Optimization of Drone Routing for Humanitarian Applications

Promoteur :
Pr. Thierry PIRONET

Lecteurs :
Pr. Pierre DENEYE
Christian CLAVIJO LÓPEZ

Travail de fin d’études présenté par Thomas LAMBERT en vue de l’obtention du diplôme de Master en sciences de gestion à finalité spécialisée en management général Année académique 2018/2019
Optimization of Drone Routing for Humanitarian Applications
Acknowledgements

I would like to express my deepest appreciation to all the people that helped me during the realization of this project.

In particular, I would like to thank my supervisor, Pr. Thierry Pironet for giving me the opportunity to work on this thesis. I am forever grateful for his guidance, his wise advice and for being available to answer my numerous questions. I am also thankful for his deep interest in the question addressed in this work. It goes without saying that without him I would not have been able to carry out this project.

I would also like to thank Thierry Boucher from MSF-Belgium and Geert de Cubber from the Royal Military Academy (and B-FAST) for accepting to give me interviews and precious information about the utilization of drones in the context of humanitarian assistance. Their insight is what makes this thesis very pragmatic and particularly relevant to the actual needs of such organizations. It was Mr Boucher that advised me to pick this particularly interesting subject. Thanks to him, I learned a lot of valuable information on the operational aspects of world-known humanitarian organizations such as MSF. I must express all my gratitude to them for giving me these few moments of their very valuable time.

Finally, I would like to thank my family and friends, especially my wife Maud and my son Maxime for their unfailing support, patience and encouragements during the two years of this Masters degree. This accomplishment would never have been possible without them.
Optimization of Drone Routing for Humanitarian Applications

by Thomas LAMBERT

The present thesis aims to determine if drones could effectively replace in-situ inspection for the collect of information in humanitarian crisis situations. This study focuses on the elaboration of optimization models and their application to route efficiently an Unmanned Aerial Vehicle for a given humanitarian mission. The four models developed were all implemented in a mixed-integer linear programming utility so the solutions for UAV routing could be compared with a land vehicle completing the same mission.

This report is divided in four main chapters. The first one introduces the drone technology environment and the humanitarian applications with these drones. A macro-environmental study is performed using a "PESTEL" analysis to better understand the reasons why drones would be useful in the humanitarian context. In the second chapter, a hypothetical mission based on a simplistic version of the transportation network of Haiti will be presented. This scenario will be used in the following chapters as a baseline case study. Chapter three concerns the elaboration of four different optimization models. The first three are a subset of node routing problems (Traveling Salesman Problem and Distance-constrained Vehicle Routing Problems), while the last one is closer to the arc routing category (Capacitated General Routing Problem).

The results obtained for all these models show that a UAV is always faster than a single land vehicle operating in normal conditions for the test network. However, due to the very large network used as a basic example, the endurance limitations of existing UAVs appear to be a major issue for the real-world applications. Some existing UAV systems could fulfill the mission but they are likely still too expensive for humanitarian organizations. Fortunately, the models elaborated here can be applied to any network, and therefore the advantages of drones with a smaller autonomy can be verified, especially in jungle or mountain environments.
Summary

1 Introduction
   1.1 Thesis subject
   1.2 Drones for humanitarian assistance
   1.3 Drone technologies
   1.4 Optimization of drone routing

2 Case Study
   2.1 Context
   2.2 Situation
   2.3 UAV selection

3 Models
   3.1 Introduction
   3.2 Traveling Salesman Problem
   3.3 Distance-constrained Vehicle Routing Problem
   3.4 DVRP with city inspection
   3.5 Capacitated General Routing Problem
   3.6 Other extensions

4 Applications
   4.1 Introduction
   4.2 Traveling Salesman Problem
   4.3 Distance-constrained Vehicle Routing Problem
4.4 DVRP with city inspection
4.5 Capacitated General Routing Problem

5 Conclusion
5.1 Conclusion
5.2 Future work

A UAV systems

B Additional results for the model applications
B.1 DVRP
B.2 CGRP

Bibliography
# List of Abbreviations and Acronyms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>BVLOS</td>
<td>Beyond Visual Line Of Sight</td>
</tr>
<tr>
<td>CARP</td>
<td>Capacitated Arc Routing Problem</td>
</tr>
<tr>
<td>CVRP</td>
<td>Capacitated Vehicle Routing Problem</td>
</tr>
<tr>
<td>DVLOS</td>
<td>Digital Visual Line Of Sight</td>
</tr>
<tr>
<td>DCVRP</td>
<td>Distance-constrained Capacitated Vehicle Routing Problem</td>
</tr>
<tr>
<td>DVRP</td>
<td>Distance-constrained Vehicle Routing Problem</td>
</tr>
<tr>
<td>ILPF</td>
<td>Integer Linear Programming Formulation</td>
</tr>
<tr>
<td>MILPF</td>
<td>Mixed Integer Linear Programming Formulation</td>
</tr>
<tr>
<td>MSF</td>
<td>Médecins Sans Frontières</td>
</tr>
<tr>
<td>NGO</td>
<td>Non-Governmental Organization</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmaned Aerial Vehicle</td>
</tr>
<tr>
<td>UAS</td>
<td>Unmaned Aerial System</td>
</tr>
<tr>
<td>VLOS</td>
<td>Visual Line Of Sight</td>
</tr>
<tr>
<td>VRP</td>
<td>Vehicle Routing Problem</td>
</tr>
<tr>
<td>VTOL</td>
<td>Vertical Take-Off and Landing</td>
</tr>
<tr>
<td>TSP</td>
<td>Traveling Salesman Problem</td>
</tr>
</tbody>
</table>
List of Symbols

Subscripts and superscripts

\( (\cdot)_i \) Relative to city \( i \)
\( (\cdot)_{ij} \) Relative to the arc from \( i \) to \( j \)
\( (\cdot)^p \) Relative to the sub-tour \( p \)

Parameters

- \( c_{ij} \) Arc length [km]
- \( d_{ij} \) Demand for inspection of an arc [h]
- \( e_{ij} \) Extra time for servicing arc \( (i, j) \) [h]
- \( h_i \) Hovering time for the inspection of a city [h]
- \( k \) Maximum number of refueling [-]
- \( n \) Number of cities (base+clients) [-]
- \( s \) Vehicle speed [km/h]
- \( t_{ij} \) Arc travel time [h]
- \( t'_{ij} \) Arc travel time (with city monitoring time) [h]
- \( t^*_{ij} \) Road time in real conditions [h]
- \( \alpha \) Time necessary to refuel the drone [h]
- \( \lambda \) Maximum flight-time of a drone [h]

Variables

- \( F_{ij}^p \) Flow variable for the arc \( (i, j) \) in sub-route \( p \) [-]
- \( L_{ij}^p \) Arc \( (i, j) \) is serviced in sub-route \( p \) [-]
- \( U_i \) Dummy variable [-]
- \( V_{ij} \) Total time traveled from the origin to the node \( j \) in \( i \rightarrow j \) direction [h]
- \( X_{ij} \) Arc \( (i, j) \) is part of the solution [-]
- \( X_{ij}^p \) Arc \( (i, j) \) is part of the solution for sub-route \( p \) [-]
1 Introduction

1.1 Thesis subject

The aim of this thesis is to determine if drones could effectively replace physical inspection to collect information in humanitarian crisis situations. To this end, various methods to route efficiently an Unmanned Aerial Vehicle (UAV)\(^1\) will be defined in the context of humanitarian assistance. These optimization procedures will be pursued in order to minimize the total time required to collect information about ground infrastructures in the aftermath of a natural disaster. The simulation results for the drone will be compared with the ones of a land vehicle in order to get some quantitative data regarding the possible improvements with airborne means.

The drone economy is currently booming [Mazur et al., 2016; Windju et al., 2015; Peasgood and Valentin, 2015]. Unmanned Aerial Systems (UAS) are no longer only confined to the military use and they have been drawing more and more attention from every branch of the society for a few years now. Many new products and services are continuously being released in the global market. Big actors are starting to develop their own solutions and a service economy related to drones is quickly emerging. Researchers from all around the world have also started to work on the optimization of these systems, in both the hardware and service domains.

Due to their astonishing capabilities for collecting information, monitoring or even delivering cargo, drones are expected to play a very important role in the logistic sector for the coming years. Logistics giants such as Amazon (Amazon Prime Air), DHL (DHL Parcelcopter) or UPS (UPS Flight Forward) are all developing solutions for parcel delivery. In parallel major defense and aerospace companies such as Boeing, Airbus or Lockheed Martin are pushing for the adoption of the technology in all sectors (defense, surveillance, law enforcement, delivery, transportation, etc.). Finally, historic drone developers that started initially with the leisure market (photography, cinematography, sports) are beginning to diversify their activities towards industry and B2B services.

---

\(^1\)In the reminder of this thesis "UAV" and "drone" will be used interchangeably even though drone could have a larger meaning.
Companies like DJI, Parrot, Yuneec or Kespry are building upon their proven experience in the sector and recently released new high-end systems aimed at commercial applications.

The overwhelming range of possibilities offered by these unmanned systems is starting to interest humanitarian agencies as well. Dozens of case studies have been realized in this sector for the last five years [Swiss Foundation for Mine Action, 2016; Samsioe et al., 2017]. In this particular context, drones are expected to become an incredible tool to collect relevant and up-to-date information about the field of operations. Drones could also be used to deliver valuable cargo faster in harsh environments, where ground infrastructures are missing or no longer practicable. Such benefits would drastically increase the capacity of action at the very beginning of an emergency response.

As the technology matures, drones are starting to get even more efficient, which leads to a non-negligible impact on their operational capacities. However, some major issues are still to solve and drone routing is one of them. In the context of humanitarian aid, every minute gained and every bit of information gathered is not only money spared, but also lives potentially saved.

Some notable research work has already been conducted in the general domain of drone routing this last decade. However, the focus of these papers was mainly on commercial deliveries such as last mile delivery or traveling sidekick situations (a drone goes along with a delivery truck to serve the farthest clients). A wide range of other situations such as arc routing problems (traffic monitoring for instance) are usually not much studied as drones are often reduced to fly in straight lines only. However, as one might expect, the humanitarian problems are a bit different than the ones encountered by commercial applications. Thus, new models are required.

Just like for their civilian counterparts, the drone applications could be separated in two main categories: the collect of information (mapping, survey of ground infrastructure, counting of resources, damage assessment, etc.) or the delivery of critical items (blood samples, medicines, defibrillator, survival food rations, specialized tooling, etc.). In both cases, additional factors need to be taken into account for humanitarian applications, such as the human factor, the level of urgency of some situations, unexpected events, etc. For instance, some new targets may require drone intervention unexpectedly, people may be forced to relocate in response to a natural disaster (such as flooding) which would add uncertainty to their position, etc. All these factors will surely impact the route planned for the drone and some dynamic routing may be required far more often than in civilian operations. The time factor, the specificity of the missions, the priority in the treatment of the demand, the state of the operational ground or the behavior of
the populations are much more exposed to unexpected variation than in classical civil-
ian situations. In addition to that, the types of mission themselves may be completely
different as well. It is therefore crucial to find new models for drone routing that take
into account those specificities. Failing to do so, could make the drones a useless bur-
den for humanitarian associations and delay the deployment of the whole logistic chain.

Firstly, this thesis will focus on a general description of the utilization of drones in a
humanitarian context. The state of the art will be reviewed for both the technology and
the optimization models for drone routing.

Then a hypothetical mission will be described, based on the actual network of Haiti
where a UAV could be used in order to assess the damages in the surroundings of Port-
au-Prince. In this hypothetical mission, only a few major cities and the roads that con-
nect them will be considered. This is obviously an over simplification of the actual sit-
uation, but it will serve to assess the good functioning of the models and to estimate the
performance of the drones in a realistic situation. This part will represent the case study
for the rest of this thesis.

The following chapter will detail the different models studied with their mathematical
formulations and the type of mission they would represent. Some additional models
will also be briefly described as possible extension to this thesis.

Finally, chapter 4 will focus on the applications of these models to the case described
in chapter 2. Some input parameters will be varied in order to analyze their impact on
the optimal solutions. For each solution, a comparison will be carried out with a Jeep
performing the same mission under normal circumstances. This comparison will serve
as a way to determine the benefits or disadvantages of UAVs for such situations.

1.2 Drones for humanitarian assistance

Surveys in Non-Governmental Organizations already concluded that the humanitarian
sector is interested in using UAVs to collect information and deliver some critical items
[Swiss Foundation for Mine Action, 2016]. The current humanitarian applications cover
these two use cases, however the former is by far preferred by the users and therefore
the most common one.\footnote{The results of these surveys were confirmed by Thierry Boucher, head of the Emergency Unit Log &
Supply at Médecins Sans Frontières Belgique during an interview that took place at MSF in Brussels on the 13th of November 2018.}

First of all, a "PESTEL" analysis could be conducted in order to identify the macro-
environmental factors the impact the use of drone in the context of humanitarian aid.
This study will help to understand why and how drones could be a real advantage in
these emergency situations. The results of this analysis will also serve as a baseline to devise a realistic mission for studying the route optimization models.

1.2.1 PESTEL analysis

Political

Most of humanitarian crises happen in developing countries that usually lack the resources to face the situation on their own. Unfortunately, a lot of these developing countries are also prone to military or civil conflicts (Sudan, Yemen, Somalia, etc.). In such situations, the humanitarian aspects of the general conflict are often neglected as the forces in power are more focused on fighting their opponents to remain in place than assisting their population. In this context, any type of humanitarian aid is already difficult.

When the rare Non-Governmental Organizations (NGOs) allowed to operate in the country want to use drones to help the population, they are often suspected by the authorities of spying on the civil populations, the military interests or on the governmental activities. This mistrust from the authorities greatly reduces the possibility of employing airborne means over such regions.

To tackle these political issues, the most preeminent humanitarian associations involved with drones assembled into a consortium (UAViators\textsuperscript{3}) devoted to facilitate the use of UAVs by NGOs. A part of their mission is to negotiate treaties to facilitate the access to the airspace for NGOs in case of a major event in these countries.

In the meantime, when operations are allowed, official representatives are often invited to observe all the mission phases. That way, they can directly verify the rightful usage of the technology. Usually, this collaboration also requires that the drone operator shares all the data collected (images, videos, telemetry, etc.) with the authorities in order to ease their concerns and provide them useful insight on their territories.

Other political concerns arise when the central government has little authority over its territory. In this case, the NGOs often need to treat directly with the village chiefs or rebel groups in order to get access to the field of operation.

Fortunately, a lot of NGOs have a very strong background of intervention in such complicated political environments. This proven utility helped them to build some fragile trust over the years and the crises. That way, making special requests for drone usage are usually a bit easier.

In more stables regions, the political climate is usually favorable to humanitarian intervention. Authorities understand their limitations and are quite happy to receive

\textsuperscript{3}uaviators.org
1.2. Drones for humanitarian assistance

assistance. Usually, they use their power to grant all necessary authorizations to the NGOs. During the infamous earthquake of 2010 in Haiti for instance, a lot of organizations received accreditation to fly drones in the national airspace, even though there were no specific law to regulate the drone operations.

**Economical**

Most of the countries that rely on humanitarian assistance have often a weak economy. A massive event such as an earthquake or a hurricane can grieve the finance of the whole region or country. Humanitarian actors are therefore working with their own funds and hardly rely on the local authorities for direct financial support.

In addition to that, the general lack of economic resources means that a large part of the infrastructure is already in poor condition prior to the crisis. As expected, this situation worsens in the aftermaths of the natural catastrophe. The financial difficulties also mean that the available information is often outdated. Maps, population registers and other similar data are either unavailable or not trustworthy. It is therefore important for NGOs to gather fresh intelligence before any further planning.

Thankfully, the international community often comes in the rescue and funds a large part of these humanitarian missions. Worldwide calls to donations are also welcomed and help tremendously the humanitarian associations during their mission.

In such large-scale events, it is common that a large part of the population loses all their belongings and the whole economic activity of the country is put on hold. In the meantime, this means that the people in good physical condition can become workforce for the first-aid missions (search and rescue) or for the reconstruction of their country. A lot of inhabitants therefore volunteer for simple missions on the behalf of NGOs.

**Social**

In general, local populations are used to see and work with humanitarian actors on the ground. In most cases this collaboration goes without any major issues. However, the use of drones is quite different than physical interactions with the populations. Most of the times, UAV are intuitively perceived as a threat and not an aid by the populations in need. This is even more true in war zones, were UAVs are systematically assimilated with combat drones that are employed to eliminate targets through surgical strikes [Swiss Foundation for Mine Action, 2016]. As one may expect, these populations are at best very suspicious and at worst completely panicked at the mere hearing of a drone. So much so that it is not rare that people from these communities shoot the drones down, destroy them when they land or show hostility towards the operators.
Chapter 1. Introduction

Just like the authorities, the local communities often raise concerns about espionage. Usually to circumvent that problem, drone operators disclose all their footage to the leaders of the communities. This provides them with good images and reliable information about their territory, while ensuring the drones (and their operators) are safe to work.

Outside the zones of conflict, drones are more susceptible to be welcomed by civilians and authorities that are glad to receive any kind of relief.

**Technological**

UAS are getting more and more common with each passing year. The technology still needs to mature properly but most systems currently available are already accessible for real operations. The whole economic context is in general favorable to innovations in autonomous (and flying) systems. Many large companies are doing a lot of research and development and new high-end drones are being brought into the market every month. Technical limitations on the autonomy of battery powered drones pushes for a shift in propulsion system. More drones are now being developed from the ground-up with a hybrid propulsion or fuel powered gas turbine instead of the classical electrical engines. Simple VTOL quad-copters are also slowly being replaced by hybrid aircrafts whose operational range is significantly larger. In addition to that, some mission-specific systems are also created such as "ambulance drones" that carry a defibrillator [Claesson et al., 2017].

A more extensive description of the drone technological aspects is presented in Section 1.3.

**Environmental**

Humanitarian operations usually take place in harsh environments, which is even more true in the aftermath of a natural disaster. In this context, drones could provide a reliable tool to circumvent some of the issues that raise on the ground transportation network. By being airborne, they are not stopped by road blockages or intense traffic of fleeing people. UAVs also present the benefit of being able to operate in many different types of environments (urban, mountain, jungle, deserts, etc.) without requiring too many adaptations.

Drones are also usually very light and efficient compared to an all-terrain Jeep operating on the ground. They have no impact on the lands as they travel through the air and they usually do not disturb the local wildlife. While on the opposite, Jeeps with lots of equipment may require to make their way out of the official roads, damaging parts of
the surrounding fields and forests. The passage of vehicles is also noisy and generates pollution near the ground, that may perturb the local wildlife. This is especially problematic if the operation needs to happen where engendered species are present.

The main environmental impact of drones is probably the noise pollution that comes with it, particularly in systems with rotary wings (helicopters, quadcopters, etc.). This added noise may be an issue in densely populated areas, especially if multiple NGOs operate their drones at the same time.

Legal

Usually the regulations of emerging countries are not up-to-date with the most recent advances in technology. In most of the cases, there is no regulation at all with respect to drone operations. Fortunately, that does not mean everyone could do anything either. When regulations are lacking, the best practice is to avoid any use of UAV until a proper ad-hoc authorization is given by the local aviation regulatory comity (or the transportation minister directly).

To tackle these regulatory issues, the humanitarian associations involved with UAViators often work as consulting experts to help local governments in the creation of proper regulations. This consortium also maintains a Global Drone Regulation Database\footnote{droneregulations.info} for every country and they publish strict guidelines for NGOs willing to operate drones in a humanitarian context.

Aside from the lack of regulations, the other main issue is when the existing ones are too restrictive. In most countries (including industrial ones), the laws related to drones were created in a hurry when the market started to take-off. As often in this case, the regulations were designed to be very restrictive by default in order to avoid the most foreseeable issues, while a better, more studied version of these laws was being worked on. Sadly, these new laws are still long-awaited in the majority of countries.

In essence, the current laws usually permit the use of very small drones (often less than 1 kg) at very low altitude (about 25 m max) and where the drone remains in Visual Line Of Sight (VLOS) of the operator at all times. While these are usually sufficient for UAV enthusiasts or amateur photographers, such restrictive framework is not suited for large-scale industrial, commercial or humanitarian applications. Depending on countries, some leeway is possible on these restrictions, such as the possibility to fly at Digital Visual Line Of Sight (DVLOS) instead of VLOS. In that case, the use of digital techniques (IR/thermal/4K cameras, First Person View, etc.) are permitted as a mean to extend the visual line of sight.

In addition to that, a lot of industrial countries now have drone licenses that allow an
accredited pilot to operate with bigger drones, over populated areas and at a higher altitude. Unfortunately, in most cases, even a licensed pilot is not allowed to operate a drone at its full technological capabilities. Indeed, last generation of UAVs are now perfectly capable of flying fully autonomously and far away from their departure point. This would mean that a (licensed) pilot is not even technically required anymore and the system should be trusted to complete the mission safely on its own. This also means that these UAVs can technically fly Beyond Visual Line Of Sight (BVLOS) without any issue. Sadly, there are currently very few countries that allow easily these types of missions for licensed professionals. Nevertheless, most of the times, the air traffic authorities can deliver special authorizations for very specific missions. Still, these special authorizations have to be renewed for every mission or after a given period of time, which complicates the use of such high-end drones in emergency situations.

1.2.2 Mission types

Information gathering

This is by far the most common application of drones in the context of humanitarian aid. Thanks to their relatively small size, light weight and astonishing image quality, drones can be deployed in a very short time frame and in almost any environment. Drones used to collect information or map regions are often very consumer-friendly and a large segment of their operation can be automated, making it a great tool for non-technical users.

The information collected with these drones can then be used to plan and tailor the deployment of the full logistic chain (large scale cargo, food supply, civil engineering equipment, etc.). These data can also pinpoint the important zones of interest and help to classify the urgency of the situation in various locations. All this results in a better allocation of the resources and an increased efficiency on the operational field. In addition, automatic image processing can help determine what is actually needed on the ground, for instance by counting the number of tents already available in a given zone and check if more are required.

Drones carry significant advantages for ground imagery over more conventional systems. Typically, satellites or classical airplanes are the most used systems for such tasks. However, when the area to map is restricted their cost becomes prohibitive. On the contrary to drones, which can fly at low altitude, satellites and airplanes are also useless when the cloud cover is important, which is typically the case up to a few days after a hurricane. Another benefit of the drones is that they are more suited to fast-changing environment such as urban centers, refugee camps or war zones. Also, drones can usually
start mapping in a very short time frame (providing that the local regulations allow it), while airplane need at the very least a crew and a runaway in decent condition (which is not always available). Obviously satellites imagery is even less flexible as it requires to wait for the right satellite to pass over the right area. Finally, the resolution of drone imagery is often of much better quality to the one offered by satellites (for a reasonable price) as shown in Figure 1.1.

![High-resolution drone images (left) and lower-resolution satellite images (right). From [Swiss Foundation for Mine Action, 2016].](image)

**Figure 1.1:** High-resolution drone images (left) and lower-resolution satellite images (right). From [Swiss Foundation for Mine Action, 2016].

### Delivery of critical items

One of the key aspect of humanitarian aid is the fast, reliable and cost-effective delivery of supplies to local populations in distress. The success of such missions is entirely based on the efficiency and the fast deployment of a reliable supply chain. This explains why NGOs typically allocate 60-80% of their expenditures on their supply chain [Tatham and Pettit, 2010]. The generalization of drone deliveries could be a great option to reduce these costs while keeping a high level of efficiency, especially for small payloads delivered on short distances and at high frequencies. A really good example of such situation can be observed in Rwanda and Ghana, where the company Zipline operates daily deliveries of blood to small field hospitals, saving precious hours when time is the enemy.

Apart for that specific example, drone package deliveries are not really employed massively in the humanitarian sector for various reasons [Swiss Foundation for Mine Action, 2016]. The most notable one is that the technology is not mature enough for such applications, and no proper off-the-shelf solution are currently available. If Zipline is able provide such a good service, it is only because they created permanent installations from which they can operate. These installations encompass a small runaway for the landing, a catapult to launch the UAV, a radio tower to extend the signal range, etc. In
Chapter 1. Introduction

emergency situations such as the ones caused by a natural disaster, the deployment of such an infrastructure is not an option. Moreover, the flight-time (and range) of a drone is tightly linked to the weight of its payload: the more items it transports, the shorter its range. Considering the current state of the technology, a high-end Vertical Take-Off and Landing (VTOL) multi-rotor drone could barely transport 4-10 kg over 100 km (in a one-way trip). Fixed-wing systems (such as the ones used by Zipline) can provide longer ranges and heavier payload, but they present their own disadvantages (see Section 1.3.1). Very large cargo drones also exist, but they are incredibly expensive and often only affordable by the military. While they are still technically unmanned aerial vehicles, these very large drones are definitely very different from the smaller ones that are relevant to the cases studied in this thesis.

The delivery of critical item bears also significant adaptations considering the nature of the payload itself. Usually, medicines, vaccines or medical samples are required to be carried in refrigerated containers. This effectively decreases the useful payload of the drone (need to account for the weight of the container), or can even decrease its autonomy if the container needs active cooling. These kinds of critical packages also need special care for the delivery. Simple dropping may not be suitable in all situations, and often requires trusted personnel on the ground to collect the goods. Parachuting packages also call for special acceptation from the air traffic regulators. On the other hand, landing the drone exposes it to pillage or deterioration and may simply not be possible due to the lack of place (clear terrain) or infrastructure (runaway). Finally, the transportation of medical samples comes also with the tremendous risk of accidental dissemination of pathogen agents in the event of a crash\(^5\). This type of event could potentially make the situation a lot worse than without any drone intervention.

Even though the added value could bear significant impact on the efficiency of the humanitarian supply chain, the risks, the costs, the technological immaturity and the large amount of requirements associated with these types of deliveries are too big to cover for NGOs operating in emergency situations. The extensive amount of funding required to develop better systems and mitigate these issues is out of reach for many humanitarian associations. It is expected that the private sector will do the most important effort in that domain, thanks to parcel delivery in urban environment for instance. Then, NGOs will probably only consider deliveries in a humanitarian context once the technology has properly matured and off-the-shelf solutions are available at a reasonable price.

\(^5\)During a case study conducted by MSF in Papua New Guinea in September 2014, a multicopter UAV carrying Tuberculosis sputum samples crashed in the forest and the samples were lost. This incident was not isolated and happened multiple times over the course of the study. Fortunately, the repeated crashes did not lead to a sprout of the infectious disease in the region [Samsioe et al., 2017].
1.3 Drone technologies

1.3.1 Types of drones

Unmanned aerial systems are usually divided into three categories: fixed-wing UAVs, rotary-wings UAVs (multirotors) and hybrid drones (which is a combination of the first two). Each of these categories present several advantages and disadvantages. Ideally, a user should choose the proper drone platform based on the mission requirements, environmental conditions or the organizational needs. However, the cost of each system is usually so large that NGOs cannot afford to buy and operate different systems. A compromise is often chosen in order to complete as many different missions as possible, even if this leads to sub-optimal results.

Fixed-wing drones

Fixed-wing systems have a two-wing design, similar to regular civilian airplanes. As for common aircrafts, the wings carry the lift and all the energy available is used for the propulsion of the drone. This usually leads to a better efficiency, a longer range and flight-time and the possibility to carry heavier payloads. It also allows for higher velocities, which can be important in case of delivery of life-saving packages or very long-range missions.

These fixed-wing drones are usually larger than the other systems, making their transportation to the field of operation a bit more tedious. Their main drawback is that they need a strip of open space (or even a proper runway in some cases) to take-off and land safely. This requirement can be hard to fulfill in mountains, forests or densely populated areas. Some small-size systems often replace the runway by a catapult. This system is shorter and allows for take-off in restricted spaces, however it is yet another part of the system to carry on site before starting the operation. Fixed-wing systems also have the inconvenience of being unable to hover above a static target. To remain airborne, they must keep moving, which forces them to circle over the same point as long as required. The images are therefore not steady and need extra-stabilization.

Rotary-wing drones

These systems are based on the presence of a rotor (helicopter) or multiple rotors (quadcopter, octo-copter, etc.). The absence of wings makes them very small and easily transportable. The main advantage of such UAVs is that they can take-off and land vertically, which allows the pilot to start the operation anywhere at any time. As a consequence, the reactivity of the NGO is greatly improved in the first hours of operation. However, as the lift is not produced by a wing, these drones rely solely on their rotor to
sustain their altitude, which drains energy at a considerably higher rate. This results in a shorter flight-time and a lower payload capacity than their fixed-wing counterparts.

**Hybrid drones**

This last type of drones is a mix of the two first ones, as they combine wings (for the cruise flight) and rotors for the VTOL capabilities. Different arrangements are possible: fixed rotors for VTOL and another propulsive system for the level-flight, or convertible systems where rotors are tilted after take-off in order to sustain the forward motion. Such systems have the benefits of the two firsts, with reduced drawbacks. They are capable of VTOL, can carry heavy payload, achieve long range and high speeds. The main drawback come from the fact that this is a compromise of the two systems, thus the efficiency of the two main flight phases (hovering and cruise) is reduced compared to an aircraft specifically designed for these missions. They are also quite large due to the presence of the wings.

The capabilities of these drones makes them the most versatile system, able to perform reasonably well in all humanitarian scenarios. Unfortunately, they are relatively new, technically more complex, more expensive and not really well represented in the off-the-shelf market yet.

![Different types of drones](image)

**Figure 1.2:** Different types of drones. (a) Fixed wing, (b) Multi-rotor, (c) Hybrid.
1.3.2 Drone main components

**Propulsion systems**

Irrespective of their type, each drone needs a propulsion system. The most common one for small-size UAVs (< 20 kg) is an electrical propulsion system composed of small motors and a set of batteries. The batteries can be recharged quite easily almost everywhere (providing the electrical network is still functioning or generators are available). However, they also suffer from a poor efficiency, which restrain their flight-time with a single charge. To compensate, typical systems have interchangeable batteries that can be quickly swapped. That way, the drone is able to resume its mission while the set of empty batteries is recharging. This also means that the drone either has to come back to the same base for the battery swap, be grounded in a remote location while its only batteries are recharging or that batteries need to be available at different locations within reach of the UAV.

For larger systems, drone manufacturers are starting to explore more complex propulsion systems such as piston engines, turboshfts and turbopropellers. These engines are basically a smaller version of the ones that have already been used in civilian aviation for decades. They rely on fuel such as kerosene or gasoline, which is usually more efficient than electricity. Therefore, the flight time is extended and can easily exceed two hours, even for rotary-wing drones. As they only need refueling, they do not necessarily have to come back to the base when the tank is empty. Instead, they can land in a remote location, get refueled there by a trusted collaborator and get back in the air in a very short time frame (easier if the fuel is gasoline, unlikely with kerosene). The longer endurance may also allow for the completion of an observation mission in a single flight. These systems are unfortunately more expensive than the smaller electric ones, they require more maintenance and fuel may be more difficult to find than electricity in remote locations. They are also considerably louder than electrical systems, which can cause issues in densely populated area or in war zones.

A solution in-between these two is also slowly appearing in the form of hybrid engines. Just like for cars, these propulsion systems ally a thermal engine (piston or turboshaft) powered by fuel with an electric battery charged by the thermal engine. This allows to use the thermal engine at its optimal point, increasing even more its efficiency. In parallel, the use of electric propulsion to drive the motors allow to change their regime very precisely and rapidly without losing energy. Such systems considerably increase the total flight time, by keeping the drone airborne for more than 3 hours. They size is often in-between fuel-powered systems and small electrical ones. However, this type of
propulsion system is quite new and still very expensive. Only a few commercial sys-
tems are currently available.

**Autonomous flight**

Thanks to the advances in computer sciences of the last decade, fully autonomous sys-
tems are now technologically possible. With the use of high-performance computing,
computer-vision and machine learning, drones are now able to make their own decision
and carry a full mission without any human intervention. Eventually, this reduces some
of the remaining burden of drone operation such as the necessity to have a certified pilot
on site, restraining the drone to VLOS of the operator, etc. Nowadays, simple systems
can determine the optimal mission parameters on the basis of a few inputs from any
operator, then directly take-off and begin the mission. They can also adapt and react
quickly to changes in the parameters or to unexpected events. Some simple consumer
drones (that only cost a few hundred dollars) are for instance capable of following a tar-
get with great precision and take a stable 4K video footing.

These different capabilities make operations beyond visual line of sight a possibility.
Therefore, this increases the range of the mission and facilitates the operation in hostile
environments like mountains, forest or urban areas.

### 1.4 Optimization of drone routing

The general field of route optimization is not new. For more than half a century engi-
neers, logistic experts and computer scientists have been working on models to fit every
possible use for the famous Vehicle Routing Problems (VRP). The recent advances in
the drone sector of the last few years constituted a new source of challenges and numer-
ous new studies were conducted in that regard.

Most of the research papers published on drone routing focus on commercial appli-
cation such as last-mile parcel deliveries, see for instance [Murray and Chu, 2015]. In
these scenarios, the UAV is usually working in collaboration with a delivery truck that
assumes the role of a moving depot for the drone. The two-vehicles system requires a
joint optimization in order to limit the total distance covered by both the truck and the
drone and therefore increase the overall efficiency of the delivery system.

A few other researches were conducted in the field of arc routing problems such as traf-
fic monitoring for instance [Chow, 2016].

Very recently, interest started to shift towards stochastic models, where the drones were
assigned areas to monitor and needed to check randomly appearing targets [Bullo et al.,
2011; Enright et al., 2015; Pavone et al., 2009; Smith et al., 2010].
1.4. Optimization of drone routing

1.4.1 Drone routing in humanitarian context

In humanitarian operations, lots of these models intersect. In a situation that requires the collection of information, it is necessary to route the drones over different points of interest (vehicle routing problems). It could also be a good idea to fly over the main roads or portion of them in order to assess the damages (arc routing problems). Finally, some parameters of the operation could vary randomly during the course of the flight: new targets could be discovered, the required monitoring time for some targets may not be known \textit{a priori} or inspecting the rest of a road could become irrelevant once it has been observed to be impassable. It is therefore crucial to develop models specifically tailored for these issues.

1.4.2 Dynamic vehicle routing problems

Many optimization strategies are now shifting towards uncertainty models. In these cases, the routing problem becomes dynamic as all the network parameters are not known at the beginning of the mission. These stochastic parameters and variables mean that multiple choices should be made during the flight. For instance, the choices could decide which are the optimal routes considering the probability of a new target appearing, how to allocate the tasks among the different UAVs, when should the UAV come back to the base for refueling, etc. Usually, these types of problem include a rolling horizon where events appear (or not) at random time. The most common way to solve those types of problem is to use a heuristic algorithm. In this case, an entire set of events is calculated from the beginning and the absolute optimal solution with perfect knowledge (\textit{a posteriori}) is calculated. The second step involves the unraveling of the mission in the rolling horizon where choices should be performed at every given interval, or immediately upon reception of the new information. At the end of the simulation, the two solutions (rolling horizon and optimal) are compared to see how good/bad the choices were. When this type of simulation is repeated a large number of time, the decision parameters become clearer and a better decision making procedure could be considered to get closer to the actual optimal solution reliably.

Note that these dynamic vehicle problems have never been studied in humanitarian context. Unfortunately, there is currently no real static models either for these types of situations, so this thesis will only focus on the static ones.
1.4.3 Optimization goals

As this thesis investigates the collect of information related to ground infrastructures (cities and roads), the objective will be to minimize the total time required to complete the full mission with one drone. This goal is very common in vehicle routing problems, even though usually it is formulated as a minimization of the distance (or cost). Obviously, under the assumption of constant traveling speed, these two formulations are equivalent.

It should be noted however, that these types of goals may not suit all humanitarian applications and some may require another definition. For instance, if the total waiting time before servicing an area is an important factor (e.g. emergency kit delivery), the objective could be the minimization of that specific time instead of the total mission time.
2 Case Study

2.1 Context

In order to pick a realistic scenario, the network of roads and cities will be directly based on the actual situation in Haiti. This specific country was chosen as it is heavily exposed to extreme natural disasters. Over the last two decades, it has been hit by more than a dozen hurricanes, tropical storms or torrential rains\(^1\) and 3 earthquakes\(^2\). These catastrophic events resulted in the death of more than 320,000 people (300,000 of those are from the infamous 2010 earthquake alone) and countless number of displaced population [The Associated Press, 2010; Kang, 2016; University of Fondwa, 2018; Wikipedia contributors, 2019].

Moreover, the tragic events of 2010 were one of the first situations where drones were employed for actual missions in a humanitarian context (mostly for inventory, survey and mapping). The following years, many other operations were conducted in Haiti with UAVs. Thankfully, this particular earthquake and the situation that resulted in the aftermath were extremely well documented and monitored. This is mostly due to the advances in monitoring technologies, big data and the massive international collaboration that resulted of this emergency situation. Since then, this earthquake has become one of the most used scenario for optimization research regarding humanitarian action, as can attest recent papers such as [Penna et al., 2018; Sakuraba et al., 2016a; Sakuraba et al., 2016b].

Events of such magnitude dramatically reshape the territories and ground infrastructure. This complicates tremendously the planning of NGOs intervention in the country. The use of drones to scout the terrain, estimate damages and determine the state of the transportation network could clearly be a great advantage in the deployment of international assistance.

\(^1\)2002, 2004 (3x), 2005 (3x), 2008 (4x), 2010, 2016.
\(^2\)2010 and 2018.
2.2 Situation

This thesis focuses on the elaboration of optimization models for the special needs of humanitarian applications. The goal is not to test the robustness of existing models against abstract giant networks. Instead, all models presented here will be tested in a relatively small set of cities and roads, as a proof of concept that they function as expected. The solutions resulting from these models, will help to decide whether or not the use of drones instead of land vehicles is justified in the depicted scenarios. To this end, a simplified network of the northern part of Haiti will be studied, where only a few major cities and the main roads are considered. The base of operations – where all routes must start and end – is set in Port-au-Prince (the capital of Haiti). Due to the flight-time limitations of existing drones, the cities to inspect are chosen within a 150 km radius around this base.

Coincidentally, this specific topology is similar to the abstract one studied in the arc routing problems for drones detailed in [Monroy et al., 2013; Chow, 2016].

It should be noted that this network was simply chosen in order to use a "realistic" topology in a situation where drones are expected to be relevant. The case studied in this thesis may not represent the actual points of interest of NGOs in Haiti nor the actual strategies they would pursue in the first hours of an intervention there.

2.2.1 Cities

The following cities (Table 2.1) have been considered for the base network:

![Map of Haiti and localization of the relevant cities and roads.](image-url)
### 2.2. Situation

#### Table 2.1: Major cities considered in the scenario.

<table>
<thead>
<tr>
<th>Id</th>
<th>Name</th>
<th>Population</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Port-au-Prince</td>
<td>897 859</td>
<td>18.547327</td>
<td>-72.3395928</td>
</tr>
<tr>
<td>2</td>
<td>Mirebalais</td>
<td>88 899</td>
<td>18.8344163</td>
<td>-72.104836</td>
</tr>
<tr>
<td>3</td>
<td>Thomonde</td>
<td>56 274</td>
<td>19.0170164</td>
<td>-71.9597698</td>
</tr>
<tr>
<td>4</td>
<td>Hinche</td>
<td>109 916</td>
<td>19.1445541</td>
<td>-72.0087527</td>
</tr>
<tr>
<td>5</td>
<td>Saint-Marc</td>
<td>242 485</td>
<td>19.1020269</td>
<td>-72.699511</td>
</tr>
</tbody>
</table>

The population of the cities is a relevant parameter that will be used for some models as a way to express the time require for a full inspection a city. The more populated the city, the more time it needs for a complete scan. The GPS coordinates will be used in order to calculate the length of the direct flight path from one city to another.

#### 2.2.2 Paths and network graphs

The complete network could be divided in two different components. The first one is the land network made of the five cities and the physical roads between them (Figure 2.2). The second one is the aerial network constituted of the cities and all the possible flight paths defined as straight lines between these cities (Figure 2.3). This distinction is quite important, as some models will be using a mix of land and aerial links to complete the route (see section 3.5).

Each arc (road or aerial) is defined by its two end points (cities $i$ and $j$) and is considered undirected (it can be crossed indifferently from $i$ to $j$ or $j$ to $i$). As the arcs are undirected, they may also be called edges in the remainder of this work. Each edge is characterized by its length, $c_{ij}$. In addition, every physical road (Figure 2.2) can also be associated with its travel time in real conditions, estimated with online tools (OpenStreetMaps). This real travel time for each road is denoted $t^\ast_{ij}$. It should be noted that the aerial paths are taken as straight lines from one city to another. This means that all obstacles are neglected for simplicity (no mountains, no flight exclusion zones, etc.). However, these obstacles could be taken into account very easily by adding a few kilometers to the path length to circumvent them.

Note that, even though some segments seem similar between the land and the air (they join the same end nodes), their actual lengths are different, as air paths are in

---

3The road length was retrieved on OpenStreetMap (openstreetmap.org).

4The aerial distance is computed directly from the GPS coordinates of the cities center, using the Haversine formula.
2.3 UA V selection

Two main types of missions will be considered in this thesis. The first one will focus on the inspection of the cities, while the second one will focus on the reconnaissance of the transportation network.
The two missions require UAVs with considerable endurance. As shown in Figure 2.3, all cities lie within a 75 km radius of the base. The first type of missions requires therefore a UAV with a flight-time of 2 to 3 hours minimum. The second type of mission involves the survey of a full roads. Thus, the autonomy of the drone should be sufficient to reach the road to survey, inspect it and come back to the base. In summary, the autonomy required for this second mission should be at least 4 to 5 hours. Fortunately, such systems are already available on the market. A list of various possibilities is presented in Table A.1 in appendix. Some of the UAV listed in this table are capable of VTOL or even hovering, which would fit perfectly the mission requirements. Note that all these long-endurance systems are build around fuel (or hybrid) propulsion systems. Hence, the applications described in the next chapters will only consider fuel-based systems and not fully electric ones.

For simplicity, extra hypotheses need to be made:

- The regulations and legal aspects are neglected. Drones are allowed to fly everywhere at any time.
- BVLOS flight is authorized.
- The drones are fully autonomous.
- The communications restrictions with the drones are neglected. A drone is never out of range of the operators.
- The drone continuously streams the data it collects. It does not need to come back at the base to unload the data.

Some of these assumptions are obviously unrealistic in normal operational conditions. However, the goal here is to study the optimal routing of the drones and their benefit in that regard in comparison with land vehicles. In that aspect, these assumptions are perfectly justified.
3 Models

3.1 Introduction

In this chapter, a few models will be developed to find the optimal route for different scenarios encountered in humanitarian operations. The complexity of the models will be gradually increased in order to better represent real-world applications. This chapter only presents the mathematical formulation of each model. Their actual application on the test case described in chapter 2 will be the subject of chapter 4. Usually routing problems rely on the minimization of the total distance traveled by the vehicle. However, as UAVs need to consume energy to remain airborne, the distance traveled may not be the best indicator. Instead, optimization with respect to the flight-time is more suited to represent the actual energy constraint of the drones. By doing so, the energy consumed by a loitering drone could easily be taken into account even though the drone remains in the same spot. Under the assumption of a constant displacement velocity, the arc distances can be translated easily into time-based units. Hence, all models presented in this thesis will in fact aim to minimize the total mission time.

The first model studied is a classical Traveling Salesman Problem (TSP). With this model, the vehicle only needs to visit all the cities once, using a single route that starts and ends at the same point (the base). In the classical formulation, no specific external constraint is applied such as flight-time restriction, etc. Obviously, a drone is expected to perform better than a land vehicle in this scenario, as it can travel in straight lines between two cities whereas the car is bounded to the irregular roads.

The second model introduces limitation on the flight-time of the drone. To that means, a Distance-constrained Vehicle Routing Problem (DVRP) model will be developed. In that case, each city is still visited only once, but multiple smaller tours will be necessary in order to comply with the flight-time constraint. Note that this model could either be used to represent a fleet of similar drones completing the mission, or a single drone that needs to refuel at the base between each sub-route.

The third model is a simple extension of the second one which introduces an inspection time overhead for each of the cities visited by the vehicle. This will likely call for more frequent refueling and cause a considerably longer mission time.
The fourth model is a bit different than the former three. Instead of being based on node routing (visiting all the cities), this one aims at inspecting all the roads (arc routing problem). In addition, the previous consideration will still be taken into account. Hence, the flight-time limit will be enforced and the inspection of the cities will be accounted. These constraints will greatly impact the routes feasible with one fuel tank and therefore modify the total mission time. Such model is quite uncommon as it aims at servicing both the arcs and the nodes. Some previous literature classifies it as a Capacitated General Routing Problem (CGRP), which is a combination of the Capacitated Arc Routing Problem (CARP) and the DVRP. In the situation studied in this thesis, only the arcs corresponding to physical roads will require inspection (in order to check if the roads are practicable). The air paths will still exist in the network, but the drones will not be forced to take them. Note that this last model also requires a modification of the network graph as the two different sub-networks should be merged into one (land network and air network).

Finally, a few extensions to these models are discussed. They all represent more complex cases relevant to humanitarian applications. Sadly, these extensions are far more complex and out of the scope of this thesis. Nonetheless, a brief description will be given for each one, without any rigorous mathematical formulation.

**General hypotheses**

Every model presented in this chapter is made under the assumption of undirected arcs (i.e. edges). This means that there is no difference in crossing an arc from $i$ to $j$ or from $j$ to $i$. Mathematically, it also means that the distance matrix $c_{ij}$ is symmetrical.

Another assumption is made regarding the velocity of the vehicles, which is assumed constant and equal for every edge of the network. In the applications of the models (chapter 4), the velocity will only differ from one vehicle type to another (drone vs car).

Lastly, some hypothesis need to be made on the fuel and energy requirements. In real applications, the progressive burning of the fuel would make the drone lighter over time. This reduces the power required to sustain the lift and the horizontal velocity. The corollary is that it also means that the drone could possibly fly faster for the same consumption at the end of the flight. Obviously, those considerations are only true for fuel powered systems and not electrical ones (the weight of the batteries remain constant, no matter their charge level).

In addition to that, the energy consumption of a UAV depends usually on a large set of parameters such as the altitude, the ambient temperature and pressure, the wind speed, the presence of gust, etc. And this is valid for both fuel or electric powered drones. In
order to simplify the modeling phase, the energy consumption is assumed to be constant over the course of the mission. Finally, in the models that require one or more refueling during the mission (DVRPs and CGRP), every refueling stop is supposed to take the same amount of time. This means that the fuel remaining from the previous tour or the fact that the next tour does not require a full tank are not taken into account.

Other hypotheses specific to the various situations will be introduced with their respective models.

**Notation**
For clarity, the following models will all respect the same notation. The parameters will be written in small letters, while the variables will be capitalized. The subscripts \(i\) and \(j\) will be used to refer to the index of the cities (see Table 2.1). Therefore, in the original network (depicted in Figure 2.1), \(i\) and \(j\) both go from 1 to \(n = 5\), were \(n\) represents the total number of cities ("clients" + base).
Some models also require to identify the different sub-routes of the solution. In that case, the superscript \(p\) indicates that the arc or the city is visited/inspected during the sub-tour \(p\).
Arcs (edges) are indicated in the form of \(ij\) to represent the fact that they join the cities \(i\) and \(j\).

A full list of all the symbols and notations used in this work is presented at the beginning of this thesis.

### 3.2 Traveling Salesman Problem

#### 3.2.1 Introduction
The traveling salesman problem is probably the simplest and most famous situation regarding route optimization. The goal is straightforward: one "salesman" (a drone) should visit a set of "clients" (the cities) by taking the shortest/fastest/cheapest route between them. Every client must be visited exactly once and the route must start and finish at the same point. In this case, the optimal route will be the one with the lowest traveling time, which is also equivalent to the shortest one under the assumption of constant speed.
3.2.2 Model

In the scenario depicted in chapter 2, the TSP model can be used to determine the total time required to visit all cities in the network and then come back to the base. Note that this first formulation is free of external constraints such as the fuel capacity of the vehicles or the state of the roads infrastructure (in the case where the model is used for a land vehicle). At first, this model does not take into account the inspection of the cities either. The drone just fly-by and continue its route. It is therefore a very idealistic situation. So far, this model does not introduce elements specific to humanitarian action, therefore it can be represented by its most famous equations using the Miller-Tucker-Zemlin formulation [Miller et al., 1960].

Parameters

Two main parameters can be set:

- \( c_{ij} \) [km]: the distance between cities \( i \) and \( j \);
- \( s \) [km/h]: the speed of the vehicle (constant).

As the problem will be solved relatively to the total mission time, these two parameters can be merged into one in order to represent the actual travel time along the arc \( t_{ij} \):

\[
  t_{ij} = \frac{c_{ij}}{s} \text{ [h]}
\]

Variables

The only variable of a classical TSP problem determines if the path between the cities \( i \) and \( j \) is included in the final optimal route:

\[
  X_{ij} = \begin{cases} 
  1 & \text{if the edge from city } i \text{ to city } j \text{ belongs to the solution;} \\
  0 & \text{otherwise.}
\end{cases}
\]

In addition to that, the Miller-Tucker-Zemlin formulation [Miller et al., 1960] relies on the introduction of a dummy variable in order to eliminate sub-tours. This variable is an integer denoted \( U_i \) with \( i = 1, ..., n \).

Formulation

The model can be written as

\[
  \min \sum_{i=1}^{n} \sum_{\substack{j=1 \atop j \neq i}}^{n} t_{ij} X_{ij} \quad (3.2a)
\]
subject to:
\[ \sum_{i=1}^{n} X_{ij} = 1 \quad j = 1, \ldots, n; \quad (3.2b) \]
\[ \sum_{j=1}^{n} X_{ij} = 1 \quad i = 1, \ldots, n; \quad (3.2c) \]
\[ U_i - U_j + nX_{ij} \leq n - 1 \quad 2 \leq i \neq j \leq n; \quad (3.2d) \]
\[ 0 \leq U_i \leq n - 1 \quad 2 \leq i \leq n; \quad (3.2e) \]
\[ X_{ij} \in \{0, 1\} \quad i, j = 1, \ldots, n; \quad (3.2f) \]
\[ U_i \in \mathbb{Z} \quad i = 1, \ldots, n. \quad (3.2g) \]

The objective function (3.2a) minimizes the travel time of the total route. The two first constraints (3.2b) and (3.2c) are the in-degree and out-degree constraints. They ensure that exactly one path comes into a city and one goes out of the city (each city is therefore visited exactly once). The following two equations (3.2d) and (3.2e) are the so called sub-tour elimination constraints. Their purpose is to force the solution to be one single large tour joining all cities and not multiple smaller isolated tours. Finally, constraint (3.2f) makes sure that the decision variable is binary and (3.2g) forces the dummy variable to be an integer.

### 3.3 Distance-constrained Vehicle Routing Problem

#### 3.3.1 Introduction

The vehicle routing problem is a generalization of the TSP. This problem aims at defining a set of routes that minimizes the total distance (or time) to visit each city once, using a fleet of vehicle that all start and stop at the same point (the base).

The classical formulation of the VRP was defined in 1959 [Dantzig and Ramser, 1959], but over the years this original model was significantly extended to more complex ones by the addition of different restrictions in order to form different types of constrained VRPs [Almoustafa, 2013], such as:

- Capacitated Vehicle Routing Problem (CVRP), where the vehicles have a finite capacity of items they can deliver;
• Distance-constrained Vehicle Routing Problem (DVRP), where the vehicles have a maximal range (distance or time);

• Distance-constrained Capacitated VRP, which is a combination of the two previous ones;

• Vehicle Routing Problem with Time Windows (VRPTW), extension of the CVRP where the items need to be delivered to the clients within some time window;

• Vehicle Routing Problem with Backhauls (VRPB), extension of the CVRP where the vehicles must also pick-up items from some clients;

• Vehicle Routing Problem with Pickup and Delivery (VRPPD), extension of the CVRP where all customers require both pickup and delivery.

In the present work, the drones are only expected to be used for monitoring. This means that the various forms of CVRP do not apply. However, it is assumed that the drones dispose of a finite reserve of fuel, which constrains their displacements. The DVRP is therefore employed to represent adequately the maximum flight-time available. As for the previous model (section 3.2), the "distance" criterion will be converted in time units under the assumption of constant traveling speed.

Note that in this thesis, the DVRP model will actually not represent the different routes assigned to a fleet of drones working in parallel. Instead, it will represent the decomposition of the mission for a single drone that needs refueling between routes. However, if the operator owns multiple similar UAVs, the same model and the optimal solution would also apply. In this situation, the multiple drones would be able to operate simultaneously, which would reduce effectively the total mission time. If multiple drones were to be employed however, the objective function should be adapted so it minimizes the individual mission time of each drone instead of the sum of all the sub-route times.

3.3.2 Model

In a DVRP problem, the number of vehicles used in the fleet (or in this case the number of refueling at the base), $k$, can either be fixed or free. In the former situation, $k$ is a simple parameter of the problem. In the latter, it becomes a variable that can be bound to specific constraint (integer with upper and lower limits). Note that a compromise is also possible where $k$ is a parameter that represents the maximum number of vehicle (or refueling) allowed. In that case, the constraints specific to $k$ will be written as inequalities instead of equalities. This is the option chosen for the DVRP model presented here after.
In addition to the traveling time of each edge, the objective function of the model can also take into account the number of refueling (as added time). In that case, the goal is then to minimize both the traveling time and the number of refueling. Most of the time the minimal traveling time will also result in the minimal number of routes. However, the opposite is not true and it is easy to find different solutions (with different total time) for the same number of vehicles (or refueling). It is however possible to determine some upper bounds on the total time or number of vehicle found with one optimization strategy or another, as detailed in [Li et al., 1992]. In the present case, the refuel "cost" ($\alpha$) is almost negligible in comparison with the traveling time of the edges. Hence, it does not impact much the results and only serves to discriminate solutions of similar time with different number of drones.

Even though the problem of DVRP (and DCVRP) is quite common, the literature on the topic is scarce [Kara, 2010]. These recent years, a few new formulations were proposed in order to properly express the model in a polynomial form, especially regarding the sub-tour elimination and distance constraints. Most of these efforts were produced by Kara, who presented multiple papers that synthetized the existing formulations of the DVRP and proposed new ones (vertex-based and flow-based) [Kara, 2008; Kara, 2010; Kara, 2011; Kara et al., 2011]. The DVRP model used in this thesis is directly derived from the flow-based, two-indexed integer linear programming formulation (ILPF) DVRP proposed by Kara in these references.

**Parameters**

This model uses the same parameters than the TSP. These are the distance between cities ($c_{ij}$) and the constant speed of the vehicle ($s$). Once again, they can be combined to express the travel time ($t_{ij}$) as described in Eq. (3.1).

A new parameter must be introduced to represent the maximal capacity of the fuel tank. As the model is expressed in the time-domain, this parameter is actually defined as the flight-time limit with a full tank. It is noted $\lambda$ and expressed in hours. The value of $\lambda$ can be considered constant for each route, which would indicate a complete refueling of the drone between each trip.

The maximal number of refueling allowed, $k$, is also a parameter of the problem. It is trivial that its value should be equal to the number of clients to serve. Indeed, this would mean that the drone comes back to the base before inspecting any city. A higher value of $k$ would therefore represent a route that does not visit any city, which is impossible. Therefore,

$$k = n - 1$$
Finally, a constant refueling cost, $\alpha$, is added to the objective function in order to represent the time "lost" for refueling and differentiate solutions with the same traveling time but different number of refueling.

**Variables**

The path decision variable used in the TSP model can be used as-is. This is due to the fact that each route is traveled by the same drone with the same characteristics (i.e. maximal flight-time and velocity). If the drone parameters were different from one route to another, this decision variable would have been dependent of the route as well, and a three-indexed ILPF should have been employed.

\[
X_{ij} = \begin{cases} 
1 & \text{if the edge from city } i \text{ to city } j \text{ belongs to the solution;} \\
0 & \text{otherwise.}
\end{cases}
\]

The formulation of the DVRP proposed by Kara [Kara, 2008] introduces also an auxiliary variable $V_{ij}$ that measures the total time traveled by a vehicle from the origin to the node $j$ when it goes from node $i$ to node $j$ and is equal to zero otherwise.

**Formulation**

\[
\begin{align*}
\min \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} X_{ij} + \sum_{j=1}^{n} \alpha X_{1j} - \alpha \right] \\
\text{subject to:} \\
\sum_{i=1}^{n} X_{ij} = 1, & \quad j = 2, \ldots, n; \quad (3.3b) \\
\sum_{j=1}^{n} X_{ij} = 1, & \quad i = 2, \ldots, n; \quad (3.3c) \\
\sum_{j=2}^{n} X_{1j} \leq k, & \quad (3.3d) \\
\sum_{i=2}^{n} X_{i1} \leq k & \quad (3.3e)
\end{align*}
\]
3.4 DVRP with city inspection

3.4.1 Introduction

In this model, an additional time ($h_i$) is added to account for the inspection of any city $i$. If this inspection time does not impact much the route of a land vehicle with unlimited range, it is definitely of utmost importance for the drone. Indeed, every additional time spent loitering over a city for its monitoring means that more fuel is consumed. This results in a lower total possible flight-time for the UAV, which could lead to an increase of the refueling frequency or change of routes. In the end, this would likely impact negatively the total mission time. In order to simplify the problem, this loitering time will be pre-determined as a function of the population of a city (a larger city requires a longer inspection time). However, in realistic application, that criterion could very well be specified by means of a stochastic parameter to denote the fact that it is not possible to know how long a city should be monitored before arriving on site.
3.4.2 Model

This model is heavily based on the previous one. The only major difference is in the expression of $t_{ij}$, which will also reflect the time spent loitering over the cities.

Parameters

A new parameter ($h_i$) is required to represent the hovering time (in hours) necessary for the inspection of a city $i$. In order to be closer to the reality, that inspection time is set to be proportional to the population of a city (a larger city requires a longer inspection). The inspection time was chosen arbitrarily to be 1 minute of monitoring per 10,000 inhabitants. This leads to a maximum of approximately 25 minutes of inspection in the worst case (Saint-Marc).

$$h_i = \frac{\text{pop}_i}{10000} \cdot \frac{1}{60} \text{[h]}$$

(3.4)

Note that the situation in the base is supposedly known, which means that it does not require monitoring and $h_1 = 0$.

In order to simplify the modeling part, the city monitoring time is added directly to the time initially required to travel an edge ($t_{ij}$). Doing so allows to reuse the previous model without any major changes. As each city could only be connected with two edges in the final solution, the new time associated with each edge now becomes

$$t'_{ij} = t_{ij} + \frac{h_i}{2} + \frac{h_j}{2} \text{[h]}$$

(3.5)

Variables

The formulation of the model is exactly the same as for the DVRP detailed in Section 3.3. Thus, no new variables need to be introduced.

Formulation

The only difference with the DVRP presented in Section 3.3, is that $t_{ij}$ must be replaced by $t'_{ij}$ (see Eq. (3.5)) in all the equations of the DVRP model (Eqs. (3.3)) in order to take the inspection time of the cities into account. As the mathematical expression of the model is the same as the previous one, it is not repeated here.
3.5 Capacitated General Routing Problem

3.5.1 Introduction

This model aims to cover another useful aspect of humanitarian operations. Indeed, it is not only important to collect valuable information about the surrounding cities; NGOs must also know how to reach them to deliver assistance. As roads are often damaged during a natural disaster, it would be tremendously helpful to know beforehand the status of the transportation network between cities. This piece of knowledge should greatly reduce the time spent in traffic or lost because the road is damaged, blocked by debris or purely non-existent anymore. In the end, the humanitarian supply chain could be deployed more efficiently and more people could benefit of the assistance.

This model is devised so that the UAV is forced to monitor all the physical roads in the network. However, the drone still keeps the possibility to fly in straight-lines if it is advantageous. This means that the ground and air network should be merged into one. The collect of information over actual roads will greatly impact the flight-time of a UAV (as physical roads are undoubtedly longer than air paths). In addition to that, the previous considerations still apply: the drone endurance is limited and it must spend time loitering over the cities to inspect them. All these will most likely require more refueling between mappings and the use of different smaller routes.

3.5.2 Model

From a mathematical point of view, the problem now requires an arc routing model similar to the ones described in [Chow, 2016; Almoustafa, 2013]. However, the scenario proposed in this thesis introduces two challenges barely discussed in the previous literature.

Usually, in arc routing problems such as the Chinese or rural postman problems or the garbage collection problem, the arcs represent the roads and the nodes represent the crossroads between them. Obviously, with this formulation, crossroads do not especially need servicing as they are just representing a junction between arcs. However, in the scenario depicted in this thesis, nodes represent cities that also require inspection. This means that some extra monitoring time should be integrated in the arc routing problem to account for their inspection. Such problem is a mix between DVRP and CARP, and is commonly referred as a Capacitated General Routing Problem (CGRP) [Corberán, 2014; Pandi and Muralidharan, 1995; Prins and Bouchenoua, 2005]. Unfortunately, the formulations presented in these references do not fit well the situation studied here and a new approach is required.
The second challenge comes from the fact that the network is initially made of physical roads and air paths. This particularity means that, sometimes, cities can be linked by two sets of valid edges (one on land, one in the air). As the physical roads must be inspected only once, the air paths must remain included in the network so they could be used if required. The fact that two valid links may exist between cities introduces modeling conflicts, as the variables and parameters expressed in $(i,j)$ could possibly refer to different arcs between the same cities.

As always, the goal of this model is still to minimize the total time required for mapping the whole network.

**Merging of the air and ground networks**

The situation depicted here is quite uncommon as two different types of paths exist in the network (see Figure 3.1). For instance, in the figure, the cities 1 and 5 are connected by road 2 and air path 4.

In order to avoid that confusion, the road network can be modified a bit by using a node insertion technique.

Consider the example illustrated in Figure 3.2 with a road between cities $A$ and $B$. If a new pseudo-city, $C$, is inserted in the middle of the road $A - B$, the direct road between $A$ and $B$ can therefore be replaced by the two smaller roads $A - C$ and $C - B$ of half the initial length. Obviously, the flight paths should remain untouched and no new possible air link should exist from $C$. That way, once the drone arrives at $C$, it is forced to continue with the second part of the road or come back on its track.
3.5. Capacitated General Routing Problem

With this technique, there are no longer two different direct link between two cities. Either they are directly connected on the air network, or indirectly connected through pseudo-cities on the road network. Note that the pseudo-cities are pure mathematical constructions, which means that no inspection time is added on those points.

The new consolidated network can be observed in Figure 3.3.

Even though this method solves efficiently the different issues caused by the fusion of the two networks, it also comes with a major drawback. Where the previous situation counted only 5 nodes, the new one has 12. Hence, the model now counts a lot of extra nodes which will make the numerical resolution more complex. A very large number
of equations will thus be required for every constraint of the model, and the total model will quickly grow in size. While problems of this size are still manageable on a regular desktop computer, a larger network would probably require specialized hardware and software.

**Parameters**

First of all, a new parameter must be introduced to differentiate edges that require inspection (roads) from the ones that could be crossed but are not a mandatory part of the solution (air paths). This demand for inspection \( d_{ij} \) is therefore represented by a binary variable defined as

\[
\begin{align*}
    d_{ij} = \begin{cases} 
        1 & \text{if the edge must be inspected (i.e. the edge is a road),} \\
        0 & \text{if inspection of the edge is not mandatory (i.e. the edge is aerial).}
    \end{cases}
\end{align*}
\]

As explained previously, there seems to be no model proposed for the CGRP that would fit the situation studied in this thesis. The key problem is that the addition to the node inspection time leads to some difficulties in comparison with a typical CARP model. Indeed, in arc routing problems, it is not uncommon to visit the same node multiple times per route or with multiple routes. Moreover, many cities are connected to more than two arcs, while their inspection time only needs to be counted once. This means that equation (3.5) is no longer valid for this model.

 Thankfully, the arc routing models can be written with a variable that determines if the arc is serviced (i.e. inspected) by the vehicle or simply crossed by it. By enforcing the fact that a road can only be inspected once (but crossed multiple times), it is therefore possible to ensure that the cities are only monitored once. The key is to decouple the time required to cross the arc \( t_{ij} \) (as initially defined in Eq. (3.1)) and the servicing time of the arc, which accounts for the inspection of the cities.

In this case, a new parameter corresponding to an additional time for inspecting an arc \( (e_{ij}) \) should be added. As all roads originating from a city \( i \) will be serviced exactly one time each, it is therefore trivial to distribute the inspection of the city equally on all these edges. Mathematically, the added time for servicing a road can then be defined as

\[
e_{ij} = \frac{h_i}{\text{nb roads from } i} + \frac{h_j}{\text{nb roads from } j} \quad \text{[h]}
\]

(3.6)

For instance, upon inspection of the new network graph in Figure 3.3, it can be observed that the city 2 is connected to 4 roads. This means that any of these edge will count a contribution of \( h_2 / 4 \) in its servicing variable \( e_{ij} \).
Note that for this to work, all cities must be connected to at least one arc that requires inspection, and each arc should be serviced precisely one time.

Along the demand for servicing \( d_{ij} \) and the edge servicing time \( e_{ij} \), the other parameters are the ones used so far

- \( t_{ij} \), the time required to travel along an edge \((i, j)\), as defined in Eq. (3.1);
- \( \lambda \), the maximal endurance of a UAV;
- \( k \), the maximum number of vehicles.

Note that the new network topology calls for a redefinition of some parameters. Indeed, the number of cities \( n \) is now 12 instead of 5, as all the pseudo-cities should be also counted. Moreover, as this problem focuses mostly on the edges of the network, the maximal number of vehicles is no longer equal to the number of cities. Instead, it could be set to the number of full-length arcs to inspect, which means in this case \( k = 7 \).

### Variables

Two new variables need to be introduced for this model. The first one, \( L_{ij}^p \) determines if the arc \((i, j)\) is serviced by the sub-route \( p \) or not, and the second one is the flow on arc \((i, j)\) for the sub-route \( p \).

\[
L_{ij}^p = \begin{cases} 
1 & \text{if the arc } (i, j) \text{ is serviced by sub-route } p, \\
0 & \text{otherwise.} 
\end{cases}
\]

The only other variable is \( X_{ij}^p \), which describes if an arc \((i, j)\) is traveled in sub-route \( p \) or not.

### Formulation

The following mixed-integer programming formulation is adapted from the one proposed by Chow in [Chow, 2016].

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{p=1}^{k} t_{ij} X_{ij}^p \tag{3.7a}
\]
subject to:

$$\sum_{m=1}^{n} X_{mi}^p - \sum_{m=1}^{n} X_{im}^p = 0 \quad i = 1,...,n, \quad p = 1,...,k; \quad (3.7b)$$

$$\sum_{p=1}^{k} \left( L_{ij}^p + L_{ji}^p \right) = d_{ij} \quad i, j = 1,...n; \quad (3.7c)$$

$$X_{ij}^p \geq L_{ij}^p \quad i, j = 1,...n, \quad p = 1,...,k; \quad (3.7d)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \left( t_{ij} X_{ij}^p + e_{ij} L_{ij}^p \right) \leq \lambda \quad p = 1,...,k; \quad (3.7e)$$

$$\sum_{m=1}^{n} F_{im}^p - \sum_{m=1}^{n} F_{mi}^p = \sum_{j=1}^{n} L_{ij}^p \quad i = 2,...n, \quad p = 1,...,k; \quad (3.7f)$$

$$F_{ij}^p \leq n^2 X_{ij}^p \quad i, j = 1,...n, \quad p = 1,...,k; \quad (3.7g)$$

$$F_{ij}^p \geq 0; \quad (3.7h)$$

$$X_{ij}^p, L_{ij}^p \in \{0, 1\} \quad i, j = 1,...n, \quad p = 1,...,k; \quad (3.7i)$$

As always, the objective function (3.7a) minimizes the total time of the mission. The in-degree and out-degree constraints of the previous models were replaced by a flow conservation equation (3.7b) as multiple routes can now pass through the same cities. This ensures that the vehicles (the routes) going in a city also go out of the same city. Eq. (3.7c) ensures that the demand is met precisely (i.e. the physical roads are inspected only once). Constraint (3.7d) ensures that an arc can only be inspected in route $k$ if it is also traversed in route $k$. The flight-time limitation is expressed by Eq. (3.7e) and the sub-tour are eliminated through constraints (3.7f) and (3.7g). Finally, the last two constraints ensure that the decision variables are respectively positive (3.7h) and binary (3.7i).

Note that the objective function proposed in (3.7a) does not explicitly take into account the time for monitoring a city. As all cities need to be monitored exactly once, that time can be added afterwards, as a constant, to find the true total time of the mission ($+ \sum h_i$). Nonetheless, the city inspection time is taken into account properly in the maximum flight-time constraint as seen in Eq. (3.7e). It is also difficult to include the refueling time directly in the objective function as the
3.6 Other extensions

The situations and improvement discussed here under could all be relevant for the use of drones in humanitarian context. Sadly, they are a bit too complex and out of the scope of this thesis. Therefore, only a quick summary is presented and no formal mathematical development is given.

3.6.1 Dynamic DVRP with city inspection

In a dynamic (D)VRP, the city inspection time could be a stochastic parameter. In that case, the drone would only determine the total time needed for inspection once it arrives on site. This is more representative of a realistic scenario, where unexpected events are likely to appear and impact the time required for full monitoring. In order to facilitate the development of this model, the "random" inspection time could be first given a few possible values. Then eventually, the complexity of the model could be increased by using a probability density function for the inspection time.

3.6.2 Dynamic CGRP

A dynamic CGRP model could represent the detection of broken roads during their survey. In these events, the drone could be redirected in order to minimize the time lost to mapping a non-usable road. A simple approach for the re-routing would be to take advantage of the presence of the pseudo-cities that were used to combine the two networks (see Figure 3.2).

In this situation, if a problem is detected in the first half of the road (from A to C), the drone could continue towards C and recalculate all the possible flight paths from C to the rest of the network. The second portion of the road (C-B) would be useless and its demand for inspection could be reduced to $d_{CB} = 0$. Obviously, the demand for the roads already scanned could be set to 0 as well so the new route will not inspect them a second time. Also, the autonomy of the drone would need to be shortened due to the already traveled portion of the network. These new set of parameters could then be used to recalculate the optimal road to continue the mission. Note that another model would be needed for this re-optimization as the drone would start at C and must finish at the base.
The comparison with a land vehicle would be very interesting in this case. Indeed, the drone would be able to resume the mission without too much trouble, while a Jeep would be forced to turn around on its tracks and use a completely different route (making multiple passes through the same cities).

A further extension of this new model would be to integrate absolutely all the roads in the network and try to find a route that connects (indirectly) all the cities to the main base.

### 3.6.3 Randomly appearing targets

A model could also implement some randomness relative to different parameters. This would make it more representative of a crisis situation. In this type of scenarios, points of interest could appear at random times and in random positions. Consider for instance a school bus trapped somewhere in a smaller road or some campers missing in the jungle that reach out to the base calling for help. These targets could appear virtually anywhere on the network. Due to its great mobility, the drone could be rerouted from its original mission in order to check on those newly acquired targets.

The randomness of such events would require a completely new form of modeling using stochastic parameters. In this scenario, different options should be taken into account for the appropriate time to recalculate the new optimal route:

- immediately
- at a given interval
- once the drone arrives at the next city
- at each refueling

The results obtained with these various options should then be compared with the true optimal solution, calculated using a perfect knowledge of all the events (solution at time $= \infty$). With enough simulations, it would then be possible to decide which is the best time frame for re-routing the drone and what results the operator could expect with respect to the true optimal solution. This model could easily be merged with the ones using a random city inspection time.

### 3.6.4 Classification of targets

Those models could be used to classify the targets to monitor based on some criterion. This criterion would then force the routes to monitor some cities before the others.
With the same train of thought, the urgency of the situation could be integrated in the problem. In that case, the model could try to minimize the waiting time before inspection as well as the total time to complete the mission.

### 3.6.5 Complex fuel model

As explained in Section 3.1, many parameters should impact the drone energy consumption (even for electrical ones). Some of these factors could be included in the model in order to be more precise. This could also lead to an optimization of the drone performances as well. The issues related to the energy consumption of drones have been almost completely neglected in the literature so far. The only reference on the topic is from last year where Venkatachalam et al. started to tackle the problem by representing the fuel consumption as a stochastic parameter [Venkatachalam et al., 2018].

### 3.6.6 Multiple bases

One of the most restrictive constraints on drones is the limited flight-time (and therefore range). By introducing new bases on key locations of the network, the total range covered by the UAVs could be greatly extended. As explained in chapter 1, these bases would not need tremendous infrastructure if the drones used are capable of VTOL. By simply stocking fuel or preparing fully charged batteries, these small deposits could have an enormous impact on the mission.

A new model could therefore try to optimize the position of these centers in order to extend the region covered by the drones as much as possible, or simply in order to avoid losing time for fly-backs to the main base.

### 3.6.7 Mobile base

By taking inspiration from the flying sidekick problem [Murray and Chu, 2015], the multiple static bases could be replaced by one or many mobile ones in the form of a fuel truck that continuously moves along the road network. This model could therefore optimize simultaneously the position of the truck and the UAV in order to completely cut the time lost to refueling trips to the bases. Obviously, the fuel trucks could also offload some of the work of the UAV and do some inspection by themselves in order to reduce furthermore the total mission time.
4 Applications

4.1 Introduction

The models presented in chapter 3 were all implemented in MATLAB and solved with the mixed-integer linear programming solver intlinprog. This chapter presents the different results (optimal routes) obtained with respect to various input parameters such as the speed of the UAV, the inspection time of the cities or the endurance of the vehicle. Each model was applied to the basic network described in chapter 2. In most cases, a comparison with a land vehicle (a Jeep forced to follow the existing roads) will be presented to showcase the actual benefit of an airborne system.

For a few interesting cases, the optimal solutions will be drawn on the map to better visualize the total trip of the vehicle. In those figures, the air paths will always be represented using dashed lines and the physical roads will be represented using continuous lines.

4.1.1 Parameters

Unless stated otherwise, the following parameters are considered for all the simulations:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Jeep</th>
<th>Drone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuel time</td>
<td>$\alpha$ [h]</td>
<td>N/A</td>
</tr>
<tr>
<td>Speed</td>
<td>$s$ [km/h]</td>
<td>75</td>
</tr>
<tr>
<td>Arcs lengths</td>
<td>$c_{ij}$ [km]</td>
<td>Fig. 2.2</td>
</tr>
</tbody>
</table>

Table 4.1: Default parameters used for the models implementation.

Note that the distance and speed are usually combined in order to form the traveling time for an arc as detailed in Eq. (3.1). However, for the land vehicle, the real travel time $t_{ij}^*$ will also be employed. The actual values of the road travel time are given in Figure 2.2.
4.2 Traveling Salesman Problem

Considering the very limited set of cities and possible paths used in this scenario, the optimal route for a TSP is trivial. The only parameter that can vary in this model is the speed of the vehicle. However, as the distances between cities are constant and the speed is also constant through the whole route, the only thing this would impact is the total time required for the completion of the tour and not the actual path taken by the vehicle.

Figure 4.1 presents the optimal route for the land and air vehicles (Jeep and UAV). Note that in both cases the routes can be crossed clockwise or anti-clockwise because the arcs are undirected. This will also be the case for every solution of all the other models.

![Figure 4.1: TSP – Optimal route for a Jeep (left) and a UAV (right).](image)

The total times obtained for each road are presented in Table 4.2. The results corresponding to the "constant speed" line are the ones obtained with $t_{ij}$ as defined in Eq. (3.1), while the ones for "real speed" are found with the values of $t^*_ij$ given in Figure 2.2.

<table>
<thead>
<tr>
<th></th>
<th>Jeep</th>
<th>Drone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant speed</td>
<td>4.67</td>
<td>3.77</td>
</tr>
<tr>
<td>Real speed</td>
<td>8.62</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 4.2: TSP – Route time (in hours) without city inspection.

The "constant speed" situation is particularly interesting. Indeed, in that case, the Jeep is supposed to travel 25% faster than the UAV. However, because the roads are longer than the air paths, it finishes the mission with a delay of approximately one hour (24% slower than the UAV). If the Jeep travels at realistic speeds along each proposed road
4.3. Distance-constrained Vehicle Routing Problem

\( t_{ij}^* \), the true total time soars and the land vehicle takes more than twice the time of the drone (+128\%) to complete the full rotation.

On top of that, the result obtained for the Jeep with the real travel speed is very optimistic. In an actual emergency scenario, the roads are expected to be damaged, less practicable and the traffic may be more important as people are fleeing the devastated areas. All these considerations would likely impact negatively the real travel time on the ground transportation network and delay furthermore the completion of the mission.

In the situation were the cities must be inspected during the tour, the inspection time of each city can simply be added to the total time. This means that the total mission would be extended by 0.83 hours, leading to the results of Table 4.3.

<table>
<thead>
<tr>
<th></th>
<th>Jeep</th>
<th>Drone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant speed</td>
<td>5.5</td>
<td>4.6</td>
</tr>
<tr>
<td>Real speed</td>
<td>9.45</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Table 4.3:** TSP – Route time (in hours) with city inspection.

As explained in chapter 3, the TSP is an oversimplified model and does not fit well UAVs that usually have a tight flight-time constraint. Nevertheless, this model is sufficient for a land vehicle which has usually a far greater range and could even carry the extra fuel required in external tanks. The total times of 8.62 and 9.45 hours will be kept as the "time to beat" for drone applications in the DVRP models respectively without and with city inspection.

4.3 Distance-constrained Vehicle Routing Problem

Upon close inspection of the results for the TSP model, it could be noted that the total flight-time of the UAV is actually longer than the flight time of entry-level drone platforms. As the TSP model does not take into account the fuel limit of the UAV, this type of result was expected. In an actual application, the drone should either refuel at some point on the route to continue its mission or a new model must be employed to allow a refueling at the base. This is precisely the limitation that should be overcome by the use of a DVRP model.

In this situation, a maximal endurance limit is enforced on the drone so it must come back at the base before running out of fuel.

The possible routes for the UAV heavily depend on this maximum endurance. Especially, it is important to impose a lower bound when varying the maximum flight-time of the drone. Indeed, the UAV must at least be capable to reach the farthest city and come
back in a single trip. In this case, the UAV flight-time must be at least 2.50 hours (see details in Table B.1 in appendix). If the UAV autonomy is lower than that, the farthest city will never be reached and there will be no solution to the model. If that was the case in real life, a few modifications are required:

- The drone could be allowed to refuel somewhere else, or
- the cities that are too far could be dropped from the mission, or
- the drone could be replaced with a better one.

4.3.1 No refueling time

In this first simulation, the refueling time is set to 0 hours (i.e. \( \alpha = 0 \)). As this refueling time only plays a minor role in the objective function (see Eq. (3.3a)), nothing particular is expected.

The routes presented in Figure show the evolution of the solution with respect to the maximal autonomy of the UAV. The solution with a maximum flight-time above 3.77 hours is not presented, as it is equivalent to the one of the TSP (see Figure 4.1).

![Figure 4.2: DVRP – Possible routes for different values of the maximum autonomy, no refueling time.](image)

Note that some arcs have been curved on the figure in order to facilitate the identification of outward and return edges of the sub-routes. In reality they are all straight lines that overlap.

Three cases can therefore be defined, depending on the maximum flight-time of the drones:
4.3. Distance-constrained Vehicle Routing Problem

1. $3.77 < \lambda$: the drone has more autonomy than the TSP solution. No refueling is required and the solution reduces to the TSP one which is illustrated in Fig. 4.1 (right);

2. $2.6 < \lambda < 3.77$: the drone has not enough autonomy to complete the mission in one tour, but has enough autonomy to complete the tour 1-4-3-2. A second trip will be required after refueling to visit the city 5. The results are presented in Figure 4.2(a);

3. $2.5 < \lambda < 2.6$: the drone has enough autonomy to reach the farthest city and come back but not enough to complete the green tour in Figure 4.2(a) anymore. The sub-route 1-4-3-2 is then split in two. This solution is shown in Figure 4.2(b).

The times of each sub-route and the total time of each mission are presented in Table 4.4 (where the roads color are taken from Figure 4.2).

<table>
<thead>
<tr>
<th>Road</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>3.77</td>
<td>2.60</td>
<td>2.2</td>
</tr>
<tr>
<td>Purple</td>
<td></td>
<td>2.42</td>
<td>2.42</td>
</tr>
<tr>
<td>Orange</td>
<td></td>
<td></td>
<td>2.5</td>
</tr>
<tr>
<td>Total</td>
<td>3.77</td>
<td>5.02</td>
<td>7.12</td>
</tr>
</tbody>
</table>

**Table 4.4:** DVRP – Time (in hours) for each route for different values of the endurance, no refueling time.

Note that there is no other optimal solution possible, as the case 3 is already the worst case scenario. Indeed, in this situation, the longest route is already equal to the minimal endurance of the drone. Therefore, it is not possible to reduce even more the autonomy and increase the number of refueling trips.

As expected, even though there is no added time for refueling, the solutions that require the more returns to the base also have the longest total mission time. Still, it is important to note that all the solutions so far are better than a Jeep doing a TSP tour with real traveling time (8.62 hours).

### 4.3.2 Refueling time

This time, the refueling time is taken into account in the objective function ($\alpha = 0.1$ hour). The results obtained were similar to the ones without the refueling penalty. Each route has exactly the same travel time. The only difference is that the total time to
complete the mission has increased of 0.1 hour for each additional route. Even with this small contribution from the refueling time, all three possible solutions complete the mission in five to one hour before the Jeep in real conditions.

Considering the simplicity of the problem studied in this example, this behavior was expected. Nonetheless, in a situation with many more nodes, the refueling cost could considerably impact the solution, especially if that cost is quite high compared to the traveling time. This would be the case, for instance, for an electrical UAV that would need to wait for its battery to recharge.

### 4.4 DVRP with city inspection

This model adds extra time in the arcs to account for the inspection of a city. This added time simply increases the sub-route time, and therefore stresses the maximum endurance constraint of the UAV, which leads to more frequent refueling or redefinition of the roads.

As it was shown previously that the refuel time does not impact much the solution in this case, only the case where it is taken into account (as $\alpha = 0.1$ hour) is considered.

The loitering time required to inspect each city is derived from the population as detailed in Eq. (3.4). This gives the following values:

<table>
<thead>
<tr>
<th>Id</th>
<th>Name</th>
<th>Population</th>
<th>$h_i$ [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Port-au-Prince</td>
<td>897 859</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Mirebalais</td>
<td>88 899</td>
<td>0.1482</td>
</tr>
<tr>
<td>3</td>
<td>Thomonde</td>
<td>56 274</td>
<td>0.0938</td>
</tr>
<tr>
<td>4</td>
<td>Hinche</td>
<td>109 916</td>
<td>0.1832</td>
</tr>
<tr>
<td>5</td>
<td>Saint-Marc</td>
<td>242 485</td>
<td>0.4041</td>
</tr>
</tbody>
</table>

**TABLE 4.5**: Inspection time for each city.

The total extra time to account for the city monitoring is therefore 0.8293 hours.

Just like before, a lower bound on the maximal autonomy of the drone must be set. This time, the UAV must be able to do the two-way trip to the farthest city, but it should also have enough endurance to inspect it before coming back. In the present situation, the minimal flight-time to complete the mission goes from 2.50 (no city inspection) to 2.82 hours (with city inspection). The details for these minimal sub-route times are presented in Table B.1 in appendix. Note that this time now corresponds to the inspection of city 5 and not 4 (as before). This is because Saint-Marc is the most populated city,
hence it requires a longer inspection time, while the distance to the base is similar for the two towns.

The Figure 4.3 represents the evolution of the different solutions with respect to the maximum flight-time of the drones.

![Figure 4.3](image)

**Figure 4.3**: DVRP with inspection – Possible routes for different values of the maximum autonomy.

The times for each route and each mission are presented in Table 4.6 (where the roads color are taken from Figure 4.3).

The solution evolves in a similar way than before. However, a new results appears in Figure 4.3(c). That specific case comes from the fact the inspection time of the city 2 is approximately 60% higher than for city 3. It is therefore better to include the city 3 in the tour with city 4 (as they are closer) and then only inspect the second city on a separate tour. If the autonomy of the drone is lowered even more, it is no longer possible to cover the extra distance and the inspection time of 3, which explains why the inspection of city 3 is paired with city 2 and not 4 anymore.
Chapter 4. Applications

<table>
<thead>
<tr>
<th>Road</th>
<th>Case (a)</th>
<th>Case (b)</th>
<th>Case (c)</th>
<th>Case (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>4.6</td>
<td>3.03</td>
<td>2.88</td>
<td>2.44</td>
</tr>
<tr>
<td>Purple</td>
<td>2.82</td>
<td>2.82</td>
<td>2.82</td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td></td>
<td>1.5</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>Refuel time</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>4.6</strong></td>
<td><strong>5.95</strong></td>
<td><strong>7.4</strong></td>
<td><strong>8.15</strong></td>
</tr>
</tbody>
</table>

Table 4.6: DVRP – Time (in hours) for each route for different values of the autonomy.

It is also interesting to note that the total time of all missions is still lower than the TSP solution for a Jeep. And this holds even if the Jeep only passes by the cities without inspecting them. If the inspection of all cities is taken into account in the Jeep total time, the total tour is still more than one hour above the worst case scenario for the UAV.

Finally, one last observation relative to the optimization strategy for the model can be made. As discussed in Section 3.3, the DVRP could be designed to minimize the number of vehicles or the total mission time. However, one does not necessarily imply the other. This is proven here with solutions (b) and (c), where in both cases the minimal number of drones possible are employed but one solution is longer than the other. This is why it is best to combine both objectives using a refueling penalty.

### 4.5 Capacitated General Routing Problem

The general routing problem is the most complex of the ones studied in this thesis. It is also the more numerically intensive to solve, as it involves more variables, equations and number of vehicles allowed. This is partially due to the fact that the model is based on a consolidated graph where extra pseudo-nodes (cities) have been added. In specifics, instead of 5 nodes and 10 edges, the new network depicted in Figure 3.3 has now 12 nodes and 24 edges. The other reason is that the model relies on a three-indexed notation, which leads to more equations for each set of constraints.

#### 4.5.1 Without city inspection

In this first situation, the city monitoring time is neglected and the problem reduces into a simple CARP. Mathematically, the only difference is that the arc servicing times $e_{ij}$ defined in Eq. (3.6) are all equal to zero.

Once again, the maximal flight-time of the UAV will be varied in order to inspect the evolution of the solutions.
As for the previous models, a lower bound should be set on the UAV endurance. In this case, it is not sufficient to check individual cities in isolation. Instead, the minimal requirement is that the UAV should be able to make a sub-route that scans only one road, for each road of the network. The longest of these sub-route time should then give the smallest autonomy possible for the UAV. For example, if a road A-B requires inspection, the shortest route possible (thanks to the triangle inequality) is 1-A-B-1, where 1 is the base. After computing each of these sub-route times (see details in Figure B.1 in appendix), it comes that the longer one-road trip is the one that inspects the road between Saint-Marc (5) and Hinche (4). This sub-route requires a total of 4.96 hours. Thus, the minimal autonomy of the UAV should be 4.96 hours as well. If it is lower than that, it will be impossible to monitor the road 4-5.

It is also worth noting that a UAV with such a high autonomy would be able to visit and inspect all the cities in one single route, as shown with the DVRP model in Figure 4.3(a). It might therefore be more interesting to decompose the mission in two separated phases in order to get first all the information about the cities then only scan the roads without inspecting the cities once again.

Depending on $\lambda$, many different solutions are possible. As the problem is numerically expensive to solve, only a few of these will be presented here. In general, three cases can be determined:

- 1 solution without refueling (1 route only);
- multiple solutions with 1 refueling (2 routes);
- multiple solutions with 2 refueling (3 routes).

The first solution (no refueling) is the optimal routing solution. It is only achieved if the endurance of the UAV is extremely large. This solution is presented in Figure 4.4. For clarity, the total route is represented as two-half missions, but everything happens with the same drone, without any refueling in the base in-between.

The second set of solutions includes different possible combinations of routes. Only a few of these are presented here after. The optimal 2-routes solution is obtained when the autonomy is just below the 1-route time ($\lambda < 11.22$ h). If the autonomy of the drone is reduced further, the routes continuously re-balance so that the longest sub-route is still below the endurance limit. Eventually after a while, the two sub-routes end up with a very similar traveling time, which is itself close to the maximal flight-time imposed. At that point, if the autonomy is reduced once again, the solution calls for a third route to complete the mission. Figures 4.5 and 4.6 present two possible solutions with 2 routes each. Another 2-routes solution is presented in Figure B.2 in appendix. Note that the roads not inspected/traveled by the drone are still shown on the graph (thin continuous
Chapter 4. Applications

Figure 4.4: CGRP without inspection – Optimal route without refueling (Sol. A).

brown lines) to better visualize the whole network and the part not covered by the subroute.

Figure 4.5: CGRP without inspection – Two-route solution (Sol. B.1).

The final set of solutions involves 3 routes each. At some point (depending on \( \lambda \)), the longest of these three routes is equal to the lower bound on the endurance. This means that the allowed flight-time of the drone cannot be decreased further and the final solution (worst case) is found. The Figure 4.7 shows the final solution, obtained when the drone autonomy cannot be reduced anymore.

No solution involving more than three routes for the total mission were found during the simulations with this data set.

The travel time of each sub-route and the total travel time of the mission are presented in Table 4.7 (where the roads and the case name are taken from the figures 4.4 to 4.7).
4.5. Capacitated General Routing Problem

**Figure 4.6**: CGRP without inspection – Two-route solution (Sol. B.2).

**Figure 4.7**: CGRP without inspection – Three-route solution (Sol. C).
Chapter 4. Applications

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>11.22</td>
<td>7.51</td>
<td>6.49</td>
<td>4.96</td>
</tr>
<tr>
<td>Purple</td>
<td>4.96</td>
<td>6.08</td>
<td>4.96</td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td></td>
<td></td>
<td></td>
<td>3.86</td>
</tr>
<tr>
<td>Refuel time</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11.22</strong></td>
<td><strong>12.57</strong></td>
<td><strong>12.67</strong></td>
<td><strong>14.02</strong></td>
</tr>
</tbody>
</table>

Table 4.7: CGRP without inspection – Time (in hours) for each route for different values of the autonomy.

The numeric data of Table 4.7 clearly show that more refueling of the UAV also means a longer total mission time. This indicates that the endurance of the UAV is one of the most important factor to take into account when looking for a new system. However, this result cannot be formerly proven and some edge-cases (with another more specific network topology) may not find the same conclusion.

It is also clear that the arc routing problem is extremely more time consuming than the simple node routing problem (DVRP). The total time of the mission is tripled for the best case scenario (3.77 hours become 11.22 hours) and doubled for the worst case scenario (7.12 hours become 14.02 hours). It is also quite interesting to note that the CGRP model can only be solved if the UAV has an autonomy that is already larger than the one required for the optimal solution of the node routing problem. It may therefore be more interesting for the drone operator to first inspect the cities as fast as possible (DVRP), and then come back with the same UAV to inspect the roads. That way, the different collaborators will already have a lot of valuable information about the cities to begin their work, while the drone is still inspecting the roads.

4.5.2 With city inspection

In this situation, an additional servicing time, $e_{ij}$ is added to each arc that represents a physical road, as detailed in Eq. (3.6). This servicing time accounts for the city monitoring by distributing the city inspection on all the roads that connect to it. Obviously, this inspection time impacts the minimum autonomy requirement of the drone. Once the new arc times are calculated, it can be found that the tour surveying only the road from Saint-Marc (5) to Hinche (4) is still the longest one, with a requirement of 5.15 hours. The calculation for each road are presented in the Table B.1 in appendix. The lowest endurance possible for a drone to complete this mission is 5.15 hours. Just like before, the available flight-time of the UAV is varied in order to observe the different solutions.
This time, the results can be divided in four categories:

- 1 solution without refueling (1 route only);
- multiple solutions with 1 refueling (2 routes);
- multiple solutions with 2 refueling (3 routes).
- multiple solutions with 3 refueling (4 routes).

As always, the first solution (no refueling) can only be obtained for drones with a very large autonomy. Unsurprisingly, the solution is exactly the same as without city inspection (see Figure 4.4). This result was expected because in this case, the city inspection could be considered as a constant of the system (all cities have to be inspected during the same trip in any case). The optimization then reduces to a simple traveling time minimization, which has only one solution in absence of flight-time limitation. Note that in this case however, the autonomy required for a single trip is larger than before (12.05 hours), because the inspection time of the cities was added (+0.83 hours).

Just like before, the 2-routes solutions are found by progressively decreasing the maximal autonomy of the vehicles lower than the 1-route solution time. Eventually when both routes have similar time and the autonomy is decreased lower than that value, the solution pushes for the creation of a third route. Two possible 2-routes solutions are presented in figures 4.8 and 4.9.

![Figure 4.8](image)

**Figure 4.8:** CGRP with inspection – Two-route solution (Sol. B.1).

It is interesting to note that these two solutions were also found without city inspection (they are respectively similar to the Figures 4.5 and 4.6). However, for the second solution ($\lambda = 6.89$), the slowest route without inspection becomes the longest one because the monitoring time of 5 is the largest of the whole network. In general, the
sub-route containing the most physical roads to (or from) 5 will usually take a longer
time to complete.

Different sets of 3-routes solutions appear if the endurance of the UAV is reduced
even more. An example is presented in Figure 4.10. Other 3-routes solutions for different
values of the autonomy are shown in Figure B.3 in appendix.

Finally, the last sets of solutions include four sub-routes. It is not possible to lower
the flight-time even more as the longer route of the solution presented here is already
equal to the minimal endurance required to complete the mission. Hence, this solution
corresponds to worst case scenario. Figure 4.11 presents the four sub-routes found with
this solution.

Just like before, the sub-route times and total time of the solutions presented here-
above are detailed in Table 4.8 (solutions C.2 and C.3 are the ones detailed in Fig-
ures B.3 in appendix).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>12.05</td>
<td>8.14</td>
<td>6.79</td>
<td>6.4</td>
<td>5.56</td>
<td>5.29</td>
<td>5.15</td>
</tr>
<tr>
<td>Purple</td>
<td>5.15</td>
<td>6.61</td>
<td>4.74</td>
<td>5.29</td>
<td>5.15</td>
<td>4.99</td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td>3.5</td>
<td>3.8</td>
<td>4.21</td>
<td>3.91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td></td>
<td></td>
<td></td>
<td>2.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refuel time</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>12.05</td>
<td>13.39</td>
<td>13.5</td>
<td>14.84</td>
<td>14.85</td>
<td>14.85</td>
<td>17.14</td>
</tr>
</tbody>
</table>

Table 4.8: CGRP with inspection – Time (in hours) for each route for
different values of the autonomy.
As it was already observed, more refueling leads to a significant increase of the total mission time. Another very interesting fact is that some solutions are very close to each other. Specifically here, solutions C, C.2 and C.3 all have a similar total time. However the autonomy requirement are quite different from one another (C needs UAVs capable of 6.4 hours flights while C.2 only needs 5.56 hours). This means that a drone with a significantly lower autonomy will not necessarily lead to an important increase of the total mission time. In this specific example, the drone of C.2 has about 20% less autonomy but the total mission time barely changes.

As the models could also be used to determine which drone to buy before a specific mission, it should be useful to include the total endurance as a variable that needs to be minimized as well, while guaranteeing a decent total mission time. Usually, drones with a lower autonomy are easier to find and less expensive than their long-range counterparts. This modification of the model could therefore provide a very useful tool to the people in charge of buying the assets of the organization.
Influence of the city inspection time

It is also interesting to analyze the evolution of the solution when the city inspection time changes. For this analysis, the inspection time of city 4 is doubled (from 0.1832 hours to 0.3664 hours, which corresponds to an increase of about 10 minutes) and the autonomy is kept unchanged. All the other inspection times remain the same. The results presented in Figure 4.12 show the solution for a drone with maximal endurance of 6.78 hours. These results should be put in perspective with the ones of Figure 4.10 that represent the same flight-time limitation but with the default city monitoring time.

The difference in the two solutions is quite significant. The three sub-routes are all redesigned just because the inspection time of city 4 was increased by 10 minutes. Although, the total mission time does not change much and goes from 14.84 hours to 15.03 hours. The difference is simply explained by the added inspection time of city 4.

This result emphasis on one of the biggest limitation of the static models. In this
case, if a major unexpected event was requiring a more throughout inspection of city 4, the initial results would have fallen short and the whole mission would have potentialiy drifted far apart from this ideal solution. When parameters of this importance are usually unknown, it is best to develop stochastic models such as the ones described in Section 3.6 to ensure that a reasonably good solution is achieved in case of unexpected delay.

4.5.3 Jeep with city inspection

Finally, the CGRP needs to be solved for the Jeep as well to provide a comparison point for the UAV performances. Obviously in this case, only the land network (described in Figure 2.2) is taken into account. This makes the problem significantly easier to solve numerically due to the limited number of variables and equations. Note that the following results all account for the extra time spent to inspect the cities.
Obviously, the arcs are supposed to be practicable by the vehicle in both direction without any issue. This represents a very idealistic scenario in the aftermath of a natural disaster.

The results for one single Jeep handling the whole mission without refueling are presented in Figure 4.13, where the total tour is detailed over two figures for clarity.

![Diagram](image)

**Figure 4.13:** CGRP with a Jeep – Optimal solution.

As the Jeep is unable to travel in straight lines between two cities, many roads need to be crossed twice. These multiple passes make the system quite inefficient. It may also cause significant issues in case of unexpected road blockages or traffic along one specific edge of the network.

The major NGOs usually dispose of multiple land vehicles for such large scale operations. It may then be interesting to check how fast the whole network could be mapped if each sub-route was short enough to be completed under one work-day. To this end, the maximal "autonomy" of the vehicles was set to the time requested to inspect the road 4-5 only, which is 9.2 hours in this case. Note when only road 4-5 is inspected, the cities crossed by the sub-route (cities 2, 3, 4) are not all inspected. Indeed, some roads are also taken by other sub-routes, and their servicing will only be accounted there. The model gives a solution with 3 Jeeps to complete the whole mission under 9.2 hours. The results are shown in Figure 4.14.

As expected, the first route (longest one) focuses on inspecting the road 4-5. The other two separate the network in West and East sub-networks.

The different times for 1 or 3 Jeeps are presented in Table 4.9. Note that the time of the two-half routes presented in Figure 4.13 were merged into one route ("green") in the table, as they are two parts of the same total route.
The first thing to notice is that, once again, the Jeep is slower than the worst case scenario for the UAV (i.e. lowest endurance, maximum number of refueling trips). And this does not even accounts for possible issues on the ground transportation network. In this example, a single Jeep takes approximately 7% more time to complete the mission than a "small-range" UAV that returns to the base three times for refueling.
In the situation where 3 different Jeeps are employed simultaneously, the mission time is bounded to the route that takes the most time, which is 9.18 hours. Such a small mission time is unfortunately not possible with one single UAV. However, all solutions involving two similar UAVs (or more) are able to beat this time by a significant margin. With two UAVs, Sol B.2 (see Table 4.8) is approximately 25% faster. Better solutions could likely be found for any number of drones. However, in that case, the objective function of the models should be rewritten so that all individual routes are minimized and not the sum of all routes.
5 Conclusion

5.1 Conclusion

Many aspects of the drone utilization in humanitarian context were addressed in this work. First, the introduction focused on a review of the whole situation related to drones in humanitarian action using a PESTEL analysis. The state of the art regarding the technology and the various optimization techniques was also discussed. Then, a realistic mission was described, based on the situation of Haiti. The different complexities of the network were introduced and the characteristics of the required UAV were discussed. Different models were presented in chapter 3, each adding more complexity on top of the previous ones. The mathematical formulation proposed matched the technicalities of the mission previously described. A few other models were introduced to extend the work to more complex situations.

Finally, the models were all implemented in a programming software and were solved with a mixed-integer linear programming solver for various input parameters (such as the endurance of the UAV or the inspection time of the cities). A throughout comparison with a land vehicle fulfilling the same mission was proposed for each model.

Various interesting observations emerged from this research. First of all, it was clearly shown that drones may not be the ideal solution for all types of humanitarian operations. For instance, deliveries of critical items are still too dangerous to handle. Also, the lack of proper regulations (or too restrictive ones) is a common issue when operating in developing countries. The existence of a wide range of technologies, each for their specific mission type, and their prohibitive pricing is also a severe restriction to the use of drones in common operations.

Secondly, the different models proposed seem to all fit quite well the various types of possible utilization for an imaging drone. Although, these models may still be too simplistic to represent accurately the vast range of challenges faced by the humanitarian actors.

Still, all these models were applied to a real-life network, and a few interesting results
emerged from that analysis.

It was demonstrated that in every situation, a single drone was always faster than a single land vehicle in normal conditions. This holds true even when the drone has the minimal autonomy possible and requires many trips to the base to refuel. In an emergency situation, the overhead of the drone would be even more significant as the Jeep would be delayed because of road issues (debris, traffic, etc.). In situations where two other Jeeps were added in order to distribute the total work, the addition of a second drone was sufficient to beat the Jeeps with a mission time 25% lower.

The autonomy of the drone is the most limiting factor to their utilization in operation. The mission designed here involved a very long-range (or long endurance) requirement that is hard to meet even with the most recent systems. In general, the drone with the longest autonomy was also resulting in the shorter total mission time. Nonetheless, some interesting phenomenon were observed where some solutions were marginally better with respect to the total flight-time, but far less efficient than their counterparts. In some situations, the total mission time with a drone with a very long autonomy was not better than the one obtained with a drone of lower endurance. If these numerical models were to be chosen to determine which drone platform to buy for a specific mission, it would be interesting to adapt the objective function in order to minimize the individual route times instead of the cumulative one. This would make it possible to buy a cheaper UAS with a lower autonomy, without sacrificing too much performances.

The autonomy constraint is especially difficult to meet for the CGRP model, as one of the longest road to inspect is located far away from the base. It was even calculated that the minimum autonomy to complete the CARP was already higher than the optimal solution time for the node inspection (DVRP). Therefore, if these scenarios were to be applied in real life, it would make sense to start first with a DVRP in order to get all the information about the cities, then do a CARP to monitor the status on the ground infrastructure. By doing so, the different experts will have more time to study the issues with the cities while the drone would scan the roads.

Fortunately, the models do not depend directly on the network and could be used just as well on smaller scale situations. In this case, the use of UAVs with significantly lower range would be a possibility. The added benefit of UAVs utilization in smaller network could especially be important in hard-to-reach areas, such as mountains, jungles, remote villages, etc.

In conclusion, this work demonstrated that the drones are definitely a great tool for mapping and survey in the humanitarian context. The different simulations showed that the advantages of using drones on large scale network was significant with respect to in situ observation on land. Although drones are considerably more expensive to buy than simple land-based solutions, the time they spare is infinitely worthy when lives depend
on the rapidity of the assistance delivered.

5.2 Future work

Some additional models and extensions were proposed in section 3.6. Unfortunately, their complexity puts them out of the scope of this thesis and no mathematical formulation was provided. These extensions include different classes of problems. The first ones include randomness (stochasticity) in various forms. In specifics, the inspection time of a city could be unknown until the vehicle arrives there, which would impact in a positive or negative way the rest of the route. Some new targets to inspect could also appear at random times and locations. And for the arc routing problems, roads could be damaged and impracticable. In this case, the drone could be rerouted towards more useful zones to inspect.

The second type of models involve joint-optimization. Such models could for instance combine one jeep for the inspection of the nearby cities and the UAV for the farthest ones. Moreover, as it was shown that the UAV autonomy was an issue, it might be useful to prepare the network by placing refueling stations on various points of the network to extend the UAV range. A model could be devised in order to optimize the position of these depots so that the total drone flight-time is kept at its minimum. Another version of the similar problem would be to use refueling mobile trucks instead of bases. Obviously, these trucks would also inspect the network and offload some of the UAV work in addition to extending its range.

The third type of models provide incremental additions to the existing ones. In emergency situations, it is common that some targets have priority over some others. This could be integrated in the models in order to minimize the waiting time of these targets. Another amelioration would be a better representation of the fuel consumption. As the autonomy is one of the key factor in these models, a better representation of the fuel consumption and of the capacities of the drone in that regard would lead to more realistic solutions.

Finally, some models could be a variation of the ones presented here but with the goal to become a tool to help during the buying process of a UAV. As it was previously showed, it is possible to obtain similar results with two drones of very different autonomy. Such model will therefore try to lower the individual route cost as well as the consolidated one.

These proposed modifications to the existing models could perfectly fit the humanitarian intervention framework. Most of these are based on pre-existing work, especially in the field of package deliveries. Although, the conversion to humanitarian models may not be so trivial.
A UAV systems

Examples of existing long-endurance UAVs

The Table A.1 presents some details about notable UAV platforms commonly used for imagery, mapping or surveillance. The models presented in this table are theoretically all usable for civilian operations. Each drone is differentiated by its type as described in Section 1.3 (F-W: fixed-wing, M: multi-copter, H: hybrid). The endurance and propulsion system technology are also given for each model.

Note that this list is not exhaustive as only a few examples of long autonomy systems are described. Moreover, many larger UAVs present an even longer endurance, but they are often confined to the defense sector or too large to be used in humanitarian applications. Therefore, they are not part of this list.

<table>
<thead>
<tr>
<th>Company</th>
<th>Model</th>
<th>Type</th>
<th>Endurance</th>
<th>Propulsion</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASC</td>
<td>CH-96 UAV</td>
<td>F-W</td>
<td>10 h</td>
<td>Fuel</td>
</tr>
<tr>
<td>Latitude</td>
<td>Latitude Hybrid Quadrotors</td>
<td>H</td>
<td>15 h</td>
<td>Hybrid</td>
</tr>
<tr>
<td>Flying-Cam</td>
<td>Discovery</td>
<td>M</td>
<td>2.5 h</td>
<td>Fuel</td>
</tr>
<tr>
<td>MartinUAV</td>
<td>V-Bat*</td>
<td>F-W</td>
<td>8 h</td>
<td>Fuel</td>
</tr>
<tr>
<td>Quaternium</td>
<td>HYBriX.20</td>
<td>M</td>
<td>4 h</td>
<td>Hybrid</td>
</tr>
<tr>
<td>Textron Systems</td>
<td>AeroSonde</td>
<td>F-W</td>
<td>14 h</td>
<td>Fuel</td>
</tr>
<tr>
<td>UAV Factory</td>
<td>Penguin B</td>
<td>F-W</td>
<td>24 h</td>
<td>Fuel</td>
</tr>
<tr>
<td>UAV Factory</td>
<td>Penguin C UAS</td>
<td>F-W</td>
<td>20 h</td>
<td>Fuel</td>
</tr>
<tr>
<td>UAV Solutions</td>
<td>Talon 240G</td>
<td>F-W</td>
<td>10 h</td>
<td>Fuel</td>
</tr>
<tr>
<td>Vanguard Defense</td>
<td>Shadowhawk</td>
<td>F-W</td>
<td>3 h</td>
<td>Fuel</td>
</tr>
<tr>
<td>Industries</td>
<td>Skyfront Perimeter 8 XLRs</td>
<td>M</td>
<td>5 h</td>
<td>Hybrid</td>
</tr>
</tbody>
</table>

*The V-Bat UAV is actually capable of vertical take-off and landing even though it is based on a fixed-wing configuration. This ability comes from the fact that it can tilt itself and land on its tail.*
B Additional results for the model applications

B.1 DVRP

Minimal UAV autonomy

The Table B.1 presents the minimum time of a route that originates from the base, reaches a city $i$ and flies back to the base. The drone speed is set to 60 km/h and the time with and without loitering over the city for inspection are presented. The table shows that to reach the farthest city (4) in the case without inspection, the drone needs a minimum autonomy of 2.5 hours. When the UAV must loiter over a city to inspect it, the longest trip is for city 5 with 2.82 hours, because it is the most populated city (hence it requires a longer inspection). A drone with an endurance lower than those values would not be able to complete the mission entirely.

<table>
<thead>
<tr>
<th>City</th>
<th>w/o inspec.</th>
<th>w/ inspec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>1.35</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>2.2</td>
<td>2.29</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
<td>2.69</td>
</tr>
<tr>
<td>5</td>
<td>2.42</td>
<td>2.82</td>
</tr>
</tbody>
</table>

Table B.1: Minimal route time (in hours) to reach each city individually, without and with inspection of the cities.

B.2 CGRP

B.2.1 Minimal UAV autonomy

The following figure and table presents the minimum time of a sub-route that include the inspection of only one physical road. Note that the inspection of the cities is also
Appendix B. Additional results for the model applications

added to the arc contribution as detailed in Eq. (3.6). These values were determined using a drone speed of 60 km/h.

It shows that the trip involving road 4-5 is the longest. Note that this trip is composed of edges (1,4) by air, then (4,5) following the physical road and then (5,1) by air.

This result implies that a drone would need at least 5.15 h of autonomy to be able to complete the Capacitated General Routing Problem. If its endurance is lower than that, the arc (4,5) would never be inspected. In a similar fashion, this table can also be used to determine which physical roads should be dropped from the inspection if only a drone with autonomy $\lambda^*$ was available.

<table>
<thead>
<tr>
<th>Id</th>
<th>City i</th>
<th>City j</th>
<th>w/o inspec.</th>
<th>w/ inspec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2.66</td>
<td>2.83</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
<td>3.7</td>
<td>3.9</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2.41</td>
<td>2.49</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4</td>
<td>4.64</td>
<td>4.79</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>5</td>
<td>3.33</td>
<td>3.5</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4</td>
<td>2.65</td>
<td>2.76</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>5</td>
<td>4.96</td>
<td>5.15</td>
</tr>
</tbody>
</table>

**Figure B.1:** Minimal route time to inspect each road individually, without and with city inspection.

**B.2.2 Examples of other possible routes**

The following graphs were obtained with other values for the maximum endurance of the UAV $\lambda$. The solutions presented here are not an extensive list and other set of sub-routes may be possible with other $\lambda$ values.

**Without city inspection**

The Figure B.2 presents the optimal solution for the CGRP without city inspection in the case of a drone endurance of 7.5 hours. The figure should be put in perspective with the other two-routes solutions presented in Figures 4.5 and 4.6.

In this case, the green sub-route takes 6.63 hours and the purple one 5.84 hours, totaling 12.57 hours when the refuel time is taken into account.
With city inspection

The two-routes solutions with city inspection are an exact match of the ones found without the city inspection. However, many more three-routes solutions were found when the city monitoring is taken into account. Two of them are listed below in Figures B.3.

The resulting times for these two routes is presented in Table B.2. Interestingly, the total time is similar between these two solutions (the difference are of the order of 0.001 hour) and the one presented in Figure 4.10.

<table>
<thead>
<tr>
<th>Road</th>
<th>Sol. C.2</th>
<th>Sol. C.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>5.56</td>
<td>5.29</td>
</tr>
<tr>
<td>Purple</td>
<td>5.29</td>
<td>5.15</td>
</tr>
<tr>
<td>Orange</td>
<td>3.8</td>
<td>4.21</td>
</tr>
<tr>
<td>Refuel time</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>14.85</strong></td>
<td><strong>14.85</strong></td>
</tr>
</tbody>
</table>

Table B.2: CGRP with inspection – Time (in hours) for each route for two other 3-routes solutions.
Appendix B. Additional results for the model applications

Figure B.3: CGRP with inspection – Two different 3-routes solutions. Sol. C.2 on the left, Sol. C.3 on the right.
## List of Figures

1.1 High-resolution drone images (left) and lower-resolution satellite images (right). From [Swiss Foundation for Mine Action, 2016].

1.2 Different types of drones. (a) Fixed wing, (b) Multi-rotor, (c) Hybrid.

2.1 Map of Haiti and localization of the relevant cities and roads.

2.2 Land network graph.

2.3 Aerial network graph.

3.1 Road (left) and air (right) networks connecting the same set of cities.

3.2 Insertion of an artificial city between two nodes.

3.3 New consolidated network graph (roads in red, air paths in green).

4.1 TSP – Optimal route for a Jeep (left) and a UAV (right).

4.2 DVRP – Possible routes for different values of the maximum autonomy, no refueling time.

4.3 DVRP with inspection – Possible routes for different values of the maximum autonomy.

4.4 CGRP without inspection – Optimal route without refueling (Sol. A).

4.5 CGRP without inspection – Two-route solution (Sol. B.1).

4.6 CGRP without inspection – Two-route solution (Sol. B.2).

4.7 CGRP without inspection – Three-route solution (Sol. C).

4.8 CGRP with inspection – Two-route solution (Sol. B.1).

4.9 CGRP with inspection – Two-route solution (Sol. B.2).

4.10 CGRP with inspection – Three-route solution (Sol. C).

4.11 CGRP with inspection – Four-route solution (Sol. D).

4.12 CGRP with inspection – Three-route solution with a new inspection time.

4.13 CGRP with a Jeep – Optimal solution.


B.1 Minimal route time to inspect each road individually, without and with city inspection.

B.2 CGRP without inspection – An other two-route solution (Sol. B.3).
B.3 CGRP with inspection – Two different 3-routes solutions. Sol. C.2 on the left, Sol. C.3 on the right.
List of Tables

2.1 Major cities considered in the scenario. 19
4.1 Default parameters used for the models implementation. 43
4.2 TSP – Route time (in hours) without city inspection. 44
4.3 TSP – Route time (in hours) with city inspection. 45
4.4 DVRP – Time (in hours) for each route for different values of the endurance, no refueling time. 47
4.5 Inspection time for each city. 48
4.6 DVRP – Time (in hours) for each route for different values of the autonomy. 50
4.7 CGRP without inspection – Time (in hours) for each route for different values of the autonomy. 54
4.8 CGRP with inspection – Time (in hours) for each route for different values of the autonomy. 56
4.9 CGRP with a Jeep – Time (in hours) for each route. 61
A.1 Examples of long-endurance UAVs for civilian applications. 67
B.1 Minimal route time (in hours) to reach each city individually, without and with inspection of the cities. 69
B.2 CGRP with inspection – Time (in hours) for each route for two other 3-routes solutions. 71
## Contents

1 Introduction  
1.1 Thesis subject ............................................. 1  
1.2 Drones for humanitarian assistance ......................... 3  
  1.2.1 PESTEL analysis ......................................... 4  
  1.2.2 Mission types ........................................... 8  
1.3 Drone technologies ........................................... 11  
  1.3.1 Types of drones .......................................... 11  
  1.3.2 Drone main components ................................. 13  
1.4 Optimization of drone routing ................................. 14  
  1.4.1 Drone routing in humanitarian context .................. 15  
  1.4.2 Dynamic vehicle routing problems ....................... 15  
  1.4.3 Optimization goals ..................................... 16  

2 Case Study  
2.1 Context ..................................................... 17  
2.2 Situation ................................................... 18  
  2.2.1 Cities .................................................. 18  
  2.2.2 Paths and network graphs ............................... 19  
2.3 UAV selection ................................................. 20  

3 Models  
3.1 Introduction ................................................. 23  
3.2 Traveling Salesman Problem ................................. 25  
  3.2.1 Introduction ............................................ 25  
  3.2.2 Model .................................................. 26  
3.3 Distance-constrained Vehicle Routing Problem ............. 27  
  3.3.1 Introduction ............................................ 27  
  3.3.2 Model .................................................. 28  
3.4 DVRP with city inspection .................................. 31  
  3.4.1 Introduction ............................................ 31  
  3.4.2 Model .................................................. 32
3.5 Capacitated General Routing Problem ........................................ 33
  3.5.1 Introduction ............................................................... 33
  3.5.2 Model ................................................................. 33
3.6 Other extensions ............................................................... 39
  3.6.1 Dynamic DVRP with city inspection ............................... 39
  3.6.2 Dynamic CGRP ......................................................... 39
  3.6.3 Randomly appearing targets ....................................... 40
  3.6.4 Classification of targets ........................................... 40
  3.6.5 Complex fuel model ............................................... 41
  3.6.6 Multiple bases ...................................................... 41
  3.6.7 Mobile base ......................................................... 41

4 Applications ................................................................. 43
  4.1 Introduction ............................................................... 43
  4.1.1 Parameters ............................................................ 43
  4.2 Traveling Salesman Problem .......................................... 44
  4.3 Distance-constrained Vehicle Routing Problem ................ 45
    4.3.1 No refueling time ................................................. 46
    4.3.2 Refueling time .................................................... 47
  4.4 DVRP with city inspection ........................................... 48
  4.5 Capacitated General Routing Problem .............................. 50
    4.5.1 Without city inspection ......................................... 50
    4.5.2 With city inspection ............................................. 54
    4.5.3 Jeep with city inspection ...................................... 59

5 Conclusion ................................................................. 63
  5.1 Conclusion ............................................................... 63
  5.2 Future work ............................................................. 65

A UAV systems ................................................................. 67

B Additional results for the model applications ....................... 69
  B.1 DVRP ................................................................. 69
  B.2 CGRP ................................................................. 69
    B.2.1 Minimal UAV autonomy ......................................... 69
    B.2.2 Examples of other possible routes ............................ 70

Bibliography ................................................................. 77
The present thesis aims to determine if drones could effectively replace *in-situ* inspection for the collect of information in humanitarian crisis situations. This study focuses on the elaboration of optimization models and their application to route efficiently an Unmanned Aerial Vehicle for a given humanitarian mission. The four models developed were all implemented in a mixed-integer linear programming utility so the solutions for UAV routing could be compared with a land vehicle completing the same mission.

This report is divided in four main chapters. The first one introduces the drone technology environment and the humanitarian applications with these drones. A macro-environmental study is performed using a "PESTEL" analysis to better understand the reasons why drones would be useful in the humanitarian context. In the second chapter, a hypothetical mission based on a simplistic version of the transportation network of Haiti will be presented. This scenario will be used in the following chapters as a baseline case study. Chapter three concerns the elaboration of four different optimization models. The first three are a subset of node routing problems (Traveling Salesman Problem and Distance-constrained Vehicle Routing Problems), while the last one is closer to the arc routing category (Capacitated General Routing Problem).

The results obtained for all these models show that a UAV is always faster than a single land vehicle operating in normal conditions for the test network. However, due to the very large network used as a basic example, the endurance limitations of existing UAVs appear to be a major issue for the real-world applications. Some existing UAV systems could fulfill the mission but they are likely still too expensive for humanitarian organizations. Fortunately, the models elaborated here can be applied to any network, and therefore the advantages of drones with a smaller autonomy can be verified, especially in jungle or mountain environments.