Three-dimensional Topology Optimization of Planar Multilayer Busbar

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***Abstract*— This paper addresses the optimal design of three-dimensional planar multilayer busbar, through the general setting of topology optimization. The optimization problem is efficiently solved with a gradient-based mathematical programming algorithm, that exploits a harmonic adjoint variational formulation sensitivity analysis for the harmonic linear magnetodynamic problem. This formulation can handle topology design variables defined on a finite element mesh.**

I. Introduction

The use of switching technology at all levels of the electrical power sector, e.g., generation, transmission and distribution, has enabled the decrease in size of hardware while maintaining high power density. The main challenge lies in defining the appropriate topology of the devices and components without compromising their performance (e.g. power losses, electromagnetic interference compatibility). We tackle these issues through the general setting of a PDE-constrained density based topology optimization [1], which aims at determining how the material should be distributed within the design domain, to reach some objectives without having to make any a priori guess about the final distribution, which offers a great flexibility in the design. Most existing sensitivity calculation approaches deal with 2D static systems, leaving aside 3D and time-harmonic cases. The extension to harmonic fields however requires adapted theoretical frameworks [2], [3]. We show in this paper how this setting allows to derive the variational sensitivity formula for a general 3D harmonic magnetodynamic problem, in both direct and adjoint approaches. The technique is successfully applied to the design of a three-dimensional planar multilayer busbar.

II. Description of the Model

A representative three-dimensional system, Fig. 1, fed with a sinusoidal current injected in a surface Σ_I, is modeled in terms of a magnetodynamic $\boldsymbol{A}-v$ formulation,

$$\mathbf{curl}\,\nu\,\boldsymbol{B} + i\omega\sigma\boldsymbol{A} + \sigma\,\mathbf{grad}\,v = 0 \qquad (1)$$

with $\boldsymbol{A}$ the magnetic vector potential, $\boldsymbol{B} = \mathbf{curl}\,\boldsymbol{A}$ the magnetic flux density, on a bounded domain Ω and v the electric potential on a conducting region $\Omega_C \in \Omega$. In (1), the conductivity σ is set to $5 \cdot 10^7$ (S/m) in the conducting domain Ω_C, and the angular frequency $\omega = 2\pi f$ is computed for $f = 500$ (Hz). The weak formulation of the problem

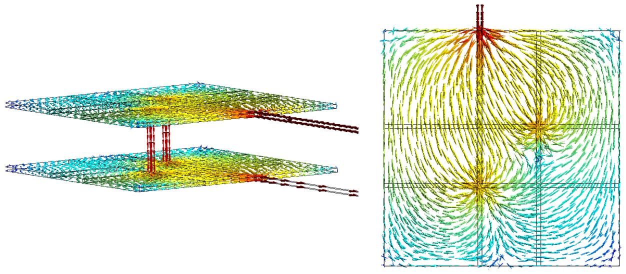

Fig. 1. Considered three-dimensional inductor (left) fed by a sinusoidal current, split into two separate currents that reunite and leave the system, through the output boundary and the resulting current distribution in the top plate (right).

reads [4]: find $\boldsymbol{A}$ and v in appropriate complex function spaces, respectively $Z_A^0 = \boldsymbol{H}_0(\mathbf{curl};\Omega)$ and $Z_v^I = \boldsymbol{H}^1(\Omega)$ and verifying appropriate boundary conditions such that the residual $r(\boldsymbol{A}, v, \bar{\boldsymbol{A}}, \bar{v})$ verifies

$$\begin{cases} \int_\Omega \left(\nu\boldsymbol{B}\cdot\bar{\boldsymbol{B}} + i\omega\sigma\boldsymbol{A}\cdot\bar{\boldsymbol{A}}\right)\,\mathrm{d}\Omega \\ + \int_{\Omega_C} \sigma\nabla v\cdot\bar{\boldsymbol{A}}\,\mathrm{d}\Omega = 0,\ \forall\bar{\boldsymbol{A}} \in Z_A^0 \\ \int_{\Omega_C} i\omega\boldsymbol{A}\sigma\cdot\nabla\bar{v}\,\mathrm{d}\Omega + \int_{\Omega_C}\sigma\nabla v\cdot\nabla\bar{v}\,\mathrm{d}\Omega \\ - \int_{\Sigma_I} I\cdot\bar{v}\,\mathrm{d}\Omega = 0,\ \forall\bar{v}\in Z_v^I \end{cases} \qquad (2)$$

with the global current I set to 1 (A). The two left-hand sides in (2) define the residual $r(\boldsymbol{A}, v, \bar{\boldsymbol{A}}, \bar{v})$. The solution of problem (2), Fig 1, is carried out using an open source finite element code GetDP/Gmsh [5], [6], we obtain the result in .

III. Topology Optimization Problem

Density-based topology optimization technique uses the finite element mesh (also used for field simulation) and defines a design variable τ_e in each finite element e of the mesh, representing a material density. The latter can vary between 0 (empty) and 1 (copper). Special care must be taken in the selection of the interpolation scheme that assigns conductivity to points of intermediate density, as the magnitude of the conductivity at $\tau_e = 1$ strongly affects the solution of (2), and thus the span of several order of magnitudes of σ in the design domain is crucial. As the conductivity varies from σ_{min} to σ_{max}, an interpolation in logarithmic scale [7] is adopted in each finite element e,

$$\log_{10}\sigma_e = \log_{10}(\sigma_{max}) - \frac{1-\tau_e}{1+\tau_e}\log_{10}(\frac{\sigma_{max}}{\sigma_{min}}), \quad (3)$$

with numerical experiments shown in Fig. 2 for various σ_{min}.

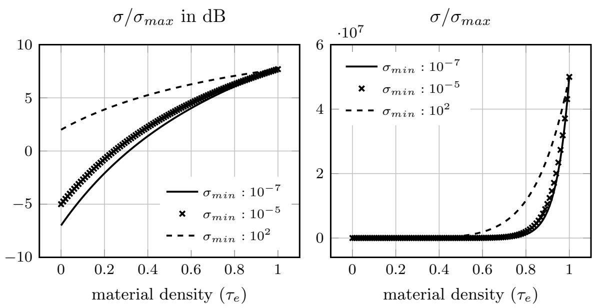

Fig. 2. Conductivity as a function of material density in logarithmic scale (3) (left) and natural scale (right) for various σ_{min}, and σ_{max} set to $5 \cdot 10^7$ (S/m).

The aim of a PDE-constrained topology optimization problem is to determine the optimal distribution τ for the two plates of the inductor, that minimizes the mismatch between the complex currents I_1 and I_2 in the vertical vias of the busbar, while filling at most a given volume fraction α of the available domain. A penalty function $P(\tau)$ as in [8] is used in order to penalize intermediate values of τ and favor a 0-1 solution. Hence, the optimization problem reads

$$\begin{aligned} \min_{\tau} \quad & ||I_1(\boldsymbol{A},v) - I_2(\boldsymbol{A},v)||_2^2 \\ s.t. \quad & V(\tau) \equiv \sum_{e=1}^{n} \tau_e \leq \alpha V(1) \\ & P(\tau) \equiv \sum_{e=1}^{n} (1-\tau_e)(\tau_e - 0.01) \leq \beta P(1) \\ & 0 \leq \tau_e \leq 1,\ e = 1,\dots,n \\ & r(\boldsymbol{A}, v, \bar{\boldsymbol{A}}, \bar{v}) = 0,\ \forall \bar{\boldsymbol{A}} \in Z_A^0, \forall \bar{v} \in Z_v^I \end{aligned} \quad (4)$$

where n is the number of finite elements. The latter uses the results $\boldsymbol{A}$ and v of problem (2) to determine the currents I_1 and I_2 in branches for a given material distribution τ. The repetition of these evaluations is time-consuming for large scale applications. A gradient-based mathematical programming algorithm, MMA [9], limits the required number of problem (2) resolutions.

The main difficulty lies in the computation of the sensitivity of the objective function with respect to the design variables τ_e in 3D. As the current depends on $\boldsymbol{A}$ and v, it requires computing the sensitivity of the state variable and thus differentiating (1), with respect to each design variable. As shown in [2] for static systems, the adjoint method provides an analytical expression of sensitivity, a tedious finite differences approach. We extend in this paper the ideas of [2] to the harmonic case based on Wirtinger's calculus [3].

Setting α to 0.5 and β to 0.01, problem (4) enables to obtain currents in phase in the two vertical branches, once the iterative optimization process is completed, Fig. 3. The full paper will detail the adjoint variational formulation.

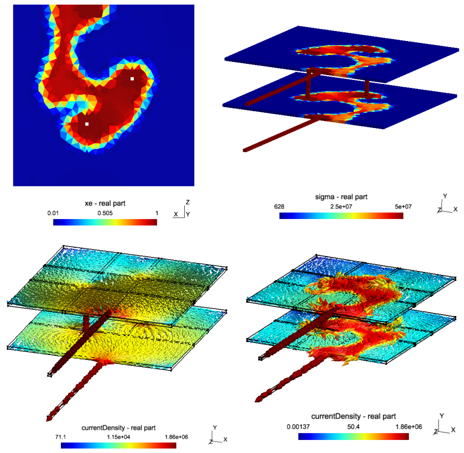

Fig. 3. Top: Optimal copper distribution (left) of inductor plates as the solution of (4) and conductivity for the optimal topology. Bottom: Distribution of current density (left) for inductor plates with respectively full copper (left) and for the optimal copper distribution (right).

Acknowledgements

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