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Topology Optimization for Impedance Mismatch Reduction of a 3D Planar Multilayer Busbar

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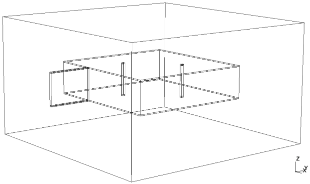
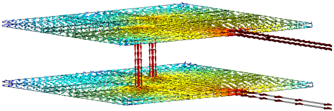
Topology optimization of a busbar

- Linear time-harmonic magnetodynamic problem (Gmsh/GetDP)





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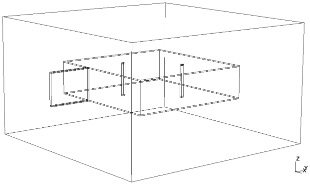
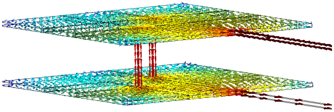
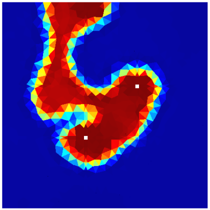
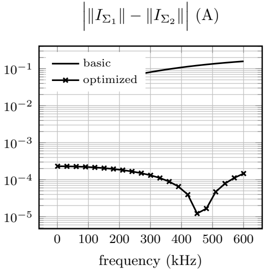
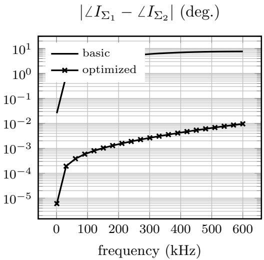




- Topology optimization: optimization parameters and objective







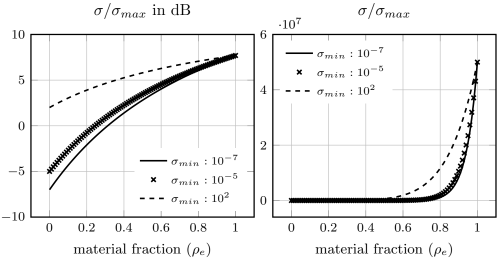
Topology Optimization with PDE constraints

Topology optimization $\rightarrow$ material interpolation scheme

- Topology optimization acts on material fractions noted ρ
  - ρ is represented by a discrete field (on a fixed region)
  - At each finite element $e \rightarrow \rho_e$ is the presence or not of copper

Topology optimization → material interpolation scheme

- Topology optimization acts on material fractions noted ρ
  - ρ is represented by a discrete field (on a fixed region)
  - At each finite element $e \rightarrow \rho_e$ is the presence or not of copper
- Interpolation scheme of conductivity σ for intermediate ρ_e



  - physical behavior strongly affected by the conductivity at $\rho_e = 1$
  - logarithmic scale → handle the span of several order of magnitudes

Current mismatch reduction → optimization problem

- Set $||\Delta I(\rho, \boldsymbol{A}^{\dagger}, v^{\dagger})||_2^2$ of the mismatch $\Delta I = I_{\Sigma_1} - I_{\Sigma_2}$ between the complex currents in the vertical vias as objective
- Set a constraint on the volume of material so as to fill at most a given volume fraction α of the available domain
- Design problem as a nonlinear PDE-constrained optimization

$$(P)\begin{cases} \min\limits_{\rho} & f_0(\rho, \boldsymbol{A}^{\dagger}, v^{\dagger}) \equiv ||\Delta I(\rho, \boldsymbol{A}^{\dagger}, v^{\dagger})||_2^2 \\ s.t. & f_1(\rho, \boldsymbol{A}^{\dagger}, v^{\dagger}) \equiv \int_{\Omega_C} \rho \, \mathrm{d}\Omega \leq \alpha \int_{\Omega_C} \mathrm{d}\Omega \\ & \rho_e^{min} \equiv 0 \leq \rho_e \leq \rho_e^{max} \equiv 1, \; e = 1, \ldots, n \\ & r(\boldsymbol{A}^{\dagger}, v^{\dagger}, \bar{\boldsymbol{A}}', \bar{v}') = 0, \; \forall \bar{\boldsymbol{A}}' \in Z_A^0, \forall \bar{v}' \in Z_v^I \end{cases}$$

Time-harmonic magnetodynamics → PDE constraint

- The complex valued PDE system in terms of the residual $r(\boldsymbol{A}, v, \boldsymbol{A}', v')$

$$\begin{cases} \int_{\Omega} \left(\nu \boldsymbol{B} \cdot \boldsymbol{B}' + i\omega\sigma \boldsymbol{A} \cdot \boldsymbol{A}' \right) \, \mathrm{d}\Omega + \int_{\Omega_C} \sigma \nabla v \cdot \boldsymbol{A}' \, \mathrm{d}\Omega = 0, \ \forall \boldsymbol{A}' \in Z_A^0 \\ \int_{\Omega_C} \left(i\omega\sigma \boldsymbol{A} \cdot \nabla v' + \sigma \nabla v \cdot \nabla v' \right) \, \mathrm{d}\Omega - \int_{\Sigma_I} I_s \cdot v' \, \mathrm{d}\Omega = 0, \ \forall v' \in Z_v^I \end{cases}$$

  with $\boldsymbol{A}$ the magnetic vector potential, $\boldsymbol{B} = \mathbf{curl}\, \boldsymbol{A}$ the magnetic flux density on Ω and v the electric potential
- All the state variables are complex-valued
- Time consuming f_j evaluation: FE resolution required as ρ changes
- Gradient (sensitivity)-based algorithms → drastic reduction of f_j evaluations

Sensitivity in time-harmonic domain

Gradient based algo $\rightarrow$ we need sensitivity

- Sensitivity aims (mainly) at obtaining $\frac{\mathrm{d}f_0}{\mathrm{d}\rho} = \frac{\mathrm{d}}{\mathrm{d}\rho}||\Delta I(\boldsymbol{\rho}, \boldsymbol{A}^\dagger, v^\dagger)||_2^2$
- Differentiation of f_0 w.r.t. ρ requires
  - differentiation of $\frac{\mathrm{d}I_{\Sigma_1}}{\mathrm{d}\rho}(\boldsymbol{\rho}, \boldsymbol{A}^\dagger, v^\dagger)$ and $\frac{\mathrm{d}I_{\Sigma_2}}{\mathrm{d}\rho}(\boldsymbol{\rho}, \boldsymbol{A}^\dagger, v^\dagger)$
  - and hence the differentiation of the solution $\boldsymbol{A}^\dagger \rightarrow \frac{\mathrm{d}\boldsymbol{A}^\dagger}{\mathrm{d}\rho}$ and $v^\dagger \rightarrow \frac{\mathrm{d}v^\dagger}{\mathrm{d}\rho}$

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- Providing an extension of (Kuci et al.) in time-harmonic domain
  - we differentiate under the integration sign of the residual
  - we obtain a unique linear formulation for scalar/vector fields in 3D
  - we avoid the simple but slow finite difference method

$\frac{\mathrm{d}\boldsymbol{A}^\dagger}{\mathrm{d}\rho}$ and $\frac{\mathrm{d}v^\dagger}{\mathrm{d}\rho}$ → differentiation under the integration sign

- Let us differentiate the residual by applying the chain rule

$$\begin{cases} \displaystyle\int_\Omega \left(\nu\,\mathbf{curl}\,\frac{\mathrm{d}\boldsymbol{A}^\dagger}{\mathrm{d}\rho}\cdot\mathbf{curl}\,\boldsymbol{A}' + i\omega\frac{\mathrm{d}\sigma}{\mathrm{d}\rho}\boldsymbol{A}^\dagger\cdot\boldsymbol{A}' + i\omega\sigma\frac{\mathrm{d}\boldsymbol{A}^\dagger}{\mathrm{d}\rho}\cdot\boldsymbol{A}'\right)\,\mathrm{d}\Omega \\ \displaystyle + \int_{\Omega_C}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\rho}\nabla v^\dagger\cdot\boldsymbol{A}' + \sigma\nabla\frac{\mathrm{d}v^\dagger}{\mathrm{d}\rho}\cdot\boldsymbol{A}'\right)\,\mathrm{d}\Omega = 0,\ \forall\boldsymbol{A}'\in Z_A^0 \\ \displaystyle\int_{\Omega_C}\left(i\omega\frac{\mathrm{d}\sigma}{\mathrm{d}\rho}\boldsymbol{A}^\dagger\cdot\nabla v' + i\omega\sigma\frac{\mathrm{d}\boldsymbol{A}^\dagger}{\mathrm{d}\rho}\cdot\nabla v'\right. \\ \displaystyle\left. + \frac{\mathrm{d}\sigma}{\mathrm{d}\rho}\nabla v^\dagger\cdot\nabla\bar{v} + \sigma\nabla\frac{\mathrm{d}v^\dagger}{\mathrm{d}\rho}\cdot\nabla\bar{v}\right)\,\mathrm{d}\Omega = 0,\ \forall\bar{v}\in Z_v^I \end{cases}$$

- We get
  - a linear system for both $\frac{\mathrm{d}\boldsymbol{A}}{\mathrm{d}\rho}$ and $\frac{\mathrm{d}v}{\mathrm{d}\rho}$
  - same jacobian matrix as the PDE → known factorization
  - terms in $\frac{\mathrm{d}\sigma}{\mathrm{d}\rho}$ → known as $\sigma(\rho)$ is explicit
  - easy to implement → add few lines in existing FEM code

Conclusions

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- FEM based and Gradient based topology optimization
  - geometry of a busbar for impedance mismatch reduction
- Sensitivity analysis based on the Lie derivative
  - extended to the time-harmonic case with vector analysis notations
  - validated with finite difference (FD)
  - much more efficient then FD (NL PDE resolutions avoided for perturbed geometries)
  - implemented in Gmsh/GetDP

Thank you
for your attention!

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