An inverse Mean-Field-Homogenization-based micro-mechanical model for stochastic multiscale simulations of unidirectional composites

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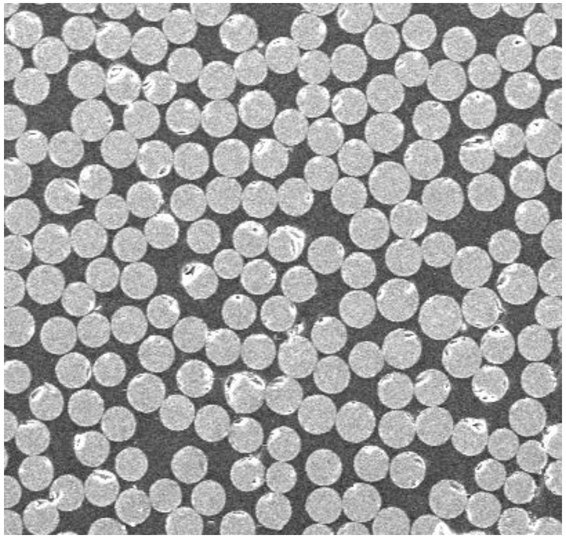
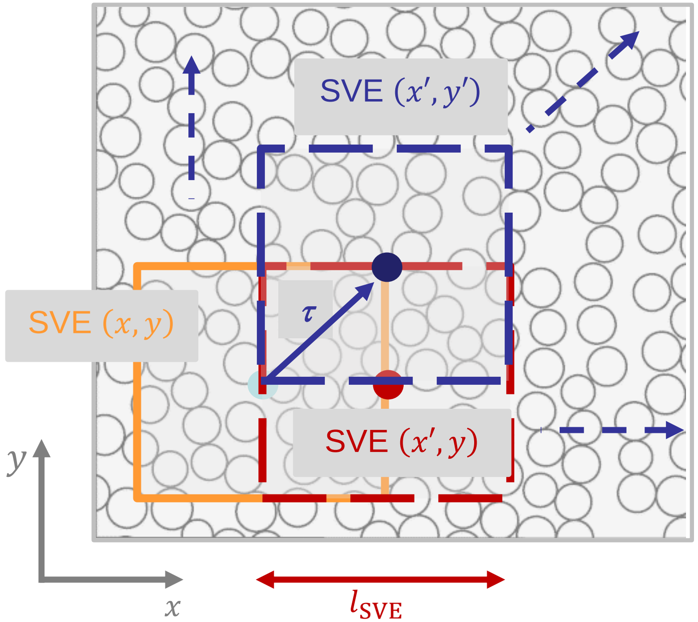
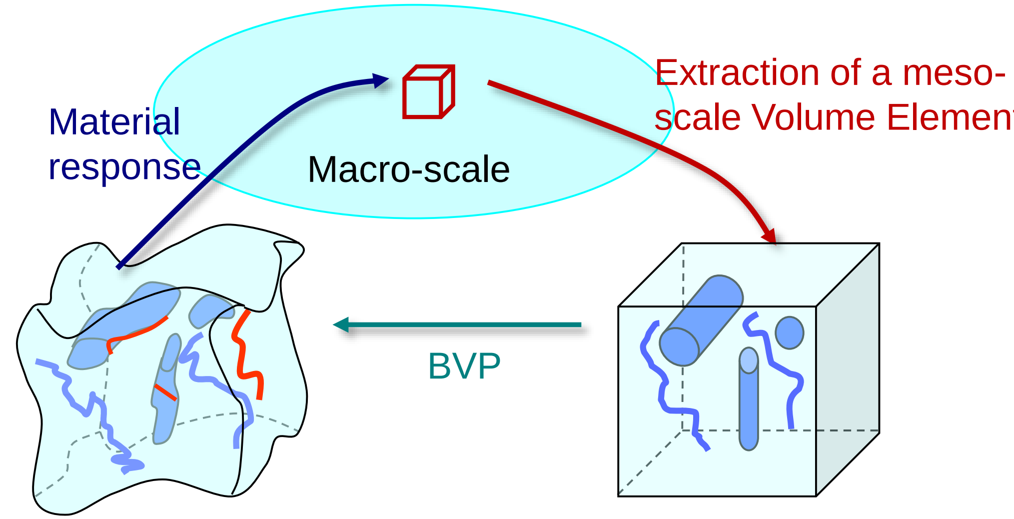




Objectives

- Multi-scale modelling
  - 2 problems are solved concurrently
    - The macro-scale problem
    - The meso-scale problem (on a meso-scale Volume Element)



P, σ, q, …

F, ε, T, ∇T, …

- Length-scales separation

$L_{macro} >> L_{VE} >> L_{micro}$

For accuracy: Size of the meso-scale volume element smaller than the characteristic length of the macro-scale loading

To be statistically representative: Size of the meso-scale volume element larger than the characteristic length of the micro-structure

Objectives

- Non-linear responses of composites
  - RVE size is too large
  - Structural response suffers from scatter
  - Requires the use of smaller VEs

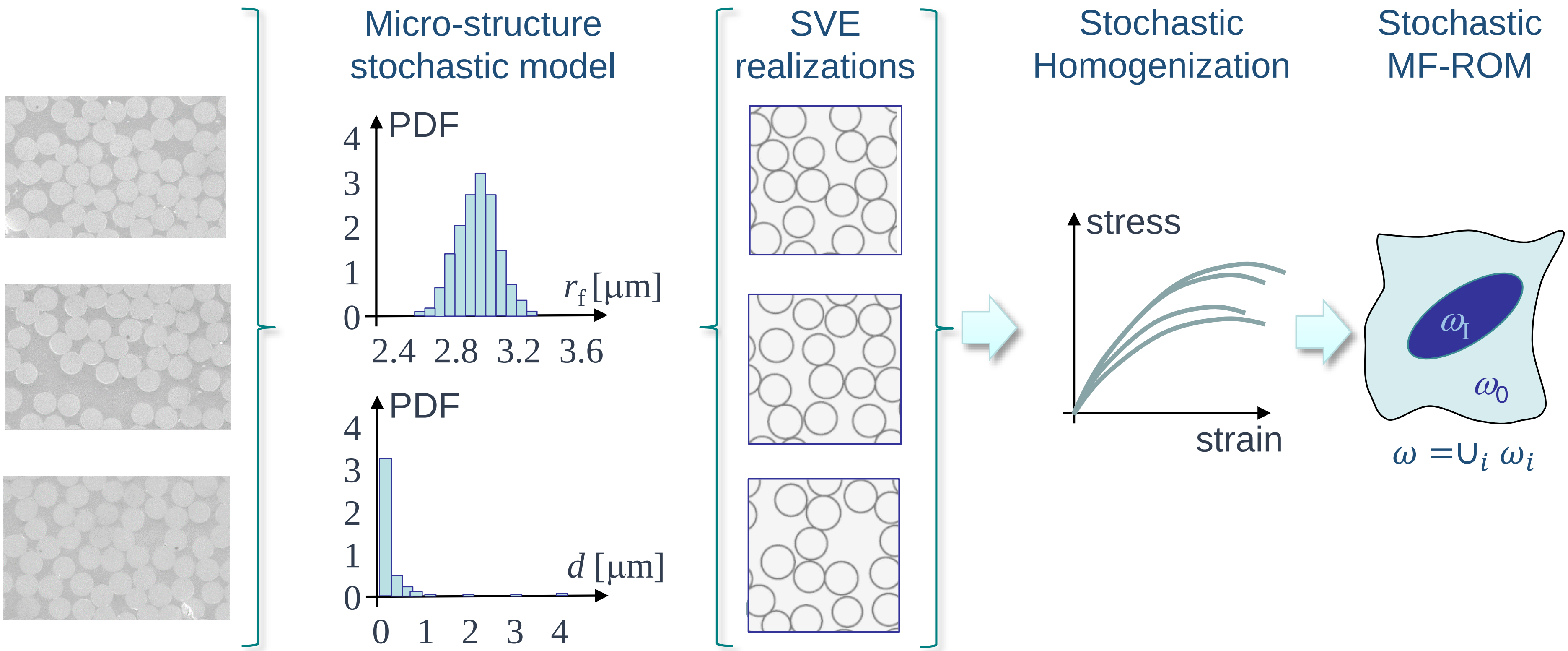
$L_{\text{macro}} >> L_{\text{VE}} >\sim L_{\text{micro}}$

For accuracy: Size of the meso-scale volume element smaller than the characteristic length of the macro-scale loading

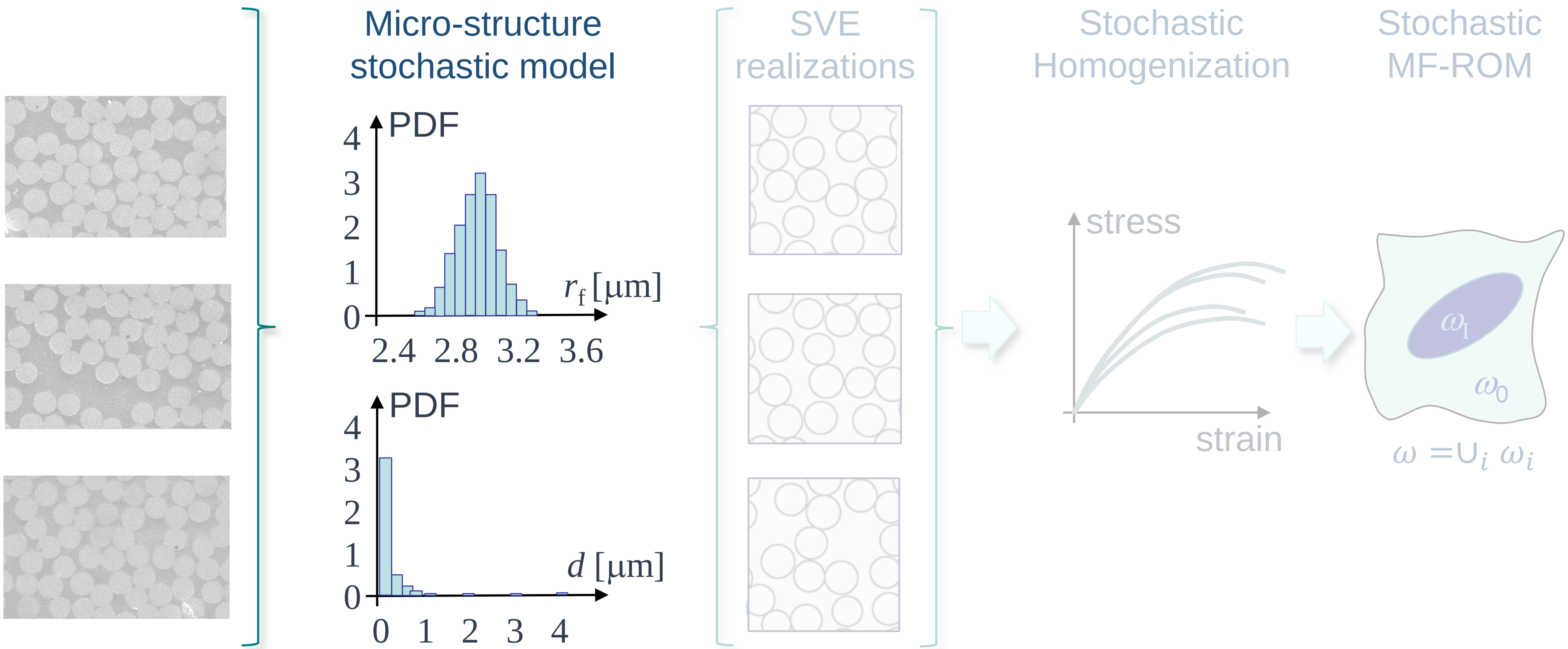
Meso-scale volume element no longer statistically representative: Stochastic Volume Elements

- Main objective
  - To develop an integrated stochastic multiscale approach to predict failure of composites

- Proposed methodology
  - To develop a stochastic Mean Field Homogenization method able to predict the probabilistic distribution of material response at an intermediate scale from micro-structural constituents characterization

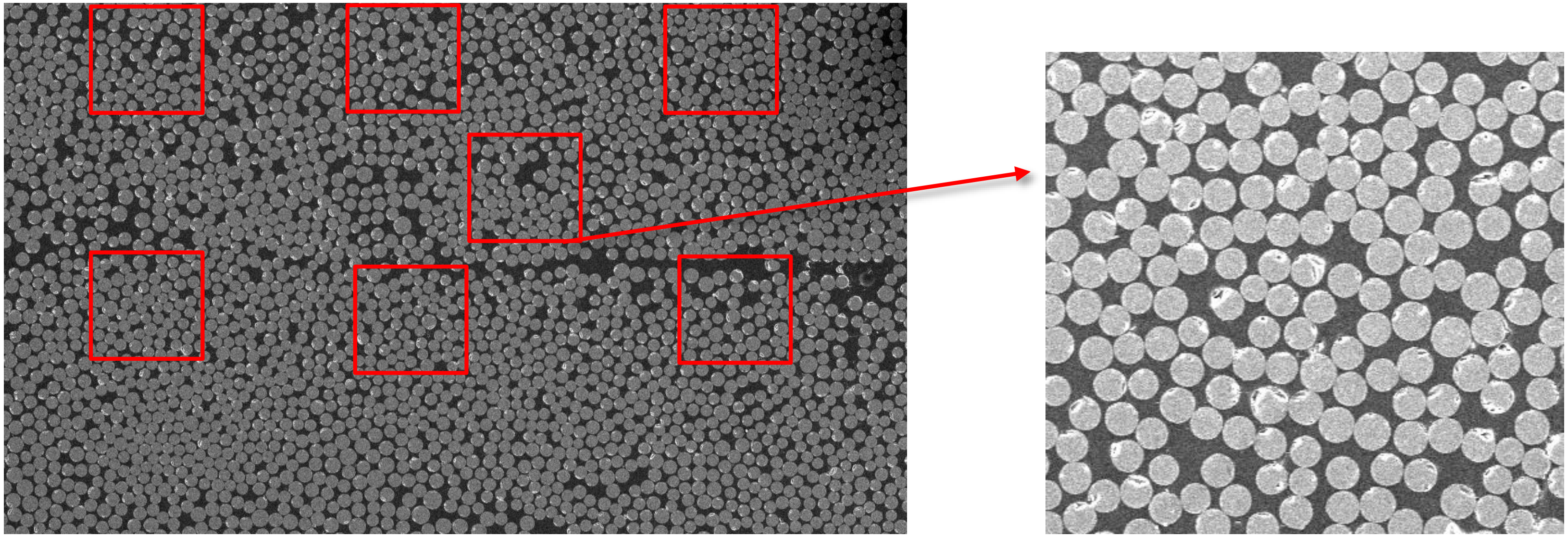


- Uncertainty quantification of micro-structure & micro-structure generator

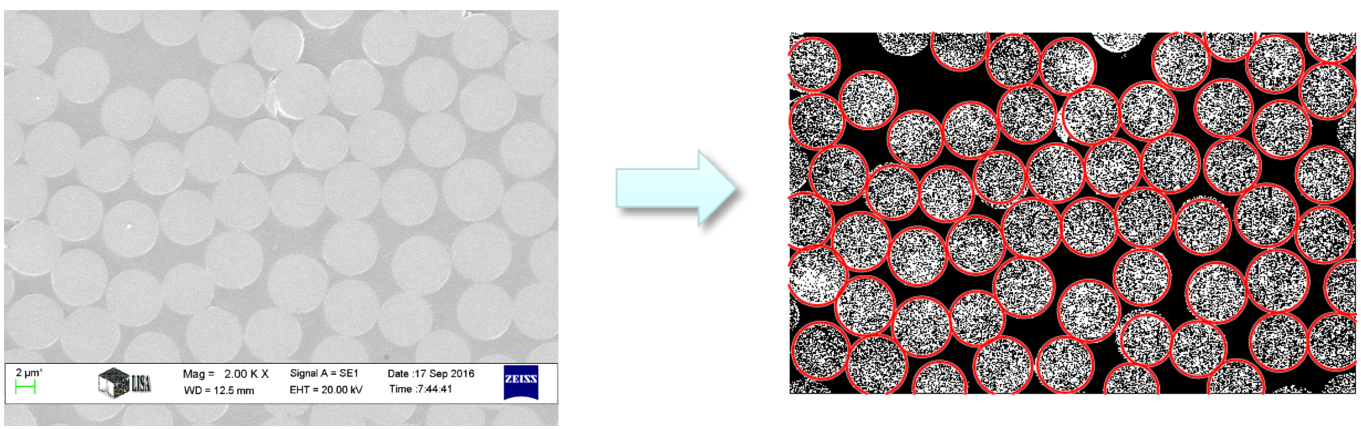


Experimental measurements

- 2000x and 3000x SEM images

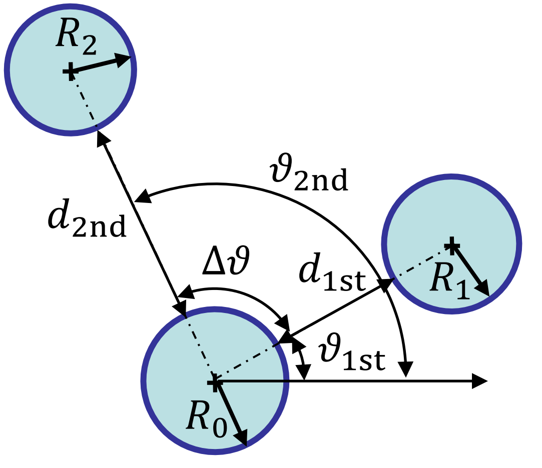


- Fibers detection



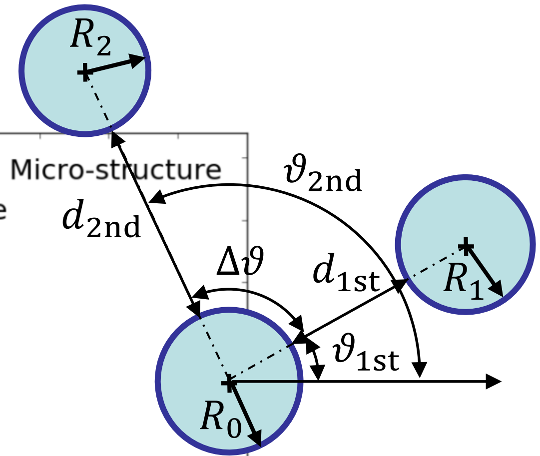
- Basic geometric information of fibers' cross sections
  - Fiber radius distribution $p_R(r)$

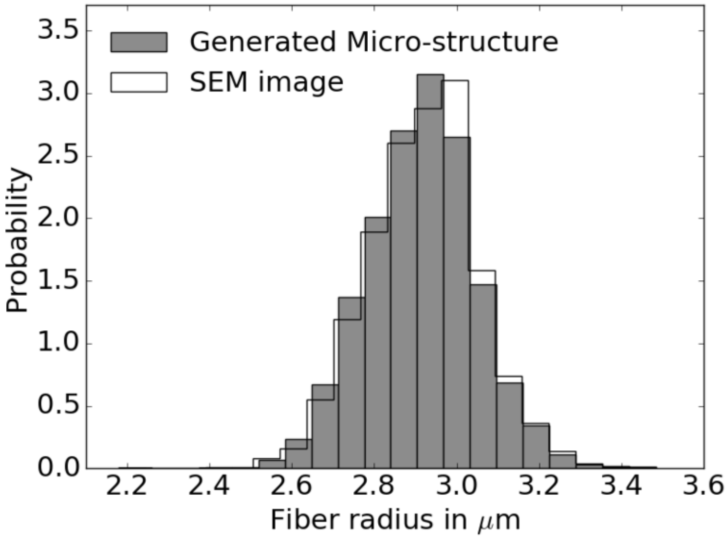
- Basic spatial information of fibers
  - The distribution of the nearest-neighbor net distance function $p_{d_{1\mathrm{st}}}(d)$
  - The distribution of the orientation of the undirected line connecting the center points of a fiber to its nearest neighbor $p_{\vartheta_{1\mathrm{st}}}(\theta)$
  - The distribution of the difference between the net distance to the second and the first nearest-neighbor $p_{\Delta d}(d)$ with $\Delta d = d_{2\mathrm{nd}} - d_{1\mathrm{st}}$
  - The distribution of the second nearest-neighbor's location referring to the first nearest-neighbor $p_{\Delta\vartheta}(\theta)$ with $\Delta\vartheta = \vartheta_{2\mathrm{nd}} - \vartheta_{1\mathrm{st}}$

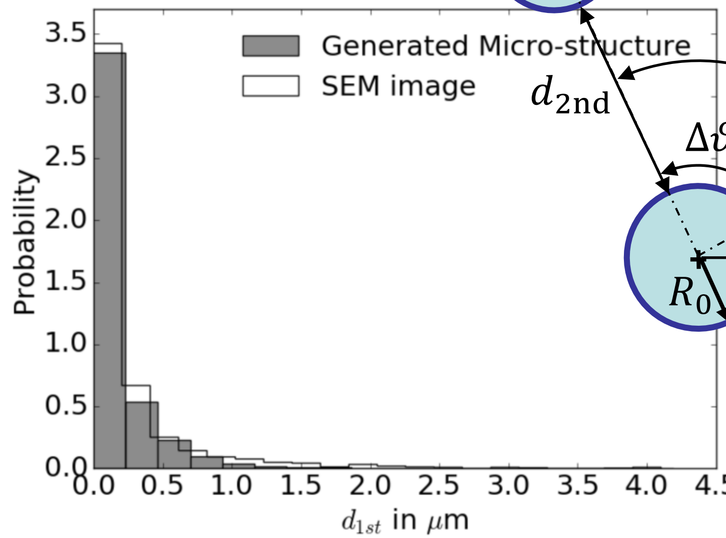


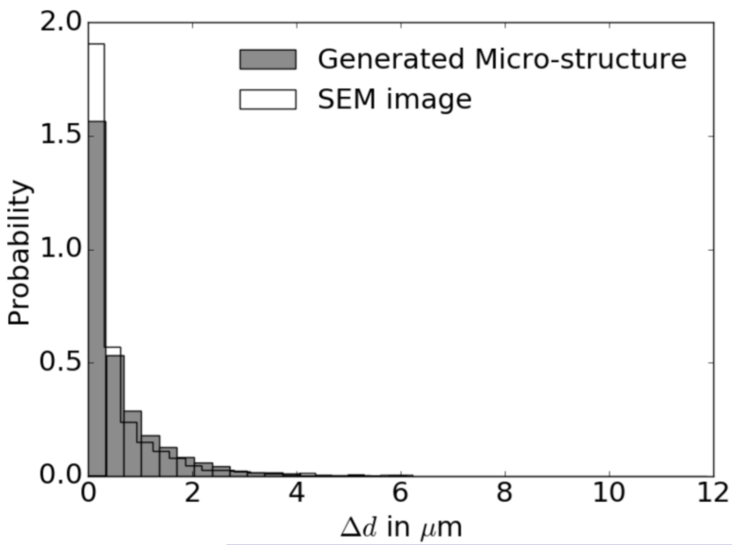
Micro-structure stochastic model

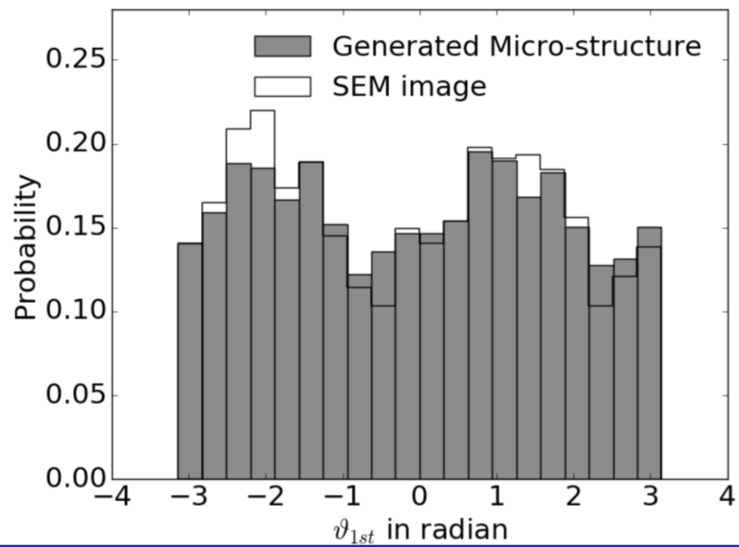
- Histograms of random micro-structures' descriptors





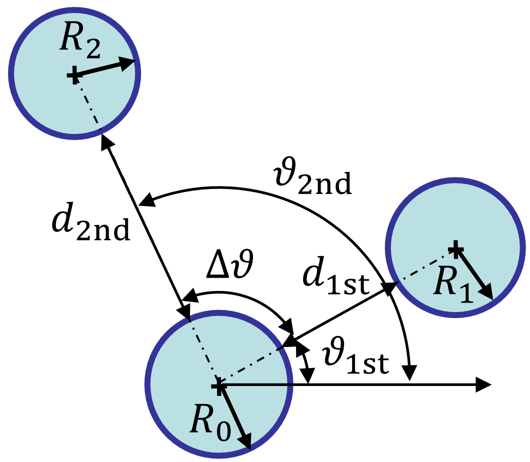






Micro-structure stochastic model

- Dependency of the four random variables $d_{1\text{st}}, \Delta d, \vartheta_{1\text{st}}, \Delta\vartheta$
- Correlation matrix



	$d_{1\text{st}}$	Δd	$\vartheta_{1\text{st}}$	$\Delta\vartheta$
$d_{1\text{st}}$	1.0	0.21	0.01	0.02
Δd		1.0	0.002	-0.005
$\vartheta_{1\text{st}}$			1.0	0.02
$\Delta\vartheta$				1.0

- Distances correlation matrix

	$d_{1\text{st}}$	Δd	$\vartheta_{1\text{st}}$	$\Delta\vartheta$
$d_{1\text{st}}$	1.0	0.27	0.04	0.08
Δd		1.0	0.05	0.06
$\vartheta_{1\text{st}}$			1.0	0.05
$\Delta\vartheta$				1.0

$d_{1\text{st}}$ and Δd are dependent

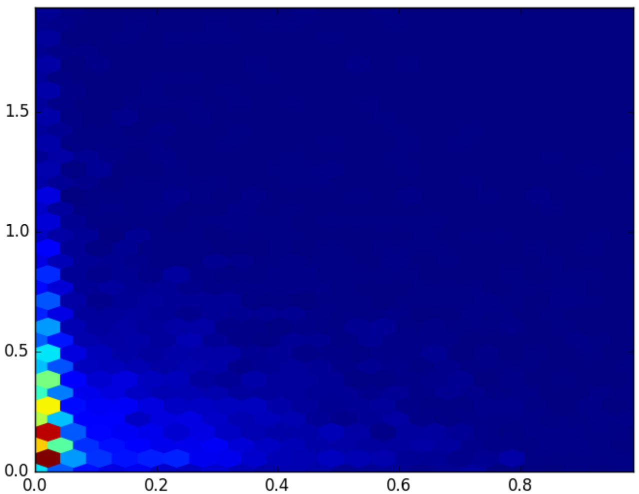
⇨ they will have to be generated from their empirical copula

Micro-structure stochastic model

- d_{1st} and Δd should be generated using their empirical copula

SEM sample

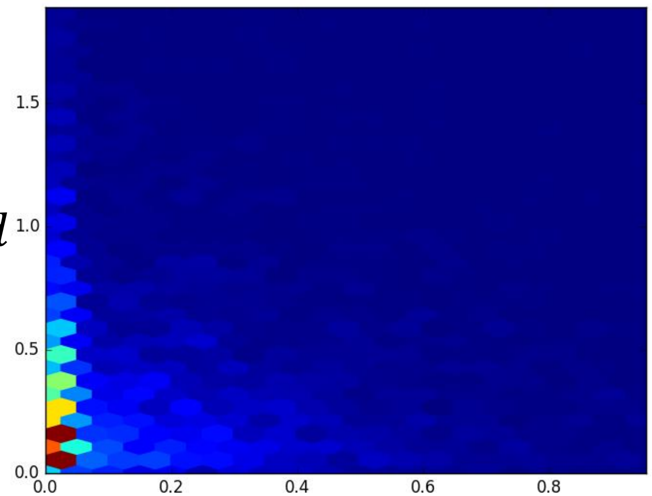
Δd



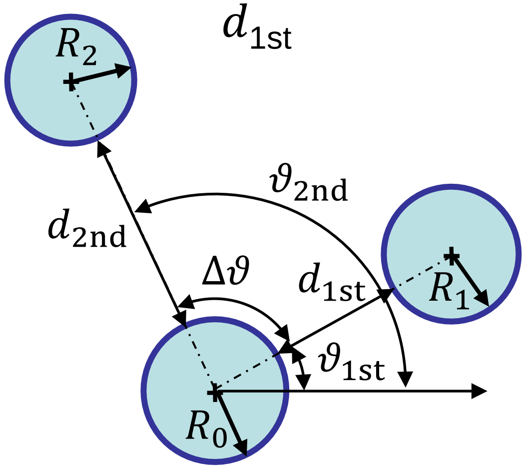
d_{1st}

Generated sample

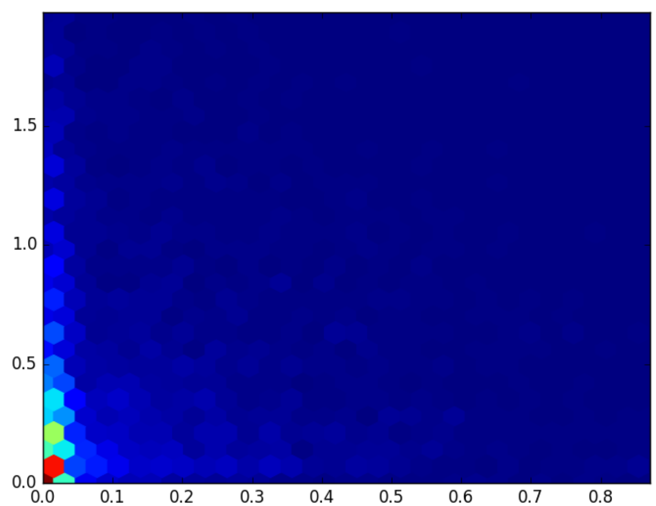
Δd



Directly from copula generator



Δd

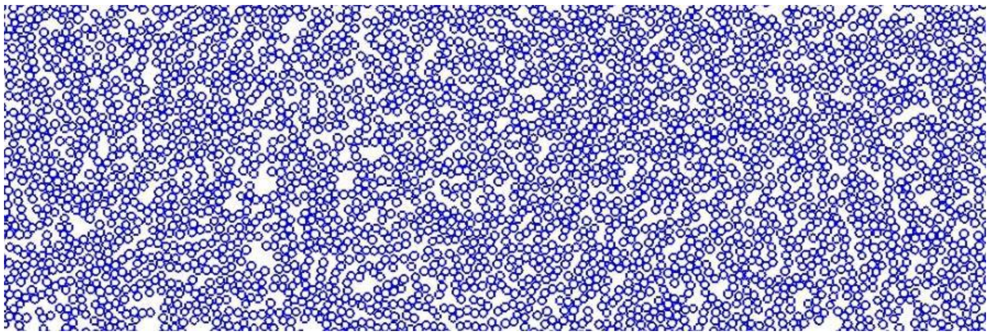


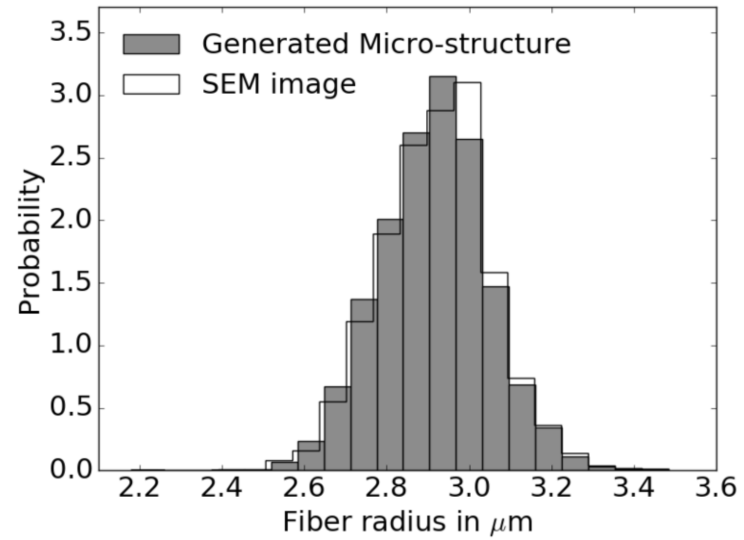
d_{1st}

Statistic result from generated SVE

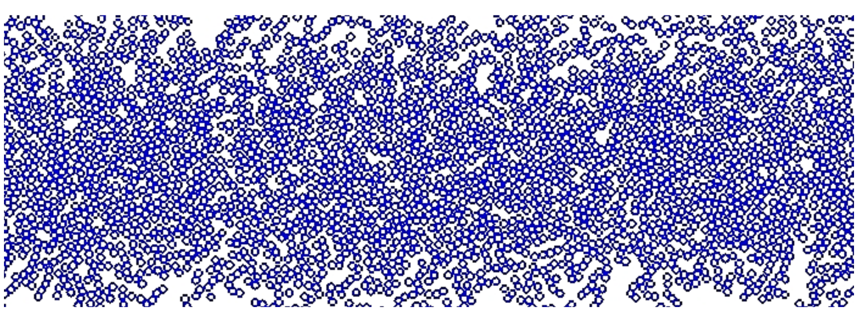
Micro-structure stochastic model

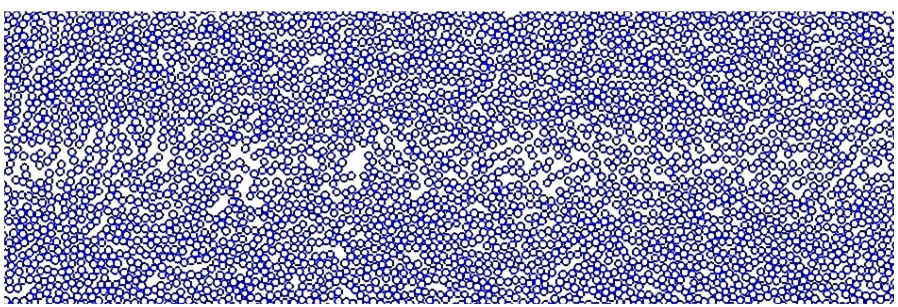
- Numerical micro-structures are generated by a fiber additive process
  - Arbitrary size
  - Arbitrary number





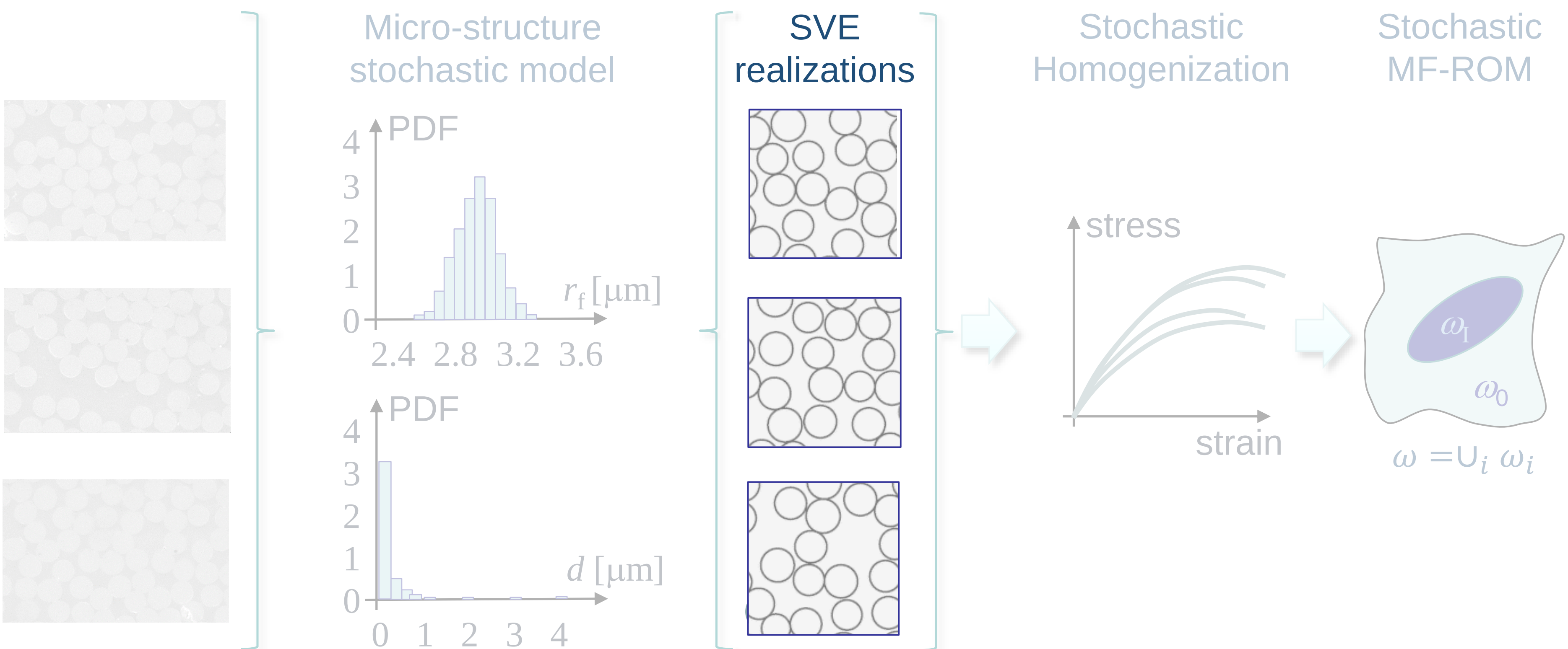
  - Possibility to generate non-homogenous distributions





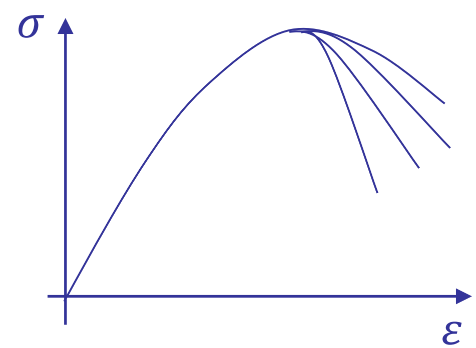
Methodology

- SVEs definition

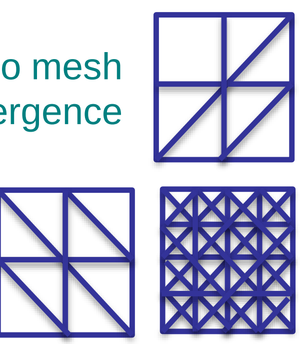


Micro-structural model of fiber-reinforced highly crosslinked epoxy

- UD Composites with RTM6 epoxy matrix
  - Identified matrix material behaviour
    - Hyperelastic viscoelastic-viscoplastic constitutive model enhanced by a multi-mechanism nonlocal damage model
  - To be used in micro-structural analysis
    - Behaviour in composite is different
    - Introduce a length-scale effect
- Resin model implementation:
  - Requires non-local form [Bažant 1988]
    - Introduction of characteristic length l_c
    - Weighted average: $\tilde{Z}(\boldsymbol{x}) = \int_{V_C} W(\boldsymbol{y};\boldsymbol{x},l_c)\ Z(\mathbf{y})d\boldsymbol{y}$
  - Implicit form [Peerlings et al. 1998]
    - New degrees of freedom: $\tilde{Z}$
    - New Helmholtz-type equations: $\tilde{Z} - l_c^2\ \Delta\tilde{Z} = Z$
  - Has a length scale effect



Local: no mesh convergence



Micro-structural model of fiber-reinforced highly crosslinked epoxy

- Resin model
  - Material changes represented via internal variables
  - Constitutive law $\mathbf{P}\left(\mathbf{F};\boldsymbol{Z}(t')\right)$
    - Internal variables $\boldsymbol{Z}(t')$
    - Multi-damage strategy

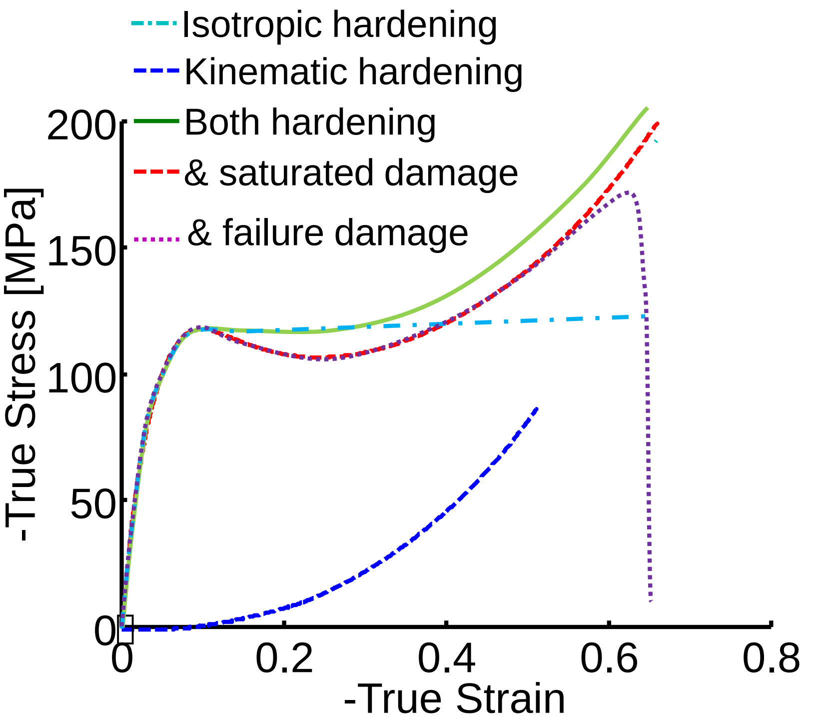
$$\mathbf{P} = (\mathbf{1} - D_s)\left(\mathbf{1} - D_f\right)\widehat{\mathbf{P}}$$

  - Non-local damage evolution laws
    - Saturation law

$$\begin{cases} \dot{D}_s = D_s(D_s, \mathbf{F}(t), \chi_s(t); Z(\tau), \tau \in [0\ t])\dot{\chi}_s \\ \chi_s(t) = \max_\tau\left(\tilde{\gamma}_s(\tau)\right) \\ \tilde{\gamma}_s - l_s^2\, \Delta\tilde{\gamma}_s = \gamma_s \end{cases}$$

    - Failure law

$$\begin{cases} \dot{D}_f = D_f\left(D_f, \mathbf{F}(t), \chi_f(t); Z(\tau), \tau \in [0\ t]\right)\dot{\chi}_f \\ \chi_f(t) = \max_\tau\left(\tilde{\gamma}_f(\tau)\right) \\ \tilde{\gamma}_f - l_f^2\, \Delta\tilde{\gamma}_f = \gamma_f \end{cases}$$



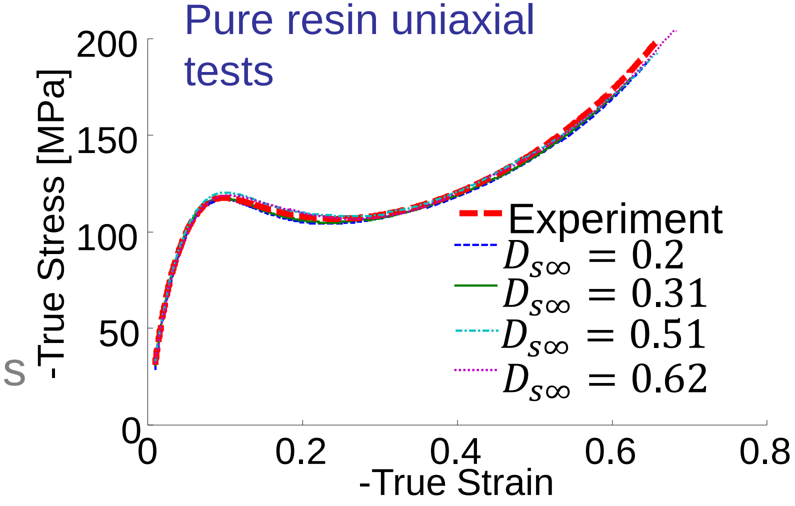
Micro-structural model of fiber-reinforced highly crosslinked epoxy

- Resin model: saturated softening
  - Saturated damage evolution

$$\begin{cases} \dot{D}_s = H_s(\chi_s - \chi_{s0})^{\zeta_s}(D_{s\infty} - D_s)\dot{\chi}_s \\ \chi_s = \max_{\tau}\left(\chi_{s0}, \tilde{\gamma}_s(\tau)\right) \\ \tilde{\gamma}_s - l_s^2\,\Delta\tilde{\gamma}_s = \gamma \end{cases}$$

  - Several hardening/softening combinations
    - Requires composite material tests

Pure resin uniaxial tests

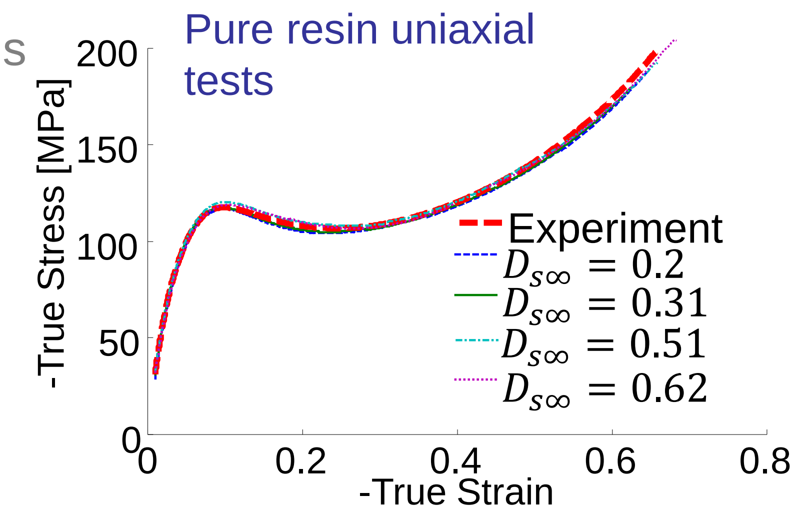


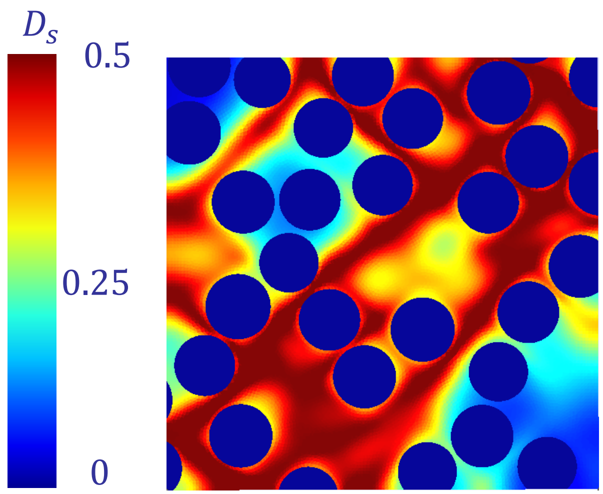
- Resin model: saturated softening
  - Several hardening/softening combinations
    - Requires composite material tests
      - Length scale effect

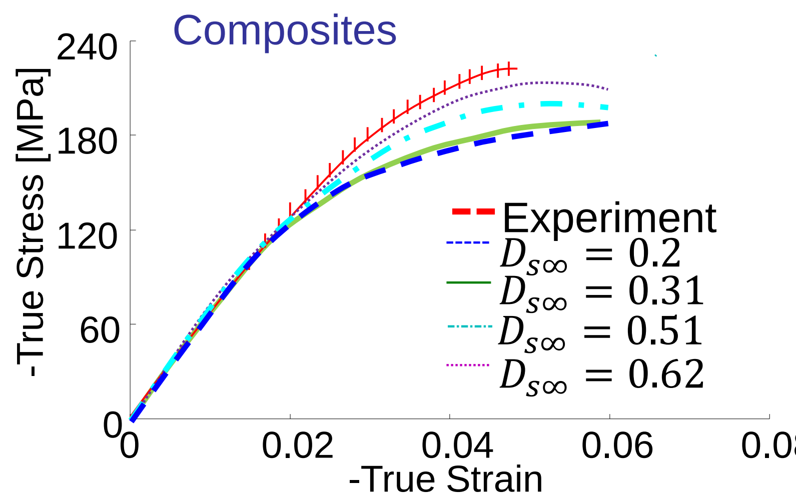
$$l_s = 3\mu\mathrm{m}\left(1 - \frac{D_s}{D_{s\infty}}\right)$$

      - Non-local BC at fibre interface

$$[\![\dot{\gamma}_s]\!] = 0$$







Micro-structural model of fiber-reinforced highly crosslinked epoxy

- Resin model: failure softening
  - Failure surface

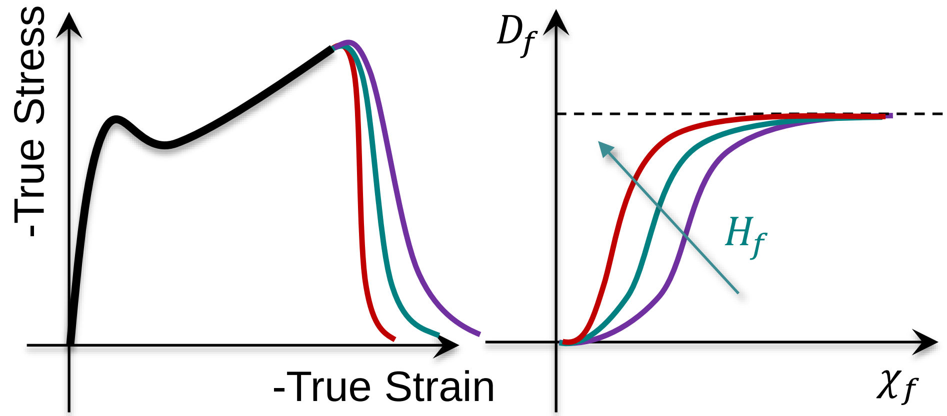
$$\begin{cases} \phi_f = \gamma - a \exp\left(-b \dfrac{\mathrm{tr}(\hat{\boldsymbol{\tau}})}{3\hat{\tau}^{eq}}\right) - c \\ \phi_f - r \leq 0;\ \dot{r} \geq 0; \text{and } \dot{r}\left(\phi_f - r\right) = 0 \\ \dot{\gamma}_f = \dot{r} \end{cases}$$

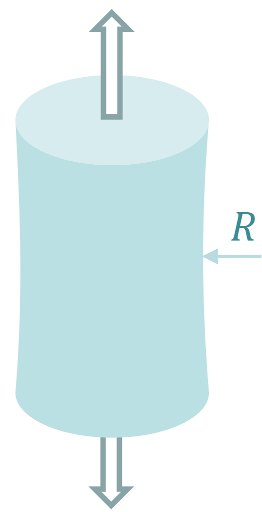
  - Damage evolution

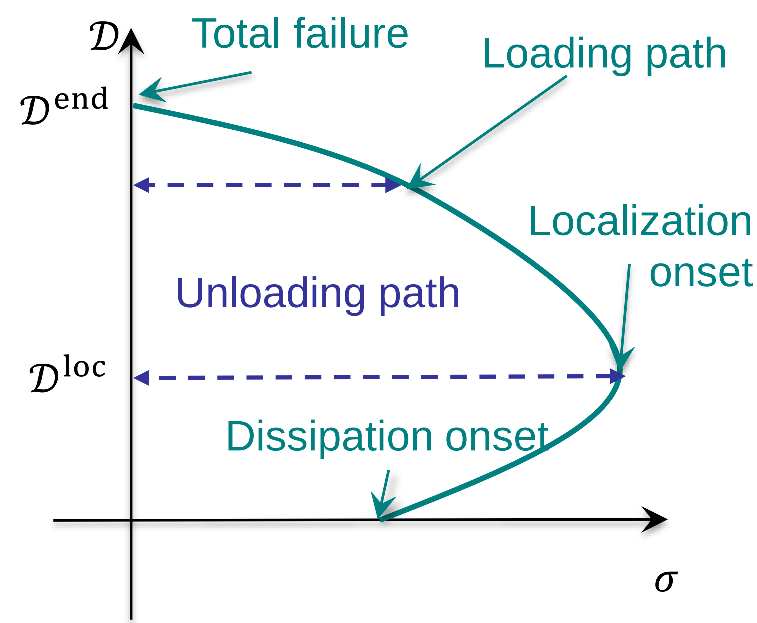
$$\begin{cases} \dot{D}_f = H_f(\chi_f)^{\zeta_f}(1 - D_f)^{-\zeta_d}\,\dot{\chi}_f \\ \chi_f = \max_{\tau}\left(\tilde{\gamma}_f(\tau)\right) \\ \tilde{\gamma}_f - l_f^2\,\Delta\tilde{\gamma}_f = \gamma_f \\ l_f = 3\ \mu\mathrm{m} \qquad \nabla_0\tilde{\gamma}_f \cdot \mathbf{N} = 0 \end{cases}$$

  - Affect ductility
  - Calibration
    - From localization simulation
    - Recover the epoxy G_c

$$G_c = \frac{\mathcal{D}^{\mathrm{end}} - \mathcal{D}^{\mathrm{loc}}}{A_0}$$

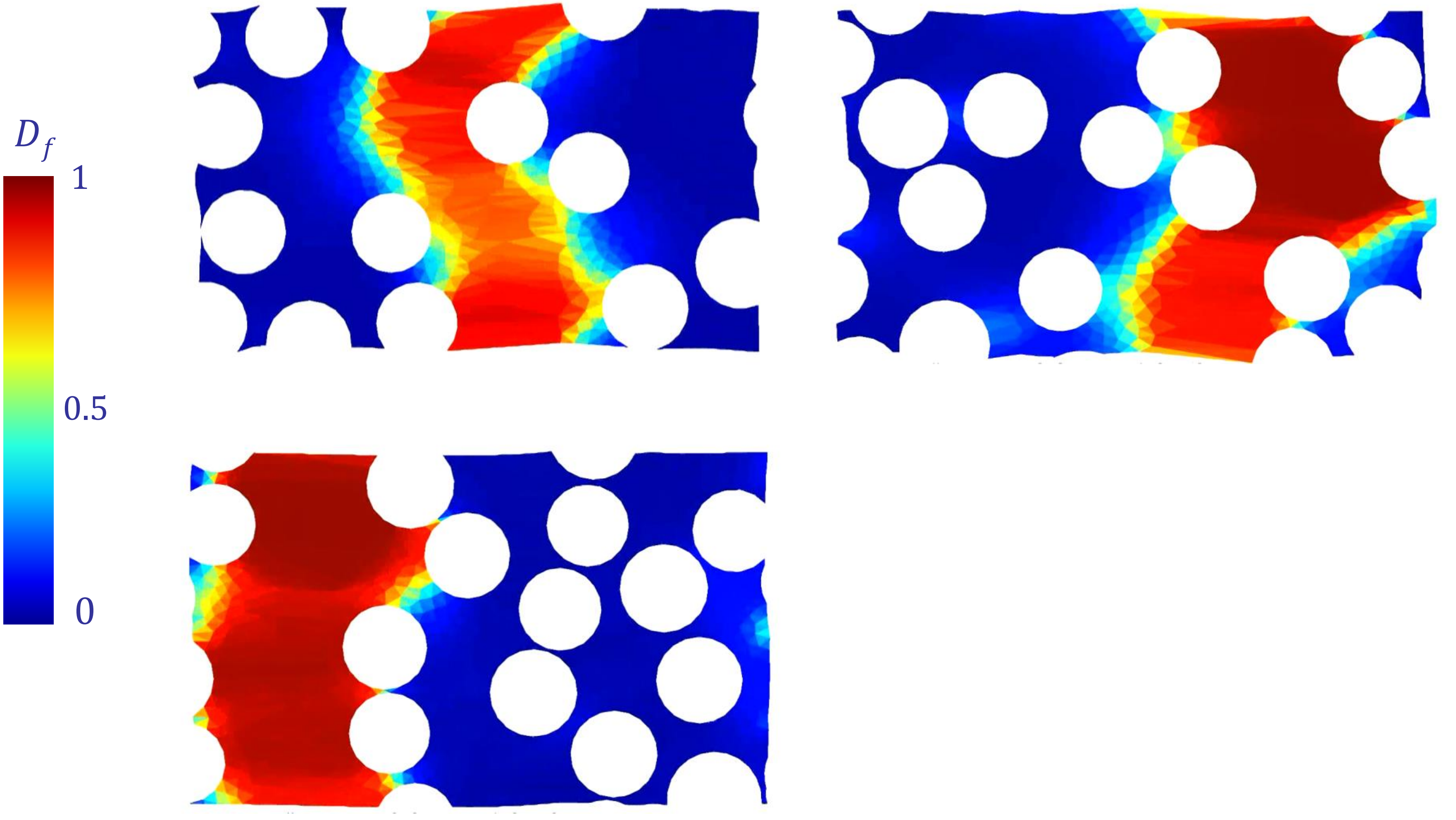




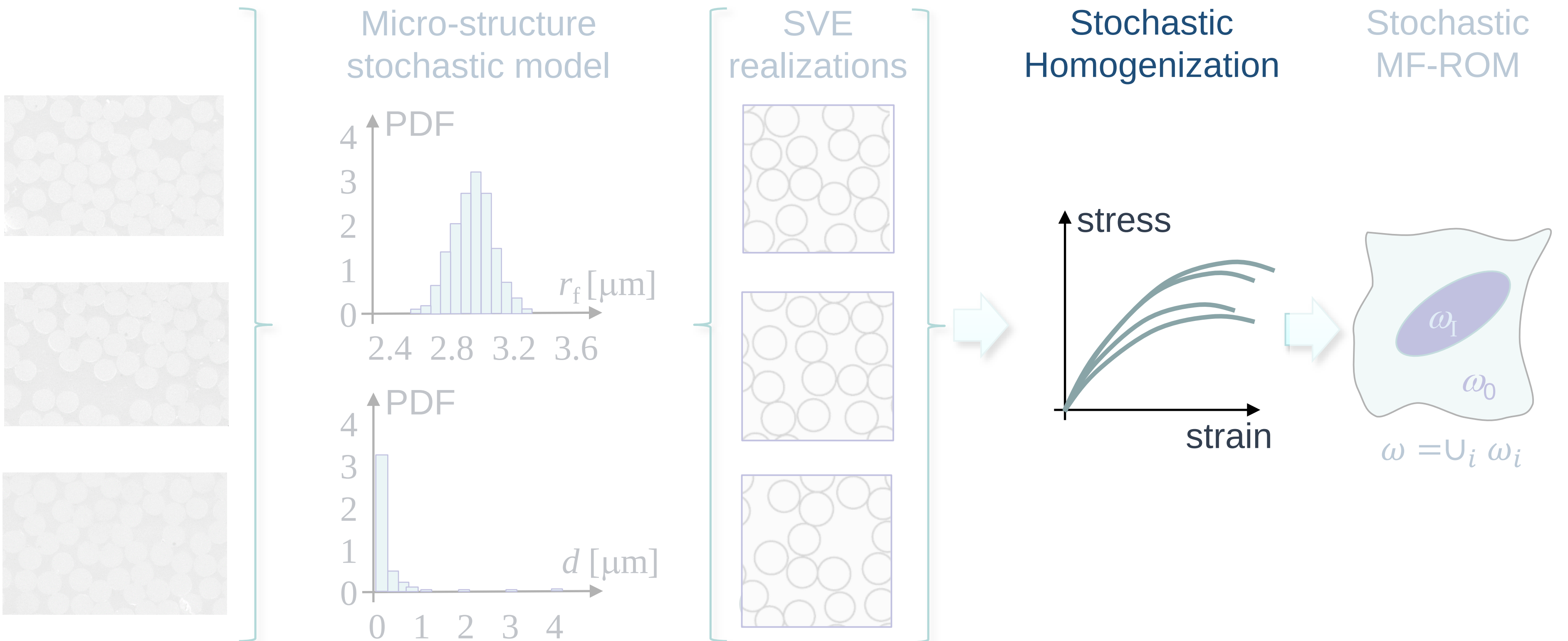


Micro-structural model of fiber-reinforced highly crosslinked epoxy

- UD Composites with RTM6 epoxy matrix
  - 2D simulations of 25 x 25 µm 40% volume fraction composite SVEs



- Extraction of apparent responses



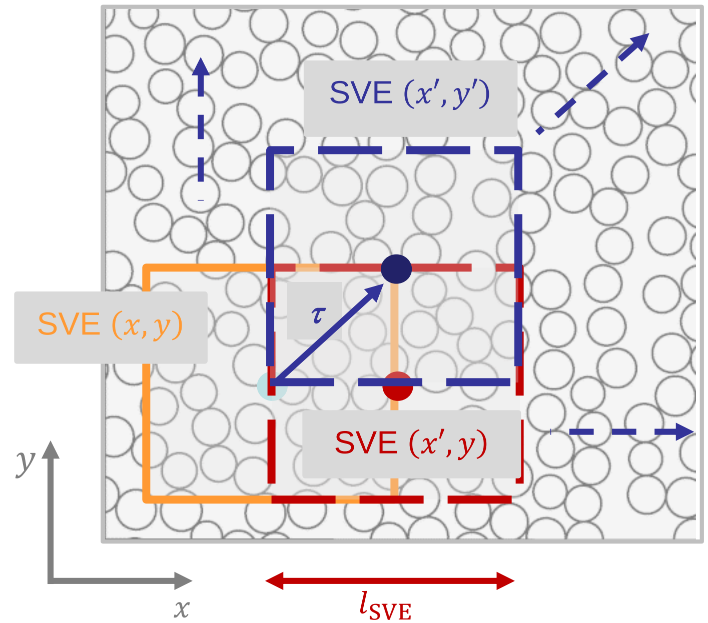
- Window technique
  - Extraction of Stochastic Volume Elements
    - $l_{\mathrm{SVE}} = 25\ \mu m$
    - Correlation

$$R_{\mathbf{rs}}(\boldsymbol{\tau}) = \frac{\mathbb{E}\left[\left(r(\boldsymbol{x}) - \mathbb{E}(r)\right)\left(s(\boldsymbol{x}+\boldsymbol{\tau}) - \mathbb{E}(s)\right)\right]}{\sqrt{\mathbb{E}\left[\left(r - \mathbb{E}(r)\right)^2\right]}\sqrt{\mathbb{E}\left[\left(s - \mathbb{E}(s)\right)^2\right]}}$$

  - For each SVE
    - Extract apparent homogenized material tensor $\mathbb{C}_{\mathrm{M}}$

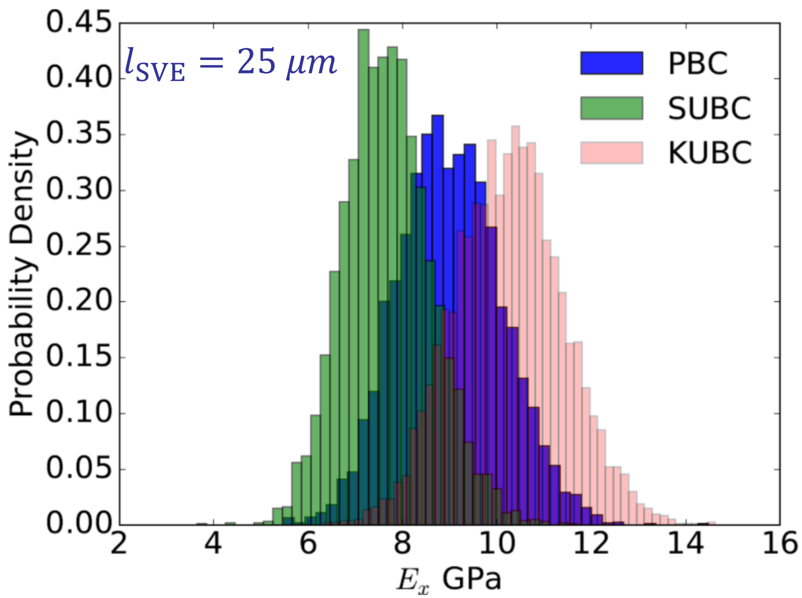
$$\begin{cases} \boldsymbol{\varepsilon}_{\mathrm{M}} = \dfrac{1}{V(\omega)} \displaystyle\int_{\omega} \boldsymbol{\varepsilon}_{\mathrm{m}} d\omega \\ \boldsymbol{\sigma}_{\mathrm{M}} = \dfrac{1}{V(\omega)} \displaystyle\int_{\omega} \boldsymbol{\sigma}_{\mathrm{m}} d\omega \\ \mathbb{C}_{\mathrm{M}} = \dfrac{\partial \boldsymbol{\sigma}_{\mathrm{M}}}{\partial \boldsymbol{u}_{\mathrm{M}} \otimes \boldsymbol{\nabla}_{\mathrm{M}}} \end{cases}$$

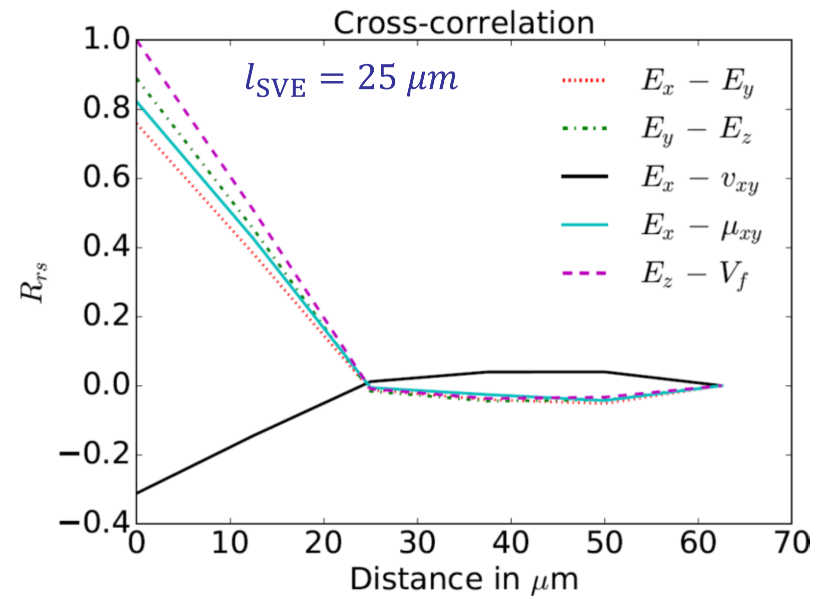
    - Consistent boundary conditions:
      - Periodic (PBC)
      - Minimum kinematics (SUBC)
      - Kinematic (KUBC)



Stochastic homogenization on the SVEs

- Apparent elastic properties: distribution & Correlation





– For large enough RVE ($l_{\mathrm{SVE}} > 10\ \mu m$)

Auto/cross correlation vanishes at $\tau = l_{\mathrm{SVE}}$

Distributions get closer to normal

Apparent properties of different SVEs are independent

However the distributions depend on

- l_{SVE}
- The boundary conditions

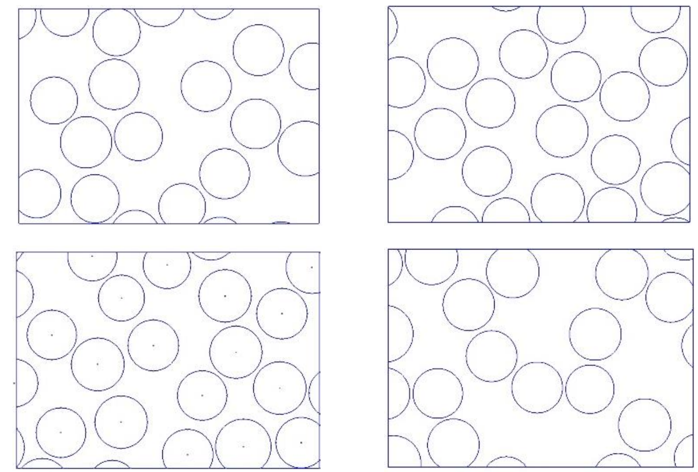
Stochastic homogenization on the SVEs

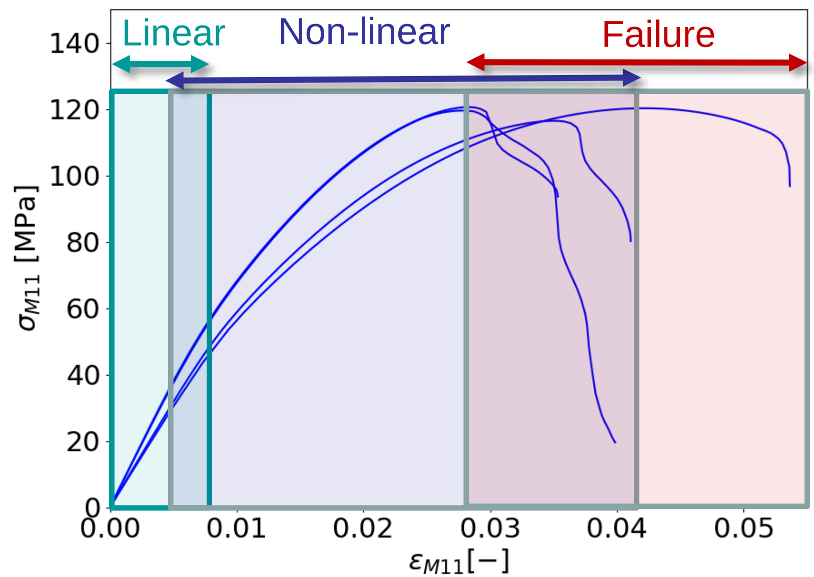
- Apparent response
  - Independent Stochastic Volume Elements
    - $l_{\mathrm{SVE}} = 25\ \mu m$
  - Stochastic homogenization
    - Extract apparent responses

$$\left\{\begin{aligned} \boldsymbol{\varepsilon}_{\mathrm{M}} &= \frac{1}{V(\omega)}\int_{\omega} \boldsymbol{\varepsilon}_{\mathrm{m}} d\omega \\ \boldsymbol{\sigma}_{\mathrm{M}} &= \frac{1}{V(\omega)}\int_{\omega} \boldsymbol{\sigma}_{\mathrm{m}} d\omega \\ \mathbb{C}_{\mathrm{M}} &= \frac{\partial \boldsymbol{\sigma}_{\mathrm{M}}}{\partial \boldsymbol{u}_{\mathrm{M}} \otimes \boldsymbol{\nabla}_{\mathrm{M}}} \end{aligned}\right.$$

  - 3 stages
    - Linear response
    - (Damage-enhanced) elasto-plasticity
    - Failure (loss of size objectivity)

SVE realizations

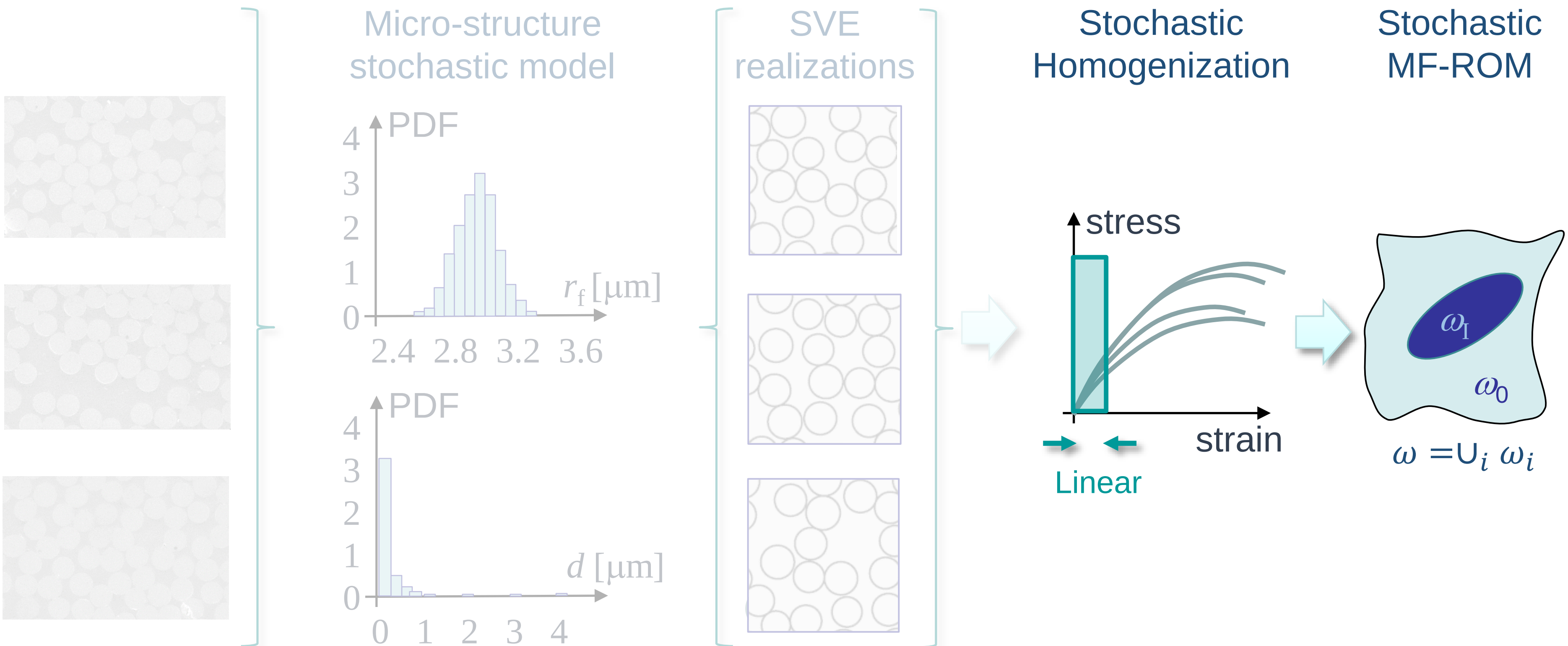




Stochastic Homogenization

Methodology

- Stochastic Mean-Field Homogenization-based model
  - First stage: linear elasticity

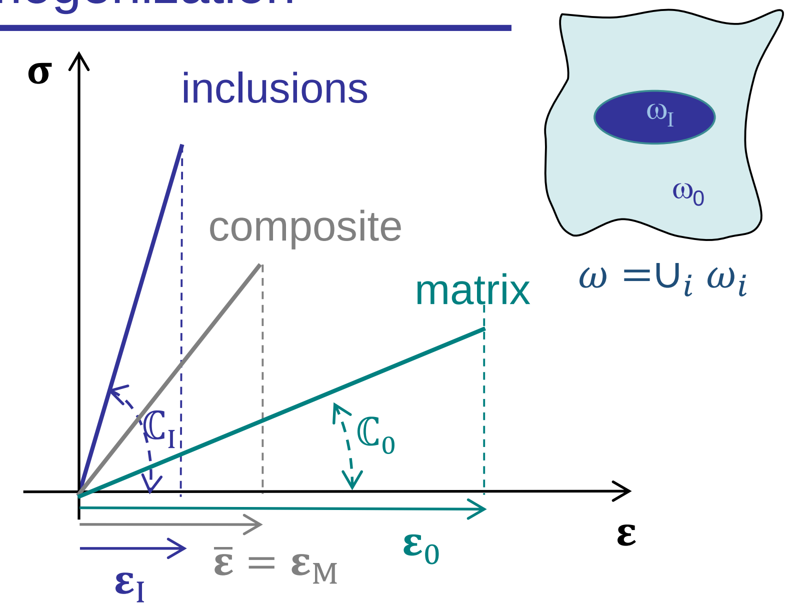


Stochastic Mean-Field Homogenization

- Mean-Field-Homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{M}} = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbb{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}) : \boldsymbol{\varepsilon}_0 \end{cases}$$

$$\Rightarrow \mathbb{C}_{\mathrm{M}} = \mathbb{C}_{\mathrm{M}}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}})$$



  - We use Mori-Tanaka assumption for $\mathbb{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}})$
- Stochastic MFH
  - How to define randomness?

L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

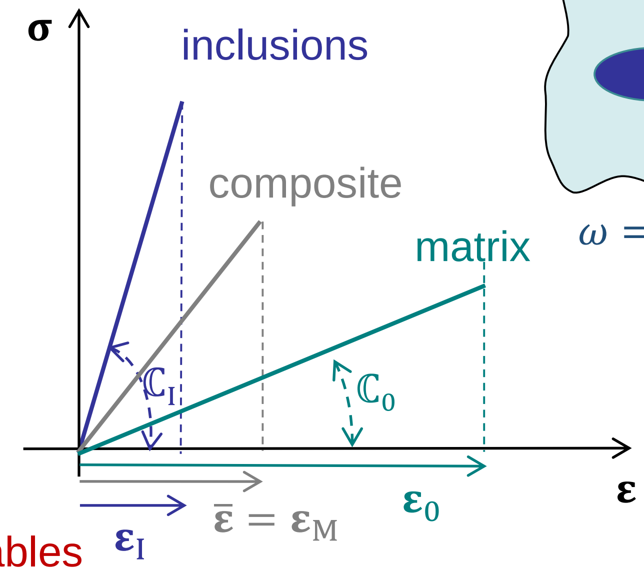
Stochastic Mean-Field Homogenization

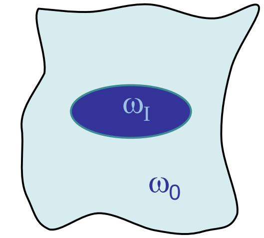
- Mean-Field-Homogenization (MFH)
  - Linear composites

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{M}} = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbb{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}) : \boldsymbol{\varepsilon}_0 \end{cases}$$

$$\Rightarrow \hat{\mathbb{C}}_{\mathrm{M}} = \hat{\mathbb{C}}_{\mathrm{M}}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}, v_{\mathrm{I}})$$

Defined as random variables

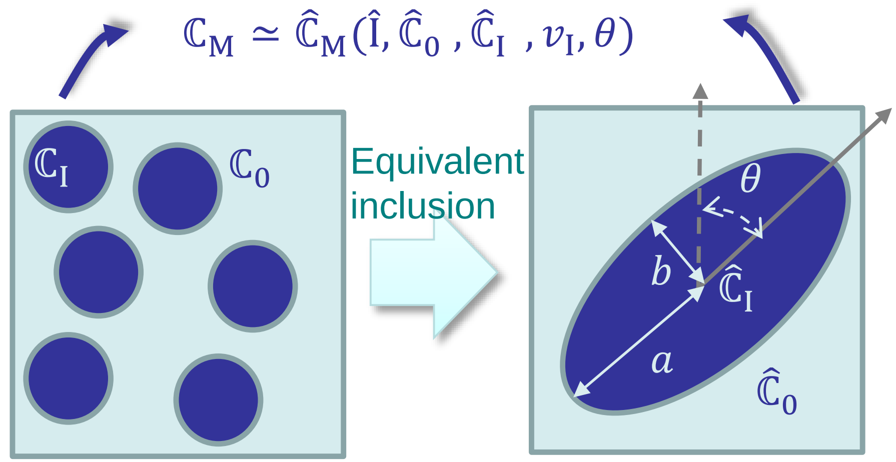




$\omega = \cup_i \, \omega_i$

- Consider an equivalent system
  - For each SVE realization i:
    - $\Rightarrow$ $\mathbb{C}_{\mathrm{M}}$ and v_{I} known
  - Anisotropy from $\mathbb{C}_{\mathrm{M}}^i$
    - $\Rightarrow$ θ is evaluated
  - Fiber behavior uniform
    - $\Rightarrow$ $\hat{\mathbb{C}}_{\mathrm{I}}$ for one SVE

$$\mathbb{C}_{\mathrm{M}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}(\hat{\mathrm{I}}, \hat{\mathbb{C}}_0, \hat{\mathbb{C}}_{\mathrm{I}}, v_{\mathrm{I}}, \theta)$$

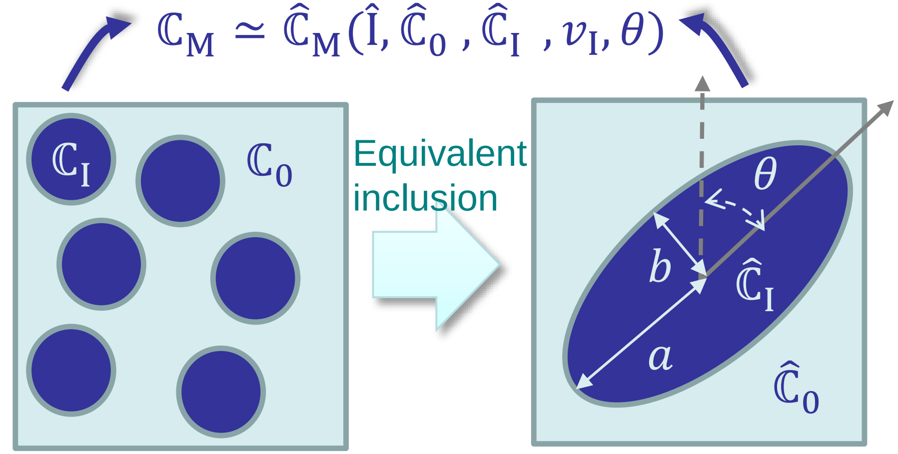


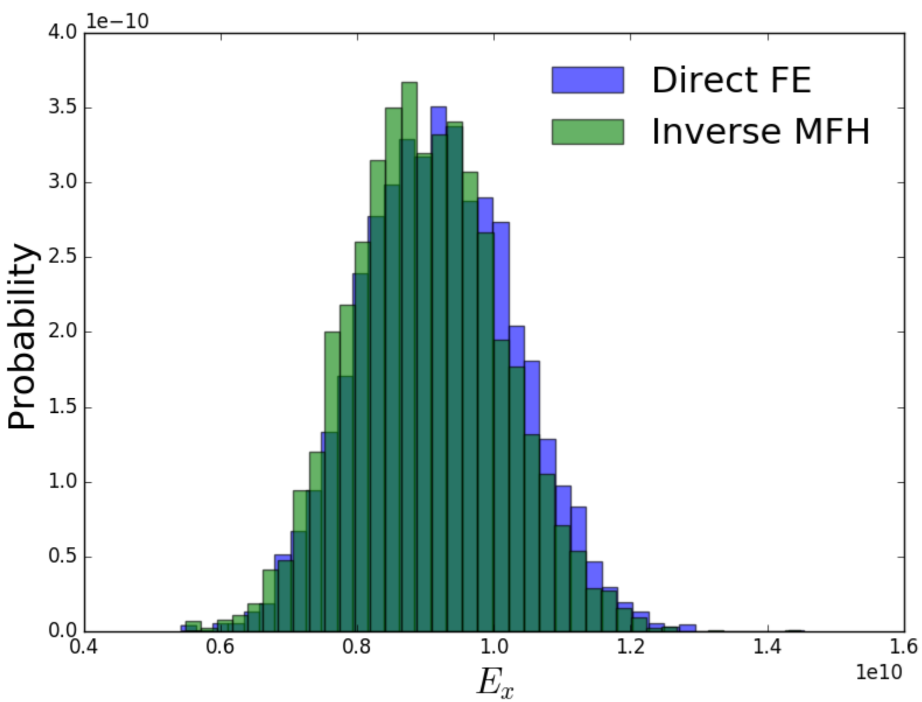
  - Remaining optimization problem: $\min_{\frac{a}{b}, \hat{E}_0, \hat{\nu}_0} \left\| \mathbb{C}_{\mathrm{M}} - \hat{\mathbb{C}}_{\mathrm{M}}\left(\frac{a}{b}, \hat{E}_0, \hat{\nu}_0 ; v_{\mathrm{I}}, \theta, \hat{\mathbb{C}}_{\mathrm{I}}\right) \right\|$

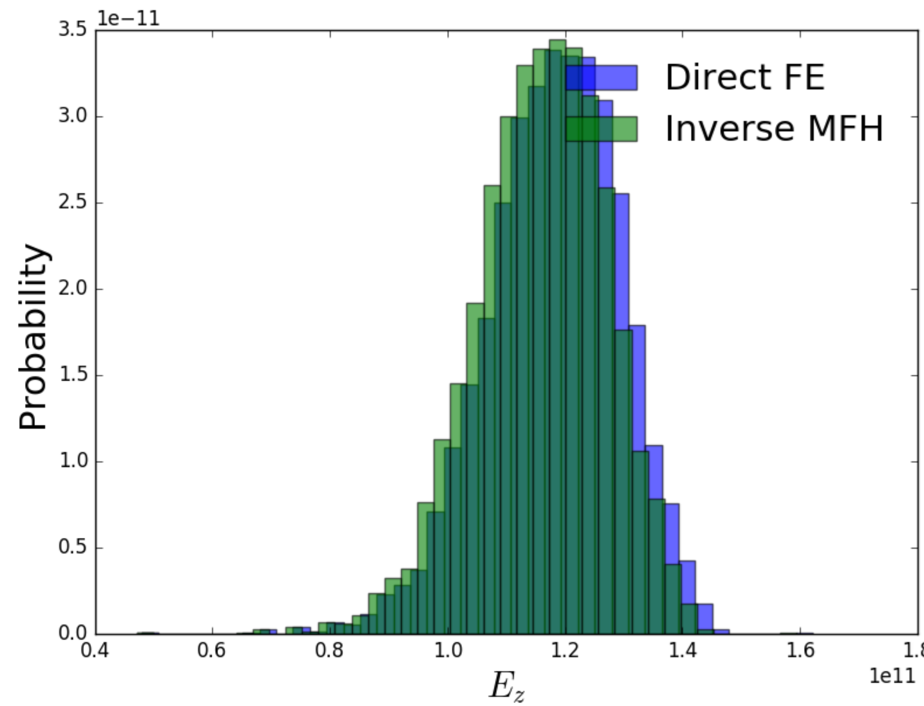
Stochastic Mean-Field Homogenization

$$\mathbb{C}_{\mathrm{M}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}(\hat{\mathrm{I}}, \hat{\mathbb{C}}_0, \hat{\mathbb{C}}_{\mathrm{I}}, v_{\mathrm{I}}, \theta)$$

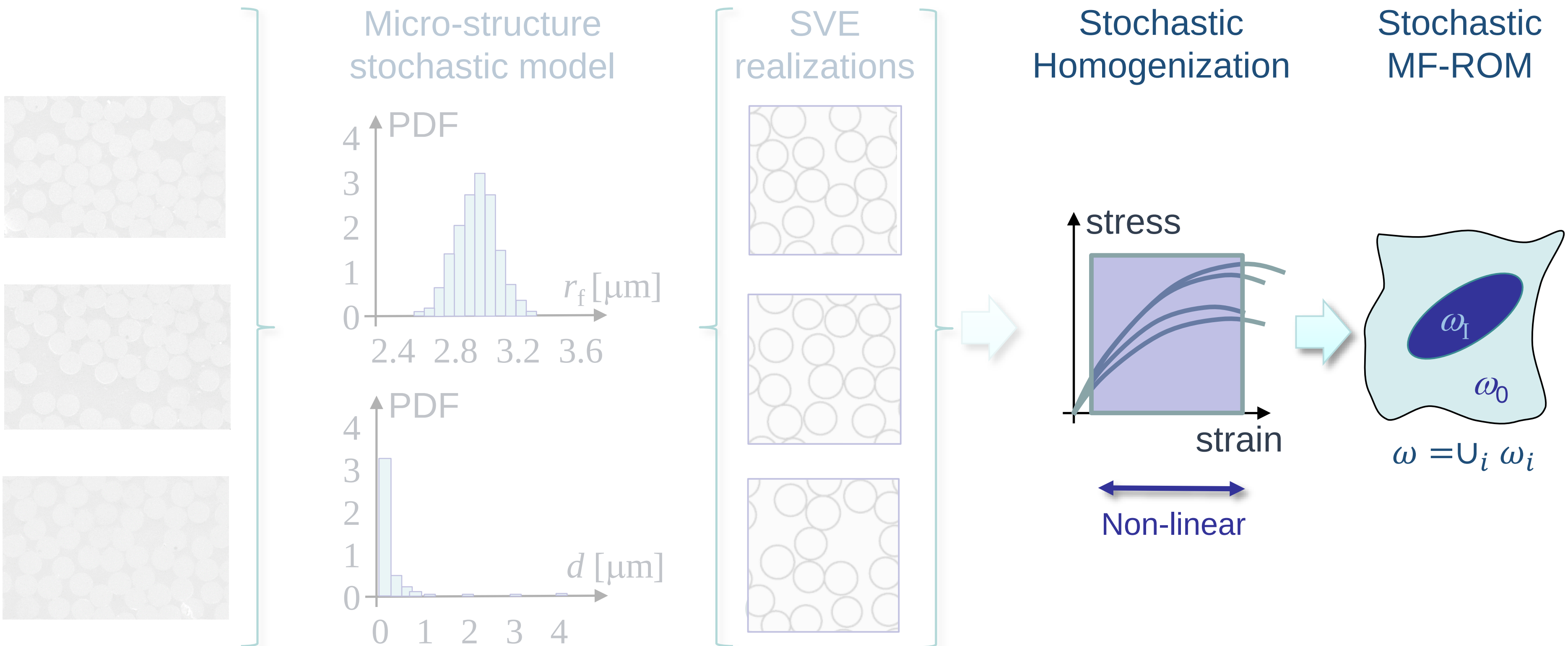
- Inverse stochastic identification
  - Comparison of homogenized properties from SVE realizations and stochastic MFH







- Stochastic Mean-Field Homogenization-based model
  - Second stage: damage-enhanced elasto-plasticity



Non-linear stochastic Mean-Field Homogenization

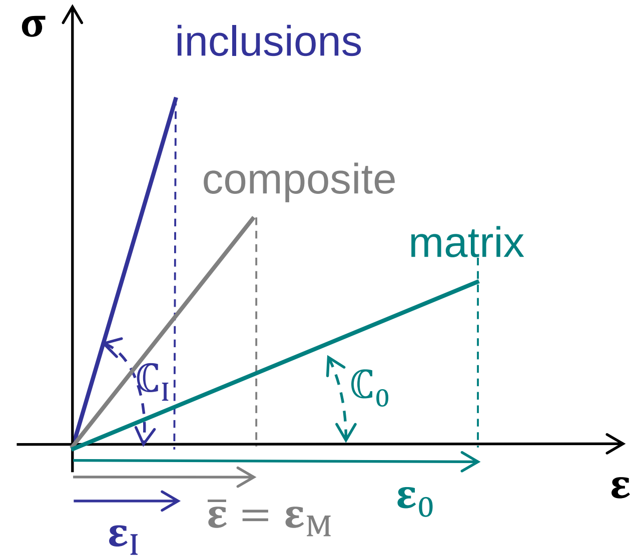
- Non-linear Mean-Field-homogenization
  - Linear composites

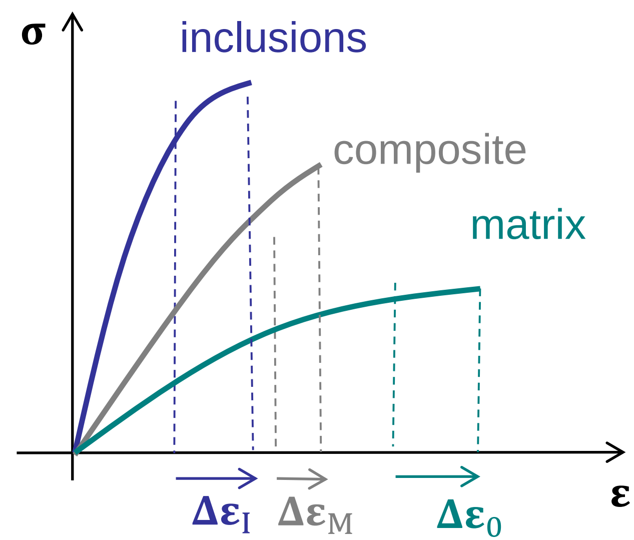
$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{M}} = \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbb{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0, \mathbb{C}_{\mathrm{I}}) : \boldsymbol{\varepsilon}_0 \end{cases}$$

  - Non-linear composites

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{M}} = \overline{\Delta\boldsymbol{\varepsilon}} = v_0 \Delta\boldsymbol{\varepsilon}_0 + v_{\mathrm{I}} \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \mathbb{B}^{\varepsilon}(\mathrm{I}, \mathbb{C}_0^{\mathrm{LCC}}, \mathbb{C}_I^{\mathrm{LCC}}) : \Delta\boldsymbol{\varepsilon}_0 \end{cases}$$

Define a linear comparison composite material

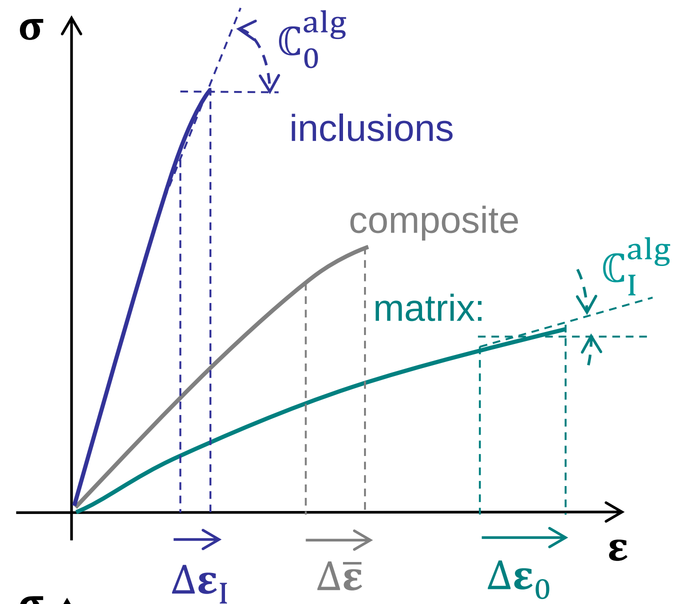




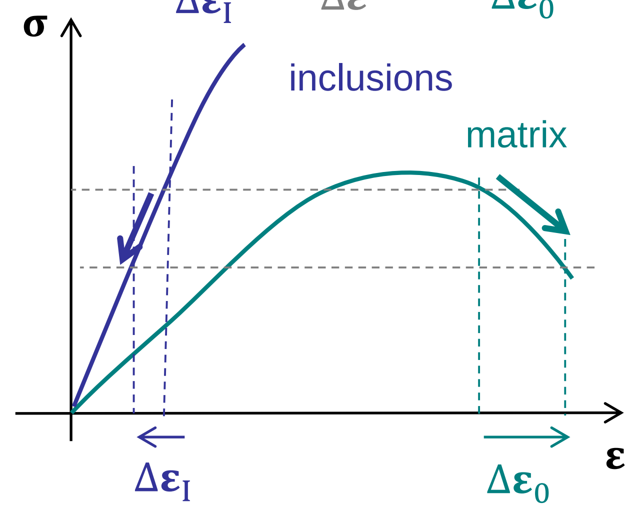
Non-linear stochastic Mean-Field Homogenization

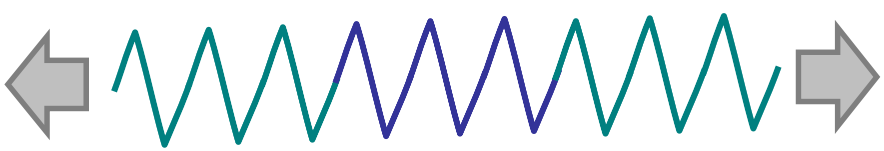
- View to damage Mean-Field-homogenization
  - Incremental forms
    - Strain increments in the same direction

$$\Delta\boldsymbol{\varepsilon}_{\mathbf{I}} = \mathbb{B}^{\varepsilon}\left(\mathrm{I}, \mathbb{C}_0^{\mathrm{alg}}, \mathbb{C}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_{\mathbf{0}}$$



  - Because of the damaging process, the fiber phase is elastically unloaded during matrix softening

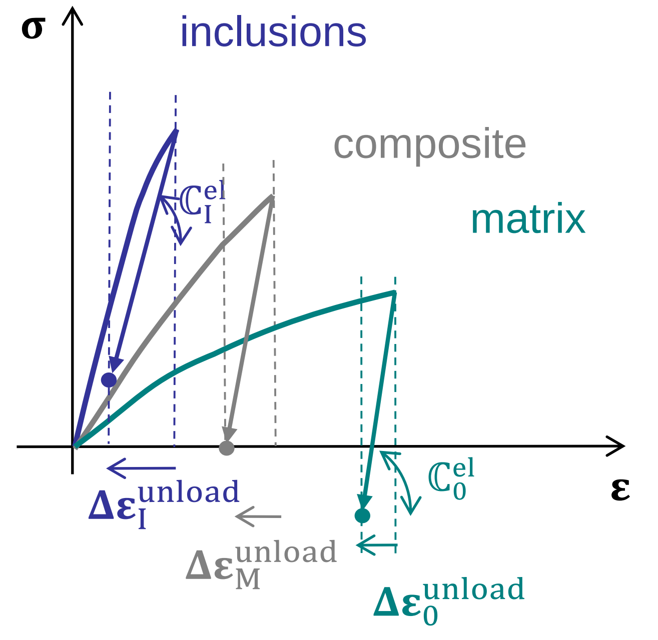




- Solution
  - We need to define the LCC from another state

Non-linear stochastic Mean-Field Homogenization

- Incremental-secant Mean-Field-homogenization
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components



- Incremental-secant Mean-Field-homogenization
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components
  - Define Linear Comparison Composite
    - From unloaded state

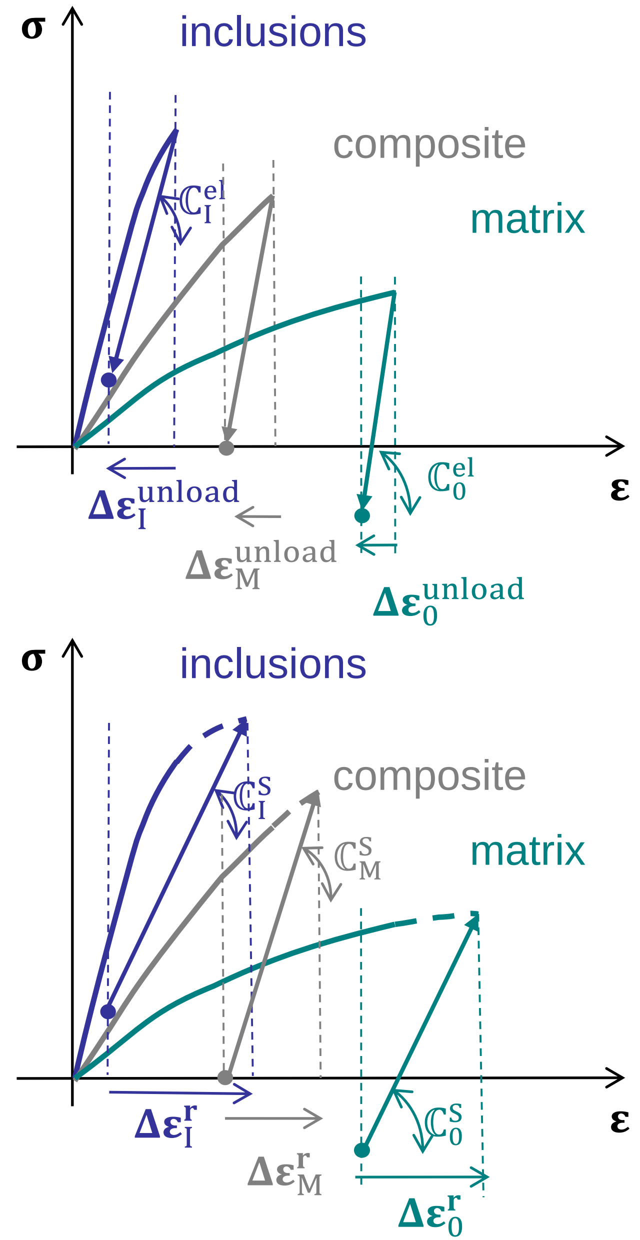
$$\Delta\boldsymbol{\varepsilon}^{\mathbf{r}}_{\mathrm{I}/0} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}/0} + \Delta\boldsymbol{\varepsilon}^{\mathrm{unload}}_{\mathrm{I}/0}$$

    - Incremental-secant loading

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0\boldsymbol{\sigma}_0 + v_{\mathrm{I}}\boldsymbol{\sigma}_{\mathrm{I}} \\ \Delta\boldsymbol{\varepsilon}^{\mathbf{r}}_{\mathrm{M}} = \overline{\Delta\boldsymbol{\varepsilon}} = v_0\Delta\boldsymbol{\varepsilon}^{\mathbf{r}}_0 + v_{\mathrm{I}}\Delta\boldsymbol{\varepsilon}^{\mathbf{r}}_{\mathrm{I}} \\ \Delta\boldsymbol{\varepsilon}^{\mathbf{r}}_{\mathrm{I}} = \mathbb{B}^{\varepsilon}\left(\mathrm{I}, \mathbb{C}^{\mathrm{S}}_0, \mathbb{C}^{\mathrm{S}}_{\mathrm{I}}\right) : \Delta\boldsymbol{\varepsilon}^{\mathbf{r}}_0 \end{cases}$$

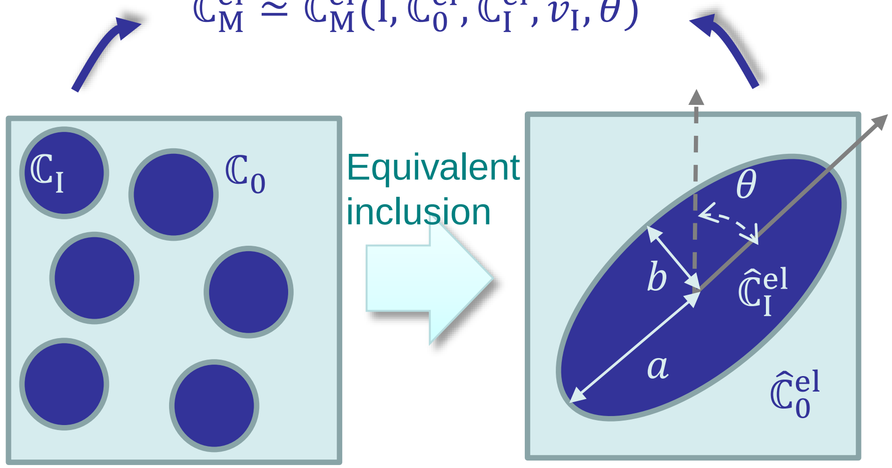
    - Incremental secant operator

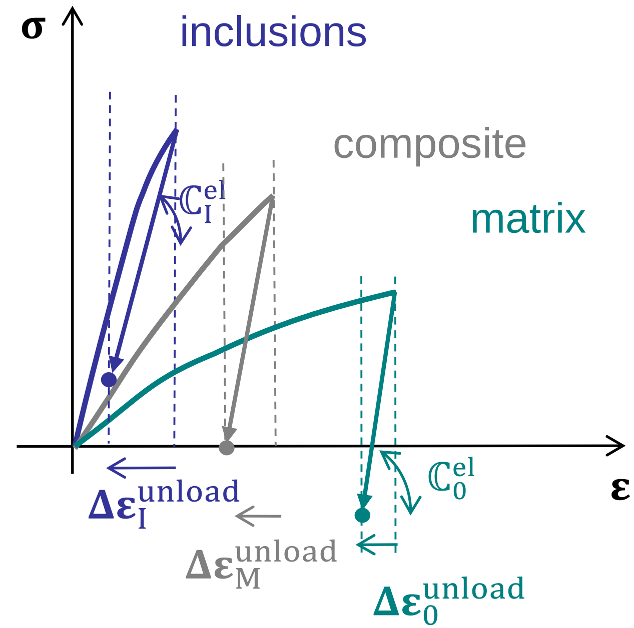
$$\Rightarrow \Delta\boldsymbol{\sigma}_{\mathrm{M}} = \mathbb{C}^{\mathrm{S}}_{\mathrm{M}}\left(\mathrm{I}, \mathbb{C}^{\mathrm{S}}_0, \mathbb{C}^{\mathrm{S}}_{\mathrm{I}}, v_{\mathrm{I}}\right) : \Delta\boldsymbol{\varepsilon}^{\mathbf{r}}_{\mathrm{M}}$$



- Non-linear inverse identification
  - First step from elastic response

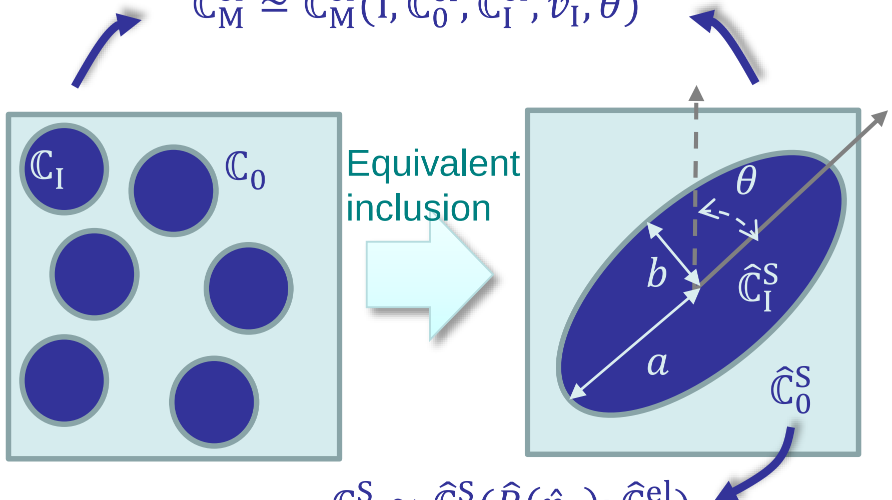
$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}} \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}(\hat{\mathrm{I}}, \hat{\mathbb{C}}_{0}^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta)$$



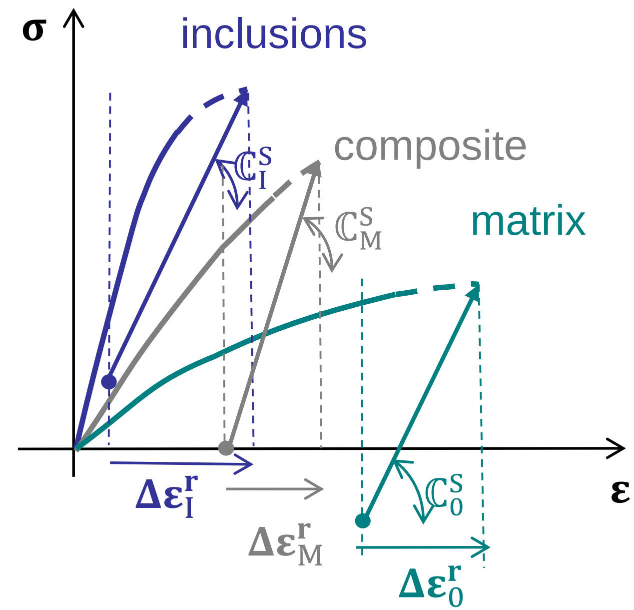


- Non-linear inverse identification
  - First step from elastic response

$$\mathbb{C}^{\mathrm{el}}_{\mathrm{M}} \simeq \hat{\mathbb{C}}^{\mathrm{el}}_{\mathrm{M}}(\hat{\mathrm{I}}, \hat{\mathbb{C}}^{\mathrm{el}}_{0}, \hat{\mathbb{C}}^{\mathrm{el}}_{\mathrm{I}}, v_{\mathrm{I}}, \theta)$$



$$\mathbb{C}^{\mathrm{S}}_{0} \simeq \hat{\mathbb{C}}^{\mathrm{S}}_{0}(\hat{R}(\hat{p}_0); \hat{\mathbb{C}}^{\mathrm{el}}_{0})$$

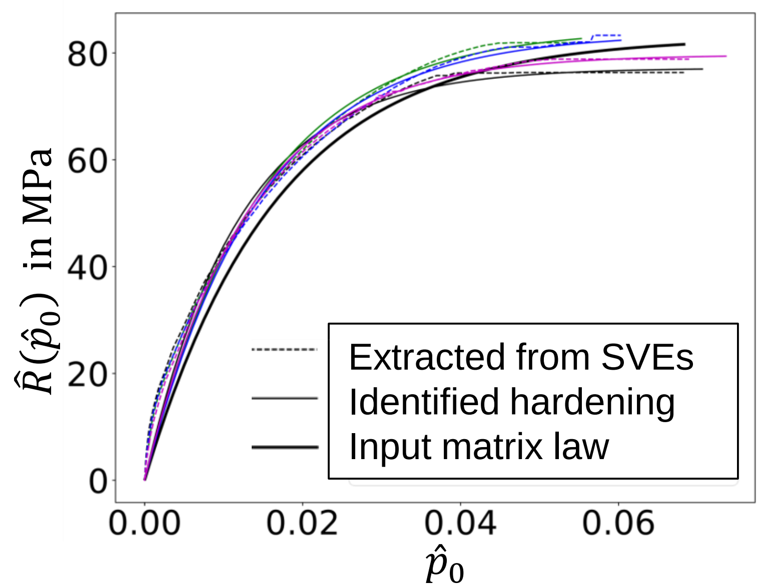


  - Second step from the LCC
    - New optimization problem

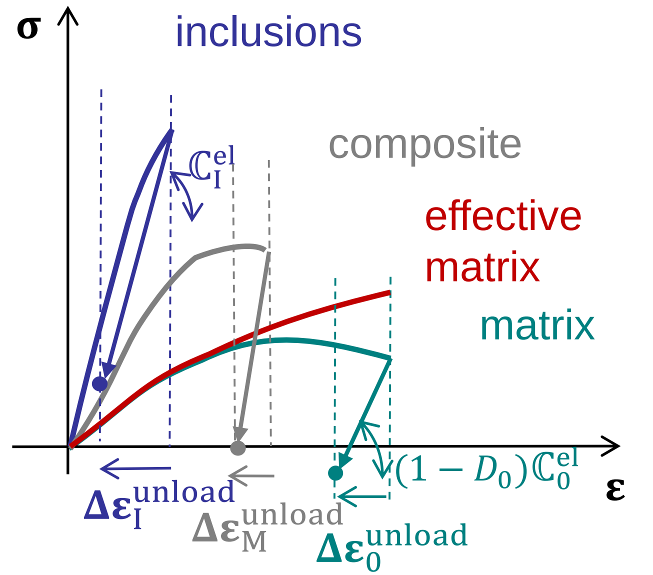
$$\boldsymbol{\Delta\sigma}_{\mathrm{M}} \simeq \hat{\mathbb{C}}^{\mathrm{S}}_{\mathrm{M}}\left(\hat{\mathrm{I}}, \hat{\mathbb{C}}^{\mathrm{S}}_{0}, \mathbb{C}^{\mathrm{S}}_{\mathrm{I}}, v_{\mathrm{I}}, \theta\right) : \boldsymbol{\Delta\varepsilon}^{\mathbf{r}}_{\mathrm{M}}$$

    - Extract the equivalent hardening $\hat{R}(\hat{p}_0)$ from the incremental secant tensor

$$\mathbb{C}^{\mathrm{S}}_{0} \simeq \hat{\mathbb{C}}^{\mathrm{S}}_{0}(\hat{R}(\hat{p}_0); \hat{\mathbb{C}}^{\mathrm{el}}_{0})$$



- Damage-enhanced Mean-Field-homogenization
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components



Non-linear stochastic Mean-Field Homogenization

- Damage-enhanced Mean-Field-homogenization
  - Virtual elastic unloading from previous state
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components
  - Define Linear Comparison Composite
    - From elastic state

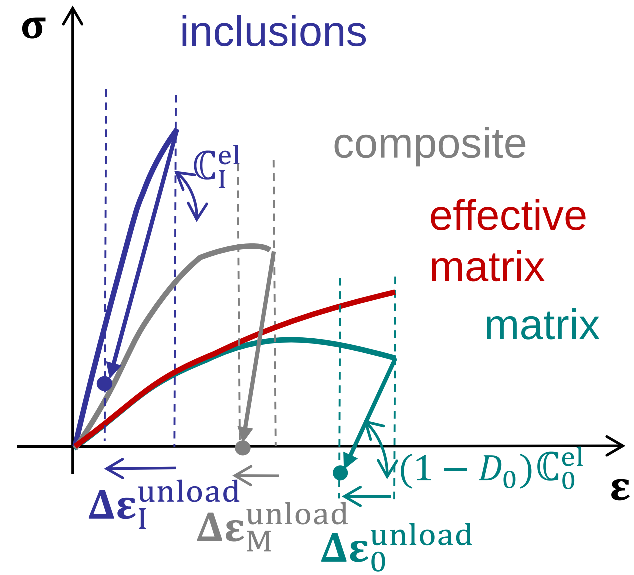
$$\boldsymbol{\Delta\varepsilon}^{\mathrm{r}}_{\mathrm{I}/0} = \boldsymbol{\Delta\varepsilon}_{\mathrm{I}/0} + \boldsymbol{\Delta\varepsilon}^{\mathrm{unload}}_{\mathrm{I}/0}$$

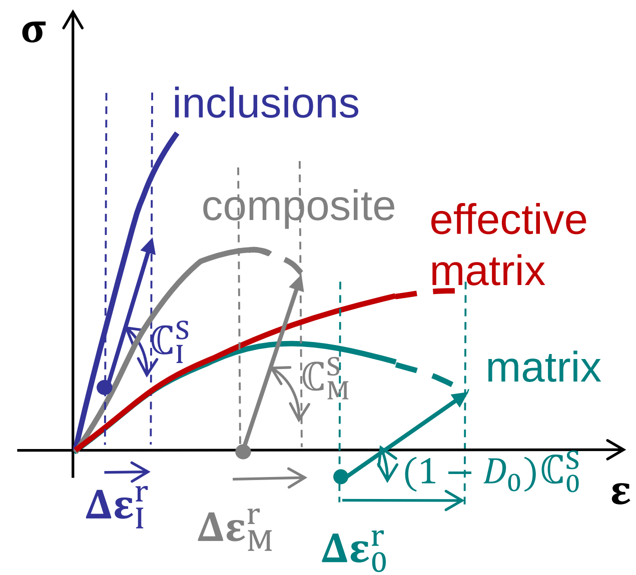
    - Incremental-secant loading

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{M}} = \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\Delta\varepsilon}^{\mathrm{r}}_{\mathrm{M}} = \overline{\boldsymbol{\Delta\varepsilon}} = v_0 \boldsymbol{\Delta\varepsilon}^{\mathrm{r}}_0 + v_{\mathrm{I}} \boldsymbol{\Delta\varepsilon}^{\mathrm{r}}_{\mathrm{I}} \\ \boldsymbol{\Delta\varepsilon}^{\mathrm{r}}_{\mathrm{I}} = \mathbb{B}^{\varepsilon}\left(\mathrm{I}, (1-D_0)\mathbb{C}^{\mathrm{S}}_0, \mathbb{C}^{\mathrm{S}}_{\mathrm{I}}\right) : \boldsymbol{\Delta\varepsilon}^{\mathrm{r}}_0 \end{cases}$$

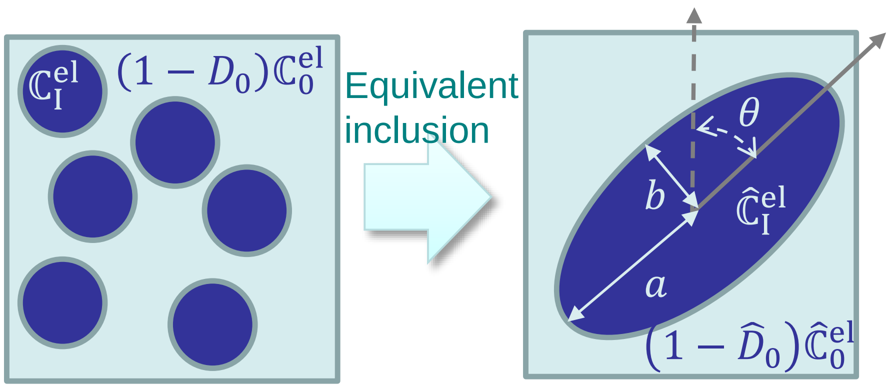
    - Incremental secant operator

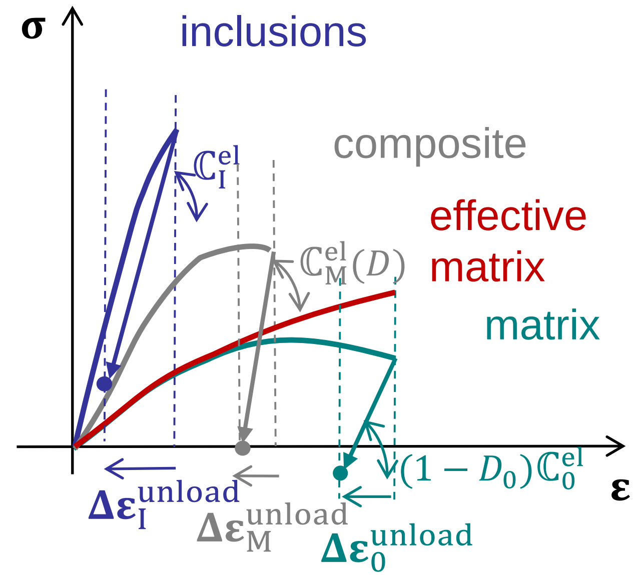
$$\Rightarrow \quad \boldsymbol{\Delta\sigma}_{\mathrm{M}} = \mathbb{C}^{\mathrm{S}}_{\mathrm{M}}\left(\mathrm{I}, (1-D_0)\mathbb{C}^{\mathrm{S}}_0, \mathbb{C}^{\mathrm{S}}_{\mathrm{I}}, v_{\mathrm{I}}\right) : \boldsymbol{\Delta\varepsilon}^{\mathrm{r}}_{\mathrm{M}}$$





- Damage-enhanced inverse identification
  - Second step: elastic unloading





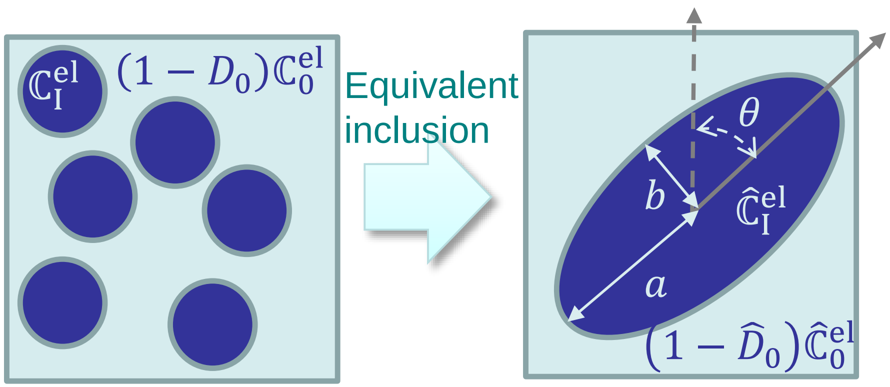
  - Identify damage evolution $\hat{D}_0$

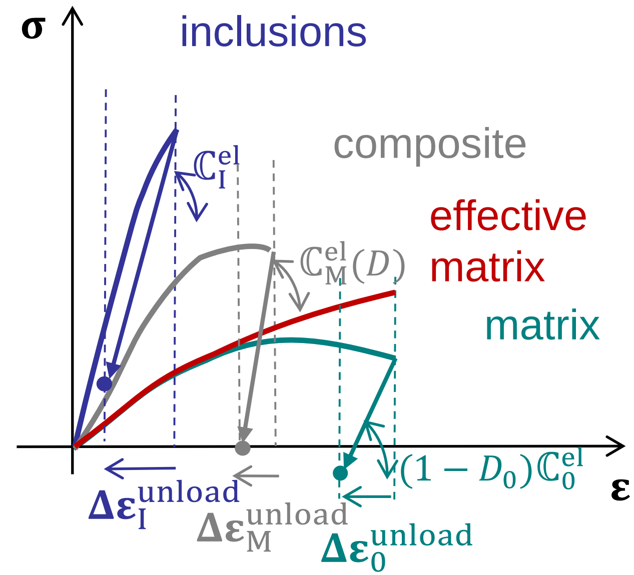
$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}}(D) \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}(\hat{\mathrm{I}}, (1-\hat{D}_0)\hat{\mathbb{C}}_0^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta)$$

L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

Non-linear stochastic Mean-Field Homogenization

- Damage-enhanced inverse identification
  - Second step: elastic unloading



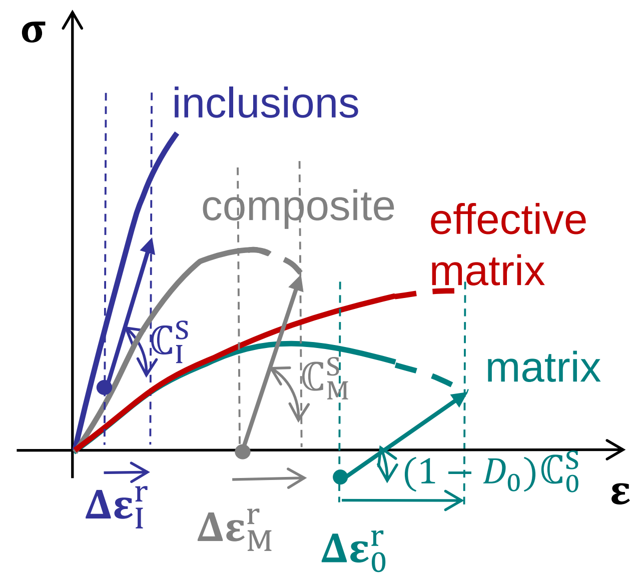


    - Identify damage evolution $\hat{D}_0$

$$\mathbb{C}_{\mathrm{M}}^{\mathrm{el}}(D) \simeq \hat{\mathbb{C}}_{\mathrm{M}}^{\mathrm{el}}\left(\hat{\mathrm{I}}, \left(1-\hat{D}_0\right)\hat{\mathbb{C}}_0^{\mathrm{el}}, \hat{\mathbb{C}}_{\mathrm{I}}^{\mathrm{el}}, v_{\mathrm{I}}, \theta\right)$$

  - Third step from the LCC
    - $\Delta\boldsymbol{\sigma}_{\mathrm{M}} = \mathbb{C}_{\mathrm{M}}^{S}\left(\mathrm{I}, (1-D_0)\mathbb{C}_0^{S}, \mathbb{C}_{\mathrm{I}}^{S}, v_{\mathrm{I}}\right): \Delta\boldsymbol{\varepsilon}_{\mathrm{M}}^{\mathrm{r}}$
    - Extract the equivalent hardening $\hat{R}(\hat{p}_0)$ & damage evolution $\hat{\mathrm{D}}_0(\hat{p}_0)$ from incremental secant tensor:

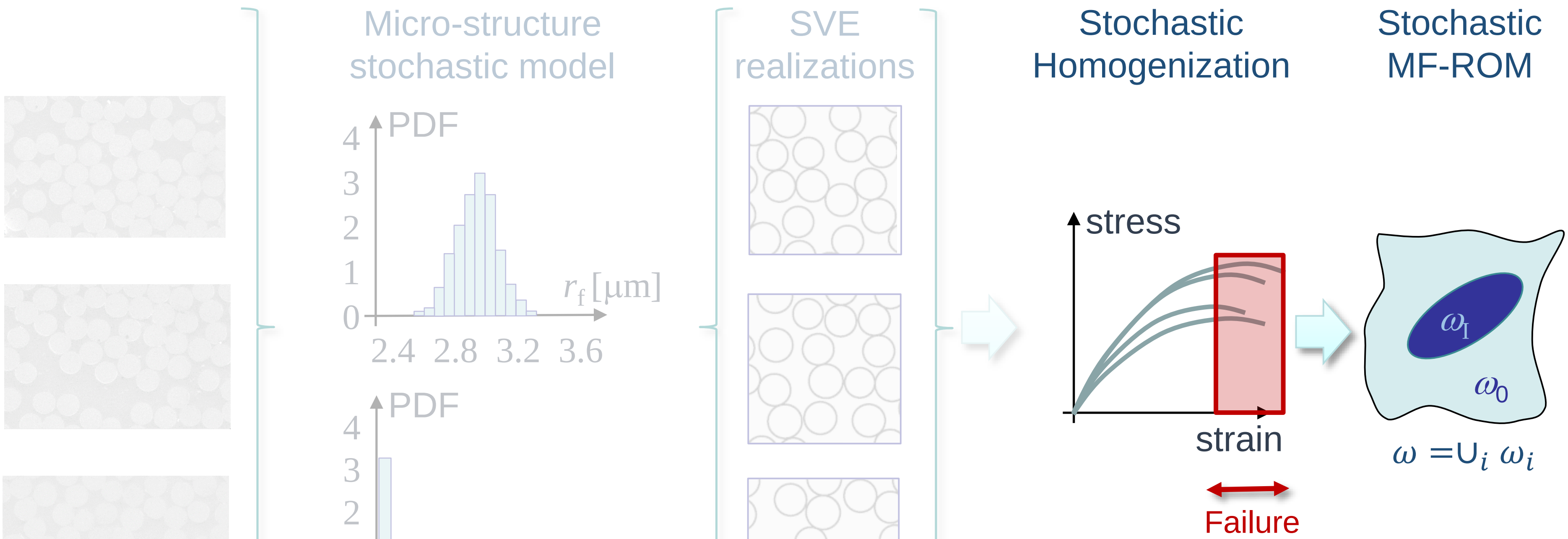
$$(1-\mathrm{D}_0)\mathbb{C}_0^{S} \simeq \left(1-\hat{\mathrm{D}}_0(\hat{p}_0)\right)\hat{\mathbb{C}}_0^{S}\left(\hat{R}(\hat{p}_0); \hat{\mathbb{C}}_0^{\mathrm{el}}\right)$$



L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites (2019)

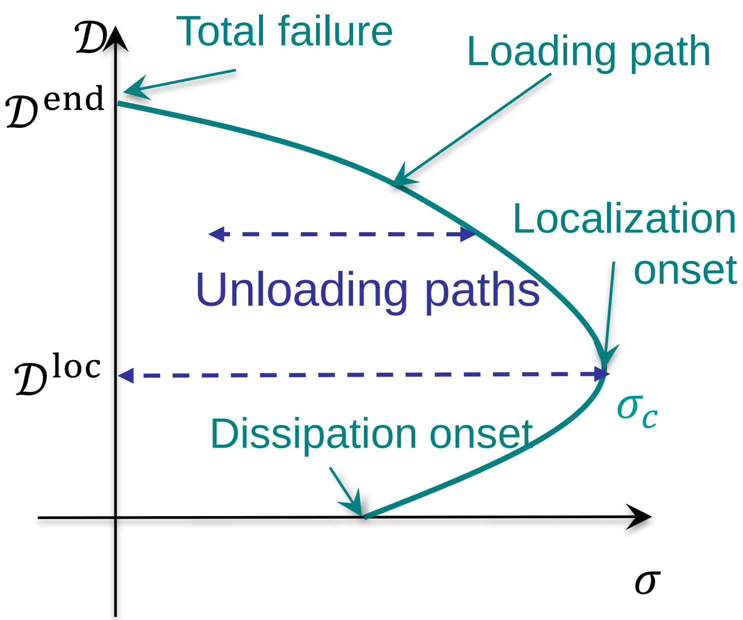
Methodology

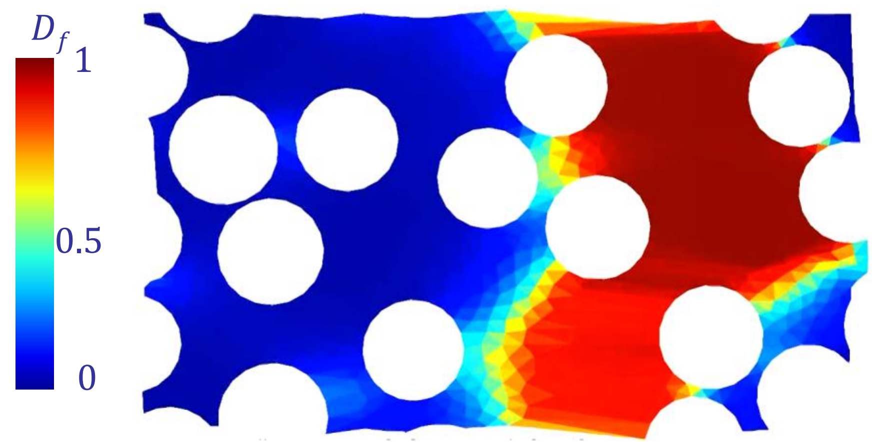
- Stochastic Mean-Field Homogenization-based model
  - Third stage: Failure



Mean-Field Homogenization with failure

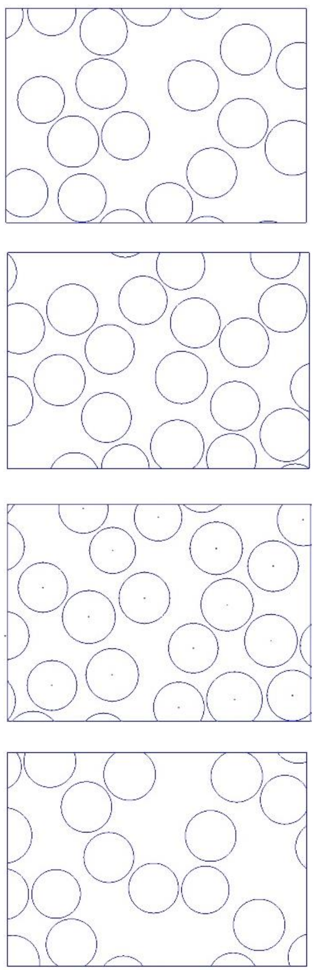
- Need to recover size objectivity
  - Strength is objective σ_c
  - Critical energy release rate is objective $G_c = \dfrac{\mathcal{D}^{\text{end}} - \mathcal{D}^{\text{loc}}}{A_0}$

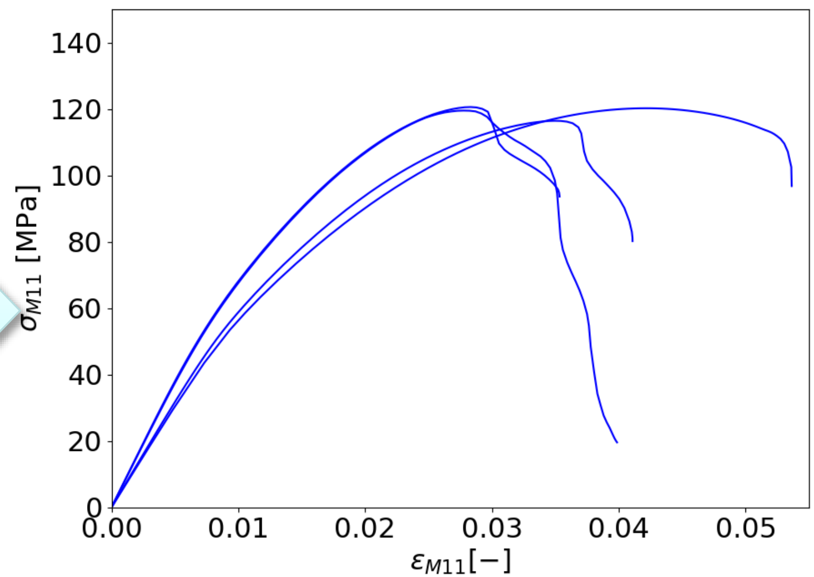




Mean-Field Homogenization with failure

- Non-linear SVE simulations





G_c [N/mm]

0.0408

0.0780

0.0654

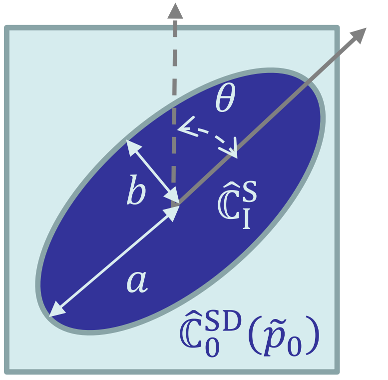
0.1019

Mean-Field Homogenization with failure

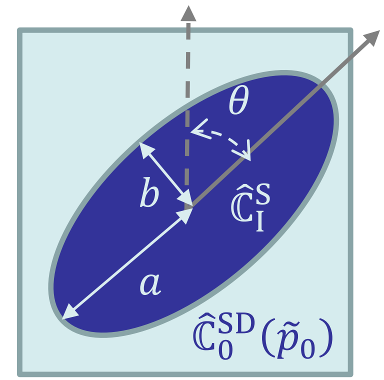
- Damage-enhanced mean-field homogenization
  - Requires non-local form for the matrix part of homogenized behavior
    - Following implicit form [Peerlings et al. 1998]
      - Apparent matrix plastic strain p_0
      - Non-local apparent matrix plastic strain $\tilde{p}_0$
      - Matrix damage evolution

$$\Delta D_0 = F_D(\boldsymbol{\Delta\varepsilon_0}, \Delta p_0) \Rightarrow \Delta D_0 = F_D(\boldsymbol{\Delta\varepsilon_0}, \Delta\tilde{p}_0)$$

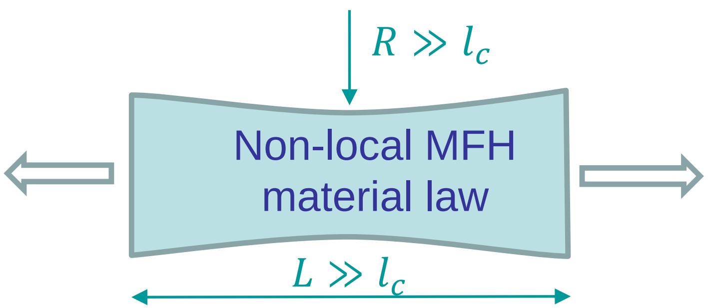
    - New Helmholtz-type equations:
      - $\tilde{p}_0 - l_c^2 \nabla \cdot (\nabla \tilde{p}_0) = p_0$
      - Definition of non-local length l_c

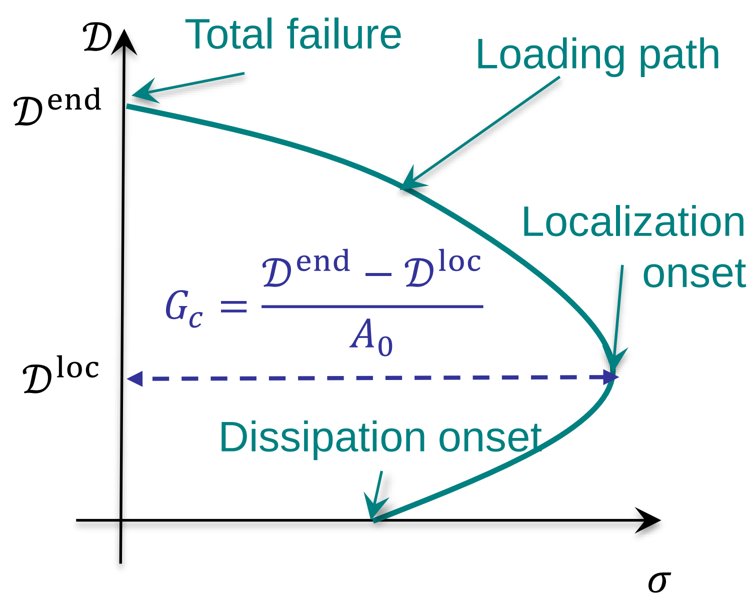


Mean-Field Homogenization with failure



- Damage-enhanced mean-field homogenization
  - Requires non-local form for the matrix homogenized behavior
    - Matrix damage evolution $\Delta D_0 = F_D(\boldsymbol{\Delta\varepsilon_0}, \Delta\tilde{p}_0)$
    - New Helmholtz-type equations:
      - $\tilde{p}_0 - l_c^2 \nabla \cdot (\nabla \tilde{p}_0) = p_0$
      - Definition of non-local length l_c
- Evaluation of non-local length
  - To recover the energy release rate of SVEs





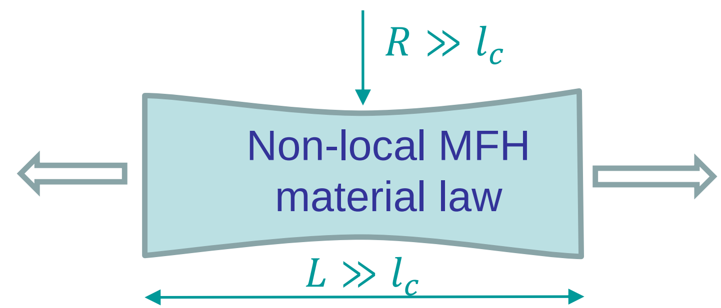
l_c (µm)	MFH G_c [N mm^{-1}]	SVE G_c [N mm^{-1}]
20	1.008	
5	0.363	0.1035
2	0.0922	

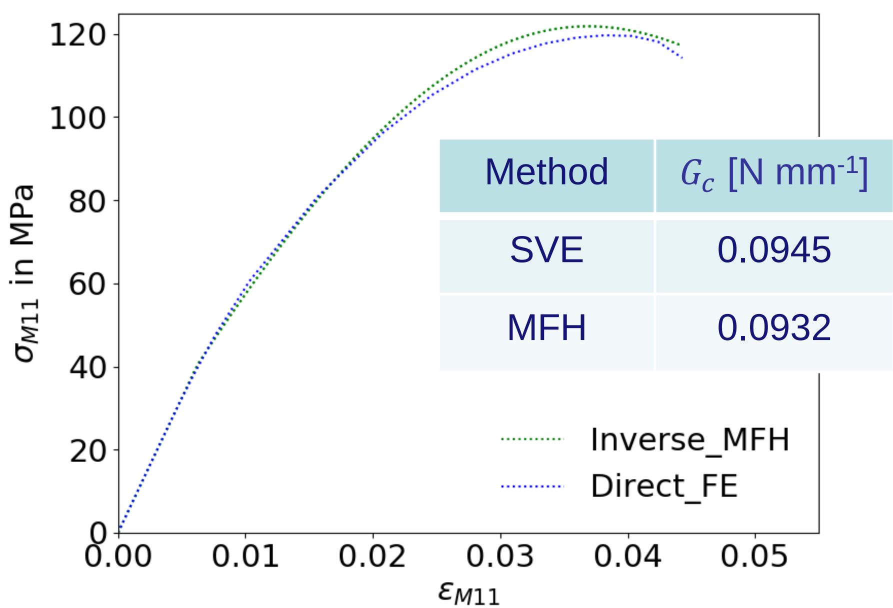
Mean-Field Homogenization with failure

- More convenient to fix non-local length
  - For macro-scale Stochastic FEM
  - To increase its length
- Modify the damage evolution to recover G_c
  - Before localization onset: identified evolution

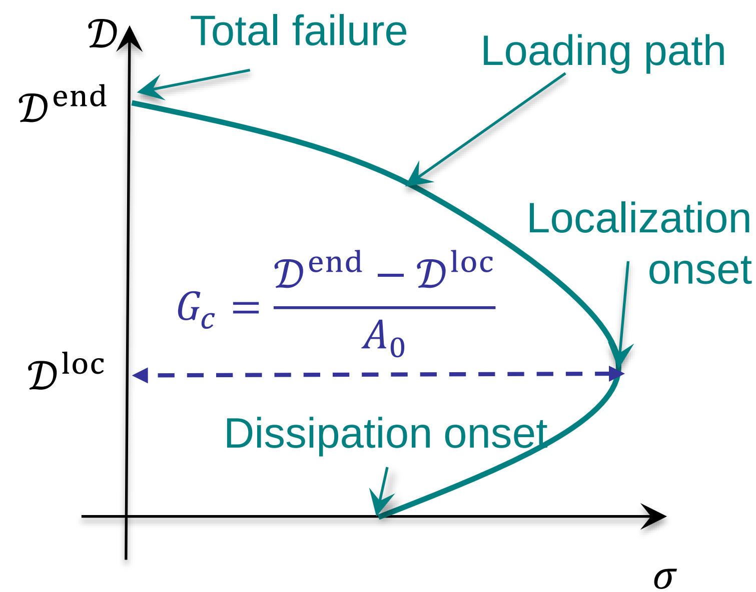
$$\Delta D_0 = F_D(\boldsymbol{\Delta\varepsilon_0}, \Delta\tilde{p}_0)$$

  - After localization onset:
    - $\Delta D_0 = \alpha(\tilde{p}_0 + \Delta\tilde{p}_0 - \tilde{p}_{0_{\text{onset}}})^{\beta}\, \Delta\tilde{p}_0$
    - Iterate on α and β

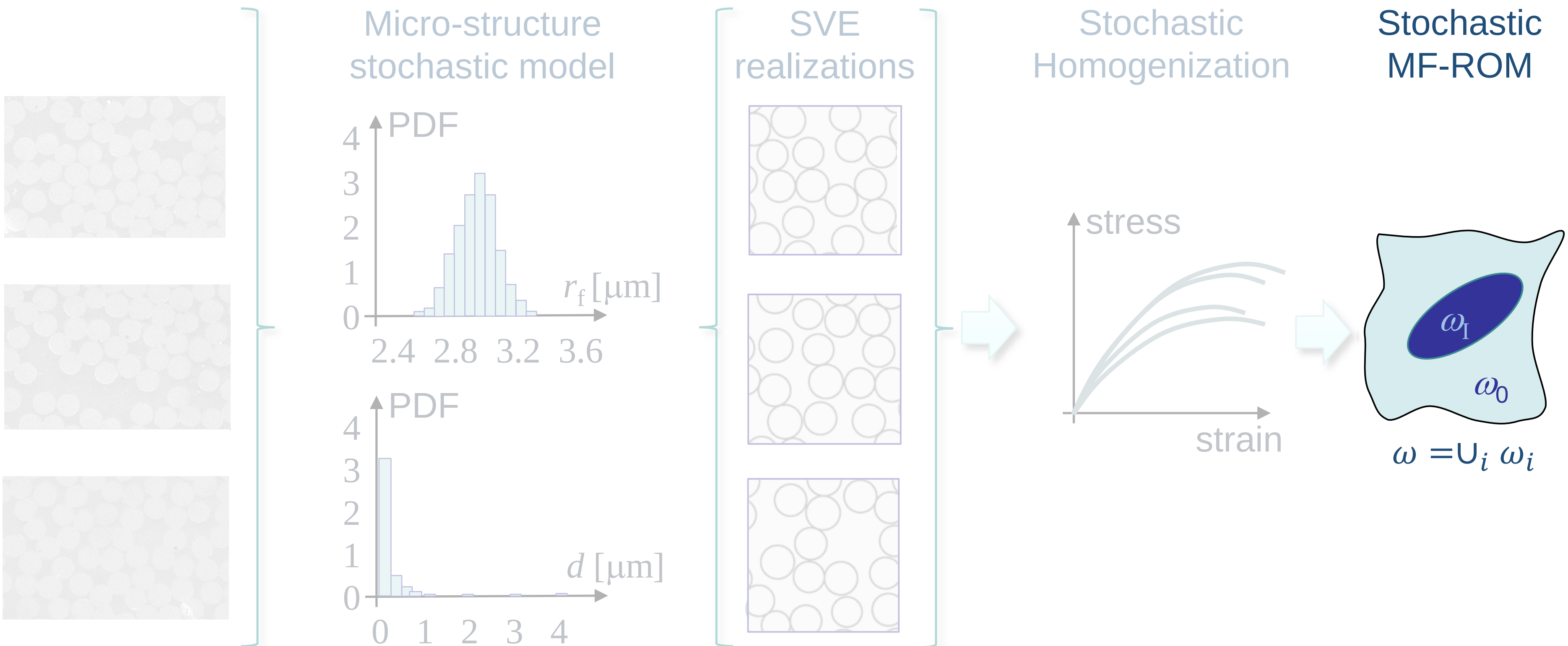




Method	G_c [N mm^{-1}]
SVE	0.0945
MFH	0.0932

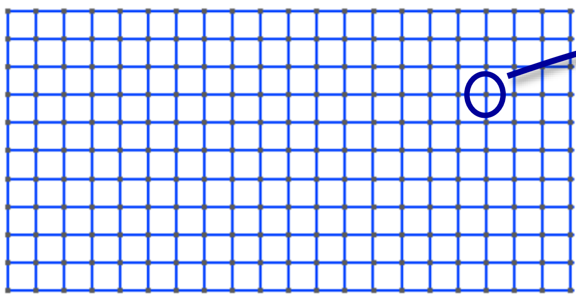


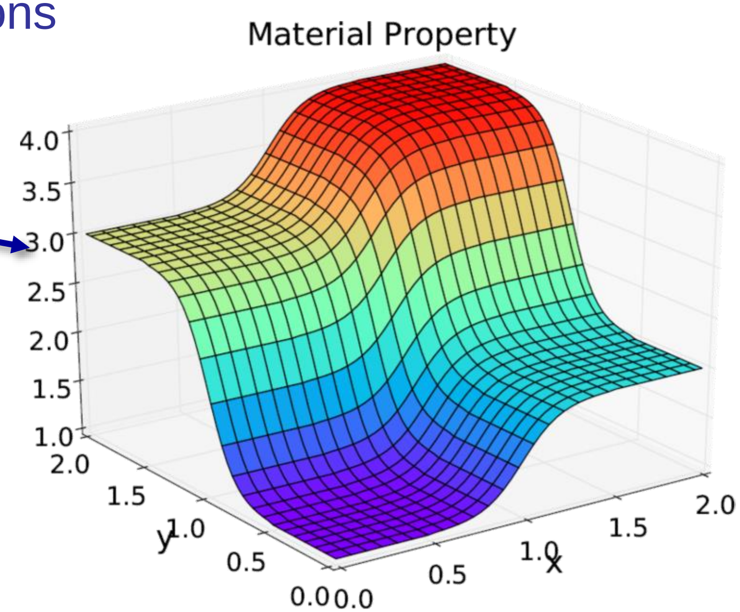
- Use Stochastic Mean-Field Homogenization as constitutive law



Use of stochastic Mean-Field Homogenization

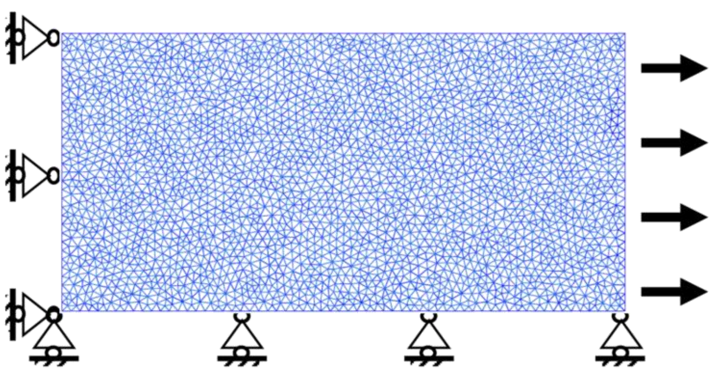
- Stochastic simulations require two discretizations
  - Random vector field discretization
    - Of MFH-model parameters



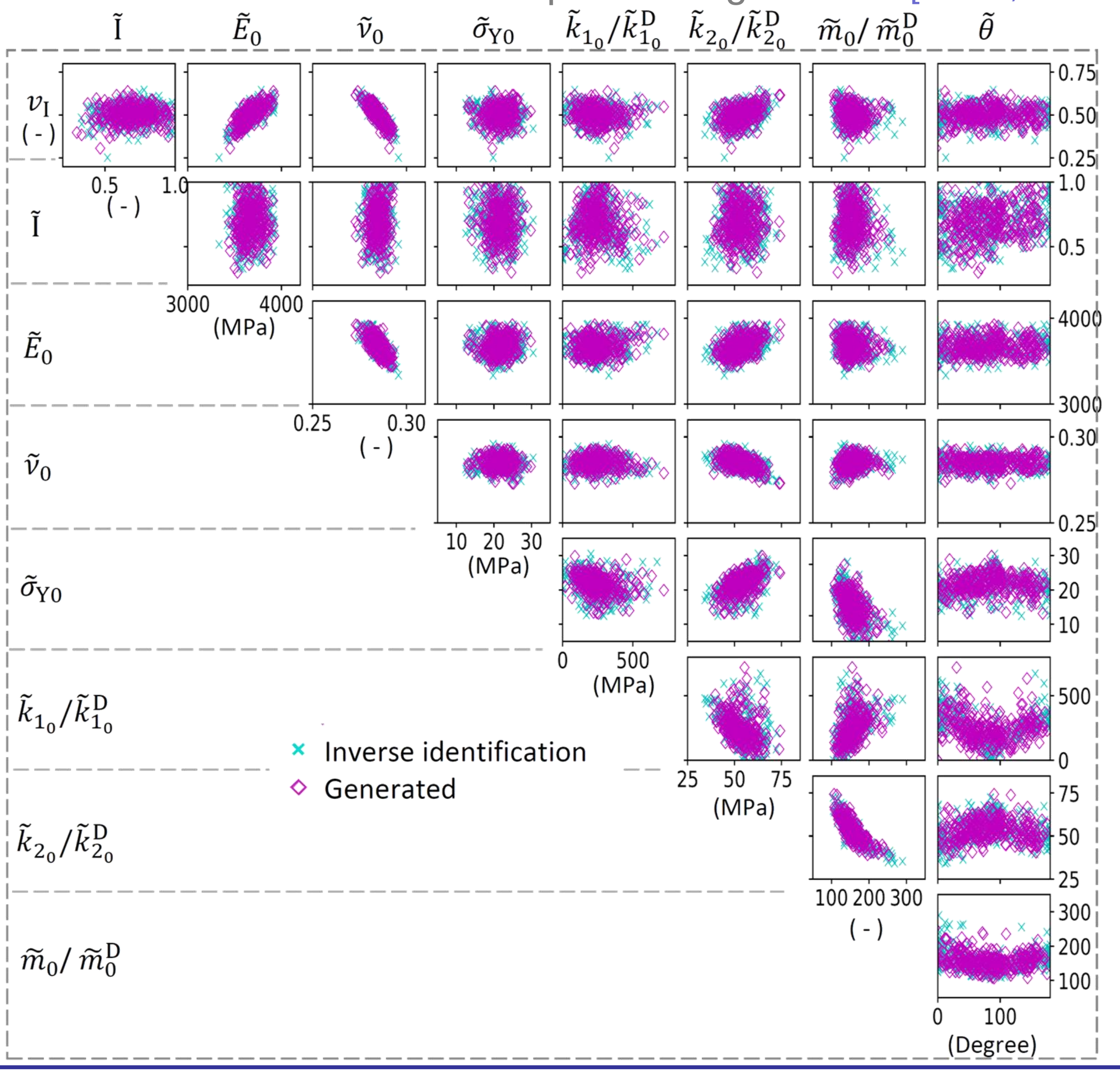


    - Stochastic MFH-model parameters identified
    - Potentially more cells than SVE realizations

      ⇒ parameters need to be generated
  - Finite element discretization
    - Finer than random field grid

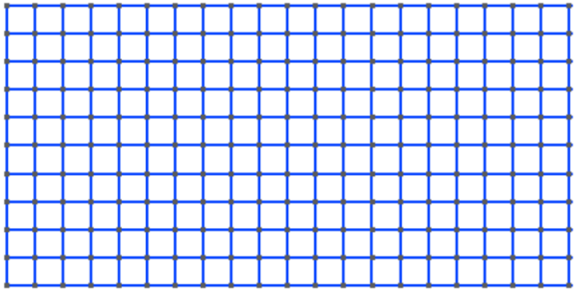


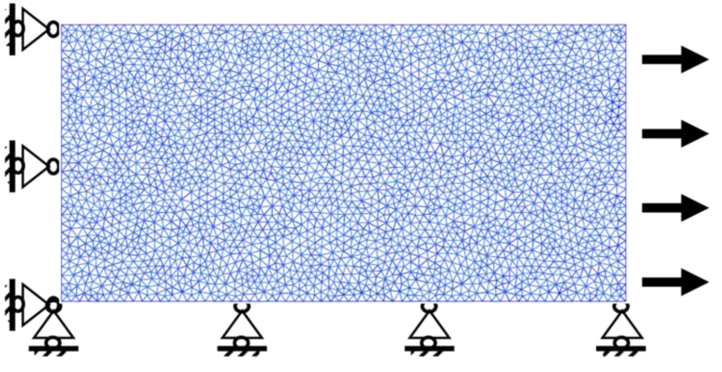
- Generation of random field
  - Inverse identification vs. diffusion map –based generator [Soize, Ghanem 2016]

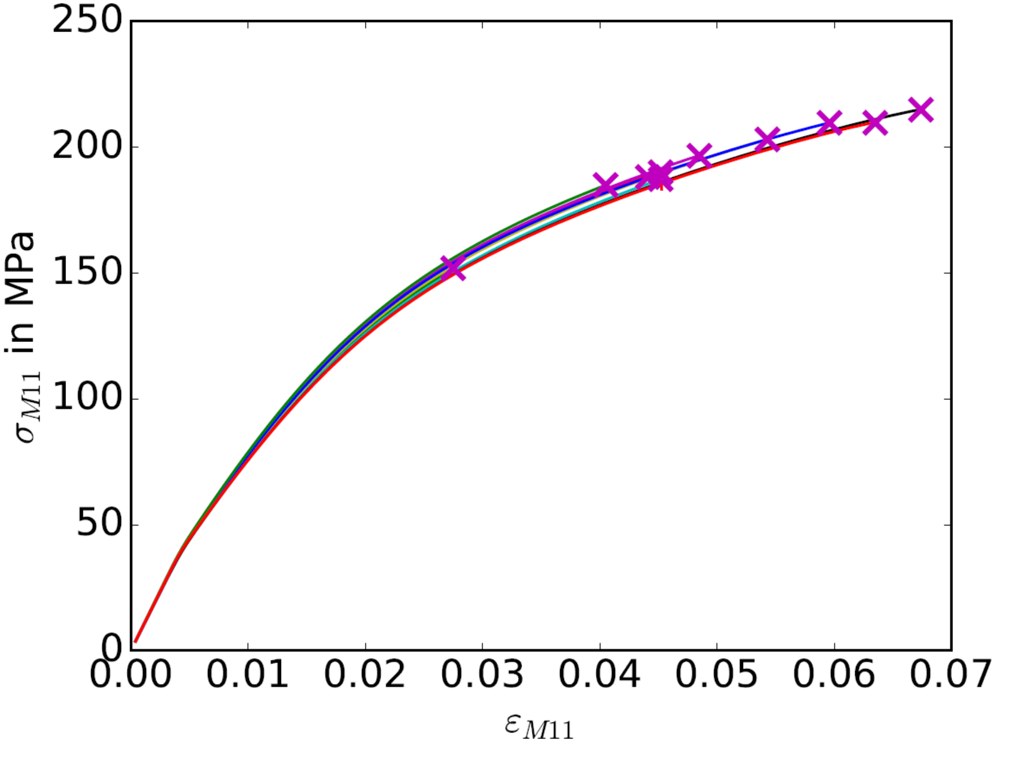


Use of stochastic Mean-Field Homogenization

- Ply loading realizations
  - Preliminary results (softening part not implemented)







Conclusions

- Stochastic micro-structures
  - Geometrical features from statistical measurements
  - Micro-structure geometry generator
  - Experimentally calibrated/validated epoxy model with length scale effect
- Inverse MFH identification
  - MFH is used as a micro-mechanics based model
  - Parameters identified from SVE simulations
  - Localization behaviour identified using objective fields
- Stochastic Finite elements
  - Stochastic MFH is used as material law
  - Random fields (MFH parameters) generated using data-driven approach
  - First ply simulations

Thank you for your attention!

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References

- L. Wu, V.-D. Nguyen, L. Adam, L. Noels, An inverse micro-mechanical analysis toward the stochastic homogenization of nonlinear random composites, Comp. Meth. in App. Mech. and Engineering (ISSN: 0045-7825) 348, (2019) 97-138, https://doi.org/10.1016/j.cma.2019.01.016
- L. Wu, C.N. Chung, Z. Major, L. Adam, L. Noels, From SEM images to elastic responses: A stochastic multiscale analysis of UD fiber reinforced composites, Compos. Struct. (ISSN: 0263-8223) 189 (2018a) 206–227, http://dx.doi.org/10.1016/j.compstruct.2018.01.051
- L. Wu, C. Nghia Chung, Z. Major, L. Adam, L. Noels, A micro-mechanics-based inverse study for stochastic order reduction of elastic UD-fiber reinforced composites analyzes, Internat. J. Numer. Methods Engrg. (ISSN: 0263-8223) 115 (2018b) 1430–1456, http://dx.doi.org/10.1016/
- T. Mori, K. Tanaka, Average stress in matrix and average elastic energy of materials with misfitting inclusions, Acta Metall. 21 (5) (1973) 571–574
- L. Wu, L. Noels, L. Adam, I. Doghri, A combined incremental–secant mean–field homogenization scheme with per–phase residual strains for elasto–plastic composites, Int. J. Plast. 51 (2013a) 80–102, http://dx.doi.org/10.1016/j.ijplas.2013.06.006
- L. Wu, L. Noels, L. Adam, I. Doghri, An implicit-gradient-enhanced incremental-secant mean-field homogenization scheme for elasto-plastic composites with damage, Int. J. Solids Struct. (ISSN: 0020-7683) 50 (24) (2013b) 3843–3860, http://dx.doi.org/10.1016/j.ijsolstr.2013.07.022.
- L. Wu, L. Adam, I. Doghri, L. Noels, An incremental-secant mean-field homogenization method with second statistical moments for elasto-visco-plastic composite materials, Mech. Mater. (ISSN: 0167-6636) 114 (2017) 180–200, http://dx.doi.org/10.1016/j.mechmat.2017.08.006.
- V.-D.Nguyen F.Lani T.Pardoen X.P.Morelle L.Noels, A large strain hyperelastic viscoelastic-viscoplastic-damage constitutive model based on a multi-mechanism non-local damage continuum for amorphous glassy polymers. Int. Journal of Sol. and Struct., (ISSN: 0020-7683) 96 (2016) 192-216, https://doi.org/10.1016/j.ijsolstr.2016.06.008
- V.-D. Nguyen, L. Wu, L. Noels, A micro-mechanical model of reinforced polymer failure with length scale effects and predictive capabilities. Validation on carbon fiber reinforced high-crosslinked RTM6 epoxy resin , Mech. Mater. (ISSN: 0167-6636) 133 (2019) 193-213, https://dx.doi.org/10.1016/j.mechmat.2019.02.017