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JOINT FLEXIBILITY EFFECTS ON THE BEHAVIOR OF STEEL PLANE FRAMES

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ABSTRACT

The flexibility of most connections behaves between the conventional perfectly rigid and pinned one but their real behavior lies between these two extremes. The effects of connection flexibility are, then, to be incorporated in the analysis and design procedure of frames as its influence change significantly not only the moment distribution among beam and column but the structural displacement as well. A simple model that takes into account these effects is proposed and a corresponding stiffness matrix element is presented. The introduction of the concept of a non-deformable element of joint describing relative displacement and rotations between the nodes and the elements of the structure in the semi-rigid connections takes all its meaning. The influence of the semi-rigid joints on steel plane frames have been shown and compared in two illustrative examples. Numerical results of different frames of two stories and two bays are compared for various situations of the joint from bilinear to tri-linear behavior.

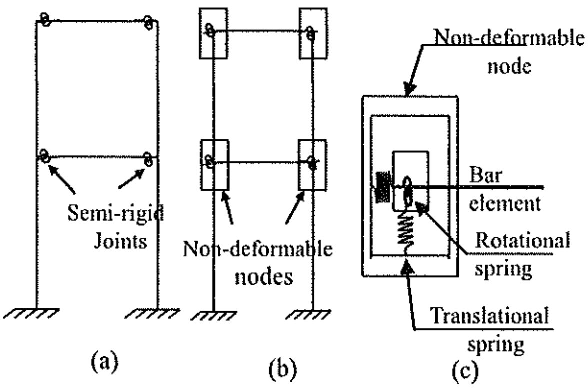
1 GENERAL INTRODUCTION

In the past, the two extreme idealizations of rigid and pinned connections have been widely used in the conventional structural steel analysis and design. However, the actual behaviour of a semi-rigid connection falls between these two situations as most connections used in common practice transmit some partial moment (Jones et al. 1983, Lui & Chen 1987, Bjorhovde et al 1990, Conception et al. 2011). Including this true behaviour of the joints in the analysis and design of steel structures changes not only the moment distribution among beams and columns (Gerstle 1988, Ihaddoudène et al. 2008) but also increases the structural displacements as well; and in particular increases the P-delta second order effects (Jaspart 1991, Ihaddoudène 2008, Ihaddoudène & Jaspart 2013).

The introduction of the concept of a non-deformable element of node describing relative displacements and rotations between the nodes and the elements of the structure in the semi-rigid connections takes all its meaning (Ihaddoudène 2008, Ihaddoudène et al. 2009).

2 MECHANICAL MODEL

To incorporate the effect of semi-rigid joints in the analyses of the steel frame shown in Figure 1a, the concept of a non-deformable node element describing relative displacements and rotations between the elements of the structure (see Fig. 1b) is introduced. The adopted model shown in Figure 1c is based on the use of three springs: two translational ones and rotational one (Ihaddoudène 2008, Ihaddoudène et al. 2009, Ihaddoudène & Jaspart 2013).



(a): Semi-rigid joints; (b): Non deformable node;
(c): Bar element and non-deformable node

Figure 1: Mechanical model adopted.

2.1 Stiffness matrix

The stiffness matrix that relates the element end forces to end displacements is formulated for all members with semi-rigid connections of the frame. To establish the modified stiffness matrix including both the effect of axial force and connection flexibility (Ihaddoudène &

Jaspart 2013), different situations are considered.

In the local reference, the stiffness matrix of a structural bar element is given by:

$$\overline{K_e} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \quad (1)$$

The nodes of the beam which are represented by non deformable frames at each ends have different flexibilities k_1 and k_2 at both ends i and j respectively.

2.2 Basic equations in buckling considerations

With the bar element of semi-rigid joints (Ihaddoudène 2008, Ihaddoudène & Jaspart 2013) subjected to both compression axial forces N and bending moments M_i and M_j at each ends as shown in Figure 2 below, the governing differential equation is:

$$EIy''(x) = -Ny - Hx + M_j \quad (2)$$

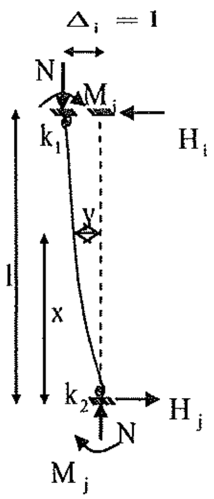


Figure 2: Bar element under unit displacement $\Delta_i = 1$

The reactions H, M_i and M_j are determined by using the boundary conditions of:

$$y'(0) = k_2 M_j \quad (3a)$$

$$y'(l) = k_1 M_i = k_1(Hl + N - M_j) \quad (3b)$$

Introducing

$$v = \alpha l \quad (3c)$$

Where

$$\alpha^2 = \frac{N}{EI} \quad (3d)$$

The solution of this system of equations is obtained as:

$$H = \frac{\omega}{l^2}\zeta_2(v) \quad (4a)$$

$$M_i = \frac{\omega}{l}\phi_4(v) \quad (4b)$$

$$M_j = \frac{\omega}{l}\phi_3(v) \quad (4c)$$

$\zeta_2(v)$, $\phi_3(v)$ and $\phi_4(v)$ which are the stability functions, presented for different type of supports.

The flexural rigidity per unit length

$$\omega = \frac{EI}{l} \quad (4d)$$

The k_{1j} expressions have been derived as:

$$k_{11} = \frac{EI}{l^3} \cdot \frac{v^3(\sin v + \Omega)}{D} \quad (5a)$$

$$k_{12} = -\frac{EI}{l^2} \cdot \frac{v^2(1 - \cos v + \eta_2)}{D} \quad (5b)$$

$$k_{13} = -\frac{EI}{l^3} \frac{v^3(\sin v + \Omega)}{D} \quad (5c)$$

$$k_{14} = \frac{EI}{l^2} \frac{v^2(1 - \cos v + \eta_1)}{D} \quad (5d)$$

Where:

$$\eta_i = k_i v w \sin v \quad (6a)$$

$$\Omega = (k_1 + k_2) v w \cos v - k_1 k_2 (vw)^2 \sin v \quad (6b)$$

$$D = (2 - 2\cos v - v \sin v) + \xi(v, k_1, k_2) \quad (6c)$$

2.3 Equations for semi-rigid joints aspects

Based on the expressions established earlier (Ihaddoudène 2008), one can report the formulas of element k_{1j} considering only semi-rigid connections and neglecting an axial force.

$$k_{11} = \frac{36\omega[1 + (k_1 + k_2)\omega]}{l^2[4(1 + 3k_1\omega)(1 + 3k_2\omega) - 1]} \quad (7a)$$

$$k_{12} = -\frac{18\omega(1 + 2k_2\omega)}{l[4(1 + 3k_1\omega)(1 + 3k_2\omega) - 1]} \quad (7b)$$

$$k_{13} = -\frac{36\omega[1 + (k_1 + k_2)\omega]}{l^2[4(1 + 3k_1\omega)(1 + 3k_2\omega) - 1]} \quad (7c)$$

$$k_{14} = -\frac{18\omega(1 + 2k_1\omega)}{l[4(1 + 3k_1\omega)(1 + 3k_2\omega) - 1]} \quad (7d)$$

In order to establish the different elements of the stiffness matrix $\overline{K_e}$ in local reference, expressions have been derived by considering only the equilibrium equations and boundary conditions for each element k_{ij} as presented in (Ihaddoudène el al. 2009).

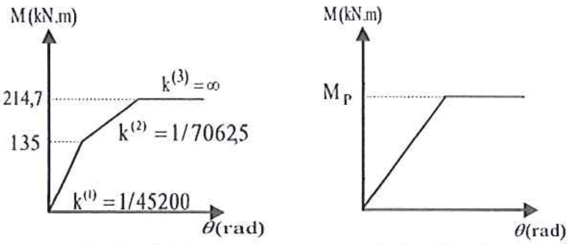
The expressions presented here (Ihaddoudène 2008) are more general as they take all type of situations, considering or neglecting the semi-rigidity of connections and axial forces or combining them in any situations.

3 EXAMPLES

The influence of semi-rigid joints on the behavior of steel structures is followed with the results obtained on two different steel frames: two story frame and portal frame with two bays. Three behavior cases of rigid, semi-rigid bilinear (with $k^{(1)}$ and $k^{(3)}$) and semi rigid tri-linear (with $k^{(1)}$, $k^{(2)}$ and $k^{(3)}$) are considered as shown in the Figure 3 below. The load is increased up to the value of P=60kN.

3.1 Two story frame

The frame of Figure 4 is analyzed where the behavior of the connections is bilinear or tri-linear. The different characteristic curves of bar element and connections are reported in the Figure 3. The IPE 400 beam elements (I_x=23130cm^4) has a plastic moment M_p=319.92 kN.m where for the HEA 280 columns (I_x=13670cm^4) the plastic moment M_p=266.88 kN.m.



a- For the joint. b- For the element.

Figure 3: Characteristic curves.

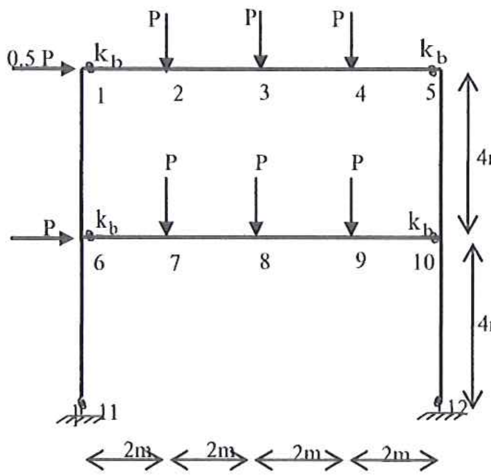


Figure 4: Two story portal frame.

The process is divided into some steps, according to the shape of the moment-rotation curve (bi-linear or tri-linear) and the state of the structure. All the joints have the same flexibility $k^{(1)}$ in the first stage of the initial portion of the moment-rotation curve. Here, the step by step process is continued until the sum of the load increments is equal to the load applied to the structure P=60kN. The calculation schemes at different stages corresponding to these cases and the bending moment diagram are respectively given in Table 1 and Figure 5 below:

Table 1: Schemes at different stages

1st Stage	The flexibility k in the critical sections					
	k_1	k_5	k_6	k_{10}	k_{11}	k_{12}
1	$k^{(1)}$	$k^{(1)}$	$k^{(1)}$	$k^{(1)}$	$k^{(1)}$	$k^{(1)}$
2	$k^{(1)}$	$k^{(1)}$	$k^{(1)}$	$k^{(2)}$	$k^{(1)}$	$k^{(1)}$
3	$k^{(1)}$	$k^{(2)}$	$k^{(1)}$	$k^{(2)}$	$k^{(1)}$	$k^{(1)}$

In table 1, k_i and $k^{(j)}$ are the flexibilities of the node i at the stage j of the characteristic curve of the joint.
The results obtained for different cases at each stage are summarized in the Table 2 below:

Table 2: Increment of load $\Delta W^{(i)}$ (kN) and bending moment $\Delta M_{ij}^{(i)}$(kN.m) values.

I^{th} Stage	1	2	$\sum_{i=1}^{2} \Delta W^{(i)}$	3	$\sum_{i=1}^{3} \Delta W^{(i)}$
$\Delta W^{(i)}$	41.62	15.2	56.79	3.2	P=60kN
$\Delta M_{15}^{(i)}$	-33.04	-5.94	-38.97	-0.39	-39.37
$\Delta M_{51}^{(i)}$	-96.2	-39.41	-135.6	-5	-140.6
$\Delta M_{16}^{(i)}$	-33.04	-5.94	-38.98	-0.39	-39.37
$\Delta M_{61}^{(i)}$	41.11	14.44	55.54	1.05	56.59
$\Delta M_{5,10}^{(i)}$	-96.2	-39.41	-135.6	-5	-140.6
$\Delta M_{10,5}^{(i)}$	61.08	11.23	72.32	2.85	75.17
$\Delta M_{6,10}^{(i)}$	-11.77	-3.18	-14.95	0.52	-14.44
$\Delta M_{10,6}^{(i)}$	-135.6	-25.83	-161.43	-5.88	-167.31
$\Delta M_{6,11}^{(i)}$	29.34	11.12	40.46	1.57	42.03
$\Delta M_{11,6}^{(i)}$	-66.17	-32.08	-97.25	-7.06	-104.31
$\Delta M_{10,12}^{(i)}$	-74.51	-14.6	-87.11	-3.03	92.14
$\Delta M_{12,10}^{(i)}$	80.47	33.28	113.75	7.55	121.3

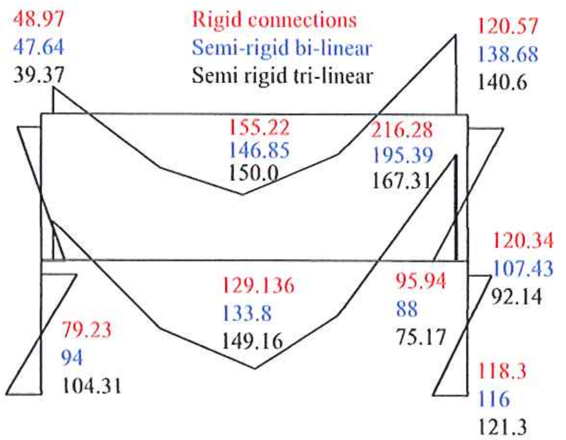


Figure 5: Bending moment diagram for different connection

Note that the bending moments in the connections (at 6 and 10 of the moment diagram) decrease from the rigid case to bilinear semi-rigid one, while they increase at mid-span (at 8 on the diagram) what it not found at the first level (in 1, 5 and 3, respectively). In this case, we should not have the same type of semi-rigid connection for the two floors of the portal frame.

3.2 Portal frame with two bays

The same procedure is applied to the portal frame with two bays of the Figure 6 and the corresponding moment diagram is shown in Figure 7 below:

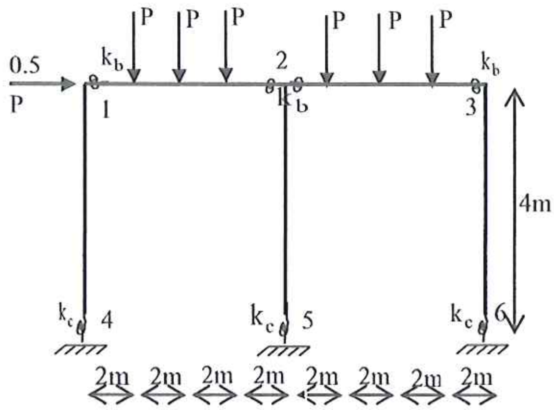


Figure 6: Two bays portal frame.

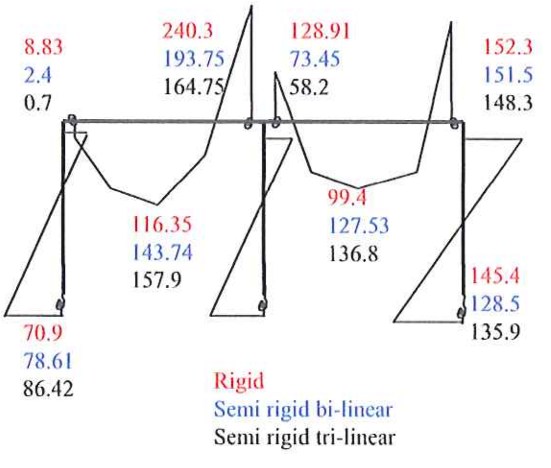


Figure 7: Bending moment diagram (kN.m) for different connections.

The influence of the flexibility of the connections on the values on bending moments is shown in Figure 7 above. Indeed, for the portal frame of two span reduced moments at the ends is accompanied by an increase at its value span.

Thus, it can be argued that the potential influence of the connections on the behavior of two story portal frame and two bays portal frame is clearly established. The bending moment diagrams for these two examples show that the beams are highly sensitive to the connection flexibility for portal frame with two stories.

4 CONCLUSIONS

Connection flexibility is known to be a major source of nonlinearity in steel structures. If this flexibility is not taken into account, it will be a source of errors and precision in the design of structures. In the proposed mechanical model, the analytical solutions are easily established and are practical in implementation. The elements of the stiffness matrix are easily obtained by the method of unit displacement. A modified stiffness matrix using analytical expressions for steel frames with semi-rigid connections is presented.

The illustrative examples of two plane steel frame and portal steel frame with two bays show the high sensitivity of the structural elements to the joint flexibility.

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