Finite Element Modeling of Thin Conductors in Frequency-Domain

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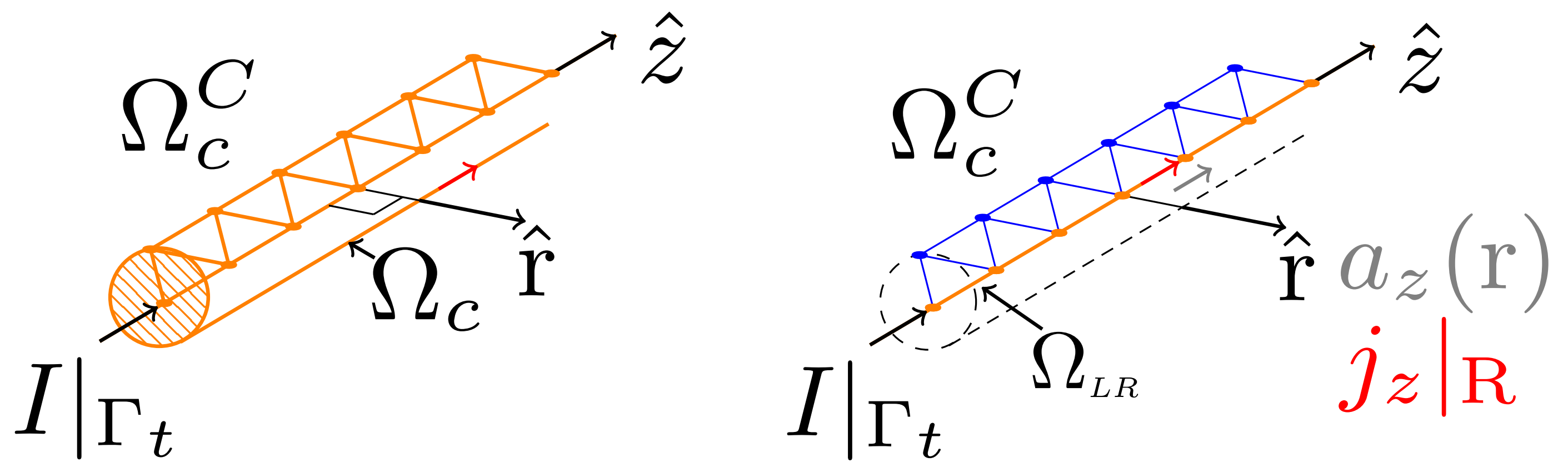
Introduction

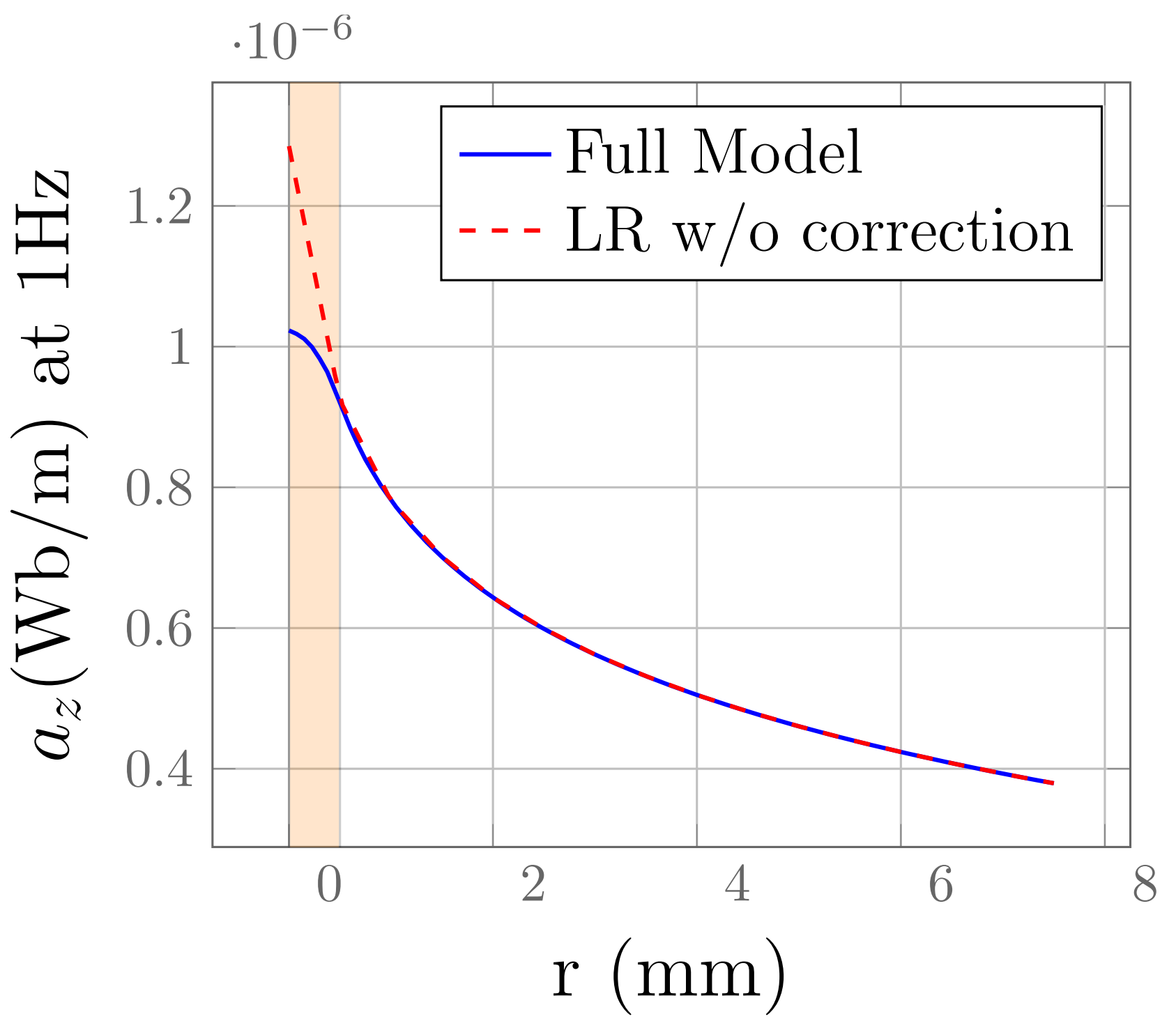
This paper describes an approach for the efficient and accurate modeling of thin conductors, based on the cancellation of the local mesh-dependent peak in the field, which follows from the 1D idealization of the wire, and the reintroduction of the details of the field distribution inside the wire, accounting for skin and proximity effect, by means of analytical solutions.

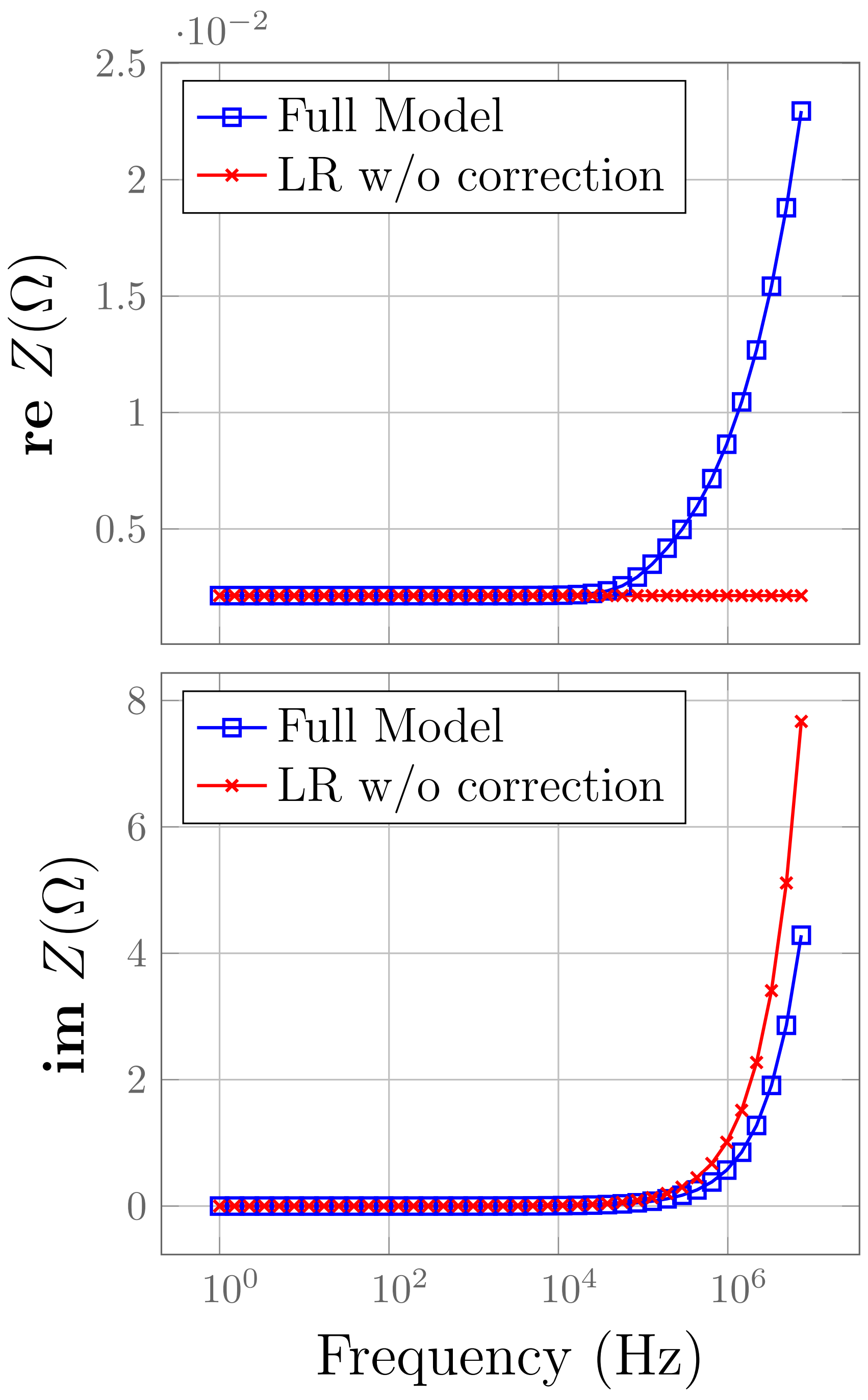
DESCRIPTION OF THE PROBLEM

The issue with the 1D thin wire idealization resides thus in that it results in the non-physical peak of the ***a***-field observed in the vicinity of the conducting edges. This mesh-dependent peak indefinitely grows in amplitude as the mesh is refined.



Furthermore, there is a substantial discrepancy in resistance, and reactance between the full model and the approximation. This manifests the need for a more accurate representation of thin wires in finite element models.



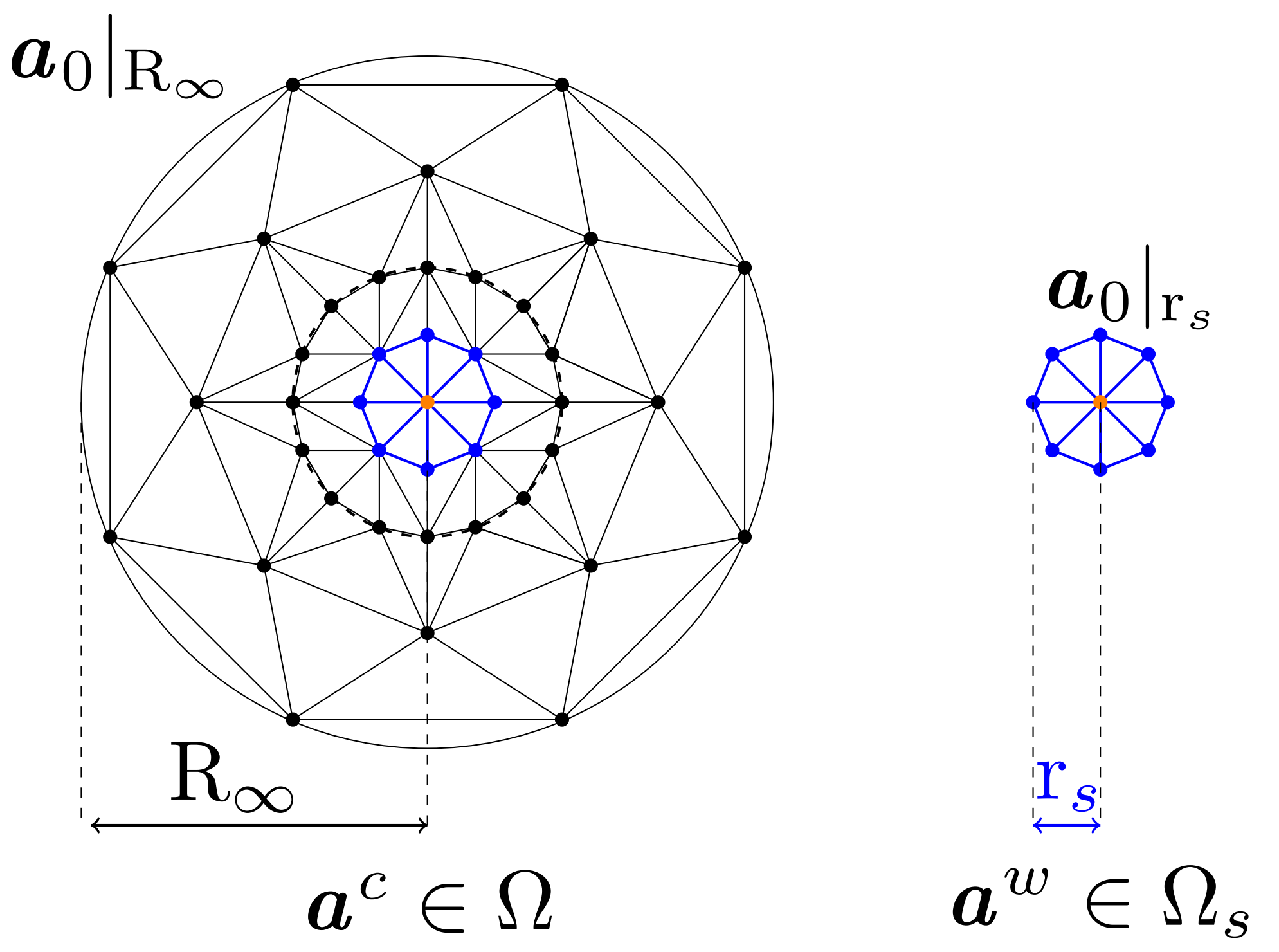


The impedance may be calculated analytically but a correction of the local field is required

$$Z = -\frac{\mu\omega l}{2\pi}\frac{i}{kR}\frac{J_0(k\,r)}{J_1(k\,R)} + i\omega\frac{\mu l}{2\pi}\left(\log(\frac{2l}{R} - 1)\right)$$

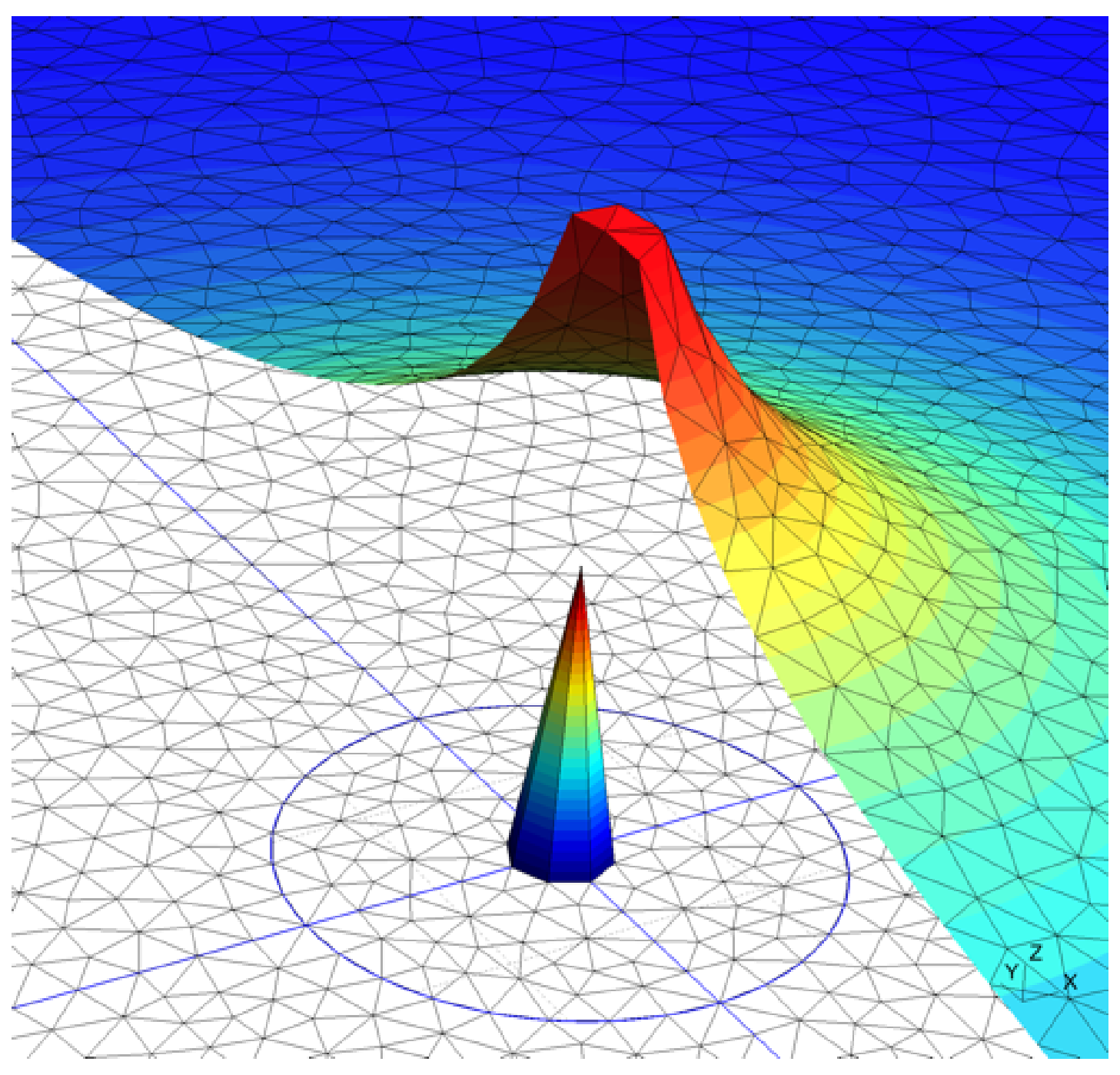
LOCAL FIELD CORRECTION

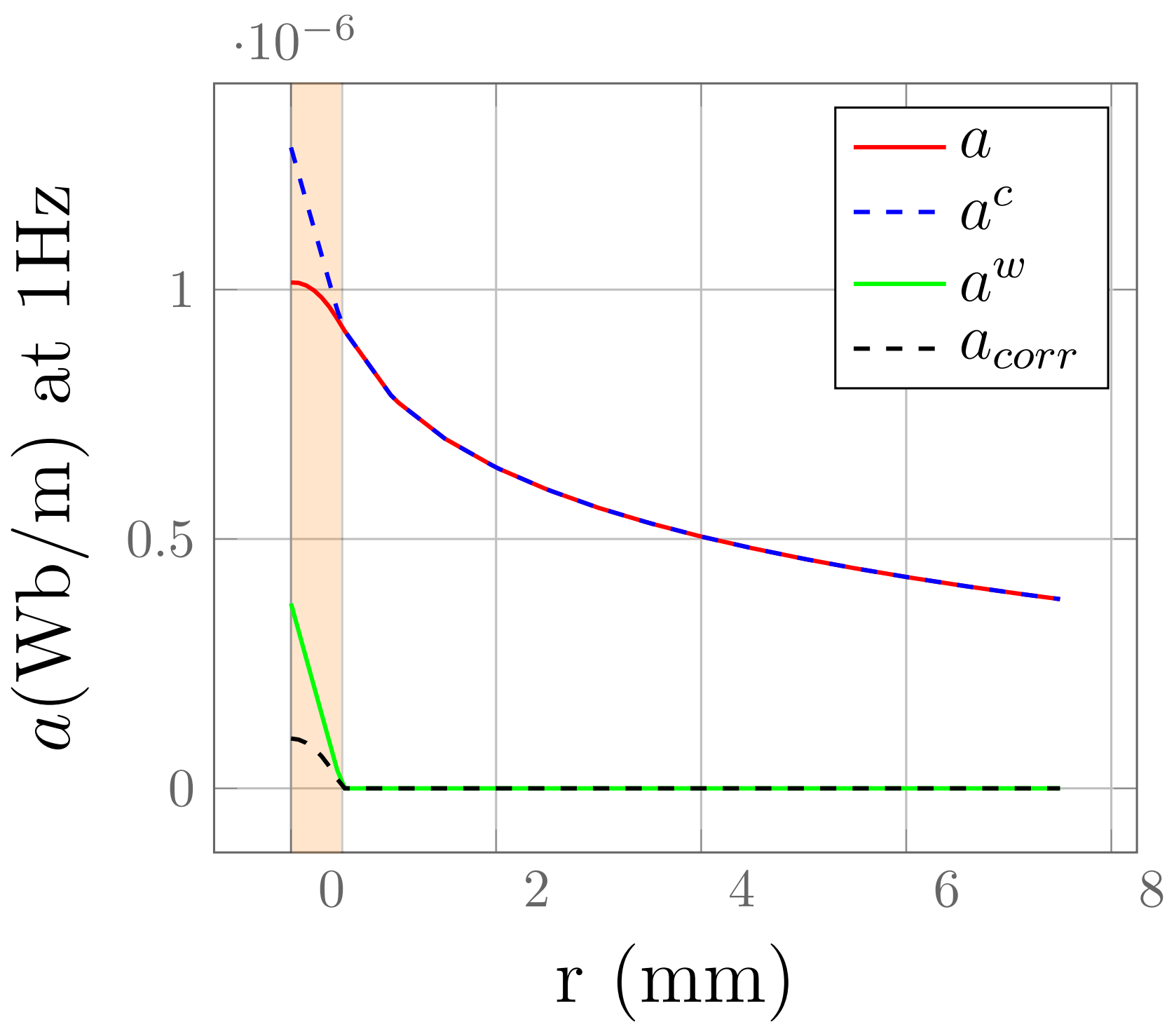
The proposed solution consists in solving an auxiliary local boundary value problem on a one-element-thick layer of finite elements adjacent to the conducting line region Ω_{LR}. We call "sleeve" this cylindrical region, and denote it by Ω_s, such that $\Omega_{LR} \subset \Omega_s$



$$\boldsymbol{a} = \boldsymbol{a}^c - \boldsymbol{a}^w + \boldsymbol{a}_{corr}$$

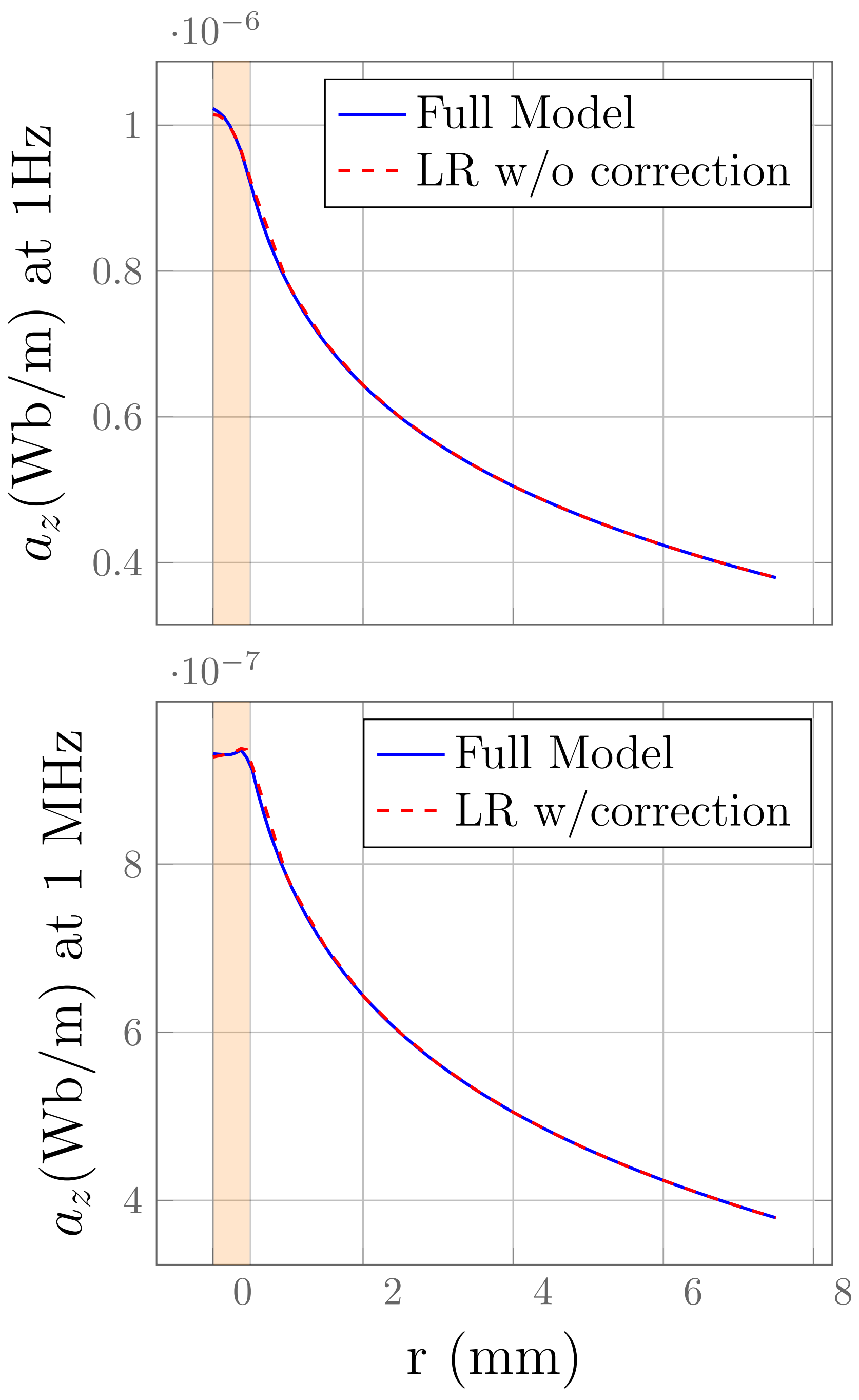
The idea behind the correction is that $\boldsymbol{a}^c$ and $\boldsymbol{a}^w$ contain the same mesh-dependent unphysical peak, and that the peaks are identical, up to a nearly uniform field, because they are computed using the same finite elements.

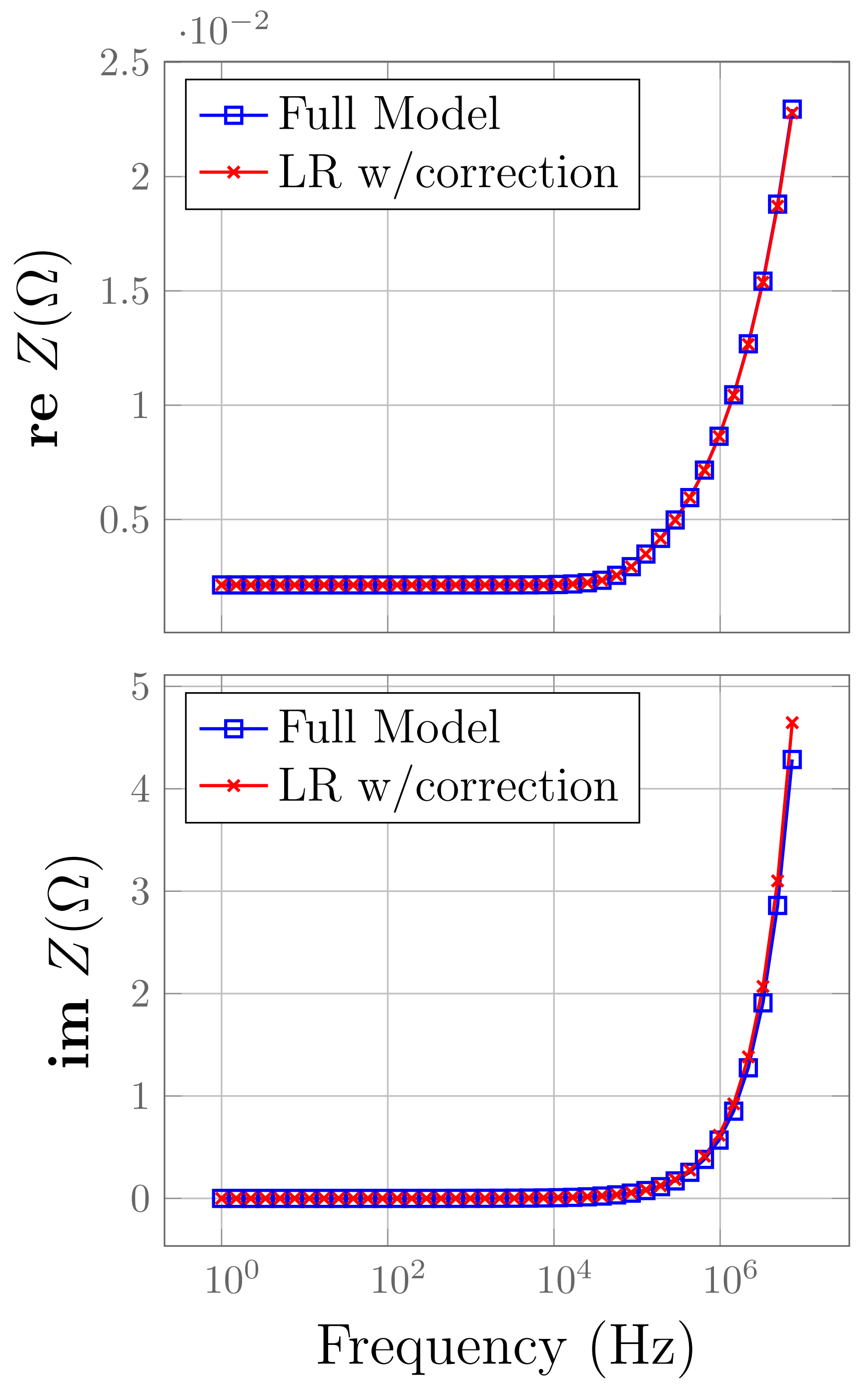




$$\boldsymbol{a}_{corr}(\mathrm{r}) = -\frac{\mu_0 \hat{I}}{2\pi}\left(\frac{\mu_r}{\tau}\frac{J_0(\tau\frac{\mathrm{r}}{\mathrm{R}}) - J_0(\tau)}{J_1(\tau)} + \log\left(\frac{\mathrm{r}_s}{\mathrm{R}}\right)\right)$$

RESULTS FOR STRAIGHT WIRE MODEL





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