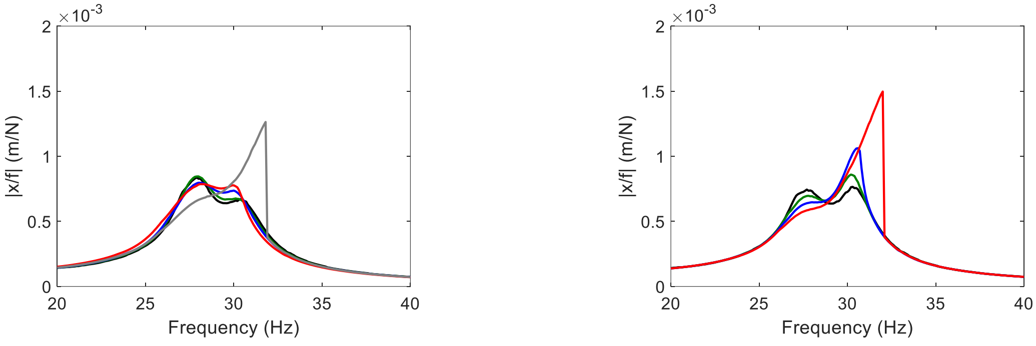
The host structure considered for the experiments is a cantilever beam covered with two arrays of piezoelectric patches. A clamped thin lamina is attached to its free end, and is responsible for an overall hardening behavior. The structure can accurately be modelled as a Duffing oscillator when considering its first bending mode [5]. A digital absorber is connected to the structure through the patches in order to mitigate its vibrations. Fig. 2 compares the performance of the absorber when a linear or a nonlinear (with cubic capacitance) shunt circuit is synthesized. The detuning phenomenon described in the previous section is clearly visible for the linear absorber: the rightmost peak grows in a high-amplitude response. Conversely, the nonlinear absorber maintains its performance (i.e., equal peaks are still observed) over the tested forcing amplitude range.



(a) (b)

Fig. 2 Experimental frequency response of the structure controlled by a linear (a) and cubic (b) digital absorber under an external forcing amplitude of 0.2 N (▬), 0.4 N (▬), 0.6 N (▬) and 0.8 N (▬).

Owing to the flexibility provided by the digital absorber, other mathematical functions can easily be synthesized. To experimentally test the abovementioned principle of similarity, quadratic ($n = 2$) and quintic ($n = 5$) capacitances were considered in the absorber. The corresponding frequency responses are shown in Fig. 3. Clearly, the amplitude-dependent character of these responses illustrates the relevance of the principle of similarity.



(a)

(b)

Fig. 3 Experimental frequency response of the structure controlled by a quadratic (a) and quintic (b) digital absorber under an external forcing amplitude of 0.2 N (—), 0.4 N (—), 0.6 N (—), 0.8 N (—) and 1 N (—).

CONCLUSION

The digital impedance eases the implementation of various types of piezoelectric absorbers. Taking advantage of this flexibility, this work compared experimentally the vibration mitigation performance on a nonlinear host structure of a linear and a nonlinear piezoelectric shunt circuit. The superiority of the latter over the former was demonstrated. The significance of the principle of similarity was also validated.

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