A nonlocal model for ductile failure incorporating void growth and coalescence

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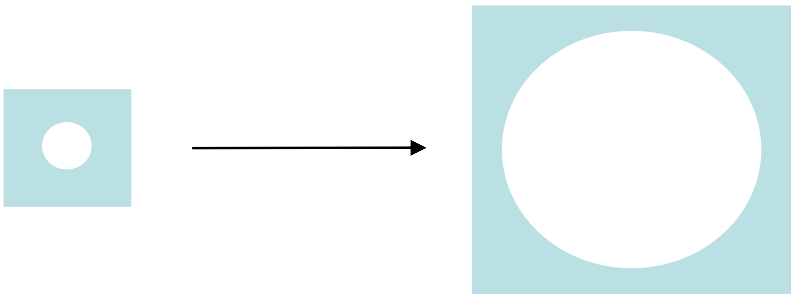
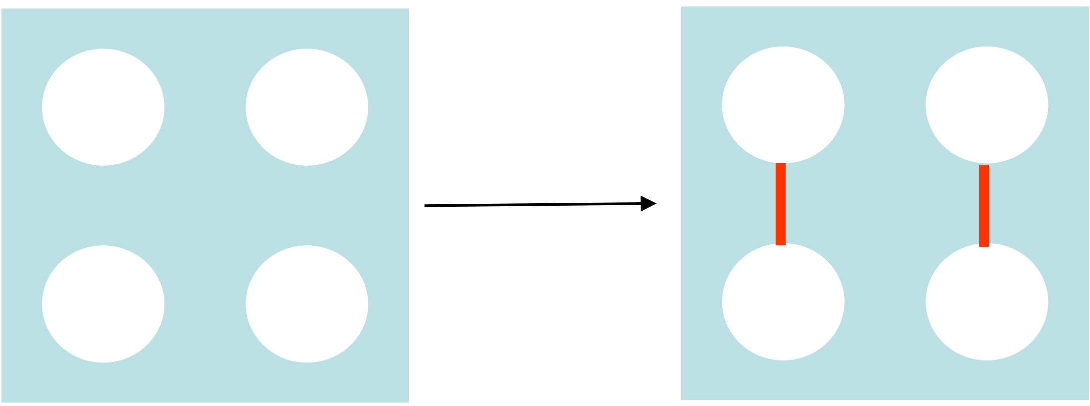
- Ductile failure: failure mechanism
  - Void nucleation (dislocation motion, particle/matrix decohesion, particle cracking, …)
  - Void growth of existing voids (because of plastic incompressibility)



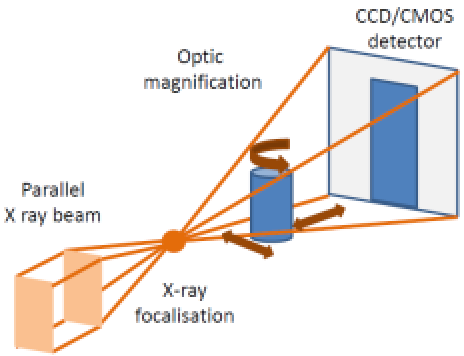
  - Void coalescence (crack growth by shrinking of ligaments between voids)

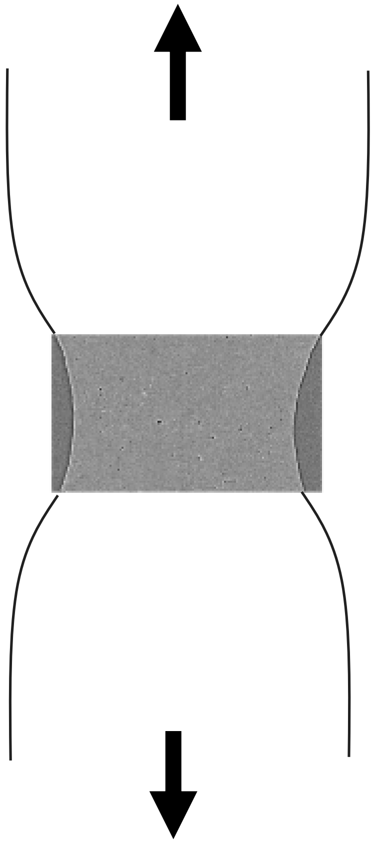


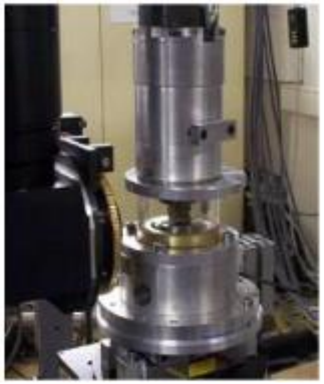
Introduction

- Ductile failure: complex coalescence scenario
  - What happens inside a « ductile » material when highly deformed ?

X-ray tomography of in-situ tensile tests
= scanner for materials







Tests performed at the ESRF synchrotron in Grenoble* by F. *Hannard (Ph. D. UCL) collaboration with Dr. E. maire INSA Lyon

(Hannard et al. 2016)

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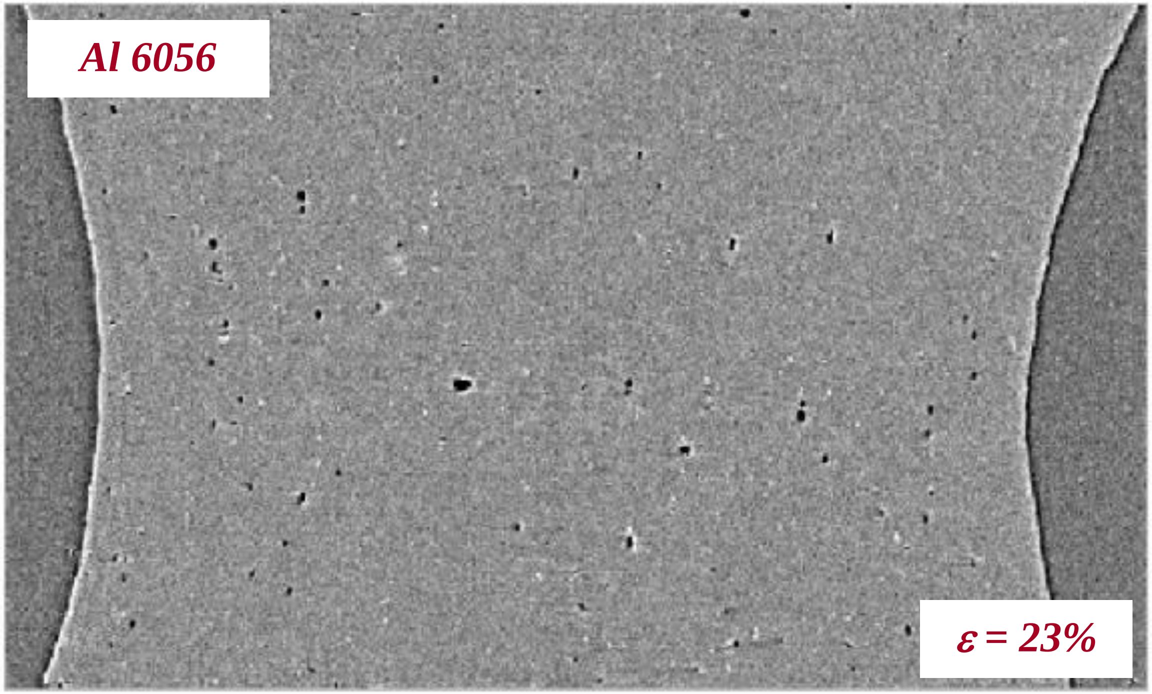
X-ray tomography of in-situ tensile tests on an aluminium alloy



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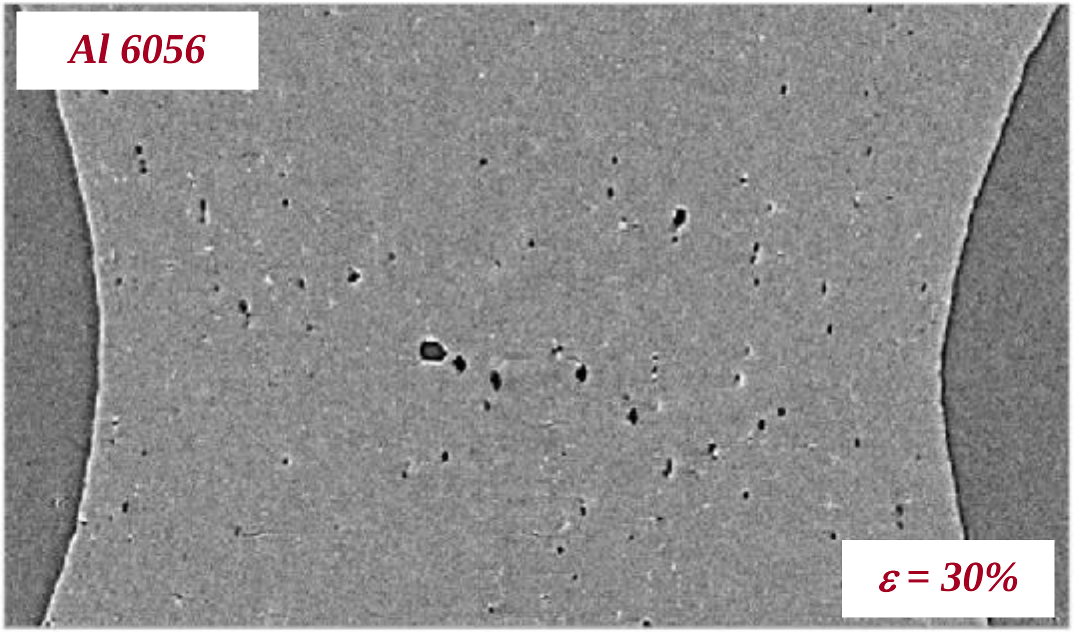

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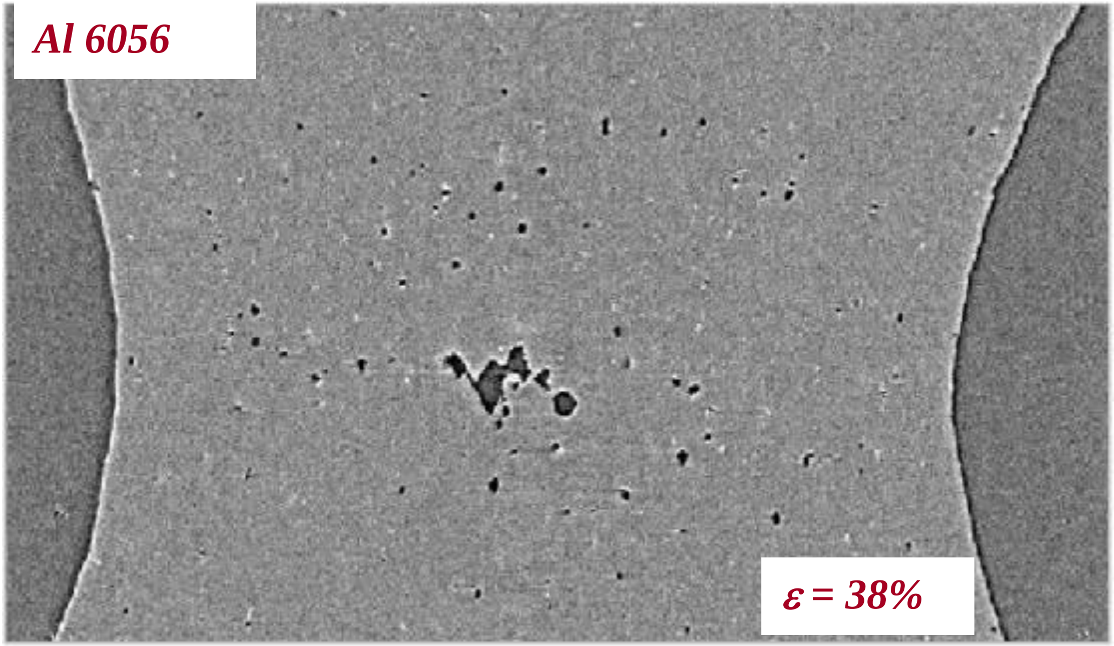

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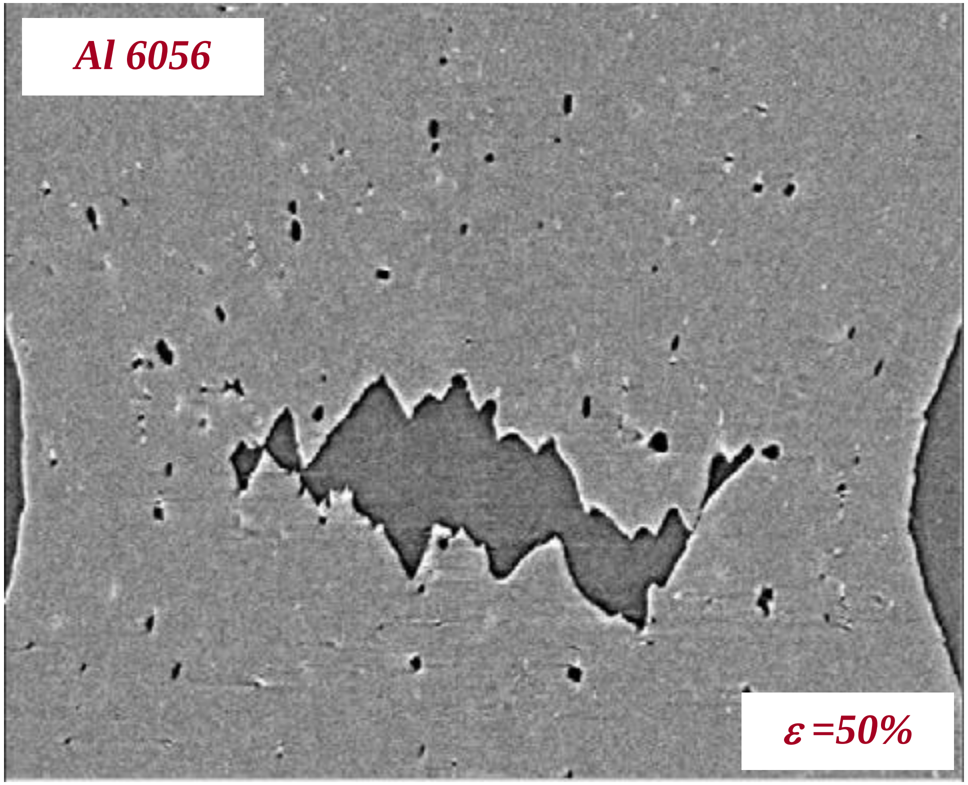

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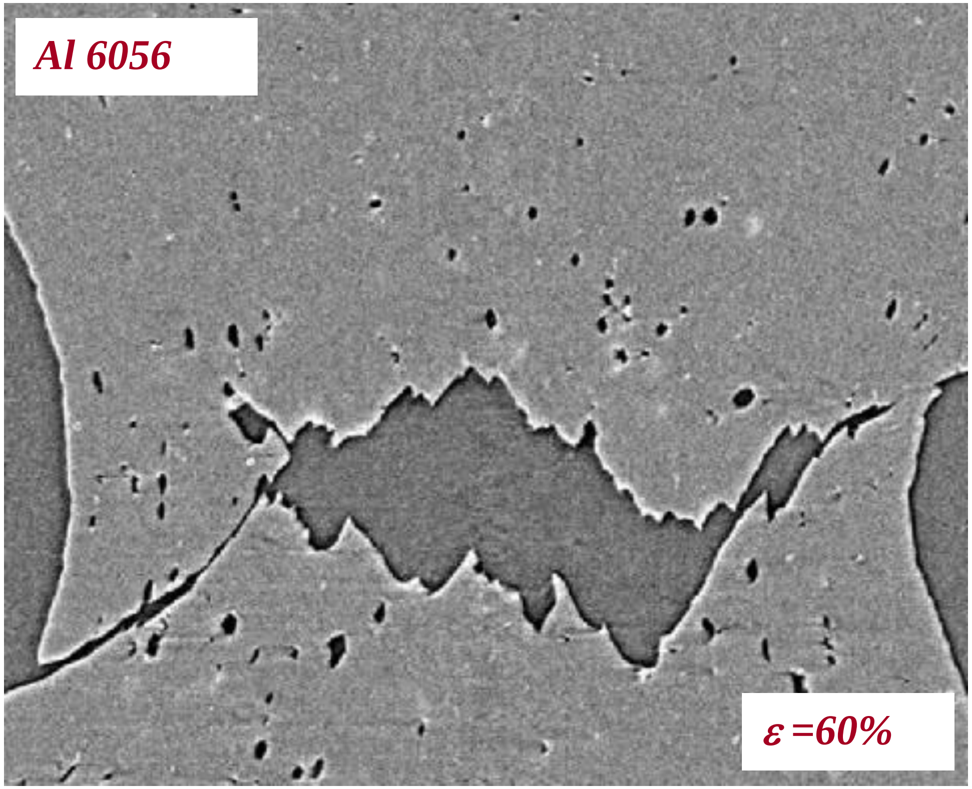
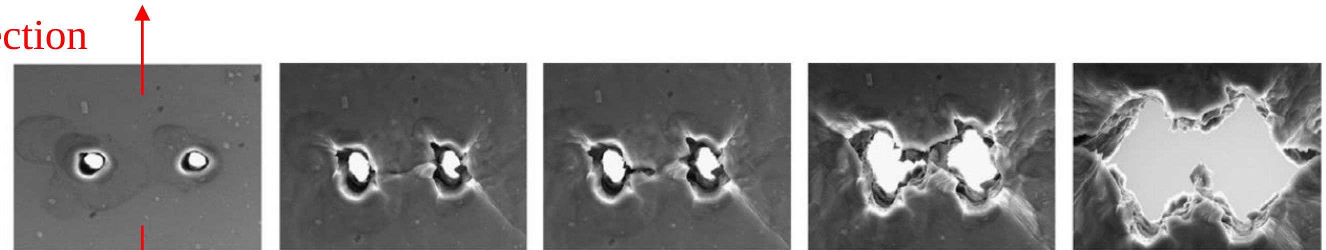


- Ductile failure: complex coalescence scenario
  - Internal necking coalescence
    - Shrinking of ligaments between voids along a localization band perpendicular to the main loading direction.

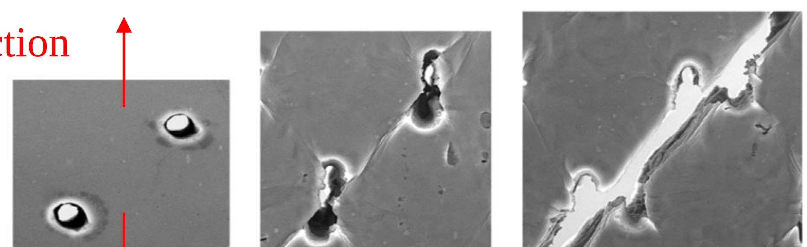
Loading direction



Internal necking coalescence

  - Shear driven coalescence
    - Formation of micro shear bands inclined to the main loading direction and joining primary voids, possibly with secondary voids nucleated in these micro bands.

Loading direction



Shear-driven coalescence

(Weck & Wilkinson 2008)

- Ductile failure: stress-state dependent fracture strain
  - Stress triaxiality dependent

$$\eta = \frac{p'}{\sigma_{\text{eq}}} \in [-\infty \ \infty]$$

  - Lode dependent

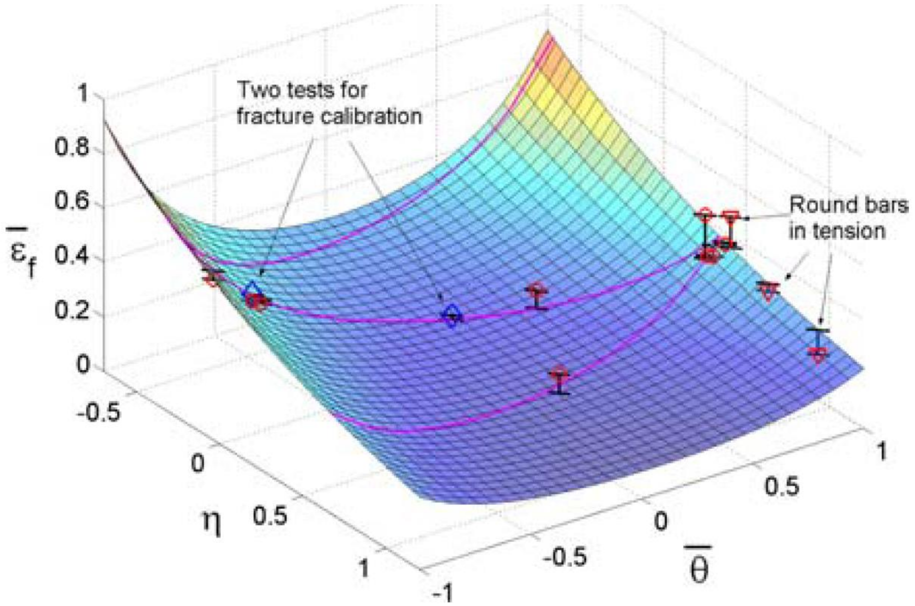
$$\bar{\theta} = 1 - \frac{6\theta}{\pi}$$

$$p' = \frac{\text{tr}(\boldsymbol{\sigma})}{3}$$

$$\sigma_{\text{eq}} = \sqrt{\frac{3}{2}\text{dev}(\boldsymbol{\sigma}) : \text{dev}(\boldsymbol{\sigma})}$$

$$J_3 = \det(\text{dev}(\boldsymbol{\sigma}))$$

$$\theta = \frac{1}{3}\arccos\left(\frac{27J_3}{2\sigma_{\text{eq}}^3}\right)$$



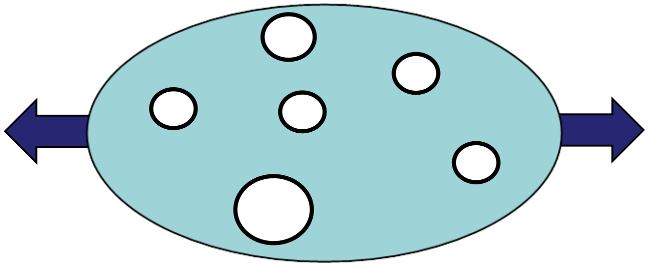
Fracture locus for 2024-T351 aluminum alloy

(Bai & Wierzbicki 2010)

- Existing modeling strategy
  - Gurson - Tvergaard - Needleman model (GTN model)
    (Gurson 1977, Tvergaard & Needleman 1984; Needleman & Tvergaard 1984)
    - Phenomenological acceleration of void growth rate when porosity is greater than a critical value
    - Enriched by the phenomenological shear enhancement to account for the failure under shear-dominated loadings
      (Nahshon and Hutchinson 2008)
  - Two-surface model
    (Pardoen & Hutchinson 2000; Benzerga 2002; Besson 2009, …)
    - Based on two solutions for the expansion of a void in an elastoplastic material:
      - Void growth solution: diffuse plasticity in the matrix around the voids
        - » Gurson-like models (GTN, …)
      - Void coalescence solution: localized plasticity in the ligament between the growing voids
        - » Yield surface extended from the Thomason criterion (Thomason 1985)
    - Effect of the Lode parameter is not considered.
  - …

- Objective:
  - Develop a constitutive model able to capture physical growth & coalescence phases

- Objective: to develop a constitutive model
  - Large strain formalism
  - Material degradation due to different stages of void expansion (nucleation, growth, and coalescence)
    - Damage indicators:
      - Porosity f_V (portion of voids)
      - Void ligament χ (min distance between two neighboring voids)

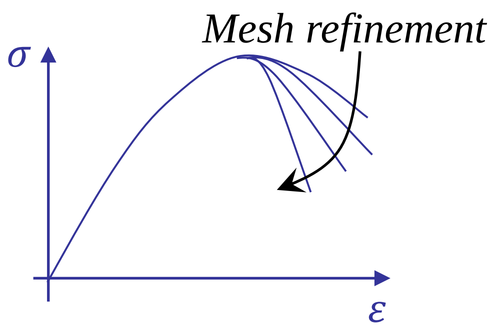


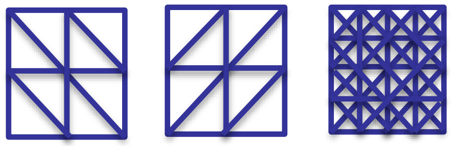
- Objective: to develop a constitutive model
  - Nonlocal form
    - Why?
      - Local forms suffer from mesh-dependency
    - Implicit formulation (Peerlings et al. 1998)
      - Introduction of a characteristic length l_c
      - New non-local degrees of freedom $\bar{Z}$
      - New Helmholtz-type equation to be solved

$$\bar{Z} - Z - l_c^2 \Delta_0 \bar{Z} = 0$$

      - Damage indicators depend on the nonlocal variable
    - Multiple nonlocal variables can be considered
      - Damage indicators depend on *N* sources

$$\bar{Z}_k - Z_k - l_{ck}^2 \Delta_0 \bar{Z}_k = 0\,,\ \text{with } k = 1, \ldots, N$$





The numerical results change without convergence

- Objective: to develop a constitutive model
  - Hyperelastic-based formulation
  - Multi-surface model incorporates
    - Void growth phase
      - GTN model
    - Internal necking coalescence phase
      - Based on the Thomason criterion of coalescence
      - Driven by maximum principal stress
    - Shear driven coalescence phase
      - Based on the Thomason criterion of coalescence
      - Driven by maximum shear stress

The maximum principal stress & maximum shear stress are Lode-dependent!

Multi-surface nonlocal porous model

- Hyperelastic-based formulation
  - Multiplicative decomposition

$$\mathbf{F} = \mathbf{F}^{\mathrm{e}} \cdot \mathbf{F}^{\mathrm{p}}$$

$$J = \det(\mathbf{F})$$

$$\mathbf{C}^{\mathrm{e}} = \mathbf{F}^{\mathrm{e}T} \cdot \mathbf{F}^{\mathrm{e}}$$

$$\mathbf{E}^{\mathrm{e}} = \ln \sqrt{\mathbf{C}^{\mathrm{e}}}$$

  - Stress definition
    - Elastic potential with logarithmic strain measure

$$\psi^e = \frac{K}{2}\left[\mathrm{tr}\left(\mathbf{E}^{\mathrm{e}}\right)\right]^2 + G\,\mathrm{dev}\left(\mathbf{E}^{\mathrm{e}}\right) : \mathrm{dev}\left(\mathbf{E}^{\mathrm{e}}\right)$$

    - Kirchhoff stress tensors
      - In corotational space

$$\boldsymbol{\tau} = \frac{\partial \psi^e}{\partial \mathbf{E}^{\mathrm{e}}} = \mathbb{C} : \mathbf{E}^{\mathrm{e}}$$

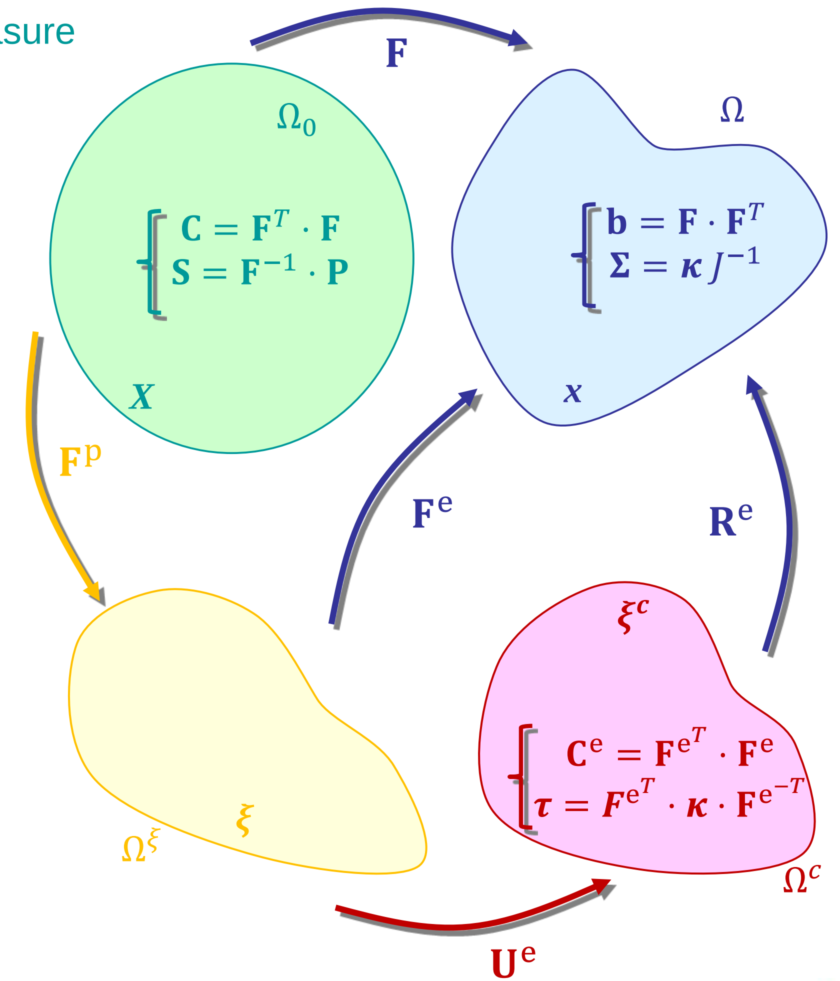
      - In current space:

$$\boldsymbol{\kappa} = \mathbf{F}^{\mathrm{e}-T} \cdot \boldsymbol{\tau} \cdot \mathbf{F}^{\mathrm{e}T}$$

    - First Piola-Kirchhoff stress

$$\mathbf{P} = \boldsymbol{\kappa} \cdot \mathbf{F}^{-T}$$

    - Cauchy stress
      - In corotational space $\boldsymbol{\sigma} = \frac{\boldsymbol{\tau}}{J}$
      - In current space $\boldsymbol{\Sigma} = \frac{\boldsymbol{\kappa}}{J}$



- Hyperelastic-based formulation
  - Porous plasticity corotational approach
    - Yield condition

$$\Phi_{\mathrm{nl}} = \Phi_{\mathrm{nl}}\left(\boldsymbol{\sigma}; \sigma_{\mathrm{Y}}, \mathbf{Y}\right) = 0$$

    - Plastic flow

$$\mathbf{D}^{\mathrm{p}} = \dot{\mathbf{F}}^{\mathrm{p}} \cdot \mathbf{F}^{\mathrm{p}-1} = \dot{\mu} \frac{\partial \Phi_{\mathrm{nl}}}{\partial \boldsymbol{\sigma}}$$

    - Evolution laws
      - Equivalent matrix plastic strain rate:

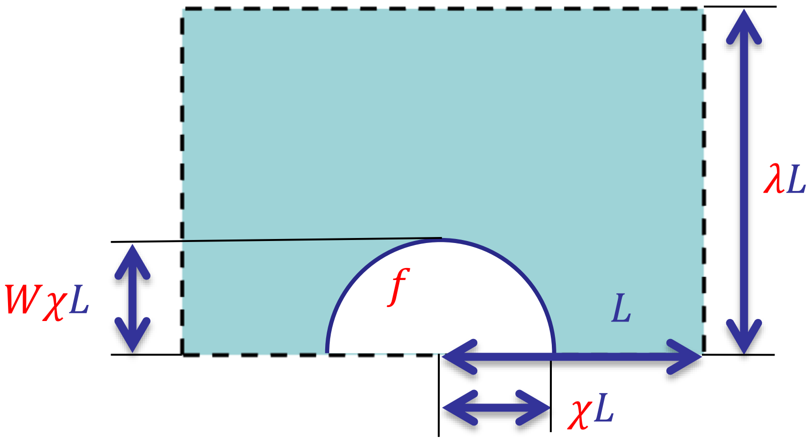
$$\dot{\varepsilon}_{\mathrm{m}} = \frac{\boldsymbol{\sigma} : \mathbf{D}^{\mathrm{p}}}{\left(1 - f\right) \sigma_{\mathrm{Y}}}$$

      - Isotropic hardening law:

$$\sigma_{\mathrm{Y}} = \sigma_{\mathrm{Y}}^{0} + R\left(\varepsilon_{\mathrm{m}}\right)$$

      - Evolution laws for void characteristics $\mathbf{Y}$

$$\mathbf{Y} = \left[f \quad \chi \quad W \quad \lambda\right]^{T}$$



Void characteristics:

Porosity : f
Void ligament ratio: χ
Void aspect ratio: W
Void spacing ratio: λ

Φ_{nl} and evolution laws for $\mathbf{Y}$ depend on the void expansion solution: (void growth, internal necking coalescence, or shear driven coalescence)

Multi-surface nonlocal porous model

- Hyperelastic-based formulation
  - Nonlocal void characteristics
    - Volumetric & deviatoric equivalent plastic strain

$$\varepsilon_{\mathrm{v}} = \mathrm{tr}\left(\mathbf{D}^{\mathrm{p}}\right) \qquad \varepsilon_{\mathrm{d}} = \sqrt{\frac{2}{3}\mathrm{dev}\left(\mathbf{D}^{\mathrm{p}}\right) : \mathrm{dev}\left(\mathbf{D}^{\mathrm{p}}\right)}$$

    - Nonlocal plastic state by implicit formulation
      - Nonlocality of the volumetric plastic deformation

$$\varepsilon_{\mathrm{v}} - \bar{\varepsilon}_{\mathrm{v}} - l^2_{\varepsilon_{\mathrm{v}}}\Delta_0\bar{\varepsilon}_{\mathrm{v}} = 0$$

      - Nonlocality of the deviatoric plastic deformation

$$\varepsilon_{\mathrm{d}} - \bar{\varepsilon}_{\mathrm{d}} - l^2_{\varepsilon_{\mathrm{d}}}\Delta_0\bar{\varepsilon}_{\mathrm{d}} = 0$$

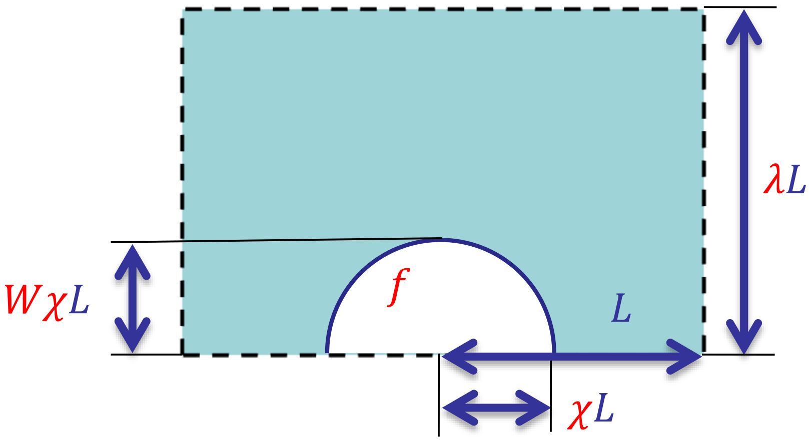
      - Nonlocality of the matrix plastic deformation

$$\varepsilon_{\mathrm{m}} - \bar{\varepsilon}_{\mathrm{m}} - l^2_{\varepsilon_{\mathrm{m}}}\Delta_0\varepsilon_{\mathrm{m}} = 0$$

    - Evolution laws for void characteristics

$$\mathbf{Y} = \mathbf{Y}\left(\varepsilon_{\mathrm{m}}, \varepsilon_{\mathrm{v}}, \varepsilon_{\mathrm{d}}, \bar{\varepsilon}_{\mathrm{m}}, \bar{\varepsilon}_{\mathrm{v}}, \bar{\varepsilon}_{\mathrm{d}}, \boldsymbol{\sigma}\right)$$

$$\mathbf{Y} = \left[f \quad \chi \quad W \quad \lambda\right]^T$$



Void characteristics:

Porosity : f
Void ligament ratio: χ
Void aspect ratio: W
Void spacing ratio: λ

Multi-surface nonlocal porous model

- Void growth phase – GTN model
  - Yield condition $\Phi_{\mathrm{nl}} = \Phi_{\mathrm{G}} = \dfrac{\hat{\sigma}_{\mathrm{G}}}{\sigma_{\mathrm{Y}}} - 1 = 0$
    - GTN effective stress

$$\hat{\sigma}_{\mathrm{G}}\left(\sigma_{\mathrm{eq}}, p', \sigma_{\mathrm{Y}}, f\right) = \frac{\sqrt{\sigma_{\mathrm{eq}}^2 + 2\sigma_{\mathrm{Y}}^2 f q_1 \left[\cosh\left(\frac{3}{2} q_2 \frac{p'}{\sigma_{\mathrm{Y}}}\right) - 1\right]}}{1 - q_1 f}$$

      - Parameters: q_1 and q_2

  - Evolution laws for void characteristics (spheroidal)
    - Porosity: $\dot{f} = \dot{f}_{\mathrm{gr}} + \dot{f}_{\mathrm{nu}} + \dot{f}_{\mathrm{sh}}$
      - Growth part
        - » Local form: $\dot{f}_{\mathrm{gr}} = (1 - f)\,\mathrm{tr}\left(\mathbf{D}^{\mathrm{p}}\right)$
        - → Nonlocal form: $\dot{f}_{\mathrm{gr}} = (1 - f)\,\dot{\bar{\varepsilon}}_{\mathrm{v}}$
      - Nucleation part
        - » Local form: $\dot{f}_{\mathrm{nu}} = A_n\left(\varepsilon_{\mathrm{m}}\right)\dot{\varepsilon}_{\mathrm{m}}$
        - → Nonlocal form: $\dot{f}_{\mathrm{nu}} = A_n\left(\bar{\varepsilon}_{\mathrm{m}}\right)\dot{\bar{\varepsilon}}_{\mathrm{m}}$
      - Shear part (Nahshon and Hutchinson 2008):
        - » Local form: $\dot{f}_{\mathrm{sh}} = k_\omega\left(1 - \omega^2\right) f \dfrac{\mathrm{dev}\left(\boldsymbol{\sigma}\right) : \mathbf{D}^{\mathrm{p}}}{\sigma_{\mathrm{eq}}}$
        - → Nonlocal form: $\dot{f}_{\mathrm{sh}} = k_\omega\left(1 - \omega^2\right) f \dot{\bar{\varepsilon}}_{\mathrm{d}}$

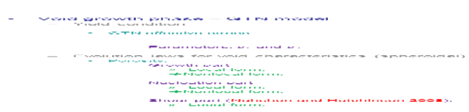
$$p' = \frac{\mathrm{tr}\left(\boldsymbol{\sigma}\right)}{3}$$

$$\sigma_{\mathrm{eq}} = \sqrt{\frac{3}{2}\mathrm{dev}\left(\boldsymbol{\sigma}\right) : \mathrm{dev}\left(\boldsymbol{\sigma}\right)}$$

$$J_3 = \det\left(\mathrm{dev}\left(\boldsymbol{\sigma}\right)\right)$$

$$\omega = \frac{27 J_3}{2{\sigma_{\mathrm{eq}}}^3}\,.$$

$$\varepsilon_{\mathrm{v}} = \mathrm{tr}\left(\mathbf{D}^{\mathrm{p}}\right)$$



$$\frac{\mathrm{dev}\left(\boldsymbol{\sigma}\right) : \mathbf{D}^{\mathrm{p}}}{\sigma_{\mathrm{eq}}} = \dot{\varepsilon}_{\mathrm{d}}$$

Multi-surface nonlocal porous model

- Void growth phase – GTN model
  - Yield condition $$\Phi_{\mathrm{nl}} = \Phi_{\mathrm{G}} = \frac{\hat{\sigma}_{\mathrm{G}}}{\sigma_{\mathrm{Y}}} - 1 = 0$$
    - Gurson effective stress

$$\hat{\sigma}_{\mathrm{G}}\left(\sigma_{\mathrm{eq}}, p', \sigma_{\mathrm{Y}}, f\right) = \frac{\sqrt{\sigma_{\mathrm{eq}}^2 + 2\sigma_{\mathrm{Y}}^2 f q_1 \left[\cosh\left(\frac{3}{2} q_2 \frac{p'}{\sigma_{\mathrm{Y}}}\right) - 1\right]}}{1 - q_1 f}$$

      - Parameters: q_1 and q_2

  - Evolution laws for void characteristics (spheroidal)
    - Porosity:

$$\dot{f} \quad = (1-f)\,\dot{\bar{\varepsilon}}_{\mathrm{v}} + A_n\left(\bar{\varepsilon}_{\mathrm{m}}\right)\dot{\bar{\varepsilon}}_{\mathrm{m}} + k_\omega\left(1-\omega^2\right) f \dot{\bar{\varepsilon}}_{\mathrm{d}}\,,$$

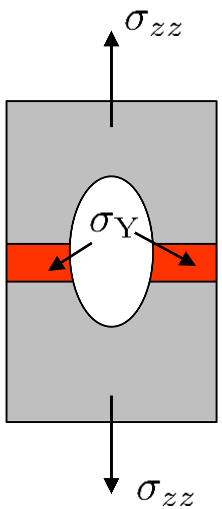
    - Other void characteristics

$$\dot{\lambda} = \kappa\lambda\dot{\bar{\varepsilon}}_{\mathrm{d}} \qquad \dot{W} = 0 \qquad \chi = \left(\frac{3f\lambda}{2W}\right)^{\frac{1}{3}}$$

      - Void distribution related parameter: κ
        - » Periodic distribution $\kappa = 1.5$,
        - » Random distribution $\kappa = 0$,
        - » Clustered distribution $0 < \kappa < 1.5$

(Benzerga et al. 2016)

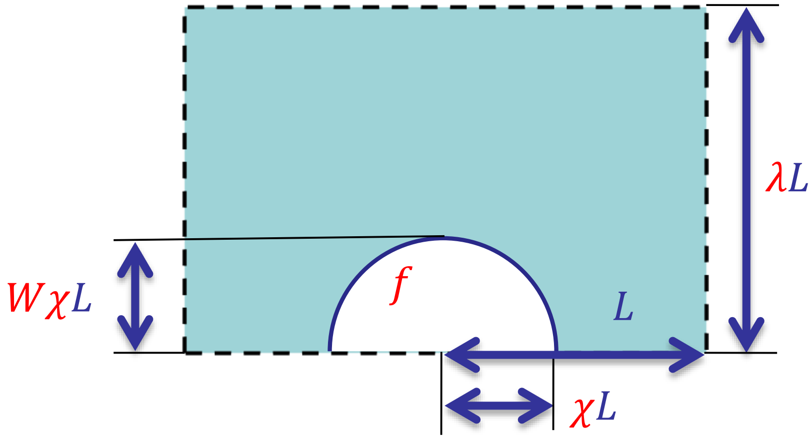
- Internal necking coalescence phase
  - Thomason condition for onset of coalescence
    - Localized plastic flow at ligament between neighboring voids
    - Limit load factor (Thomason 1985)



$$C_{\mathrm{Tf}}(\mathbf{Y}) = \frac{\sigma_{zz}}{\sigma_{\mathrm{Y}}} = (1-\chi^2)\left[h\left(\frac{1-\chi}{W\chi}\right)^2 + g\sqrt{\frac{1}{\chi}}\right]$$

  - Parameters: $g = 0.1, \ h = 1.24$ are generally adopted

$$\mathbf{Y} = [f \quad \chi \quad W \quad \lambda]^T$$



Void characteristics:

Porosity : f
Void ligament ratio: χ
Void aspect ratio: W
Void spacing ratio: λ

Multi-surface nonlocal porous model

- Internal necking coalescence phase
  - Yield surface
    - Plasticity inside void ligament remains in a limit load state
    - Driven by maximum principal stress (MPS)

$$\sigma_{zz} = \max \mathrm{eig}\left(\boldsymbol{\sigma}\right)$$

    - MPS-based Thomason yield surface

$$\Phi_{\mathrm{nl}} = \Phi_{\mathrm{T}} = \frac{\hat{\sigma}_{\mathrm{T}}}{\sigma_{\mathrm{Y}}} - 1 = 0$$

      - Thomason effective stress

$$\hat{\sigma}_{\mathrm{T}} = \frac{1}{C_{\mathrm{Tf}}} \left(\frac{2}{3} \sigma_{\mathrm{eq}} \cos\theta + |p'| \right)$$

  - Evolution laws for void characteristics

$$\begin{cases} \dot{\chi} & = \frac{3}{4}\frac{\lambda}{W}\left(\frac{3}{2\chi^2} - 1\right)\dot{\bar{\varepsilon}}_{\mathrm{d}} \,, \\ \dot{W} & = \frac{9}{4}\frac{\lambda}{\chi}\left(1 - \frac{1}{2\chi^2}\right)\dot{\bar{\varepsilon}}_{\mathrm{d}} \,, \\ \dot{\lambda} & = \kappa\lambda\dot{\bar{\varepsilon}}_{\mathrm{d}} \,, \end{cases}$$

$$f = \frac{2\chi^3 W}{3\lambda}$$

(Benzerga 2002)

$$p' = \frac{\mathrm{tr}\left(\boldsymbol{\sigma}\right)}{3}$$

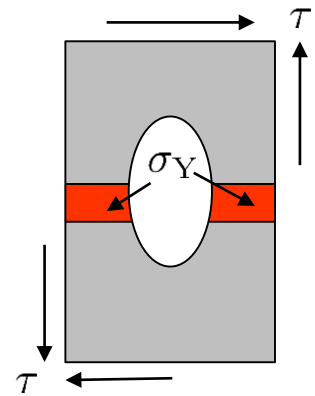
$$\sigma_{\mathrm{eq}} = \sqrt{\frac{3}{2}\mathrm{dev}\left(\boldsymbol{\sigma}\right) : \mathrm{dev}\left(\boldsymbol{\sigma}\right)}$$

$$\omega = \frac{27 J_3}{2{\sigma_{\mathrm{eq}}}^3} \,.$$

$$\theta\left(\sigma_{\mathrm{eq}}, J_3\right) = \frac{1}{3}\arccos\omega$$

$$\max(\mathrm{eig}(\boldsymbol{\sigma})) = \frac{2}{3}\sigma_{\mathrm{eq}}\cos\theta + p'$$

- Shear driven coalescence phase
  - Onset condition
    - Thomason-like condition
    - Limit load factor



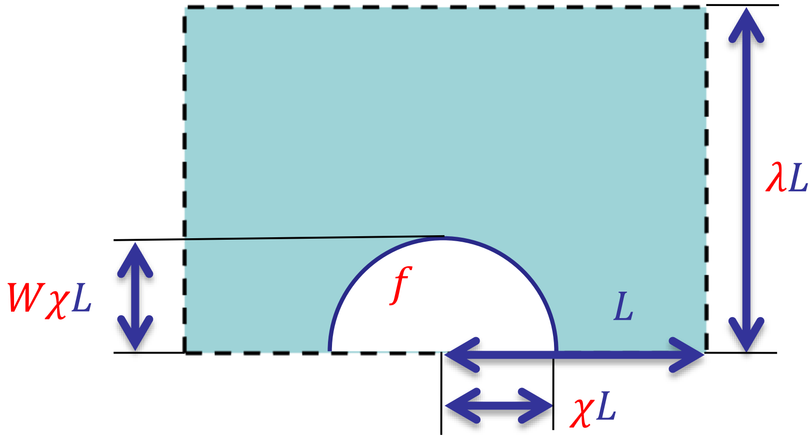
$$C_{\mathrm{Sf}}\left(\mathbf{Y}\right) = \frac{\sqrt{3}\tau}{\sigma_{\mathrm{Y}}} = \xi\left(1-\chi^2\right)$$

– Parameter ξ

» Theoretically $\xi = 1$ if σ_Y is uniformly distributed inside localization band.

» But $\xi > 1$ is used as the real distribution is not perfect.

$$\mathbf{Y} = \left[f \quad \chi \quad W \quad \lambda\right]^T$$



Void characteristics:

Porosity : f
Void ligament ratio: χ
Void aspect ratio: W
Void spacing ratio: λ

- Shear driven coalescence phase
  - Yield condition
    - Driven by maximal shear stress

$$\tau = \frac{\max \text{eig}(\boldsymbol{\sigma}) - \min \text{eig}(\boldsymbol{\sigma})}{2}$$

    - MSS-based yield condition

$$\Phi_{\text{nl}} = \Phi_{\text{S}} = \frac{\hat{\sigma}_{\text{S}}}{\sigma_{\text{Y}}} - 1 = 0$$

      - Effective shear stress

$$\hat{\sigma}_{\text{S}} = \frac{\sqrt{3}\tau}{C_{\text{Sf}}} = \frac{\sigma_{\text{eq}}}{C_{\text{Sf}}} \left(\frac{\sin\theta}{2} + \frac{\sqrt{3}\cos\theta}{2} \right)$$

  - Evolution laws for void characteristics

$$\begin{cases} \dot{\chi} & = K_{\chi} \dot{\bar{\varepsilon}}_{\text{d}} \,, \\ \dot{W} & = 0 \,, \\ \dot{f} & = 0 \end{cases} \qquad \lambda = \frac{2\chi^3 W}{3f}$$

$$\sigma_{\text{eq}} = \sqrt{\frac{3}{2} \text{dev}(\boldsymbol{\sigma}) : \text{dev}(\boldsymbol{\sigma})}$$

$$\omega = \frac{27 J_3}{2{\sigma_{\text{eq}}}^3} \,.$$

$$\theta(\sigma_{\text{eq}}, J_3) = \frac{1}{3} \arccos \omega$$

$$\max(\text{eig}(\boldsymbol{\sigma})) = \frac{2}{3} \sigma_{\text{eq}} \cos\theta + p'$$

$$\min(\text{eig}(\boldsymbol{\sigma})) = \frac{2}{3} \sigma_{\text{eq}} \cos\left(\frac{2\pi}{3} + \theta \right) + p'$$

- Shear driven coalescence phase
  - The meaning of ξ
    - Failure under pure shear ($p' = 0$ and $\theta = \frac{\pi}{6}$)
      - Void growth solution
        - » Yield condition $\Phi_{\mathrm{G}} = \frac{\hat{\sigma}_{\mathrm{G}}}{\sigma_{\mathrm{Y}}} - 1 = \frac{\sigma_{\mathrm{eq}}}{\sigma_{\mathrm{Y}}\left(1 - q_1 f\right)} - 1 = 0$
        - » Void characteristics

$$\begin{cases} \dot{f} = k_\omega f \dot{\bar{\varepsilon}}_{\mathrm{d}} \\ \dot{\lambda} = \kappa \lambda \dot{\bar{\varepsilon}}_{\mathrm{d}} \\ \dot{W} = 0 \\ \chi = \left(\frac{3 f \lambda}{2W}\right)^{\frac{1}{3}} \end{cases} \Rightarrow \begin{cases} f = f_0 \exp\left(k_\omega \bar{\varepsilon}_{\mathrm{d}}\right) \\ \lambda = \lambda_0 \exp\left(\kappa \bar{\varepsilon}_{\mathrm{d}}\right) \\ W = W_0 \\ \chi = \chi_0 \exp\left[\frac{2}{3}\left(\kappa + k_\omega\right)\bar{\varepsilon}_{\mathrm{d}}\right] \end{cases}$$

      - Shear driven coalescence onset $\Phi_{\mathrm{G}} = \Phi_{\mathrm{S}} = \frac{\hat{\sigma}_{\mathrm{S}}}{\sigma_{\mathrm{Y}}} - 1 = \frac{\sigma_{\mathrm{eq}}}{\sigma_{\mathrm{Y}}\xi\left(1 - \chi^2\right)} - 1 = 0$
        - » Estimation of ξ

$$\xi = \frac{1 - q_1 f_0 \exp\left(k_\omega \varepsilon_{\mathrm{d}s}\right)}{1 - \chi_0^2 \exp\left[\frac{2}{3}\left(\kappa + k_\omega\right)\varepsilon_{\mathrm{d}s}\right]}$$

          - » $\varepsilon_{\mathrm{d}s}$ is the onset of failure under a pure shear loading condition.
          - » $\varepsilon_{\mathrm{d}s}$ and other parameters allows estimating ξ,
          - » As $\varepsilon_{\mathrm{d}s} > 0$ in general, one has $\xi > 1$

Multi-surface nonlocal porous model

- Competition between different modes
  - Yield surface

$$\Phi_{\mathrm{e}} = \frac{\hat{\sigma}}{\sigma_{\mathrm{Y}}} - 1 = 0$$

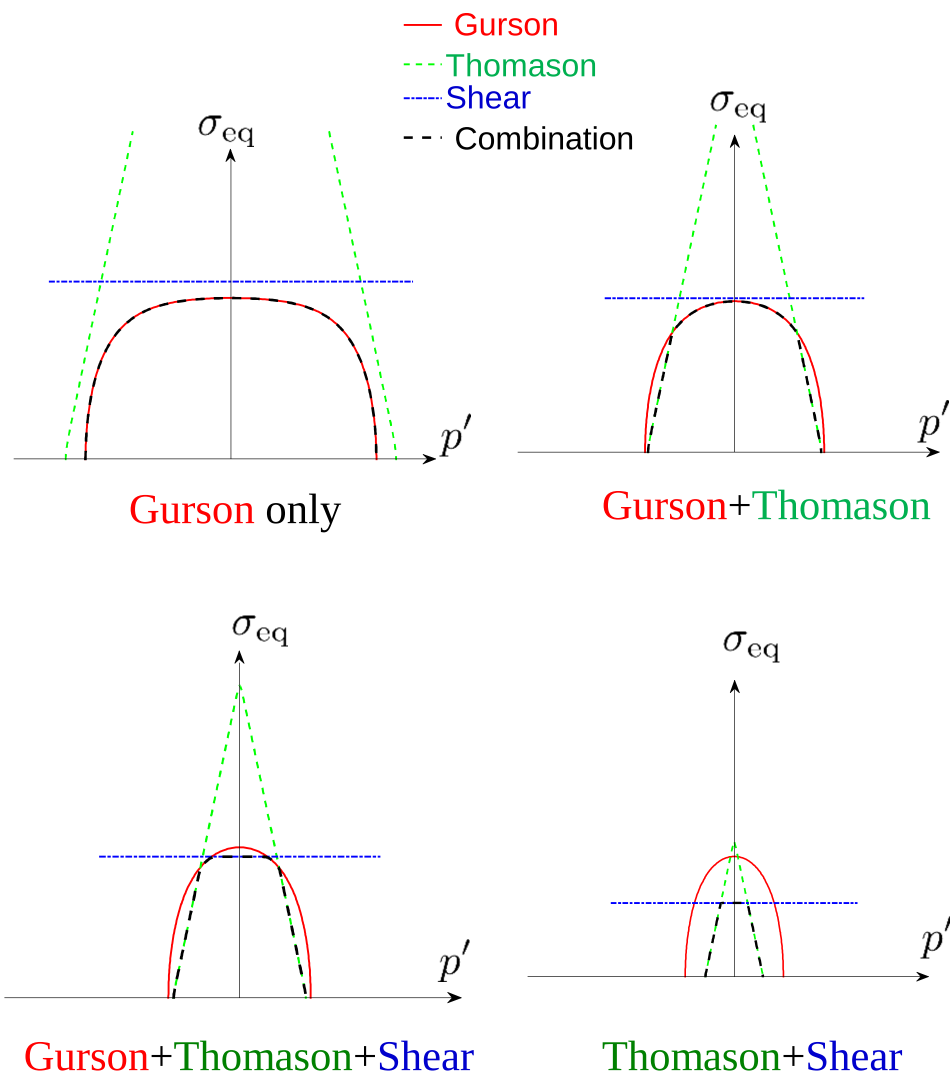
- Effective stress

$$\hat{\sigma} = \max\left(\hat{\sigma}_{\mathrm{G}}, \hat{\sigma}_{\mathrm{T}}, \hat{\sigma}_{\mathrm{S}}\right)$$

- Approximated form

$$\hat{\sigma} = \left(\hat{\sigma}_{\mathrm{G}}^{m} + \hat{\sigma}_{\mathrm{T}}^{m} + \hat{\sigma}_{\mathrm{S}}^{m}\right)^{\frac{1}{m}}$$

  - To avoid singularity of the yield normal at the intersections between different surfaces
  - User-defined parameter $m \gg 1$



  - Onset of void necking coalescence

$$\dot{\varepsilon}_{\mathrm{m}} > 0\,, \text{ and } \hat{\sigma}_T > \max\left(\hat{\sigma}_G, \hat{\sigma}_S\right)$$

  - Onset of void shear coalescence

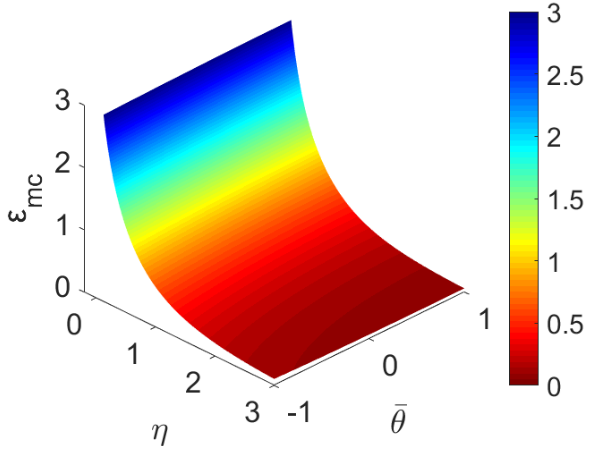
$$\dot{\varepsilon}_{\mathrm{m}} > 0\,, \text{ and } \text{ and } \hat{\sigma}_S > \max\left(\hat{\sigma}_G, \hat{\sigma}_T\right)\,.$$

Multi-surface nonlocal porous model

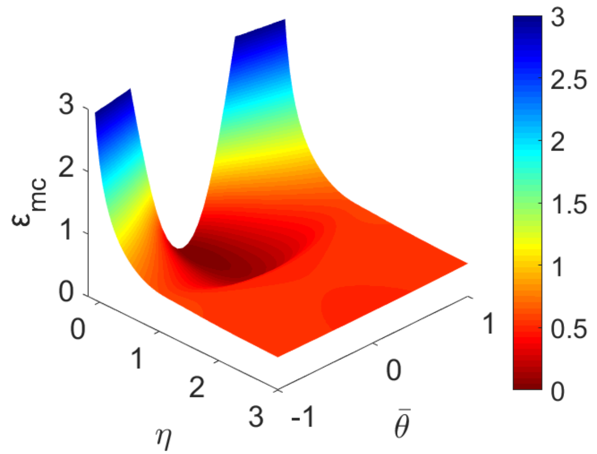
- Solution under proportional loadings
  - Constant stress triaxiality (η) and normalized Lode angle ($\bar{\theta}$)
  - $\varepsilon_{\mathrm{mc}}$- ductility = equivalent plastic deformation at the onset of coalescence

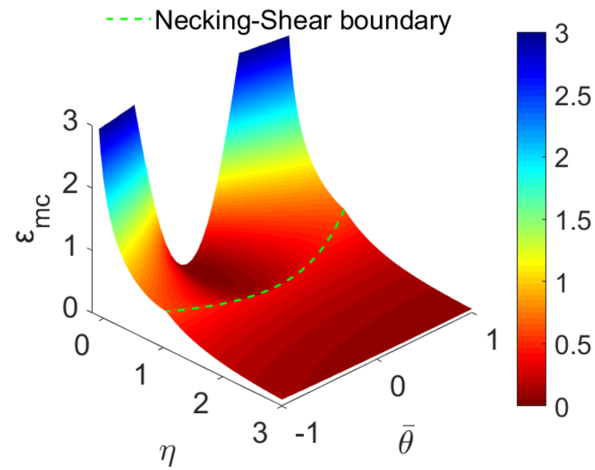
$$\bar{\theta} = 1 - \frac{6\theta}{\pi}$$

Internal necking coalescence model



Shear driven coalescence model





Multi-surface model

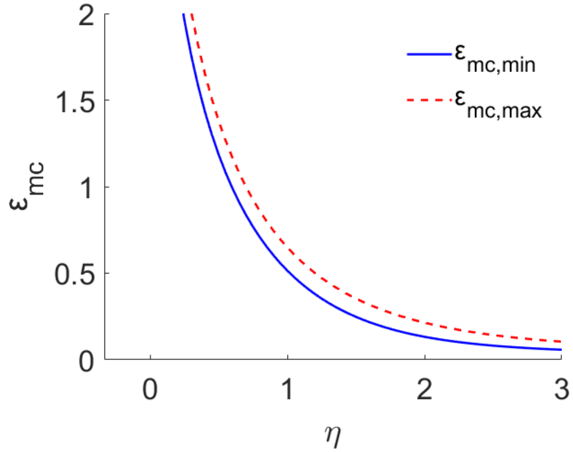
Multi-surface nonlocal porous model

- Solution under proportional loadings
  - Constant stress triaxiality (η) and normalized Lode angle ($\bar{\theta}$)
    - ε_{mc}- upper-bound and lower-bound

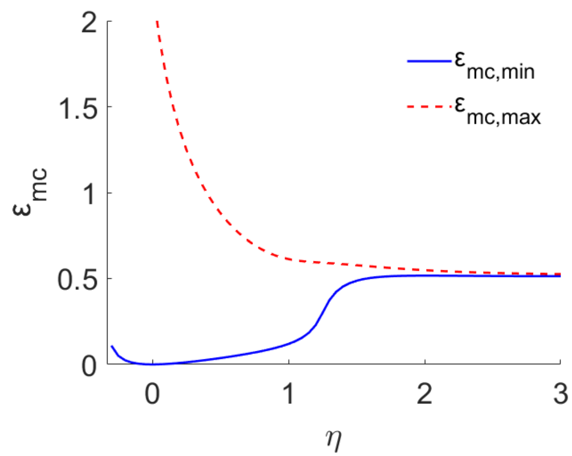
$$\varepsilon_{mc,\min}(\eta) \leq \varepsilon_{mc}(\eta, \bar{\theta}) \leq \varepsilon_{mc,\max}(\eta)$$

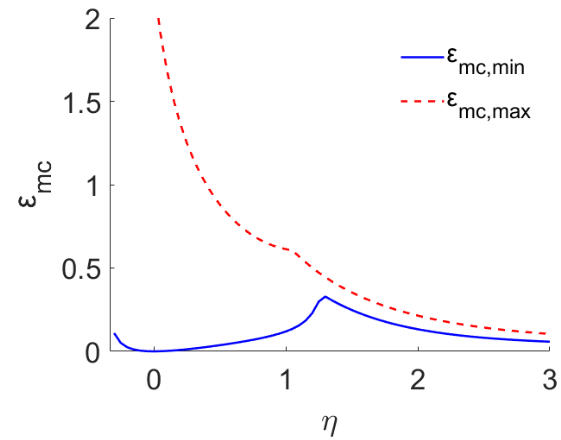
$$\begin{aligned} \varepsilon_{mc,\min}(\eta) &= \min\left(\varepsilon_{mc}(\eta, \bar{\theta}), \bar{\theta} \in [-1\ 1]\right) \\ \varepsilon_{mc,\max}(\eta) &= \max\left(\varepsilon_{mc}(\eta, \bar{\theta}), \bar{\theta} \in [-1\ 1]\right) \end{aligned}$$

Internal necking:
min: $\bar{\theta} = 1$
Max: $\bar{\theta} = -1$



Shear:
min: $\bar{\theta} = 0$
max: $\bar{\theta} = \pm 1$

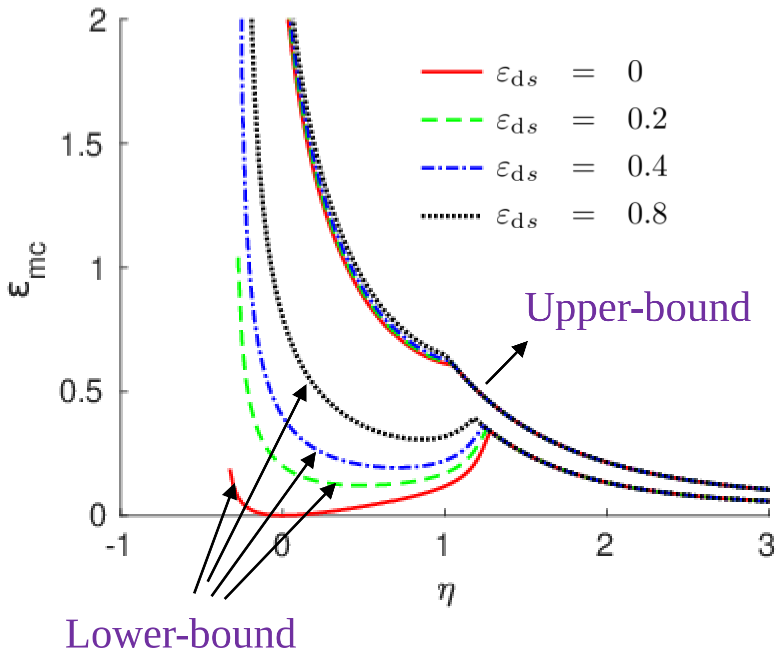




Multi-surface

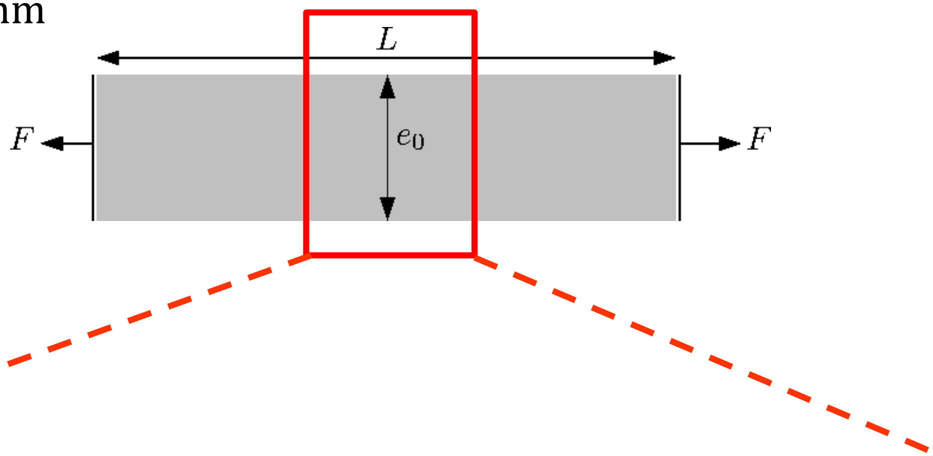
Multi-surface nonlocal porous model

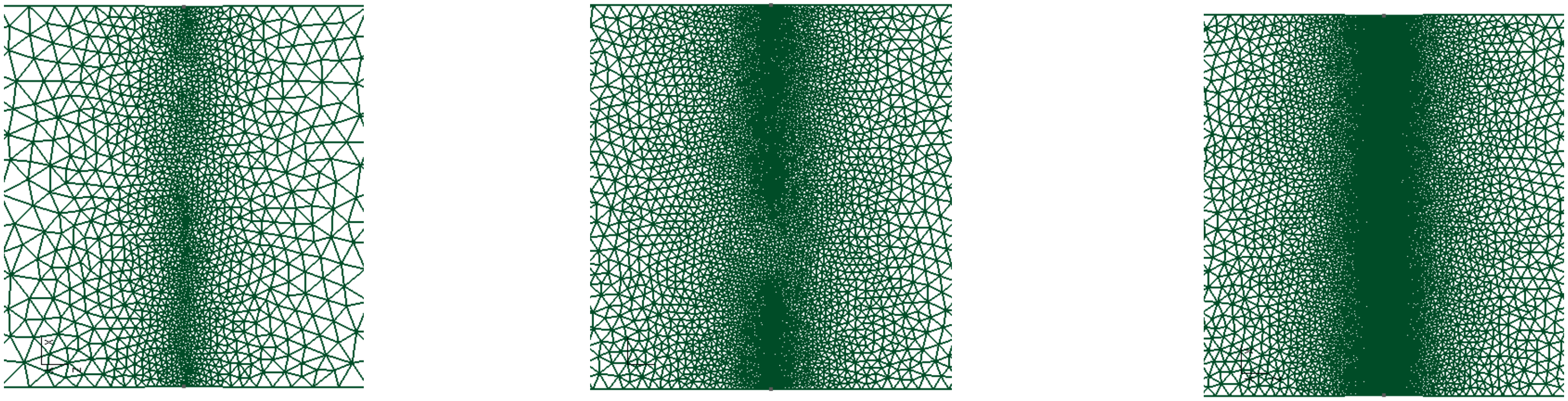
- Solution under proportional loadings
  - Constant stress triaxiality (η) and normalized Lode angle ($\bar{\theta}$)
    - $\varepsilon_{\mathrm{mc}}$- upper-bound and lower-bound
    - Influence of $\varepsilon_{\mathrm{d}s}$ → ξ



Numerical examples

- Plane strain smooth specimen under tensile loading
  - Verification of the nonlocal model
    - $L = 12.5$ mm $e_0 = 3$ mm

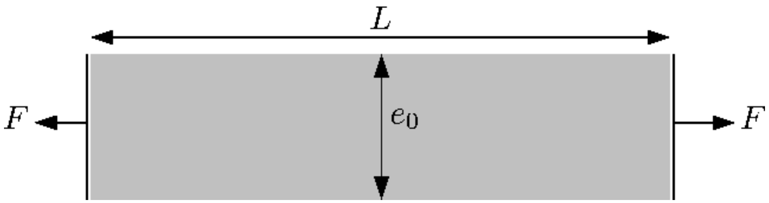


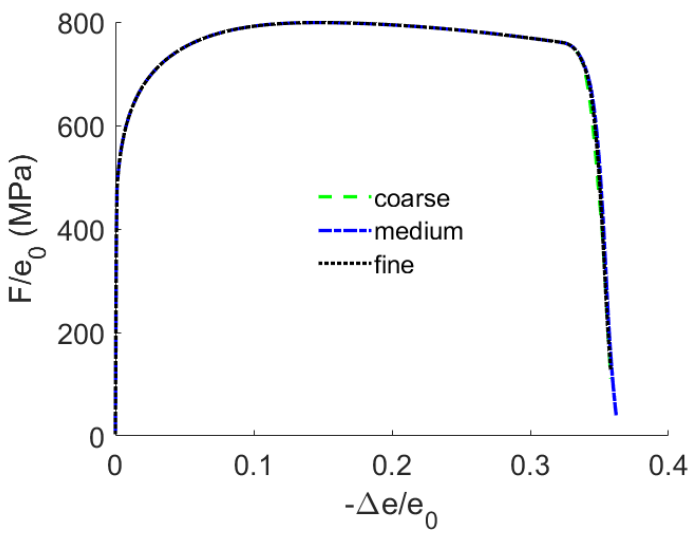


Coarse ~3250 elements *Medium ~9100 elements* *Fine ~17800 elements*

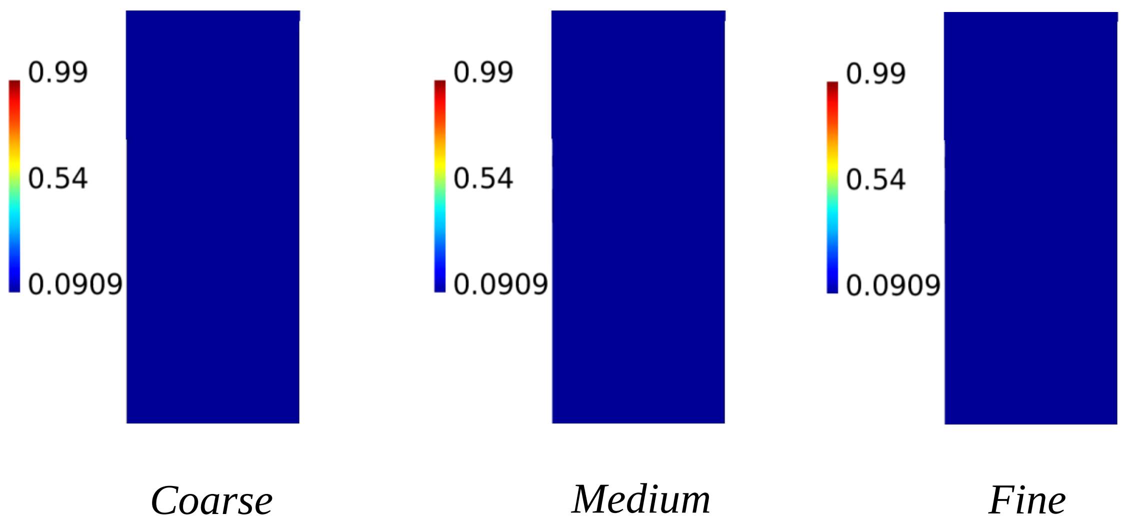
- Plane strain smooth specimen under tensile loading
  - Verification of the nonlocal model
    - $L = 12.5$ mm $e_0 = 3$ mm
    - Use $\xi = 1.015 \ \ (\varepsilon_{ds} = 0.95)$
  - Effect of mesh size

→capture slant failure



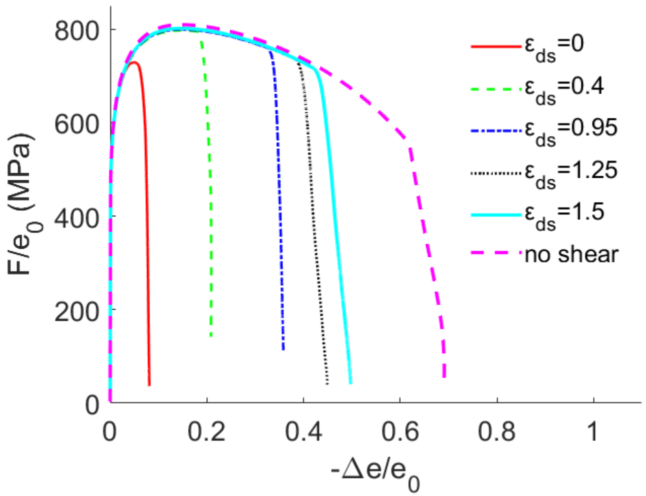


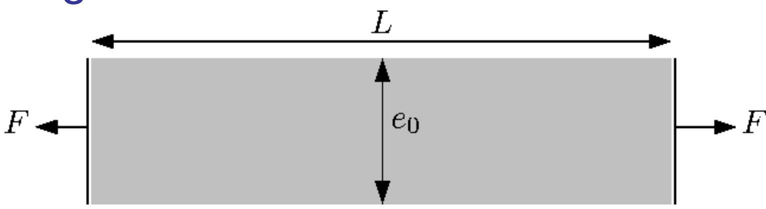
Distribution of void ligament ratio



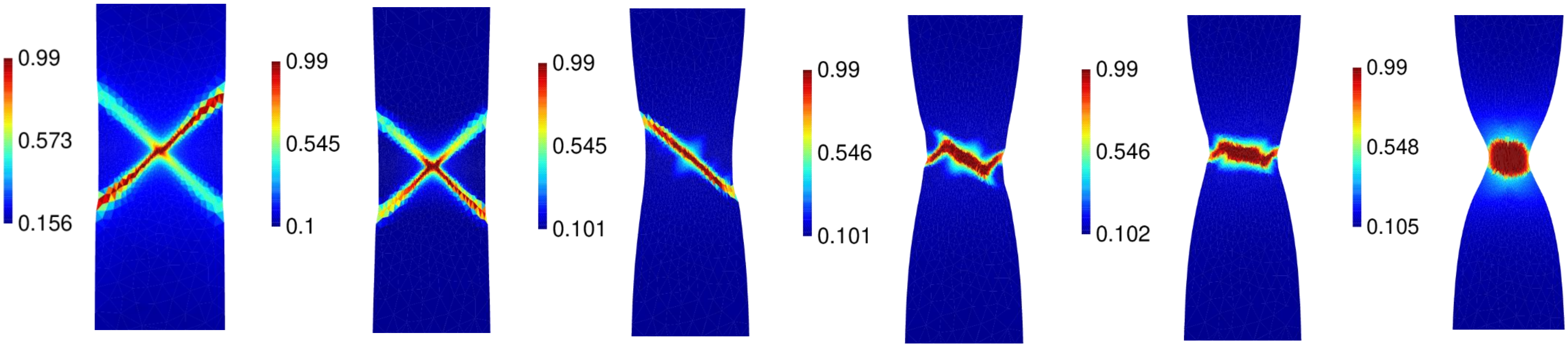
Numerical examples

- Plane strain smooth specimen under tensile loading
  - Effect of ξ





Distribution of void ligament ratio

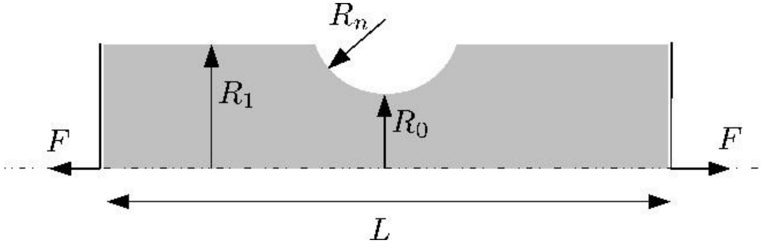


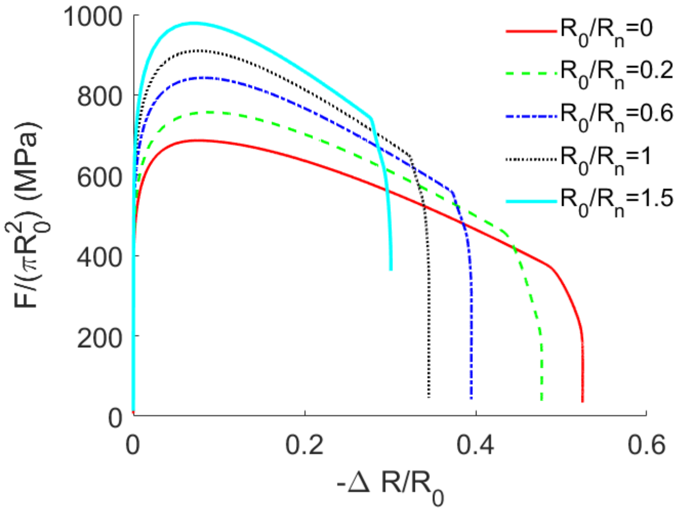
$\varepsilon_{ds} = 0$ | $\varepsilon_{ds} = 0.4$ | $\varepsilon_{ds} = 0.95$ | $\varepsilon_{ds} = 1.25$ | $\varepsilon_{ds} = 1.5$ | no shear

Numerical examples

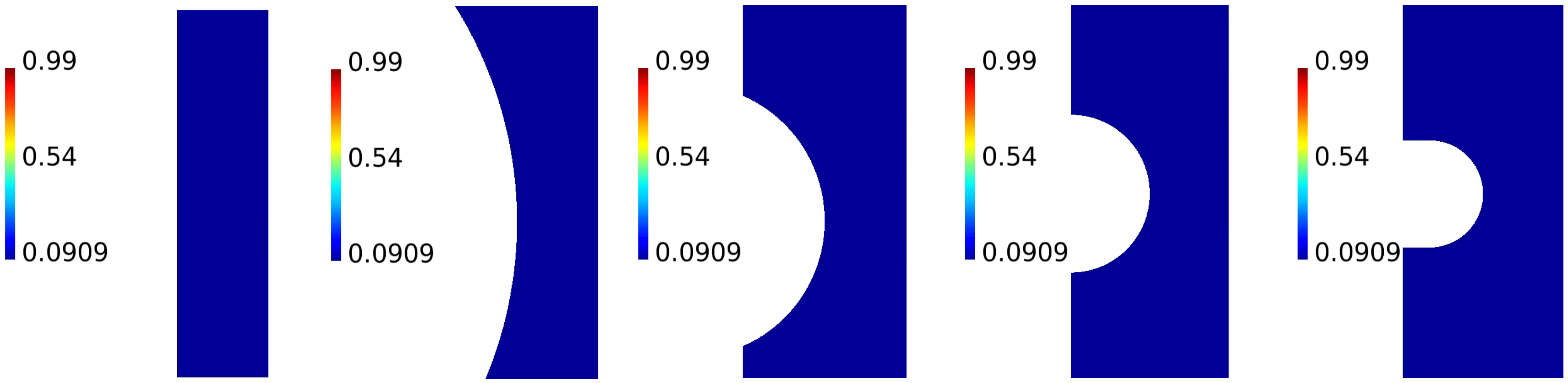
- Axisymmetric specimens under tensile loading
  - $R_0 = 3\ mm, R_1 = 6\ mm, L = 25\ mm$
  - Different notched radius: $R_0/R_n = 0, 0.2, 0.6, 1, 1.5$
  - Using $\xi = 1.015\ \ (\varepsilon_{\mathrm{d}s} = 0.95)$

→capture cup-cone failure





Distribution of void ligament ratio



Conclusion

- Objective
  - Simulation of ductile failure incorporating void growth & coalescence deformation modes
- Methodology
  - Nonlocal porous plasticity
  - Multi-surface model incorporating void growth, internal necking coalescence, and shear driven coalescence
- Results
  - The proposed framework is able to model
    - The slant fracture mode in plane strain smooth specimens
    - The cup-cone fracture mode axisymmetric smooth & notched specimens
- Upcoming tasks
  - Validation/Calibration with literature/experimental tests

Thank you for your attention

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Predictor-corrector scheme

- Elastic predictor

$$\mathbf{F}^{\mathrm{p\,pr}} = \mathbf{F}^{\mathrm{p}}_n \qquad \mathbf{F}^{\mathrm{e\,pr}} = \mathbf{F} \cdot \mathbf{F}^{\mathrm{p\,pr}-1}$$

- Plastic corrector (fully implicit radial return)

$$\begin{cases} \boldsymbol{\tau} = \boldsymbol{\tau}^{\mathrm{pr}} - \mathbb{C} : \Delta \mathbf{E}^{p} \,, \\ \boldsymbol{\sigma} = J^{-1} \boldsymbol{\tau} \,, \\ \sigma_{\mathrm{Y}} = \sigma_{\mathrm{Y}} \left(\varepsilon_{\mathrm{m}n} + \Delta \varepsilon_{\mathrm{m}} \right) \,, \\ \mathbf{Y} = \mathbf{Y}_n + \Delta \mathbf{Y} \left(\Delta \bar{\mathbf{Z}}, \boldsymbol{\sigma} \right) \,, \\ \Phi_{\mathrm{nl}} \left(\boldsymbol{\sigma}; \sigma_{\mathrm{Y}}, \mathbf{Y} \right) = 0 \,, \\ \Delta \mathbf{E}^{p} - \Delta \mu \mathbf{N}^{\mathrm{p}} \left(\boldsymbol{\sigma}; \sigma_{\mathrm{Y}}, \mathbf{Y} \right) = \mathbf{0} \,, \text{ and} \\ \boldsymbol{\sigma} : \Delta \mathbf{E}^{p} - (1 - f) \, \sigma_{\mathrm{Y}} \Delta \varepsilon_{\mathrm{m}} = 0 \,. \end{cases}$$

Unknowns: $\boldsymbol{\tau}$, $\boldsymbol{\sigma}$, σ_{Y}, $\Delta \varepsilon_{\mathrm{m}}$, $\mathbf{Y}$, $\Delta \mathbf{E}^{p}$, and $\Delta \mu$