Finite Element Modeling of Thin Conductors in Frequency-Domain

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This paper describes a method for the modeling of thin wires in the context of time-harmonic finite element simulations. The approach exploits the exact solution of the infinitely long round wire problem to correct the edge element discretization of a magnetic vector potential formulation, which enables edges in a finite element mesh to be modeled directly as thin conductors.

***Index Terms*—Boundary conditions, conducting domains, edge elements, massive inductors, thin wire**

I. Introduction

In the presence of domains with high aspect ratio, the mesh element size is constrained by the smallest dimensions of the geometrical regions within the model. Hence, the conventional finite element modeling of thin conducting regions necessarily requires dense discretizations. But the challenge in modeling thin conducting domains arises not only from the geometrical constraints, but also from the field problem itself. In the case of thin wires made out of linear isotropic materials it is safe to assume that as frequency increases, the electric current flows mainly on the outer surface of the conductor due to the skin effect. Mesh refinement within the conductor solves the inaccuracies in the solution but, at higher frequencies the problem becomes computationally prohibitive. Due to the computational cost of modeling thin conducting structures, it is appealing to formulate new techniques that address the aforementioned challenges by substituting the detailed problem by a known one.

This paper describes an approach for the efficient and accurate modeling of thin conductors, based on the cancellation of the local mesh-dependent peak in the field, which follows from the 1D idealisation of the wire, and the reintroduction of the details of the field distribution inside the wire, accounting for skin and proximity effect, by means of analytical solutions.

II. Description of the Thin Wire Problem

The problem addressed to explain the method is that of a thin wire composed by a sequence of edges of the tetrahedral mesh connected by their end-nodes, known as Line Regions (LR) [1]. The computational domain $\Omega := \Omega_c^C \cup \Omega_{LR}$ is defined as the union of the conducting domain Ω_{LR}, and the complementary non-conducting domain Ω_c^C. The wire is supplied with a sinusoidal current source $I = \hat{I}\cos(\omega t + \theta)$, where $\hat{I}$ is the current peak value in amps, ω the angular frequency in rad/s, t the time in seconds, and θ the phase shift in radians.

In Fig. 1 a line plot presenting the distribution of the magnetic vector potential $\boldsymbol{a}_z$ is shown. The curve labelled "Full Model" is obtained with a fine conventional discretisation of the wire with volume finite elements, whereas the "LR w/o correction" implies the use of idealized conducting edges. The orange rectangle highlights the area where the conductor is supposed to be placed.

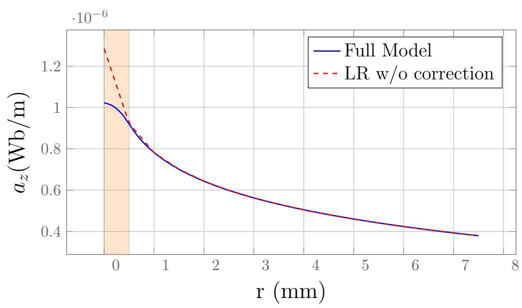

Fig. 1. Magnetic vector potential along the r-axis at 1Hz.

The issue with the 1D thin wire idealisation resides thus in that it results in the non-physical peak of the $\boldsymbol{a}$-field observed in Fig. 1 in the vicinity of the conducting edges. This mesh-dependent peak indefinitely grows in amplitude as the mesh is refined, and makes the computation of the flux embraced by the wire and therefore of the impedance (which are both mesh-independent quantities in principle) extremely inaccurate. Figure 2 showcases the discrepancy in resistance (because of the neglection of skin effect), and reactance (because of the presence of the peak explained above) between these two models. This manifests the need for a more accurate representation of thin wires in finite element models.

III. Local Field Correction

The proposed solution consists in solving an auxiliary local boundary value problem on a one-element-thick layer of finite elements adjacent to the conducting line region Ω_{LR}. We call "sleeve" this cylindrical region, and denote it by Ω_s. The formulation solved on Ω_s is identical to the formulation of the general problem, except that a homogeneous Dirichlet boundary conditions $\boldsymbol{a} = 0$ is imposed on the external boundary of the sleeve.

The details of the final $\boldsymbol{a} - v$ formulations will be given in the extended paper. In this abstract it is enough to say that the corrected $\boldsymbol{a}$-field writes

$$\boldsymbol{a} = \boldsymbol{a}^c - \boldsymbol{a}^w + \boldsymbol{a}_{corr}, \qquad (1)$$

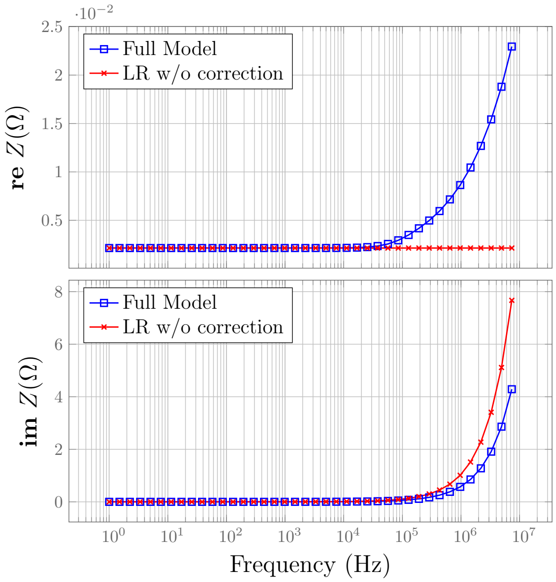

Fig. 2. Impedance of straight wire: Resistance (Top), Reactance (Bottom).

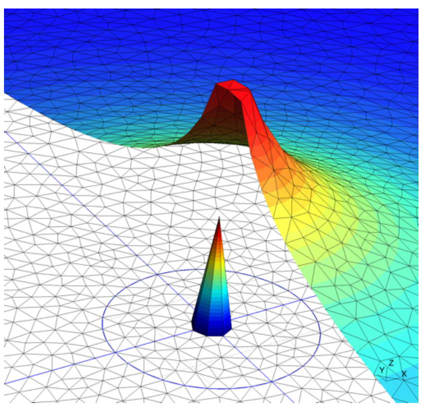

Fig. 3. Solution $\boldsymbol{a}^w$ of the local auxiliary problem, and truncated $\boldsymbol{a}^c - \boldsymbol{a}^w$ field.

where $\boldsymbol{a}^c$ is the solution of the global problem, $\boldsymbol{a}^w$ is the local solution of the same problem on the sleeve Ω_s, and $\boldsymbol{a}_{corr}$ is a correction term depending on the analytical solution around and inside a straight thin wire, of infinite length and radius R, assuming zero field at an arbitrary distance $\mathrm{R}_\infty >> \mathrm{R}$ from the wire.

The idea behind the correction (1) is that $\boldsymbol{a}^c$ and $\boldsymbol{a}^w$ contain the same mesh-dependent unphysical peak, and that the peaks are identical, up to a nearly uniform field, because they are computed using the same finite elements. The peak in the computed field, whose support is Ω_s, is thus cancelled out by subtraction, and the resulting field $\boldsymbol{a}^c - \boldsymbol{a}^w$ is exact, up to discretization errors, outside Ω_s. Fig. 3 shows the solution $\boldsymbol{a}^w$ of the local auxiliary problem, and the truncated $\boldsymbol{a}^c - \boldsymbol{a}^w$ field. The third term in (1) restores the local distribution of the field, knowing the real radius of the wire (not represented in the mesh), the value of the current, and the local radius r_s of the sleeve, assumed here to be larger than the actual wire radius, $r_s > \mathrm{R}$. The analytical solution to be used in the expression of $\boldsymbol{a}_{corr}$ depends on the problem solved. One has, for a magnetodynamic problem,

$$\boldsymbol{a}_{corr}(\mathrm{r}) = -\frac{\mu_0 \hat{I}}{2\pi}\left(\frac{\mu_r}{\tau}\frac{J_0(\tau\frac{\mathrm{r}}{\mathrm{R}}) - J_0(\tau)}{J_1(\tau)} + \log\left(\frac{\mathrm{r}_s}{\mathrm{R}}\right)\right)\hat{z}, \quad (2)$$

for $r \leq \mathrm{R}$, where $\tau = \sqrt{2} i^{3/2}\mathrm{R}/\delta$, and where δ is the skin depth.

The calculations have been performed for a radius R=0.5mm and a current $\hat{I}$=1A. All calculations have been performed using ONELAB software (Gmsh [5] and GetDP [4]).

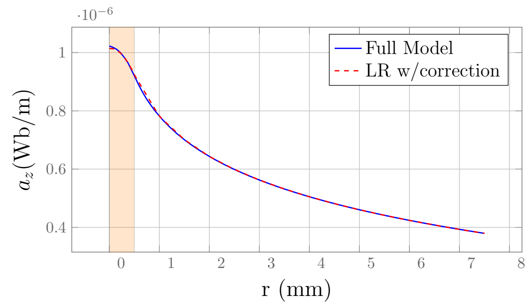

Fig. 4. Magnetic vector potential along the r-axis at 1Hz.

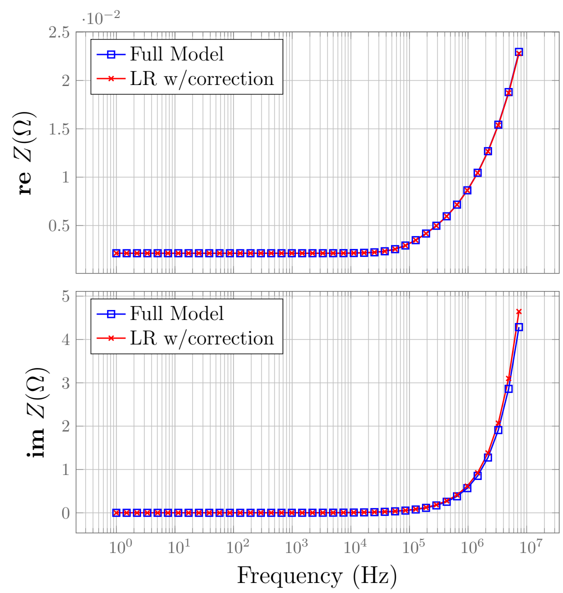

Fig. 5. Impedance of straight wire: Resistance (Top), Reactance (Bottom).

References

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