Evaluation of ground reaction forces using a rigid foot/ground contact model

Romain Van Hulle, Cédric Schwartz, Olivier Brüls

Laboratory of Human Motion Analysis
University of Liège
Allée de la Découverte 9, 4000 Liège, Belgium
Romain.VanHulle@uliege.be

EXTENDED ABSTRACT

1 Introduction

Inverse dynamics analysis is an important method to study and understand human motion. It leads to the estimation of joint torques and muscles forces based on experimental motion data and information about the external forces acting on the human body, in particular, gravity and ground reaction forces (GRF). If gravity forces can be evaluated using a model, GRF have to be measured or estimated. Force plates give an accurate estimation of the GRF along three axis. However, their implementation in the laboratory and their cost tend to complicate some studies. Instead a numerical foot/ground model can be used.

In the literature, one generally distinguishes compliant and rigid foot models. Compliant models use the deformation of the foot sole and the ground to estimate the GRF with simple equations. However, measuring this deformation is often beyond the capabilities of the current motion capture technologies, and the use of compliant model is mostly restricted to predictive analysis. Rigid models do not require the local compliance since they rely on a non-penetration condition. With rigid models available in the literature, the GRF are usually estimated from the global acceleration of the body and empirical laws are necessary to define the repartition of the forces between both feet.

The presented work provides a general numerical approach to solve the inverse dynamics problem using a foot model composed of two rigid segments. The contact between the feet and the ground is modelled using unilateral contact constraints and the Lagrange multipliers are defined by an inversion of the equation of motion in a least square sense. To determine the position of the center of pressure (COP), the model postulates that it evolves as a function of the foot segments rotation angles.

2 Methods

The dynamic equilibrium of a mechanical system, representing the balance between internal, external and inertial forces, can be written in the following matrix form [1]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q},t)\boldsymbol{\lambda} - \mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{0} \tag{1}$$

where $\mathbf{q}$ are the coordinates of the system (in our case, absolute coordinates are chosen), $\mathbf{M}(\mathbf{q})$ is the symmetric mass matrix, $\mathbf{g}$ represents the bilateral and unilateral constraints, $\mathbf{g}_{\mathbf{q}}(\mathbf{q},t) = \frac{\partial \mathbf{g}(\mathbf{q},t)}{\partial \mathbf{q}}$ is the matrix of bilateral and unilateral constraints gradients, $\boldsymbol{\lambda}$ is the vector of bilateral and unilateral Lagrange multipliers and $\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})$ is the vector of gravity, gyroscopic and centrifugal forces.

In this study, the unknowns are the GRF and reaction forces in the joints represented by the Lagrange multipliers.

In Equation (1), we separate the set of inactive constraints $\overline{\mathscr{A}}$ whose Lagrange multipliers are zero from the active ones $\mathscr{A}$ whose Lagrange multipliers are undetermined. To solve Equation (1) for the unknowns $\boldsymbol{\lambda}$, and to handle the possible over/underdetermination (depending on the number of active constraints and their degree of redundancy), we use the least square method and the pseudo-inverse [2]:

$$\boldsymbol{\lambda}^{\mathscr{A}} = \left(\mathbf{g}_{\mathbf{q}}^{\mathscr{A},\mathrm{T}}(\mathbf{q})\right)^{+}(\mathbf{f} - \mathbf{M}\ddot{\mathbf{q}}), \qquad \boldsymbol{\lambda}^{\overline{\mathscr{A}}} = \mathbf{0} \tag{2}$$

The least square distribution of the reaction forces postulates that the human body minimizes those forces during motion. This hypothesis seems reasonable during many day-to-day activities.

The classification of the contact constraints relies on the signed distance between the foot and the ground (measured experimentally), and implicitly on the unknown estimated reaction forces (computed by our inverse dynamics scheme). Both values can be affected by some uncertainties, therefore, we propose a mixed criterion relying on both values and inspired by the work of Alart and Curnier [3]. The j^{th} constraint will be considered active if, on the normal axis n:

$$\lambda^{j,n} - r g^{j,n} \geq 0 \tag{3}$$

The classification criterion depends on a parameter of activation r whose value can be selected to obtain robust results with respect to measurement errors.

Finally, two contact points per the foot are determined (one between the heel and the metatarsal bone, and one between the metatarsal bone and the hallux). To prevent irregularities in the position of COP, and then in the ground reaction moment (GRM), these points move on the segments according to a sinusoidal law on the absolute angle of the segments, and a parameter δ representing the boundaries of this motion.

Four healthy male subjects (23.8 ± 2.17 years old, 177.2 ± 6.5 cm, 74.4 ± 7.47 kg) have taken part in this study, and have performed six gait tests each (walking speed 1.39 ± 0.19 m/s).

3 Results

In Figure 1, results are shown for one particular gait test. In total, for the 24 tests, the relative root mean square error (rRMSE) of the vertical GRF is 4.1%,the rRMSE of the horizontal GRF is 11.2% and the rRMSE of the GRM is 5.3%.

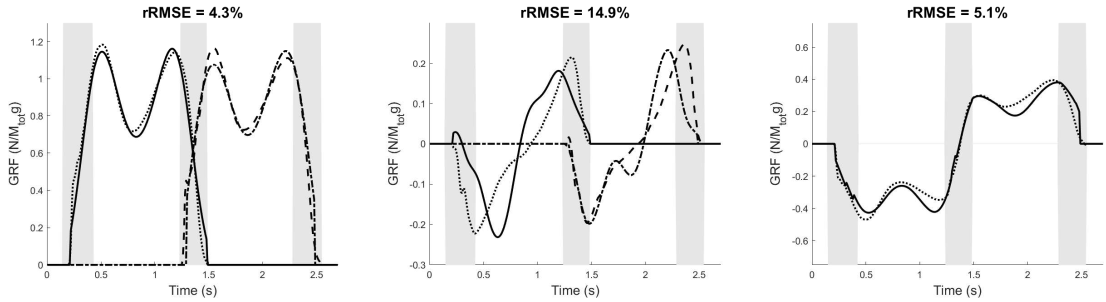

Figure 1: Results: gait test. Solid line: numerical right, dotted line: experimental right, dash-dotted line: numerical left, dashed line: experimental left, grey area: double-support phase, white area: single-support phase.

4 Discussion

The proposed model is based on two assumptions: the reaction forces are computed in the least square sense and the position of COP only depends on the segment angle. Also, three main parameters have been used: the activation parameter r, the limit angle δ and the cut-off frequency ω of the low-pass filters.

The least square repartition seems reasonable and is confirmed by our results for gait, and other activities not mentioned in this extended abstract. The assumption that the COP evolves as a function of the segment angle is consistent with a variety of human motion (gait, run, jump,...). However, when the motion is impacted by some pathologies or when high performance is pursued, these two assumptions should be verified in a case-by-case basis.

The parameter r ensures the robustness of the results even in the presence of errors in the measurements or in the calibration of the motion capture devices. With a high value of r, the criterion will be more sensitive to errors on the measurements, and conversely, with a low value, the criterion will be more sensitive to errors on the evaluation of the forces. The parameter δ will impact the smoothness of the load transfer. A low value will lead to irregularities and jumps in the COP position, and the estimated position with a too high value might not reach the boundaries of the foot. The cut-off frequency ω also affects the smoothness of the result sand its definition depends on the trade-off between the loss of information and the filtering of the noise.

5 Conclusion

We propose a method to compute the GRF and GRM for healthy gait tests. The method is based on a simple geometrical model of the foot, unilateral constraints and their Lagrange multipliers are used to estimate them alongside a mixed criterion for the activation of the constraints.

The method produces reliable results, and future work will address 3D problems and other cases like pathological gait or running.

References

[1] B. Brogliato. Nonsmooth Mechanics. Springer-Verlag, London, 1999.

[2] J. García de Jalón and M.D. Gutiérrez-Lopez. Multibody dynamics with redundant constraints and singular mass matrix: existence, uniqueness, and determination of solutions for accelerations and constraint forces. Multibody System Dynamics 30, 311-341, 2013.

[3] P. Alart and A. Curnier. A mixed formulation for frictional contact problems to Newton like solution. Computer Methods in Applied Mechanics and Engineering 92, 353-375, 1991.