Modelling of the permeability evolution of coal due to sorption

François BERTRAND

Supervised by Prof. Olivier BUZZI (UoN)
and Dr Frédéric COLLIN (ULiege)

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Introduction

2 distinct phenomena affecting permeability:

Pressure depletion → Reservoir compaction → Cleat permeability ↓

Gas desorption → Coal matrix shrinkage → Cleat permeability ↑

[Gray et al., 1987]

Unconventional models

Coalbed methane (CBM) = unconventional resource
Introduction

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[Gray et al., 1987]

Unconventional models
Microscale Model
Tomography imaging

[Jing et al., 2016]
Microscale Model

Introduction

Tomography imaging

[Jing et al., 2016]

Direct Modelling
Microscale Model

Introduction

Tomography imaging

[Jing et al., 2016]

Cleat permeability alteration due to sorption
**Microscale Model**

**Permeability**

**Navier-Stokes** between two parallel plates

(Laminar flow, Steady state conditions & No body force)

Non-slip boundary conditions at the walls:

\[ v \left( x_2 = \pm \frac{h_b}{2} \right) = 0 \]

\[ \Rightarrow v_1(x_2) = \text{Parabolic profile} \]

\[ q = \langle v_1 \rangle = \frac{1}{h_b} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}} v_1 \, dx_2 = -\frac{h_b^2}{12} \cdot \frac{1}{\mu} \int dx_1 \]

\[ \Rightarrow \text{Darcy permeability} \quad k = \frac{h_b^2}{12} \]

+ Relative permeability curves to take into account **multi-phases flow** (retention curve required)
Microscale Model
Fracture aperture

Darcy permeability $k = \frac{h_b^2}{12}$

The hydraulic aperture ($h_b$) is related to the mechanical aperture ($h$) as:

$$h_b = h_{\text{min}} + h$$

$\text{Modified from [Marinelli et al., 2016]}$
Variation of the **stress state** \((\dot{\sigma}')\) impacts the mechanical **fracture aperture** \((h)\):

\[
\dot{h}(x) = \frac{\dot{\sigma}'_{(xx)}}{K_{n(x)}}
\]

The **hydraulic aperture** \((h_b)\) is related to the **mechanical aperture** \((h)\) as:

\[
h_b = h_{\text{min}} + h
\]

*Modified from [Marinelli et al., 2016]*
Variation of the stress state ($\dot{\sigma}'$) impacts the mechanical fracture aperture ($h$):

$$\dot{h}(x) = \frac{\dot{\sigma}'_{(xx)}}{K_n(x)}$$

where $K_n$ is the normal stiffness of the fracture:

$$K_n = \frac{K_n^0}{\left(1 - \frac{h_0 - h}{h_0}\right)^2}$$


Modified from [Cerfontaine et al., 2015]
A swelling/shrinkage of the matrix is induced by the sorption/desorption.

Volumetric sorption-strain in the matrix assumed proportional to the adsorbed gas content [Cui and Bustin, 2005]:

\[ \varepsilon_{Vs} = \beta \varepsilon \cdot V_{g,Ad} \]

Constitutive mechanical model for the matrix: Isotropic elastic law (2 parameters: e.g. \( E_m, \nu_m \))
The adsorbed gas content in the matrix $V_{g,Ad}$ depends on the pressure in the cleats $p$. 
The **adsorbed gas content** in the matrix $V_{g,Ad}$ depends on the pressure in the cleats $p$.

**Langmuir’s isotherm** (equilibrium equation):

$$V_{g,Ad} = \frac{V_L \cdot p}{P_L + p} [m^3/t]$$

[Langmuir, 1918]
Microscale Model

Adsorbed gas content

**Transversal flow** (Matrix ↔ Cleat) \( \propto p_{Ad}^b - p_{Ad} \)

**Gas diffusion** in the matrix (Fick’s law) \( \propto \nabla p_{Ad} \)

**Langmuir’s isotherm** (equilibrium equation):

\[
V_{g,Ad} = \frac{V_L \cdot p}{P_L + p} \quad \text{[m}^3\text{/t]} \quad \rightarrow \quad p_{Ad}^b
\]

[Langmuir, 1918]
Microscale Model
Summary

- Matrix
  - Mechanical model
    - Isotropic elastic law: $E_m, \nu_m$
  - Hydraulic model
    - Fick’s diffusion law: $D^q_m$
  - Hydro-mechanical coupling
    - Sorption strain: $\beta_\varepsilon$
Matrix

- Mechanical model
  - Isotropic elastic law: $E_m, \nu_m$
- Hydraulic model
  - Fick’s diffusion law: $D_m^q$
- Hydro-mechanical coupling
  - Sorption strain: $\beta_\varepsilon$

Cleats

- Mechanical model
  - (Stick state) $K_n(h), K_s$
  - (Slip state) $+c, \mu$
- Hydraulic model
  - Darcy’s law with $k = \frac{h_b^2}{12}$
  - Hydro-mechanical couplings
    - $h_b = h^{min} + h$
    - $\dot{M}_g(h_b)$
    - $\sigma' = \sigma + p$
Microscale Model

Summary

Matrix
- Mechanical model
  Isotropic elastic law: $E_m, \nu_m$
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  Fick’s diffusion law: $D_m^g$
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  Sorption strain: $\beta_e$

Cleats
- Mechanical model
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  $h_b = h_{\text{min}} + h$
  $\dot{M}_g(h_b)$
  $\sigma' = \sigma + p$

Matrix $\rightarrow$ Cleats
- Hydraulic model
  Langmuir’s isotherm $V_L, P_L$

Model implemented in the **FE Lagamine code**
Laboratory Modelling

**Objective** = validation of the microscale model by comparison between the prediction of the evolution of the permeability and its measurement.
Boundary conditions and loading:

- **Free displacements**
- **Gas pressure increased by steps** (imposed dof)
- **Corresponding total stress applied** (imposed force)
Laboratory Modelling

Swelling test

Model calibration

Evolution of the volumetric strain with time

Evolution of the stabilized volumetric strain with pressure
Model calibration

Evolution of the volumetric strain with time

![Graph showing the evolution of the volumetric strain with time.]

Evolution of the stabilized volumetric strain with pressure

![Graph showing the evolution of the stabilized volumetric strain with pressure.]

François BERTRAND
Laboratory Modelling

Permeability test

**Permeability test**

2 x 1 x 3cm

Boundary conditions and loading:

- **Constant volume** (fixed boundaries)
- Constant gas pressure at the top (fixed dof)
- **Gas pressure increased** by steps at the **bottom**

![Graph showing pressure increase over time](image)
Laboratory Modelling

Permeability test

Adsorbed pressure [Pa]

Boundary pressure

P [kPa]

Time [Days]

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Coal behaviour modelling
Laboratory Modelling

Permeability test

Adsorbed pressure [Pa]

Swelling + Constant volume

Fracture closure
Fracture aperture evolution

Fracture permeability evolution

Laboratory Modelling

Permeability test

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Development of a numerical model at the scale of the fractures and matrix blocks.

Being validated by an experimental laboratory campaign.

As is, the model only usable for laboratory tests modelling (due to computational expense).

What about the reservoir scale?
Macroscale model
Mass exchange matrix → cleats:

\[ E = \frac{1}{\tau} \frac{M_g}{RT} (p_{g,m} - p_{g,m}^{\text{lim}}) \]

Sorption time:

\[ \tau = \frac{1}{\Psi D_m^g} \]

- Diffusion coefficient in the matrix \( D_m^g \)
- Shape factor \( \Psi(w) \)

\[ \Psi = \pi^2 \left( \frac{1}{w_1^2} + \frac{1}{w_2^2} + \frac{1}{w_3^2} \right) \]

[Lim and Aziz, 1995]
Mass exchange matrix → cleats:

\[ E = \frac{1}{\tau} \frac{M_g}{RT} (p_{g,m} - p_{g,m}^{\text{lim}}) \]

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[Lim and Aziz, 1995]
Hydraulic equivalent medium

Permeability Homogenization

\[ k = \frac{h_b^2}{12} \]

\[ k = \frac{h_b^3}{12w} \]
Macroscale model
Mechanical equivalent medium

\[ \dot{\sigma}_{ij} = C_{ijkl} \dot{\epsilon}_{kl} \]

\[ E_x = \frac{1}{E_m} \left( 1 + \frac{1}{K_n \cdot w_x} \right) \]

\[ K_n = \frac{K_n^0}{\left( 1 - \frac{h_0 - h}{h_0} \right)^2} \]

Isotropic elastic matrix: \( E_m, \nu_m \)
Nonlinear elastic fractures: \( K_n, K_s \)

Orthotropic nonlinear elastic equivalent medium

[Bandis et al., 1983]
Horseshoe Canyon case (Dry reservoir) [Gerami et al., 2007]
Macroscale Model

Conclusions

**Consistent macroscale model** enriched with microscale aspects

[Bertrand et al., 2017]

**Remarkable features:**

- **Dual-continuum** approach for both mechanical and hydraulic behaviours.
- Not instantaneous gas desorption from the matrix.
- Kinetics of the gas transfer based on **shape factor** and **Langmuir**'s isotherm.
- **Desorption strain** not necessarily fully converted into a fracture opening.
- **Permeability evolution** directly linked to the fracture aperture.
- **Multiphase** flows in the fractures.

But could we go further **avoiding macroscale laws**?
Multiscale Model
Multiscale Model
Overview

$FE^2$ approach

1. Localization
2. Microscale BVP
3. Homogenization
4. Macroscale BVP
Conclusions

**Microscale**
Highly accurate but **computationally expensive**
- Laboratory modelling only

**Macroscale**
Suitable for reservoir modelling but **less accurate**

**Multiscale**
**Compromise** solution
Thank you for your attention!

Modelling of the permeability evolution of coal due to sorption

Researches supported by the FNRS - FRIA and the WBI


*Synthèse des propriétés chimiques et physiques des houilles.*
Institut National de l’Industrie Charbonnière.

Volumetric strain associated with methane desorption and its impact on coalbed gas production from deep coal seams.

Type curves for dry cbm reservoirs with equilibrium desorption.

*Methods of geological engineering in discontinuous rocks.*


Microscale
## Triaxial/UCS test

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Matrix</strong></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_c , (kg/m^3)$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_m , (Pa)$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$v_m$</td>
</tr>
<tr>
<td>Width</td>
<td>$w , (m)$</td>
</tr>
<tr>
<td><strong>Cleat</strong></td>
<td></td>
</tr>
<tr>
<td>Initial normal stiffness</td>
<td>$K_n^0 , (Pa/m)$</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>$K_s , (Pa/m)$</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Cohesion</td>
<td>$c , (Pa)$</td>
</tr>
<tr>
<td>Initial mechanical aperture</td>
<td>$h_0 , (m)$</td>
</tr>
</tbody>
</table>
Swelling & Permeability tests

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<thead>
<tr>
<th>Parameters</th>
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<tbody>
<tr>
<td><strong>Matrix</strong></td>
<td></td>
</tr>
<tr>
<td>Langmuir volume</td>
<td>$V_L \ (m^3/kg)$</td>
</tr>
<tr>
<td>Langmuir pressure</td>
<td>$P_L \ (Pa)$</td>
</tr>
<tr>
<td>Swelling coefficient</td>
<td>$\beta_\varepsilon \ (kg/m^3)$</td>
</tr>
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<td>$h_0 \ (m)$</td>
</tr>
<tr>
<td>Minimal hydraulic aperture</td>
<td>$h_{b}^{min} \ (m)$</td>
</tr>
</tbody>
</table>
\[ \sum_{k=1}^{2} \left[ \int_{\Omega^k} \sigma_{ij} \frac{\partial v_i^*}{\partial x_j} \, d\Omega \right] = \sum_{k=1}^{2} \left[ \int_{\Gamma_{t_i}^k} t_i \, v_i^* \, d\Gamma + \int_{\Gamma_{c}^k} T_{i}^k \, v_i^* \, d\Gamma \right] \]
Hydraulic problem

Matrix:
\[ \frac{\partial}{\partial t} (\rho \gamma, Ad) + \frac{\partial}{\partial x_i} (J_{mi}^g) = 0 \]

Cleats:
\[ \frac{\partial}{\partial t} (\rho g, f (1 - S_r) h_b) + \frac{\partial}{\partial x_1} (\rho g, f h_b q_{gL} + (1 - S_r) h_b J_{g1}^g) + \frac{\partial}{\partial x_2} \left( \rho g, f h_b (q_{gT}^1 - q_{gT}^2) \right) = 0 \]
\[
\sum_{k=1}^{3} \left[ \int_{\Omega^k} \dot{S}_g \ p_g^* - f_{gi} \ \frac{\partial p_g^*}{\partial x_i} \ d\Omega \right] = \sum_{k=1}^{3} \left[ \int_{\Gamma_{\tilde{q}_g}^k} \tilde{q}_g \ p_g^* \ d\Gamma + \int_{\Gamma_{\delta p_g}^k} \tilde{q}_g \ \delta p_g \ d\Gamma \right]
\]
Gas slippage?

**Slip** boundary conditions [Kundt and Warburg, 1875]:

\[ v_1 \left( \frac{h}{2} \right) = -c \bar{l} \frac{dv_1}{dx_2} \bigg|_{x_2 = \frac{h_b}{2}} \]

\[ \Rightarrow v_1(x_2) = -\frac{1}{2\mu} \left( c \bar{l} h_b + \left( \frac{h_b}{2} \right)^2 - x_2^2 \right) \frac{dp}{dx_1} \]

(Laminar flow, Steady state conditions & No body force)
Fractures

Hydraulic behaviour - Longitudinal permeability

Gas slippage?

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(Laminar flow, Steady state conditions & No body force)

\[ q = \langle v_1 \rangle = \frac{1}{h_b} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}} v_1 dx_2 = -\frac{h_b^2}{12} \left( 1 + \frac{6c\bar{I}}{h_b} \right) \cdot \frac{1}{\mu} \frac{dp}{dx_1} \Rightarrow k = \frac{h_b^2}{12} \cdot f_c \]
Fractures
Hydraulic behaviour - Longitudinal permeability

\[
f_c = \left( 1 + \frac{6c\bar{l}}{h_b} \right)
\]

- **Constant** $c \approx 1$

**Gas mean free path** $\bar{l}$

\[
\bar{l} = \frac{k_B T}{\sqrt{2 \pi} d_g^2 p}
\]

- $d_g = 380 \cdot 10^{-12} m$ (Methane)
- $p = 1 MPa$
- $T = 303 K$

$\rightarrow \bar{l} = 6.52 \cdot 10^{-9} m$

**Hydraulic aperture** $h_b$

- $h_b = 1 \cdot 10^{-5} m$

where $p [Pa]$ is the gas pressure, $d_g$ the collision diameter of the gas molecule, $k_B$ the Boltzmann constant and $T[K]$ the temperature.
Fractures
Hydraulic behaviour - Longitudinal permeability

\[ f_c = \left(1 + \frac{6c\bar{l}}{h_b}\right) \]

- Constant \( c \approx 1 \)

- **Gas mean free path** \( \bar{l} \)
  \[ \bar{l} = \frac{k_B T}{\sqrt{2} \pi d_g^2 p} \]
  - \( d_g = 380 \cdot 10^{-12} m \) (Methane)
  - \( p = 1 MPa \)
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  - \( h_b = 1 \cdot 10^{-5} m \)

where \( p [Pa] \) is the gas **pressure**, \( d_g \) the **collision diameter** of the gas molecule, \( k_B \) the **Boltzmann** constant and \( T [K] \) the **temperature**.
Fractures
Hydraulic behaviour - Longitudinal permeability

\[ f_c = \left( 1 + \frac{6c\bar{l}}{h_b} \right) = 1.004 \]

- Constant \( c \approx 1 \)

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\[ \bar{l} = \frac{k_B T}{\sqrt{2} \, \pi \, d_g^2 \, p} \]

- \( d_g = 380 \cdot 10^{-12} \text{m} \) (Methane)
- \( p = 1 \text{MPa} \)
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- **Hydraulic aperture** \( h_b \)

- \( h_b = 1 \cdot 10^{-5} \text{m} \)
Two-phase flow model?

Integration of Navier-Stokes in each stratum (considering the same velocity at the interface) [Yuster et al., 1951].

Relative permeabilities:

\[ k_{rw} = \frac{S_r^2}{2} (3 - S_r) \quad \quad \quad k_{rg} = (1 - S_r)^3 \]
Saturation degree $S_r$?

\[ s = p_e \cdot (S_r^*)^{\frac{1}{\lambda}} \]

[Brooks and Corey, 1964]
Saturation degree $S_r$?

$$s = p_e \cdot (S_r^*)^{-\frac{1}{\lambda}}$$

[Brooks and Corey, 1964]
Saturation degree $S_r$?

Fractality geometry of the wall

$\rightarrow$ Fractal distribution of the number of units $N$ whose radius is larger than $r$:

$$N(r) = a \cdot r^{-D_f}$$

where $D_f$ is the fractal dimension and $a$ is a constant of proportionality.

$\rightarrow$ $p_c = p_e \cdot (S_r)^{-\frac{1}{3-D_f}}$
Fractures
Hydraulic behaviour - Retention curve

Saturation degree $S_r$?

**Fractal geometry** of the wall

→ Fractal distribution of the number of units $N$ whose radius is larger than $r$:

$$N(r) = a \cdot r^{-D_f}$$

where $D_f$ is the **fractal dimension** and $a$ is a constant of proportionality.

→ $p_c = p_e \cdot (S_r)^{-\frac{1}{3-D_f}}$
Macroscale
Hydraulic model

Mass balance equation

\[
\begin{align*}
\frac{\partial}{\partial t} (\rho_{g,f} (1 - S_r) \phi_f) &+ \frac{\partial}{\partial x_i} (\rho_{g,f} q_{gi} + (1 - S_r) J_{g_i}^g) \\
+ \frac{\partial}{\partial t} (\rho_{g,f}^d S_r \phi_f) &+ \frac{\partial}{\partial x_i} (\rho_{g,f}^d q_{li} + S_r J_{g_i}^g) = E
\end{align*}
\]

Gas phase

Dissolved gas in water phase

\[\frac{\partial}{\partial t} (\rho_{g,Ad}) = -E\]
Hydraulic model
Matrix - Langmuir’s isotherm

\[ G \propto p \]
\[ O^- \propto 1 - \theta \]
\[ \Rightarrow O^- - G \propto \theta \]
\[ \theta: \text{surface coverage of adsorbed molecules} \]
\[ p: \text{pressure of gas} \]

\[ K = \frac{[O - G]}{[G] \cdot [O^-]} \Rightarrow k = \frac{\theta}{(1 - \theta) \cdot p} \Rightarrow \theta = \frac{k \cdot p}{1 + k \cdot p} \]

\[ V_{g,Ad} = \theta \cdot V_L, \text{where } V_L \text{ is the monolayer adsorption capacity} \]

\[ \Rightarrow V_{g,Ad} = \frac{V_L \cdot p_{res}}{P_L + p_{res}} \]

Figure: Data published by [Coppens, 1967].
Hydraulic model
Matrix → Cleats - Analytical solution

\[ \dot{p}_{g,m}(t) = -\frac{1}{\tau} \cdot (p_{g,m}(t) - p_{g,m}^{\lim}(t)) \]

Solution for constant \( p_{g,m}^{\lim} \):

\[ p_{g,m}(t) = (p_{g,m}^0 - p_{g,m}^{\lim}) \cdot \exp\left(\frac{-t}{\tau}\right) + p_{g,m}^{\lim} \]

Solution for the linear evolution of \( p_{g,m}^{\lim} \) (slope \( a \)):

\[ p_{g,m}(t) = -a \tau \exp\left(\frac{-t}{\tau}\right) + a (\tau - t) + p_{g,m}^0 \]
Hydraulic model
Matrix → Cleats - Analytical solution

\[ \dot{p}_{g,m}(t) = -\frac{1}{\tau} \cdot (p_{g,m}(t) - p_{g,m}^{\text{lim}}(t)) \]

Hyperbolic evolution of \( p_{g,m}^{\text{lim}} \)
Hydraulic model

Matrix → Cleats - Analytical solution

\[ V_L = 0.02 \text{m}^3/\text{kg} \]

\[ P_L = 1.5 \text{MPa} \]

\[ \tau = 3 \text{days} \]
2 distinct phenomena affecting permeability:

- Pressure depletion $\rightarrow$ Reservoir compaction $\rightarrow$ Cleat permeability $\downarrow$
- Gas desorption $\rightarrow$ Coal matrix shrinkage $\rightarrow$ Cleat permeability $\uparrow$

\[
\phi_f = \phi_{f_0} \exp\{-c_f (\sigma - \sigma_0)\}
\]

where $c_f$ is the cleat compressibility.

\[
\Rightarrow k_f = k_{f_0} \exp\{-3c_f (\sigma - \sigma_0)\}
\]

[Seidle et al., 1992]
Hydraulic model
Cleats - Unsaturated conditions

\[ k_e = k_r(S_r) \cdot k \]

\[ k_{rw} = (S^*_r)^{\eta + 1 + \frac{2}{\lambda}} \]

\[ k_{rg} = (1 - S^*_r)\eta \cdot \left[ 1 - (S^*_r)^{1 + \frac{2}{\lambda}} \right] \]

with \( S^*_r = \frac{S_r - S_{w, res}}{1 - S_{w, res} - S_{g, res}} \)

\[ = \frac{S_r - S_{w, res} \left( \frac{\phi}{\phi_0} \right)^{-1}}{1 - S_{w, res} \left( \frac{\phi}{\phi_0} \right)^{-1} - S_{g, res} \left( \frac{\phi}{\phi_0} \right)^{-1} \left( \frac{\rho_g}{\rho_{g0}} \right)^{-1}} \]
Hydraulic model
Cleats - Unsaturated conditions

\[ k_e = k_r(S_r) \cdot k \]

\[ k_{rw} = (S_r^*)^\eta \int_0^{S_r} \frac{dS_r}{s^2} \int_0^1 \frac{dS_r}{s^2} \]  
[Mualem, 1976]

\[ s = p_e \cdot (S_r^*)^{-\frac{1}{\lambda}} \]
[Brooks and Corey, 1964]

\[ k_{rg} = (1 - S_r^*)^\eta \cdot \left[ 1 - (S_r^*)^{1 + \frac{2}{\lambda}} \right] \]

\[ k_{rg} = \frac{S_r - S_{w,рез}}{1 - S_{w,рез} - S_{g,рез}} \]
\[ = \frac{S_r - S_{w,рез} \left( \frac{\phi}{\phi_0} \right)^{-1}}{1 - S_{w,рез} \left( \frac{\phi}{\phi_0} \right)^{-1} - S_{g,рез} \left( \frac{\phi}{\phi_0} \right)^{-1} \left( \frac{\rho_g}{\rho_{g,0}} \right)^{-1}} \]
Reservoir modelling

Synthetic reservoir

Initially in the cleats:

\( P_{w0} = 2 \text{ MPa} \)

\( P_{g0} = 2 \text{ MPa} \)

In the matrix:

\( P_g \) on the Langmuir's isotherm

\[
\begin{align*}
 f_w &= T \cdot \rho_w \cdot \frac{k_{rw}}{\mu_w} (P_w - P_{\text{well}}) \\
 f_g &= T \cdot \rho_g \cdot \frac{k_{rg}}{\mu_g} (P_g - P_{\text{well}}) + H \cdot \rho_g \frac{f_w}{\rho_w}
\end{align*}
\]

[Peaceman et al., 1978]
Reservoir modelling

Synthetic reservoir - Reference case

- Water production [m$^3$/day]
- Gas production [m$^3$/day]

Years

0 5 10 15 20 25 30

0 1000 2000 3000 4000

- Water
- Gas
Influence of the depletion rate on the permeability evolution

- Reference case
- 5–years depletion
- 5–years depletion; 90% max. capacity
- 5–years depletion; 10–days first drop; 90% max. capacity
Reservoir modelling
Synthetic reservoir - Production scenario influence

![Graph showing gas production and relative permeability changes over time for different depletion scenarios.](image)
Reservoir modelling

Synthetic reservoir - Production scenario influence

\[ p_{res} = 2 \text{MPa} \]

\[ p_{g,m}^{\max} = \frac{RT}{M_g} \cdot \rho_{g,\text{std}} \cdot \rho_c \cdot \frac{V_L \cdot p_{res}}{P_L + p_{res}} = 1.897 \text{ MPa} \]

\[ p_{\text{res}}^{\text{crit}} = \frac{0.9 \cdot p_{g,m}^{\max} \cdot P_L}{\left( \frac{RT}{M_g} \cdot \rho_{g,\text{std}} \cdot \rho_c \cdot V_L - 0.9 \cdot p_{g,m}^{\max} \right)} = 1.588 \text{ MPa} \]
Reservoir modelling

Synthetic reservoir - Production scenario influence

![Graph showing gas production and relative permeability changes over years for different depletion scenarios.]

**Graph Details:**
- **Reference case**
- 5-years depletion
- 5-years depletion; 90% max. capacity
- 5-years depletion; 10-days first drop; 90% max. capacity

**Gas Production [m$^3$/day]:**
- Years: 0, 5, 10, 15, 20, 25, 30
- Gas Production: 0, 1000, 2000, 3000, 4000, 5000

**Relative Permeability ($k/k_0$):**
- Years: 0, 10, 20, 30
- Values: 0.5, 1, 1.5, 2
## Reservoir modelling

### Synthetic reservoir - Reference case parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seam thickness (m)</td>
<td>5</td>
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<tr>
<td>Reservoir radius (m)</td>
<td>400</td>
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<tr>
<td>Temperature (K)</td>
<td>303</td>
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<tr>
<td>Overburden pressure (Pa)</td>
<td>5E6</td>
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<tr>
<td>Well transmissibility $T$ (m$^3$)</td>
<td>1E-12</td>
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<tr>
<td>Penalty coefficient $\kappa$ (m$^2$.s/(kg.Pa))</td>
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<tr>
<td>Coal density $\rho_c$ (kg/m$^3$)</td>
<td>1500</td>
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<tr>
<td>Matrix Young’s modulus $E_m$ (Pa)</td>
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<tr>
<td>Matrix Poisson’s ratio $\nu_m$</td>
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<tr>
<td>Matrix width $w$ (m)</td>
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<td>Cleat aperture $h$ (m)</td>
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<tr>
<td>Cleat normal stiffness $K_n$ (Pa/m)</td>
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<tr>
<td>Cleat shear stiffness $K_s$ (Pa/m)</td>
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<tr>
<td>Maximum cleat closure ratio</td>
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<tr>
<td>Joint Roughness coefficient $JRC$</td>
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<tr>
<td>Parameters</td>
<td>Values</td>
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<tr>
<td>------------------------------------------------</td>
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<tr>
<td>Sorption time $\tau$ (days)</td>
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<tr>
<td>Langmuir volume $V_L$ ($m^3/kg$)</td>
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<td>Langmuir pressure $P_L$ (Pa)</td>
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<td>Matrix shrinkage coefficient $\beta_e$ ($kg/m^3$)</td>
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<td>Entry capillary pressure $p_e$ (Pa)</td>
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<td>Cleat size distribution index $\lambda$</td>
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<td>Tortuosity coefficient $\eta$</td>
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<td>Initial residual water saturation $S_{r,\text{res}}$</td>
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<td>Residual water saturation exponent, $n_{wr}$</td>
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<tr>
<td>Residual gas saturation</td>
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</tr>
</tbody>
</table>
Reservoir modelling
Synthetic reservoir - Parametric and couplings analysis

- Reference case
- Constant aperture
- Constant permeability
- Constant porosity

Gas Production [$m^3$/day]

Years
Reservoir modelling

Synthetic reservoir - Parametric and couplings analysis
Reservoir modelling

Synthetic reservoir - Parametric and couplings analysis

Days

$\tau = 10$ days

$\tau = 3$ days

$\tau = 1$ day

$\frac{k}{k_0}$
Reservoir modelling
Synthetic reservoir - Parametric and couplings analysis

![Graph showing the relationship between K/k0 and years for different values of beta (β = 0.4 kg/m³, β = 0.2 kg/m³, β = 0.0 kg/m³).](image)

![Graph showing gas production over years for different values of beta (β = 0.4 kg/m³, β = 0.2 kg/m³, β = 0.0 kg/m³).](image)
Multiscale
Multiscale model

Macroscopic scale

Microscopic scale

REV
1. **Macroscopic structure** discretised by finite elements

2. **Macroscopic deformation gradient tensor** computed for each IP from the estimation of the macroscopic nodal displacements relative to the external load

3. **REV** assigned at each macroscopic IP

4. **Localization**: apply appropriate displacements to the REV from the macroscopic deformation gradient tensor

5. **Microscale FE computation**: stress and deformation distributions in the REV

6. **Homogenization**: REV averaged stress returned to the macroscopic IP

7. Macroscopic **internal nodal forces**

8. **Macroscopic stiffness matrix**

9. **Balance** between external load and internal load?

   - **Next time increment evaluated**
   - **Updated estimation** of the nodal displacements required (via macroscopic stiffness matrix)