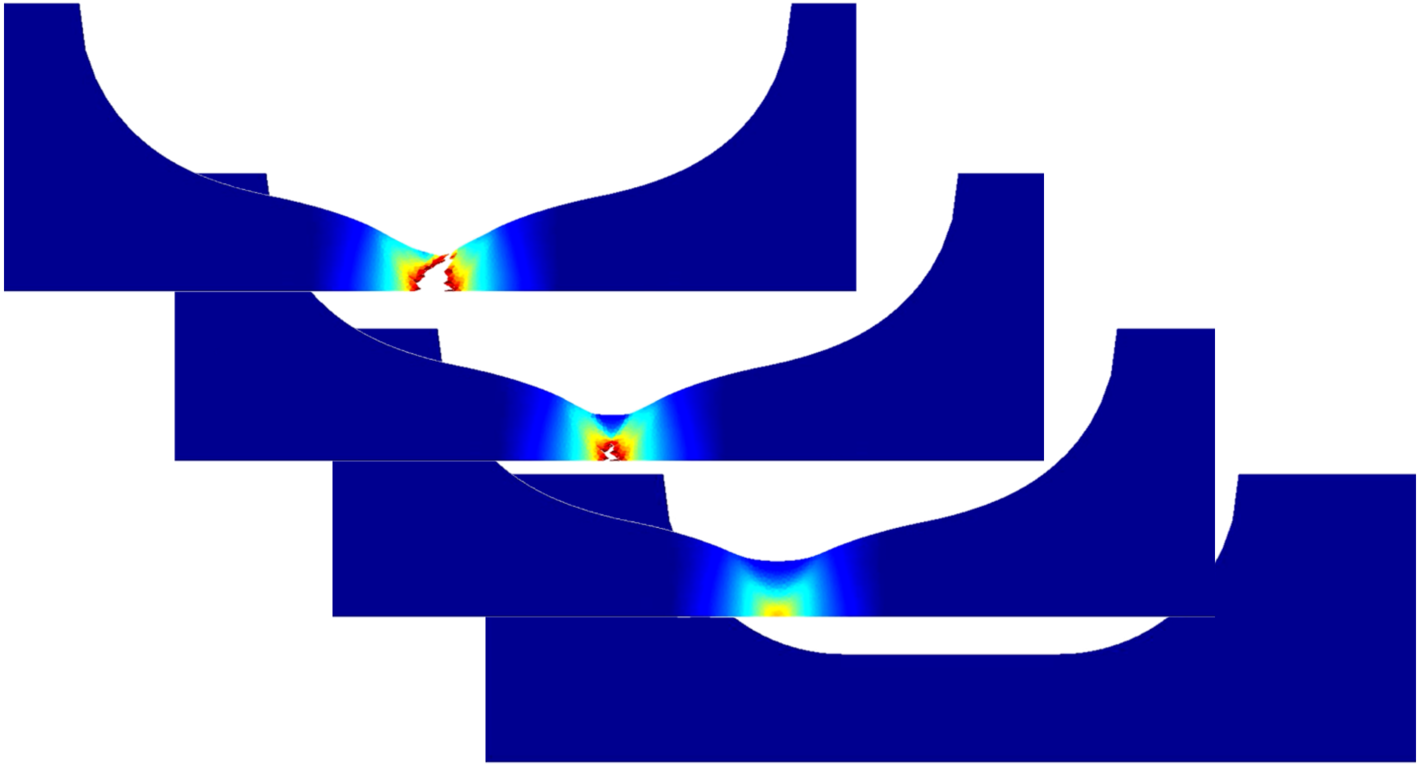
A Damage to Crack Transition Framework for Ductile Failure

Julien Leclerc, Van-Dung Nguyen, Ludovic Noels

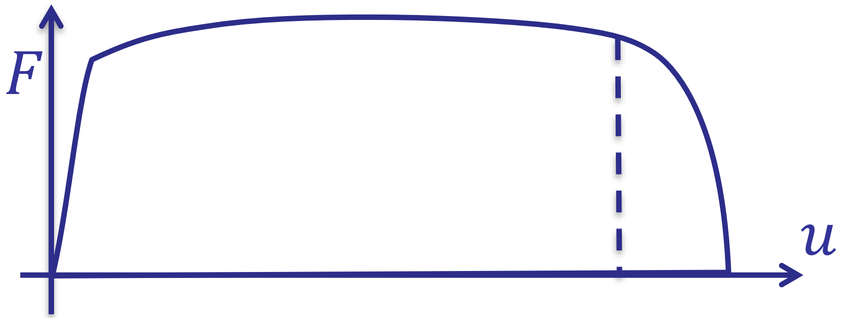


The research has been funded by the Walloon Region under the agreement no.7581-MRIPF in the context of the 16th MECATECH call.

- Goal:
  - Develop a predictive numerical framework to capture the whole ductile failure process

- Divided in two parts:

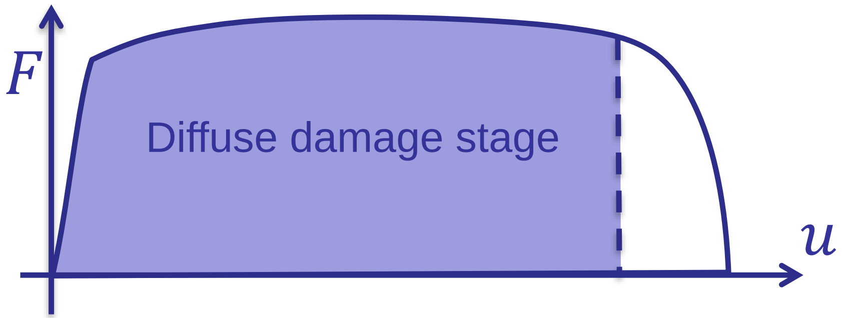
Physical process



Physical process

- Divided in two parts:
  - A **diffuse** damage stage with voids/damage **nucleation** and **growth**

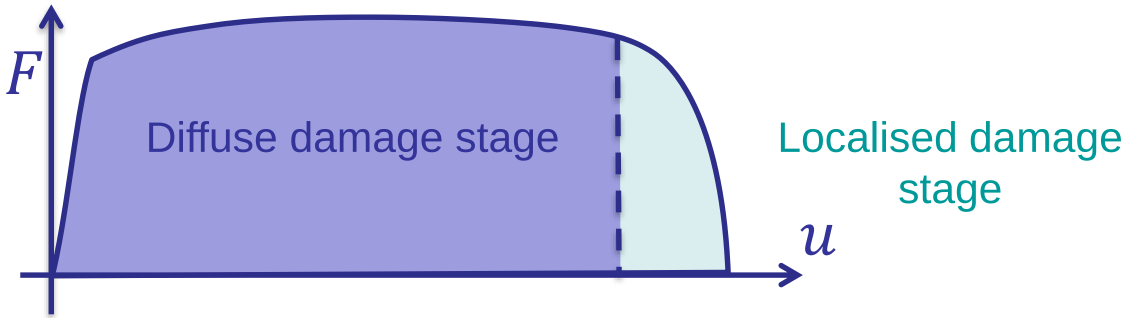
Physical process



Physical process

- Divided in two parts:
  - A **diffuse** damage stage with voids/damage **nucleation** and **growth**
    followed by
  - A **localised** stage with damage **coalescence** and **crack** initiation / propagation

Physical process



Physical process

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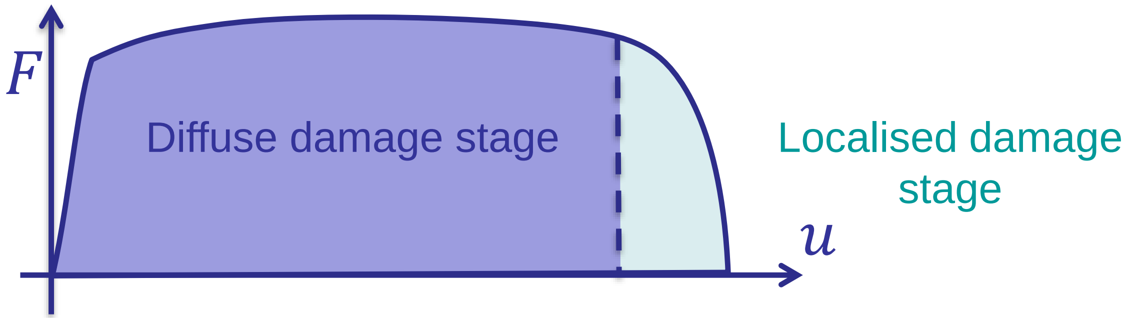
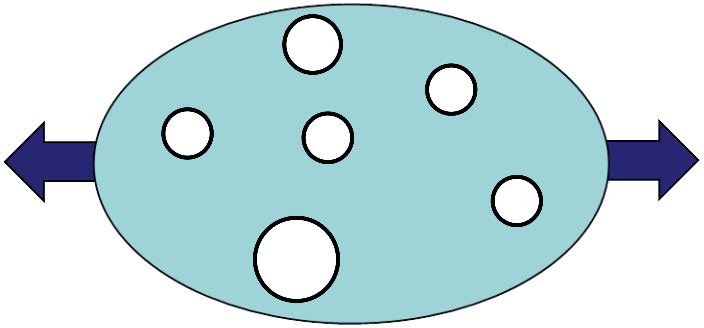
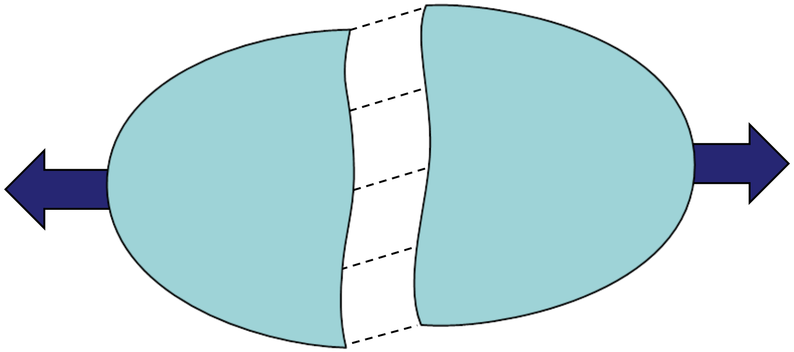
Physical process	
Numerical model	

- State-of-the-art
  - 2 approaches modeling material failure:
    - **Continuous** Damage Models (CDM)

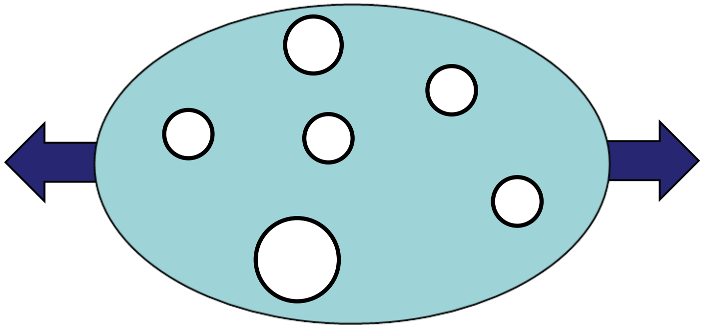


    - **Discontinuous**: Fracture mechanics



State of art: two main approaches – 1. Continuous approaches

- Material properties degradation modelled through internal variables evolution (= damage)
  - Lemaitre-Chaboche model,
  - Gurson model [Gurson1977]
  - …

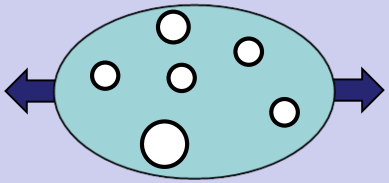


- Continuum Damage Model (CDM) implementation:
  - Local form
    - Mesh-dependency
  - Non-local form needed
    - Implicit non-local model [Peerlings et al. 1998]

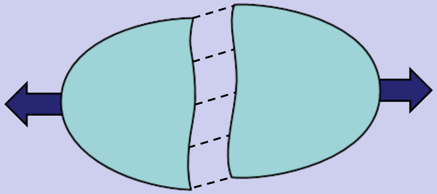
State of art: Comparison (1)

Continuous:

Continuous Damage Model (CDM)



Discontinuous:

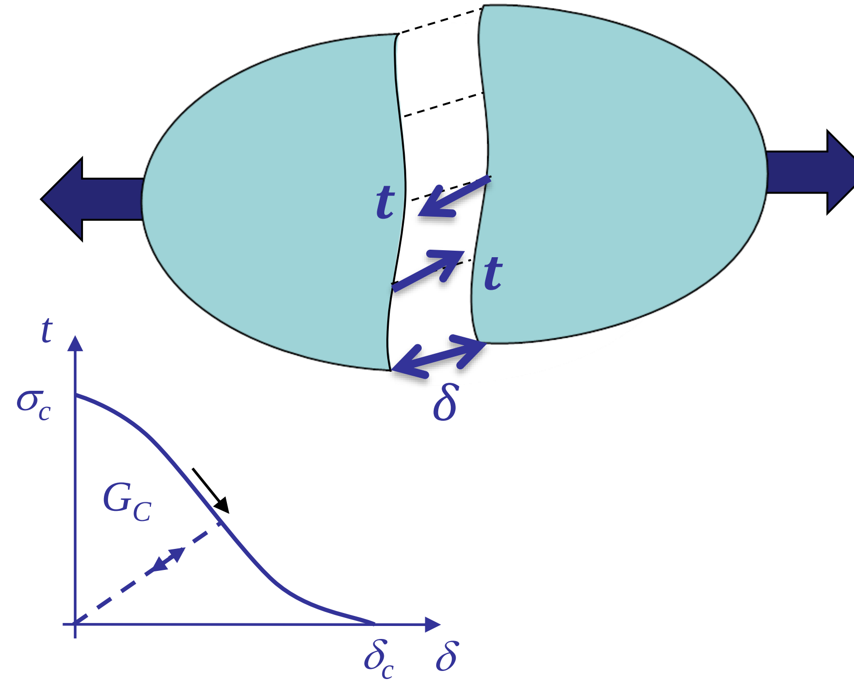


+ Capture the **diffuse damage stage**
+ Capture stress **triaxiality** and **Lode** variable effects

- **Numerical problems** with highly damaged elements
- **Cannot represent cracks** without remeshing / element deletion at $D \rightarrow 1$ (loss of accuracy, mesh modification ...)
- Crack initiation observed for lower damage values

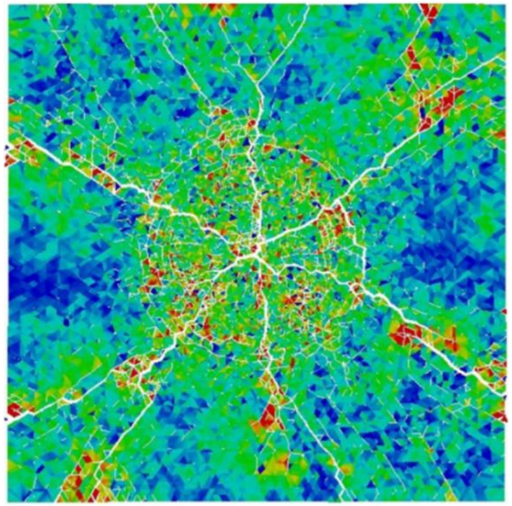
State of art: two main approaches – 2. Discontinuous approaches

- Similar to fracture mechanics
- One of the most used methods:
  - Cohesive Zone Model (CZM) modelling the crack tip behaviour inserted by:
    - Interface elements between 2 volume elements [Mergheim2004]
    - Element enrichment (EFEM) [Armero et al. 2009]
    - Mesh enrichment (XFEM) [Moes et al. 2002]
    - …



- Consistent and efficient hybrid framework for brittle fragmentation: [Radovitzky et al. 2011]
  - Extrinsic cohesive interface elements

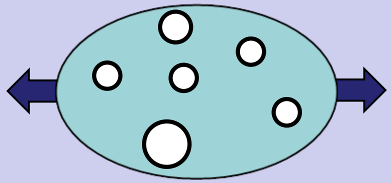
    \+
  - Discontinuous Galerkin (DG) framework (enables inter-elements discontinuities)



State of art: Comparison (2)

Continuous:

Continuous Damage Model (CDM)



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+ Capture stress **triaxiality** and **Lode variable effects**

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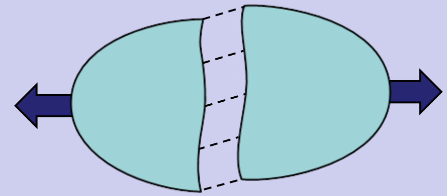
Discontinuous:

Extrinsic Cohesive Zone Model (CZM)



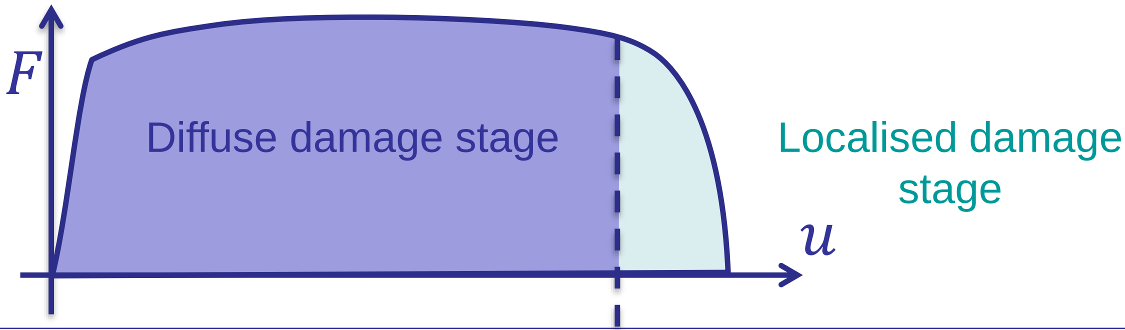
+ **Multiple crack initiation** and propagation naturally managed

- **Cannot capture diffuse damage**
- **No triaxiality** effect
- Currently valid for brittle / small scale yielding materials

Numerical model

- Main idea = combination of 2 complementary methods :

Physical process

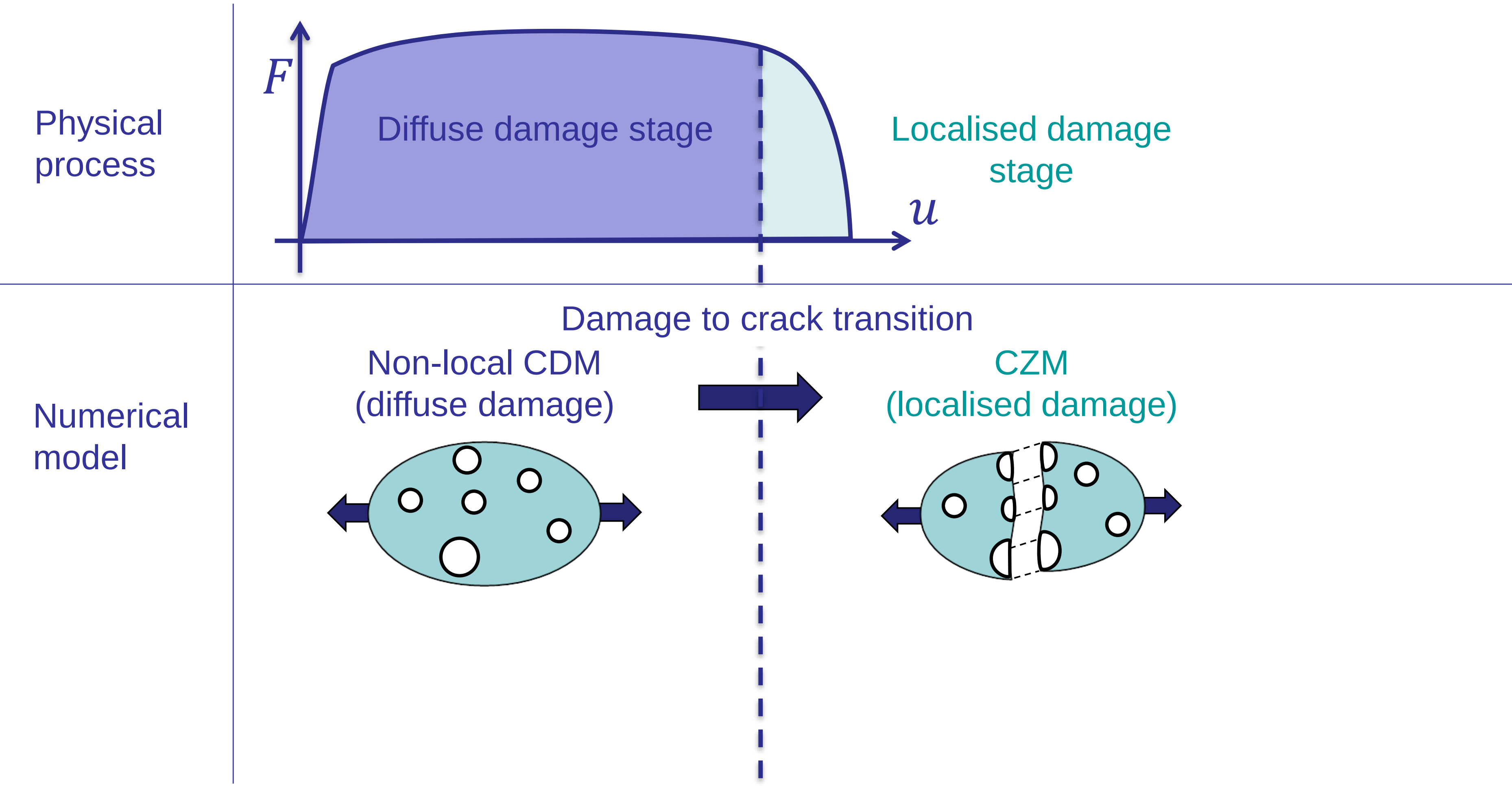


Numerical model

Numerical model

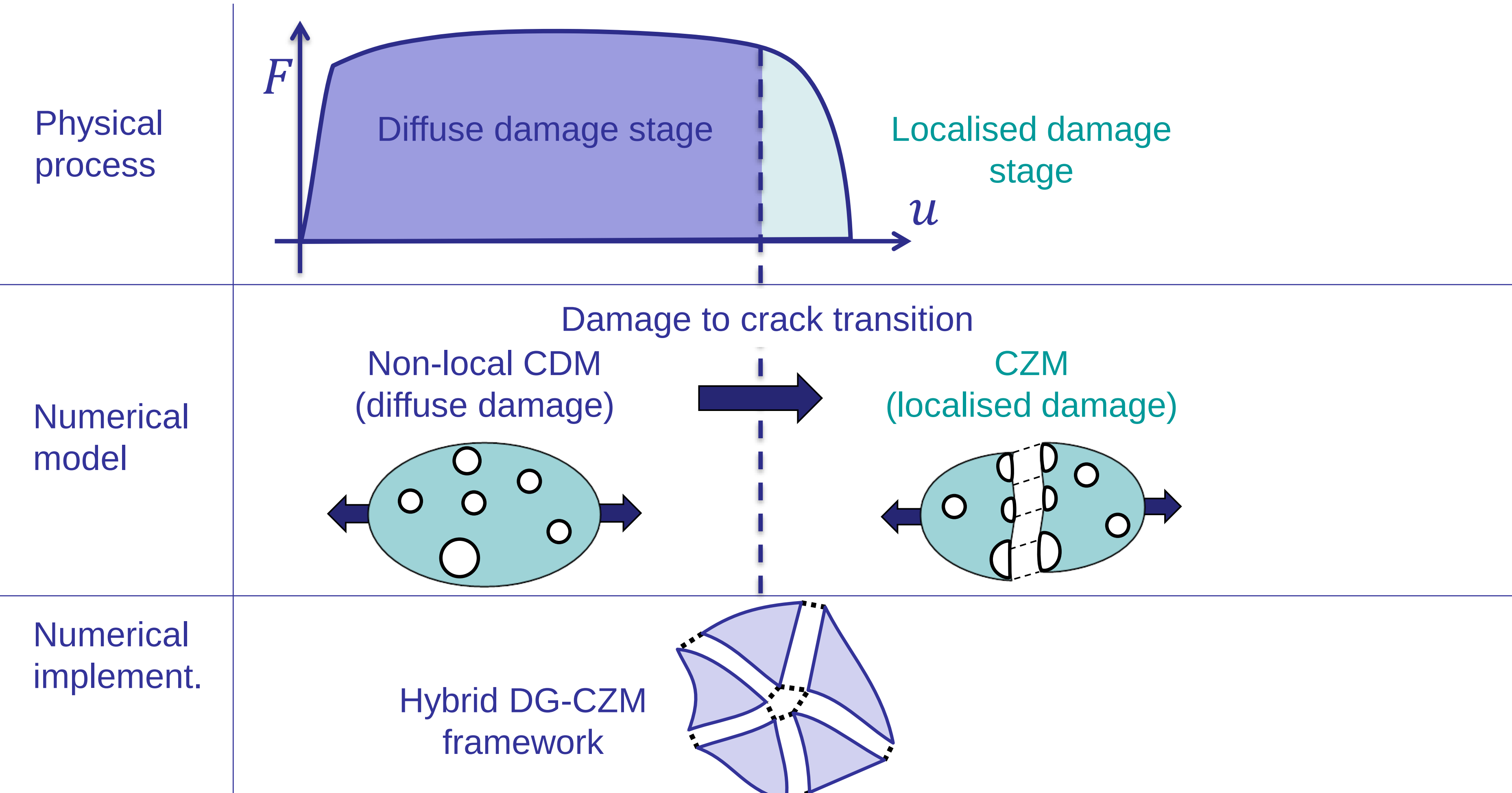
- Main idea = combination of 2 complementary methods :
  - Continuous (non-local damage model)

    + transition to
  - Discontinuous (cohesive model)



Numerical implementation

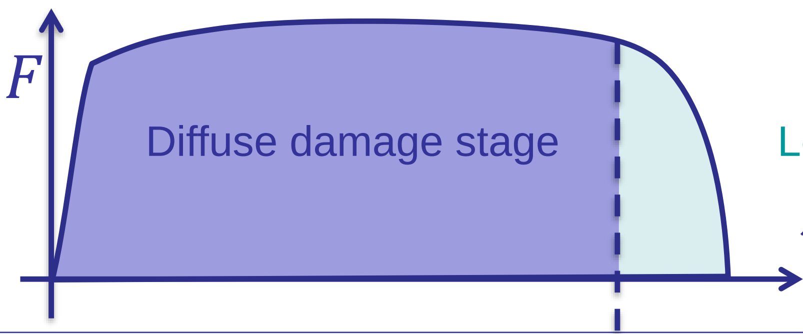
- Implementation of the damage to crack transition:
  - within a Discontinuous Galerkin (DG) framework



Numerical implementation

- How are included triaxiality effects during crack propagation ?

Physical process

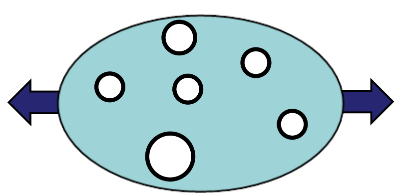


Localised damage stage

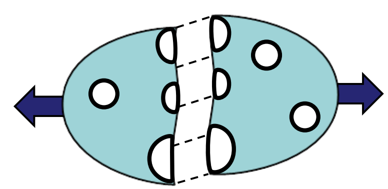
Damage to crack transition

Numerical model

Non-local CDM (diffuse damage)

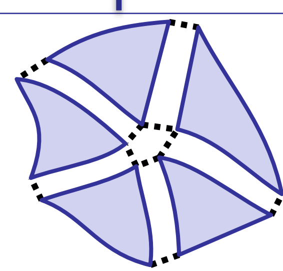


CZM (localised damage)



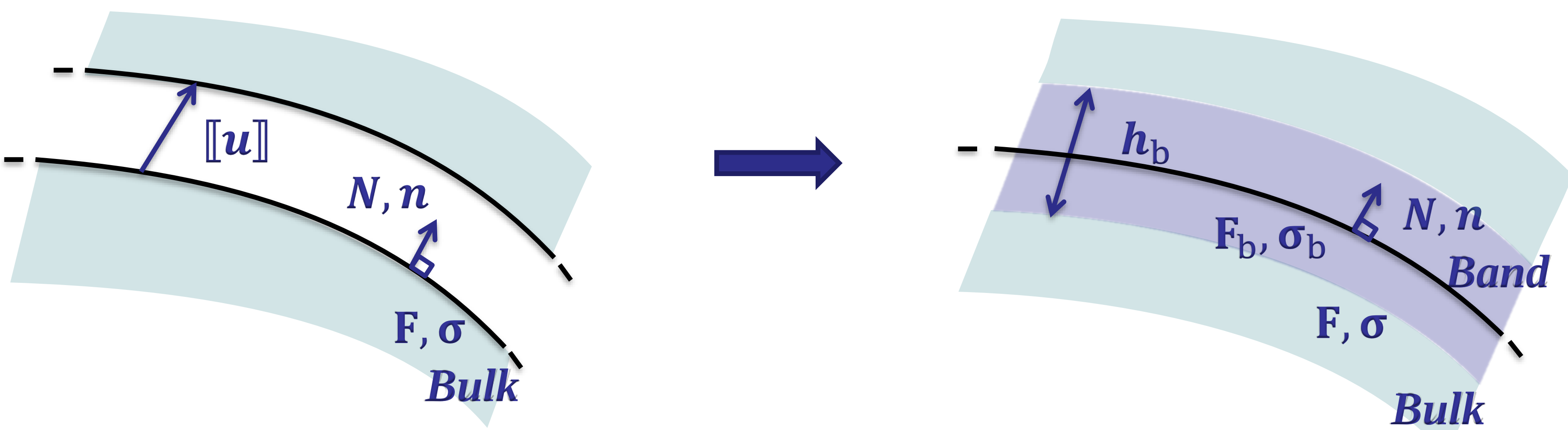
Numerical implement.

Hybrid DG-CZM framework



Cohesive zone with triaxiality – Principles

- Discontinuous model here = Cohesive Band Model (CBM):
  - Hypothesis
    - In the last stage of failure, all damaging process occurs in an uniform thin band
  - Principles
    - Replacing the traction-separation law of a cohesive zone by the behaviour of a uniform band of given thickness h_b [Remmers et al. 2013]
  - Methodology [Leclerc et al. 2018]
    1. Compute a band strain tensor $\mathbf{F}_\mathrm{b} = \mathbf{F} + \frac{[\![\boldsymbol{u}]\!] \times \boldsymbol{N}}{h_\mathrm{b}} + \frac{1}{2}\nabla_T [\![\boldsymbol{u}]\!]$
    2. Compute then a band stress tensor $\boldsymbol{\sigma}_\mathrm{b}$
    3. Recover traction forces $\boldsymbol{t}([\![\boldsymbol{u}]\!], \mathbf{F}) = \boldsymbol{\sigma}_\mathrm{b}.\boldsymbol{n}$



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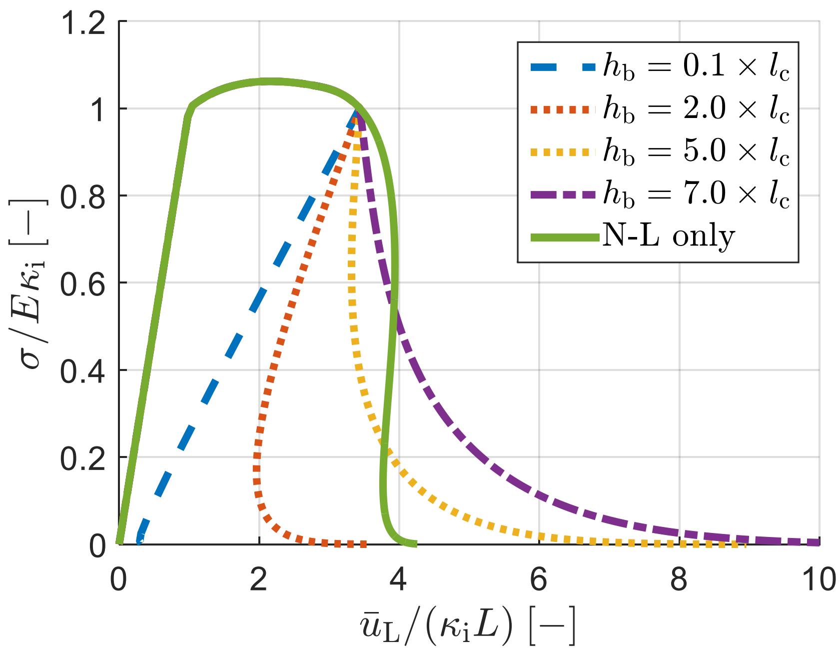
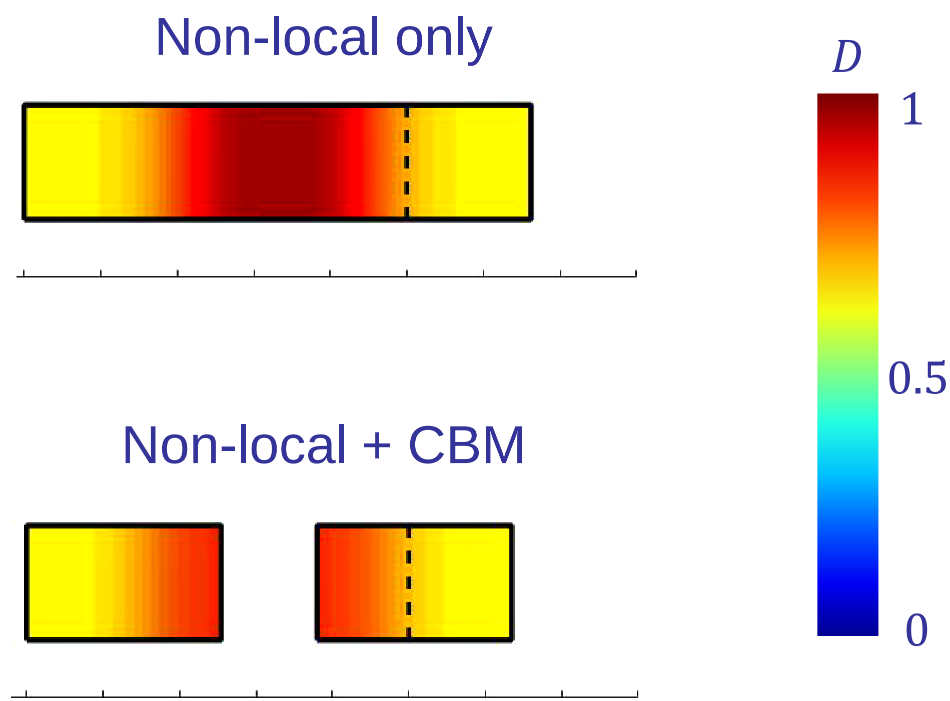
  - At crack insertion, framework only dependent on h_b (band thickness)
    - h_b controls the failure energy dissipation

Damage to crack transition for elasticity – Proof of concept

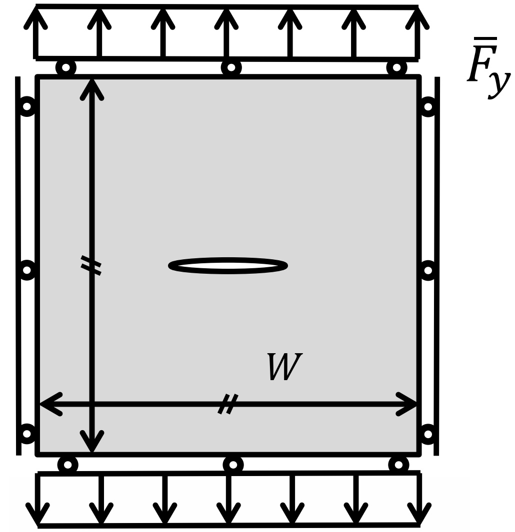
- Influence of h_{b} on response in a 1D elastic case [Leclerc et al. 2018]:
  - Total dissipated energy Φ:
    - Has to be chosen to conserve energy dissipation (physically based)

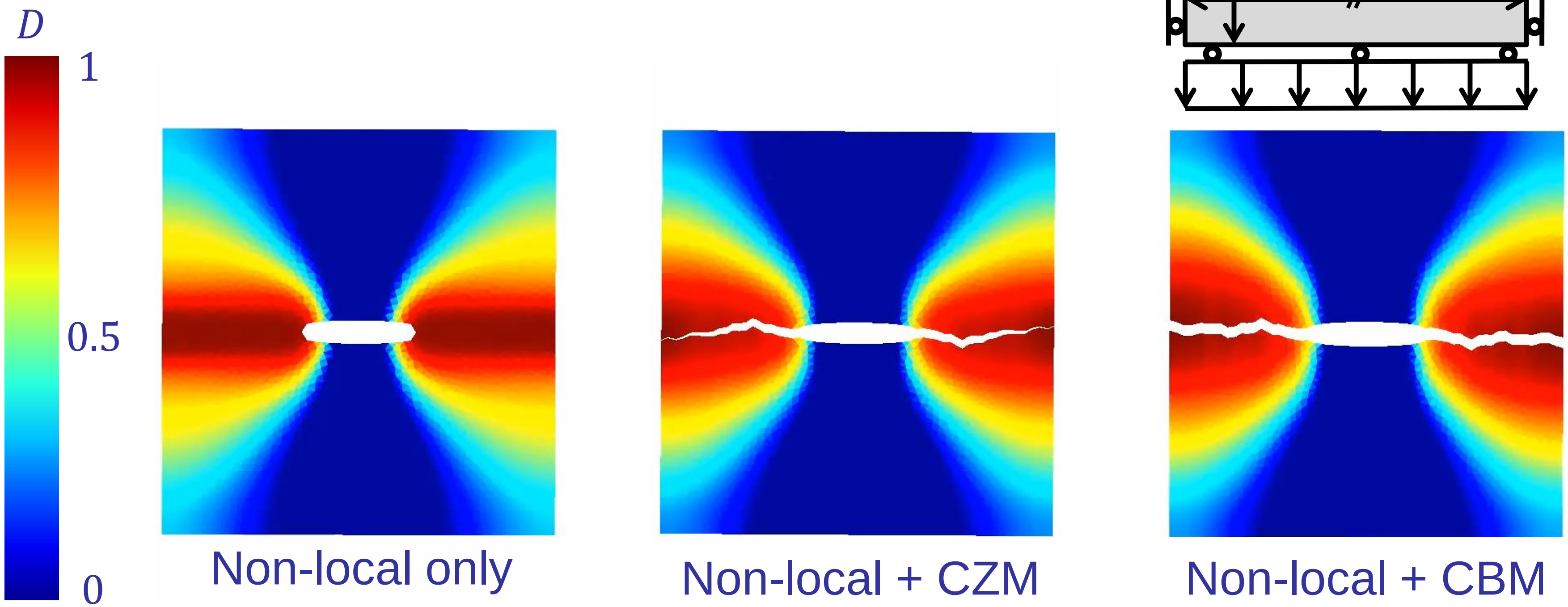




Damage to crack transition for elasticity – Proof of concept

- 2D elastic plate [Leclerc et al. 2018]:
  - With a defect
  - In plane strain





Non-local only | Non-local + CZM | Non-local + CBM

no crack insertion

cohesive models calibrated on 1D bar in plane stress

Damage to crack transition for elasticity – Proof of concept

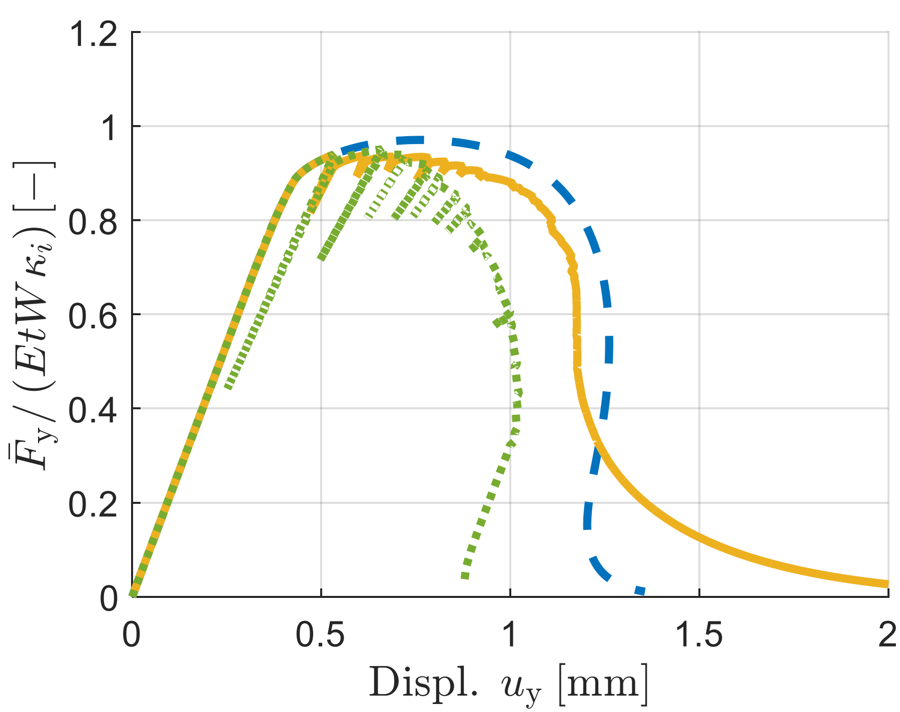
- 2D elastic plate [Leclerc et al. 2018]:

Non-Local only

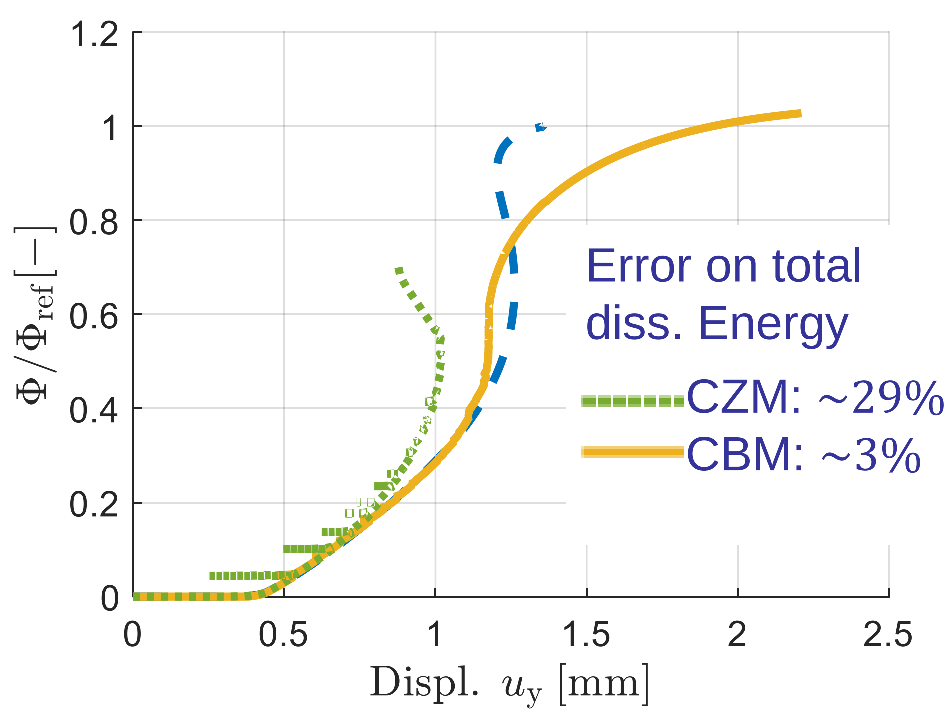
Non-Local + CZM

Non-Local + CBM

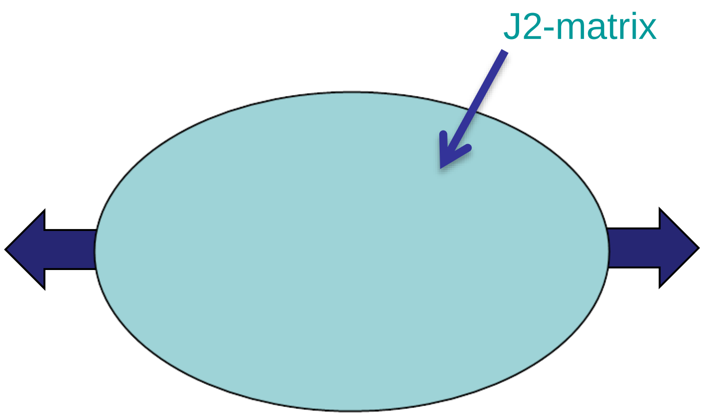
- Force evolution



- Dissipated energy evolution

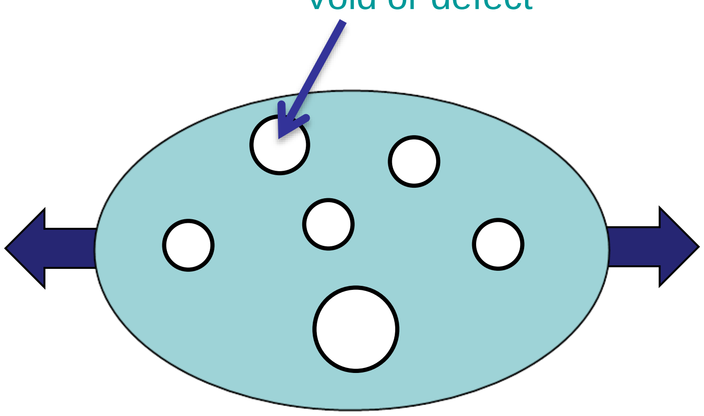


- Porous plasticity (or Gurson) approach
  - Assuming a J2-plastic matrix



Damage to crack transition in porous elasto-plasticity

- Porous plasticity (or Gurson) approach
  - Assuming a J2-plastic matrix
  - Including effects of void/defect or porosity on plastic behavior
    - Apparent macroscopic yield surface $f(\tau_{\text{eq}}, p) \leq 0$ due to microstructural state:



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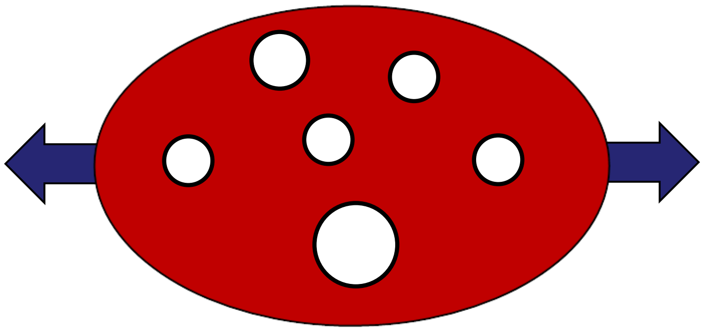

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      - Competition between two deformation modes:
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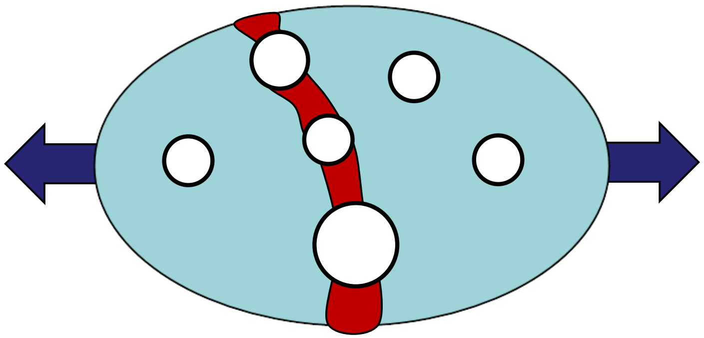
          - » Gurson model
        - » Before failure: coalescence or localized plastic flow between voids
          - » GTN or Thomason models



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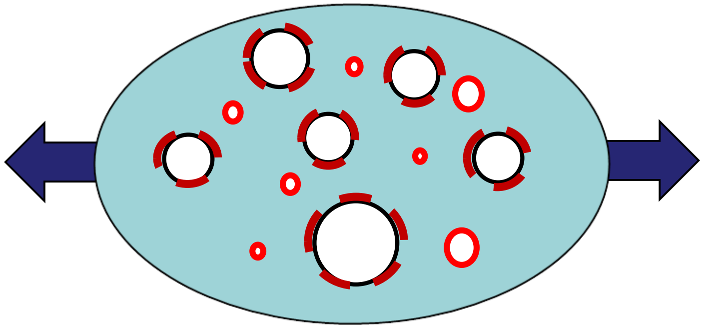
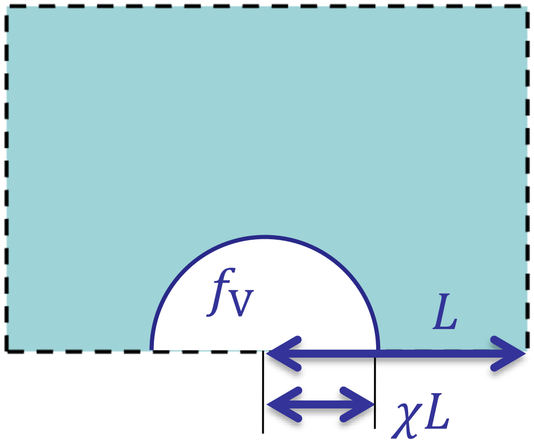
        - » Diffuse plastic flow spreads in the matrix
          - » Gurson model
        - » Before failure: coalescence or localized plastic flow between voids
          - » GTN or Thomason models
  - Including evolution of microstructure during failure process
    - Nucleation / appearance of new voids
    - Void growth by diffuse plastic flow
    - Apparent growth by shearing



Damage to crack transition in porous elasto-plasticity

- Porous plasticity (or Gurson) approach
  - Non-local form: $f\left(\tau_{\mathrm{eq}}, p, \tau_{\mathrm{y}}, \boldsymbol{Z}, \tilde{f}_{\mathrm{V}}\right) \leqslant 0$ with $\tilde{f}_{\mathrm{V}} - l_{\mathrm{c}}^{2}\Delta\tilde{f}_{\mathrm{V}} = f_{\mathrm{V}}$
  - Normal plastic flow
  - Hyperelastic formulation
  - Microstructure (= spherical voids [Besson2009])
    - τ^{eq} is the von Mises equivalent Kirchhoff stress and p the pressure
    - $\tau_{\mathrm{Y}} = \tau_{\mathrm{Y}}(\hat{p}, \dot{\hat{p}})$ is the viscoplastic yield stress
    - f_{V} is the porosity and $\tilde{f}_{\mathrm{V}}$, its non-local counterpart
    - χ is the cell ligament ratio
    - $\boldsymbol{Z}$ is the vector of internal variables
    - l_{c} is the non-local length

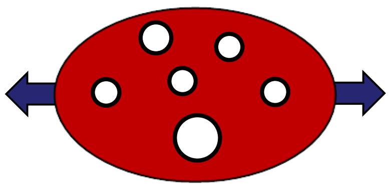


Damage to crack transition in porous elasto-plasticity

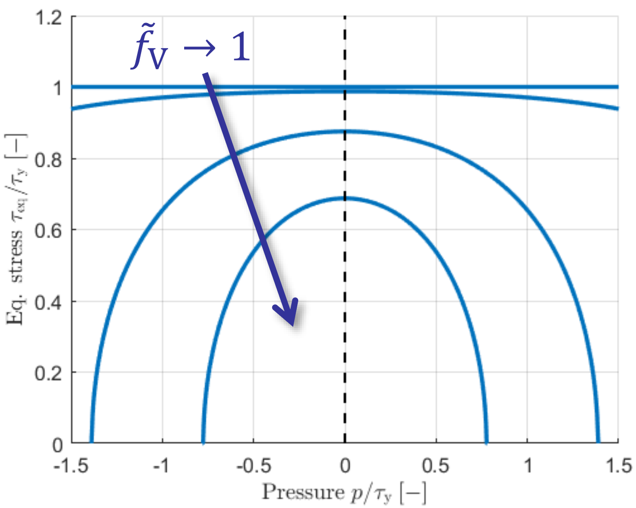
- Porous plasticity (or Gurson) approach
  - Competition between 2 plastic modes:

$$f\left(\tau_{\rm eq},\, p,\, \tau_{\rm y},\, \tilde{f}_{\rm V},\, \chi\right) = \max\left(f_{\rm G},\, f_{\rm T}\right) \leqslant 0$$

Growth mode: Gurson model

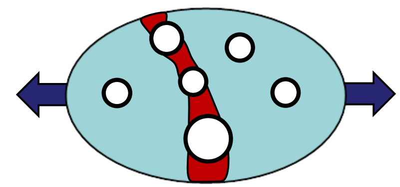


$$f_{\rm G} = \frac{\tau_{\rm eq}^2}{\tau_{\rm y}^2} + 2q_1\tilde{f}_{\rm V}\cosh\left(\frac{3}{2}q_2\frac{p}{\tau_{\rm y}}\right) - 1 - q_3^2\tilde{f}_{\rm V}^2 \leqslant 0$$

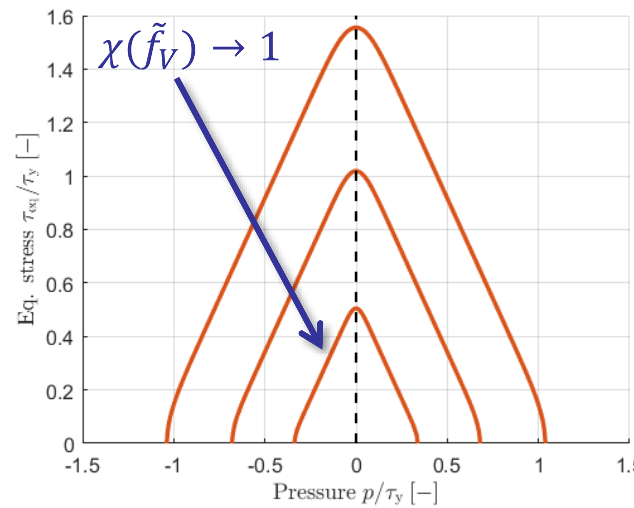


vs

Coalescence mode: Thomason model

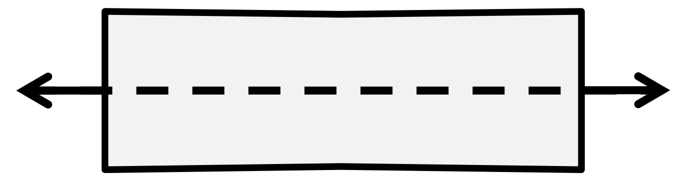


$$f_{\rm T} = \frac{2}{3}\tau_{\rm eq} + |p| - C_{\rm T}^{\rm f}\left(\chi\right)\tau_{\rm y} \leqslant 0$$

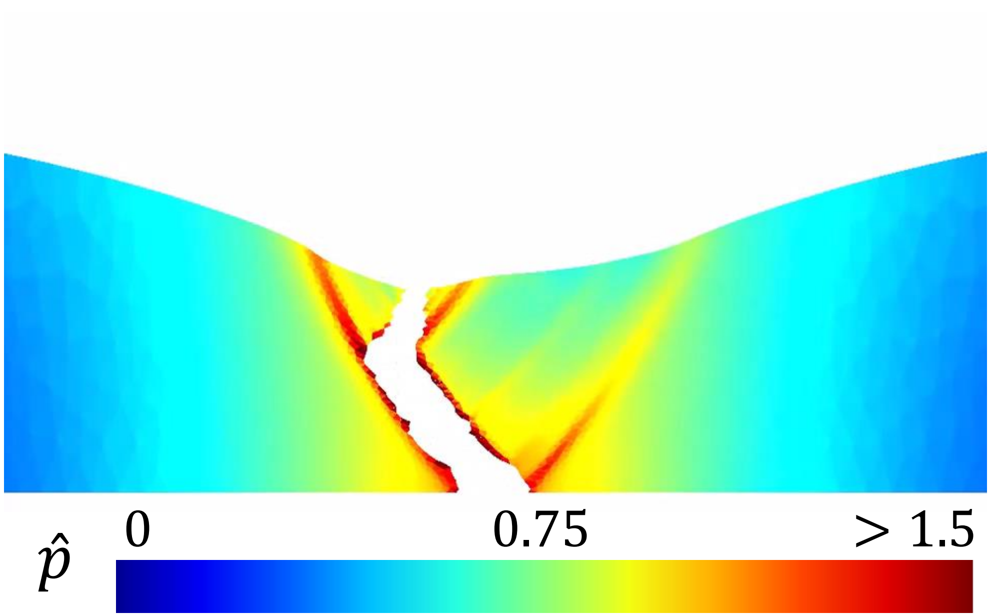


Damage to crack transition in porous elasto-plasticity

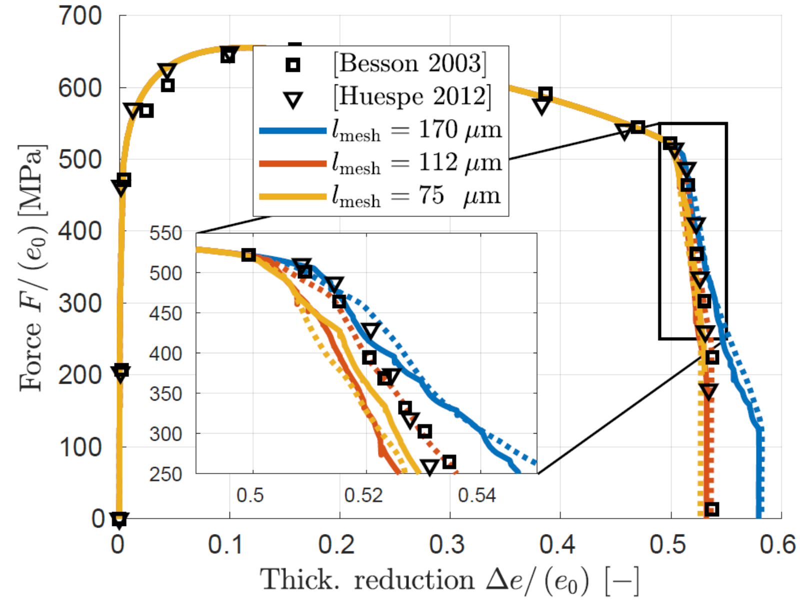
- Comparison with literature [Huespe2012,Besson2003]
  - Slanted crack in plane strain specimen



  - Crack insertion at ellipticity loss: $\det\left[\boldsymbol{N}\cdot\frac{D\mathbf{P}}{D\mathbf{F}}\cdot\boldsymbol{N}\right]\leqslant 0$
    - \+ No mesh dependency
    - \+ Energy dissipated by CBM small but mandatory
    - \- Unphysical bifurcation due to numerical crack insertion criterion



Force vs. striction



Damage to crack transition in porous elasto-plasticity

- Comparison with literature [Huespe2012,Besson2003]
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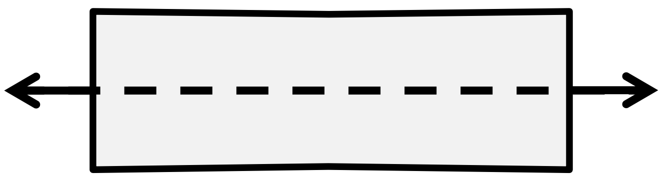
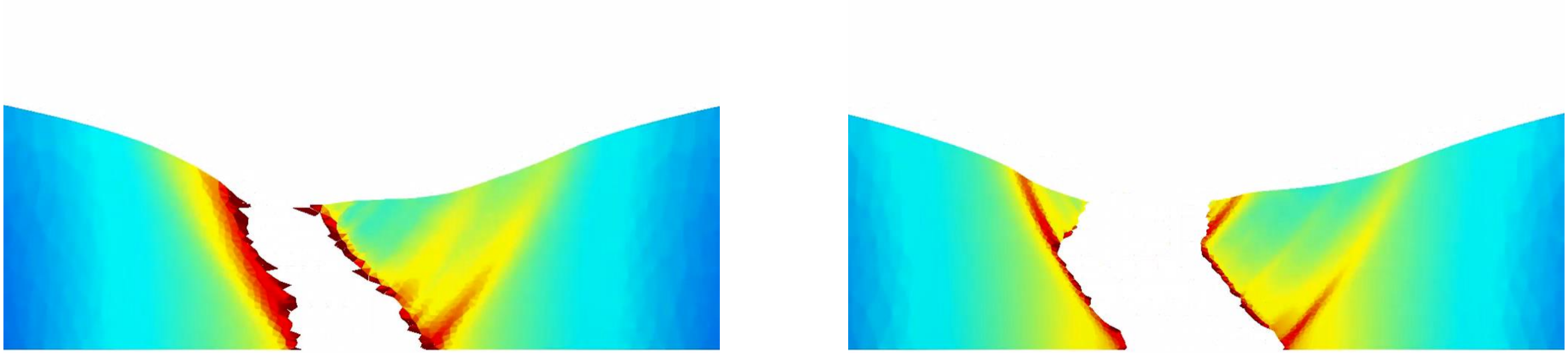
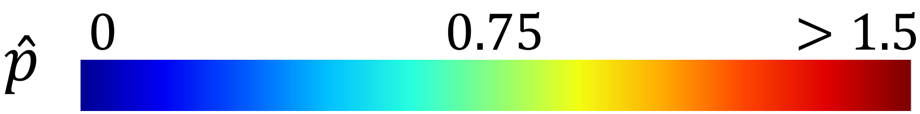


  - Comparison with developed framework : $\boldsymbol{N} \cdot \boldsymbol{\tau} \cdot \boldsymbol{N} - C_{\mathrm{T}}^{\mathrm{f}} \tau_y \geqslant 0$

Coalescence onset

Loss of ellipticity





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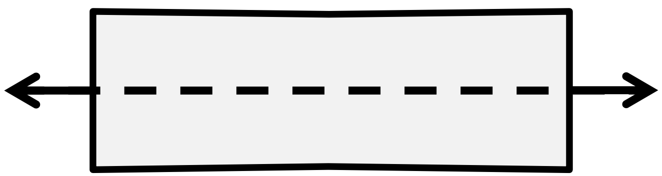
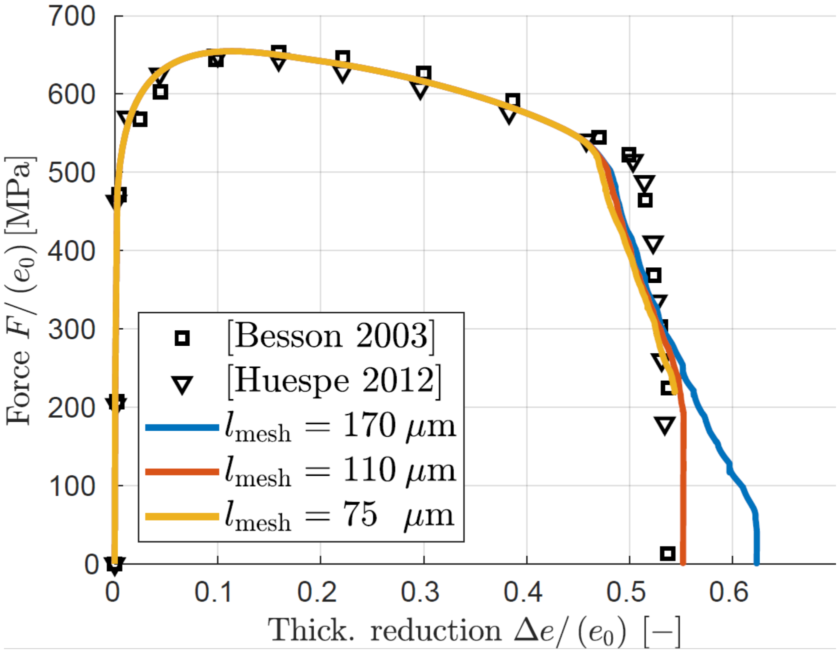
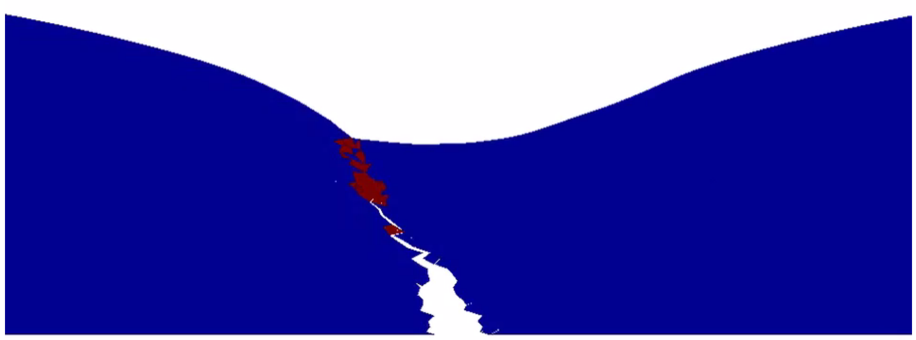


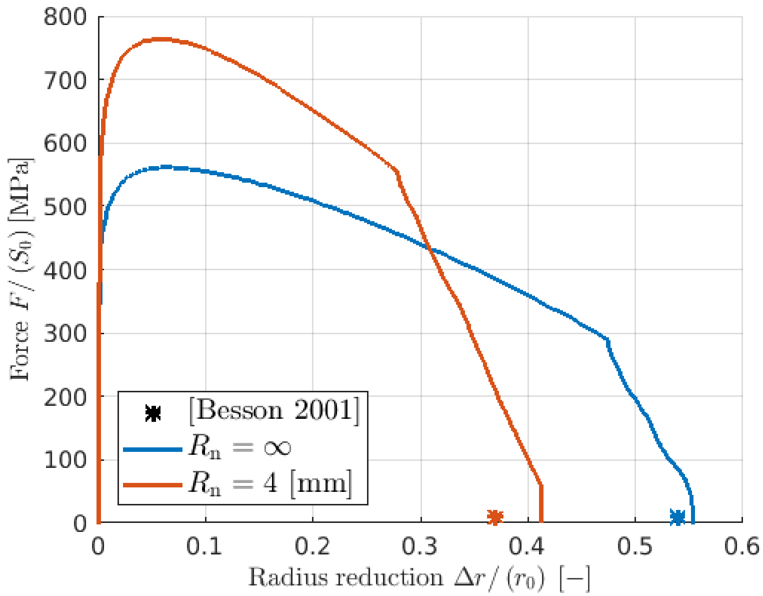
  - Comparison with developed framework: $\boldsymbol{N} \cdot \boldsymbol{\tau} \cdot \boldsymbol{N} - C_{\mathrm{T}}^{\mathrm{f}} \tau_y \geqslant 0$
    + No more unphysical crack bifurcation
      - Crack insertion beyond loss of ellipticity
      - Non-local model mandatory

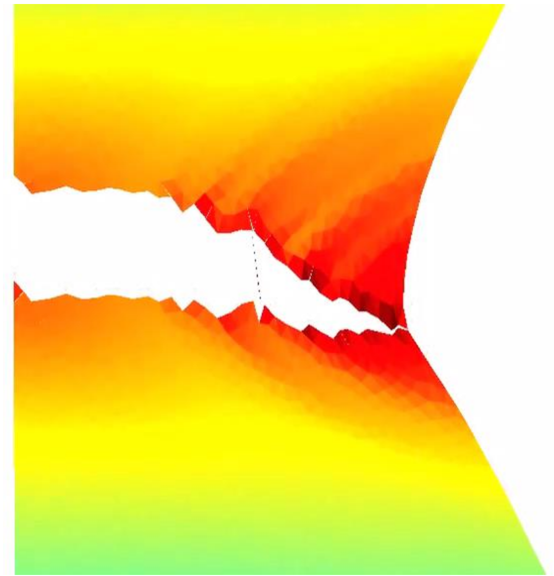




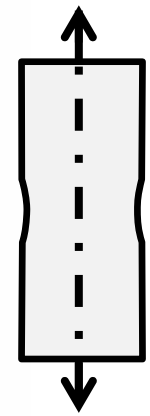
Damage to crack transition in porous elasto-plasticity

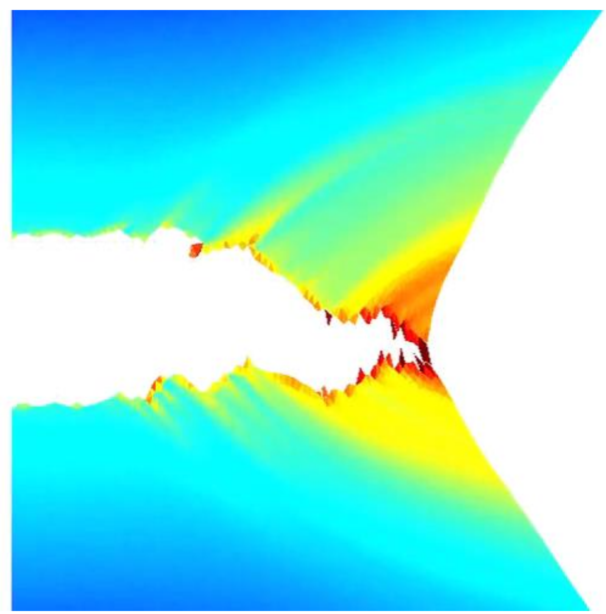
- Comparison with literature
  [Huespe2012,Besson2003]
  - Cup-cone fracture in smooth and notched round bars



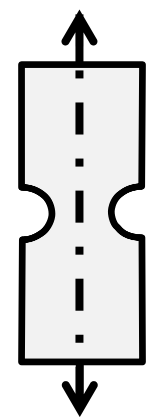


Smooth bar





Notched bar



Conclusions

- Objective:
  - Simulation of material degradation and crack initiation / propagation
- Methodology
  - Combination of
    - a non-local Continuum Damage Model (CDM)
    - And a Cohesive Band Model (CBM)
  - Integrated in a Discontinuous Galerkin framework
- Proof of concept
  - On elastic damage material model
- Ductile materials
  - Implementation of hyperelastic non-local porous-plastic model
    - Coupled Gurson-Thomason model
  - Proof on concept by comparison with literature
  - Upcoming tasks:
    - Enrichment of nucleation model and coalescence model
    - Calibration of the band thickness
    - Validation/Calibration with literature/experimental tests

I hope you enjoyed this presentation

Thank you for your attention

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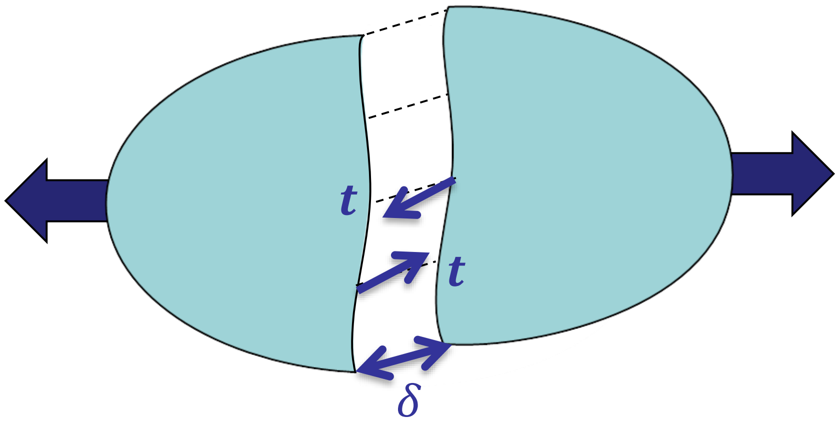
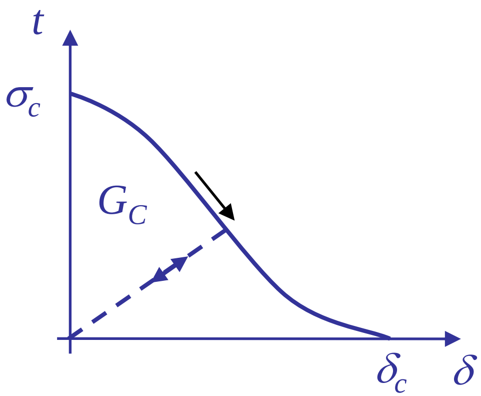
State of art: Discontinuous approaches

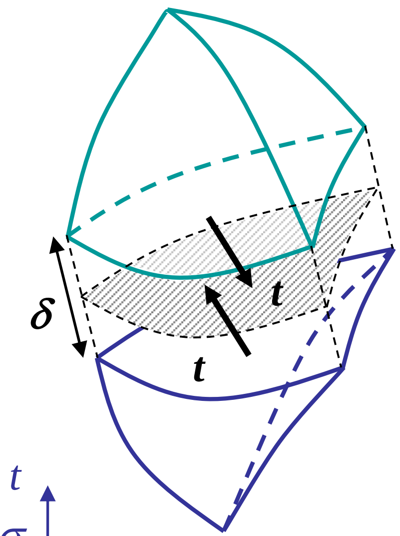
- Based on fracture mechanics concepts
  - Characterized by
    - Strength σ_c &
    - Critical energy release rate G_C



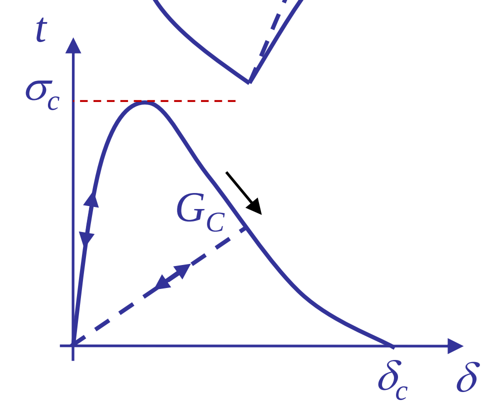
- One of the most used methods:
  - Cohesive Zone Model (CZM) modelling the crack tip behavior
  - Integrate a Traction Separation Law (TSL):
    - At interface elements between two elements
    - Using element enrichment (EFEM) [Armero et al. 2009]
    - Using mesh enrichment (xFEM) [Moes et al. 2002]
    - …



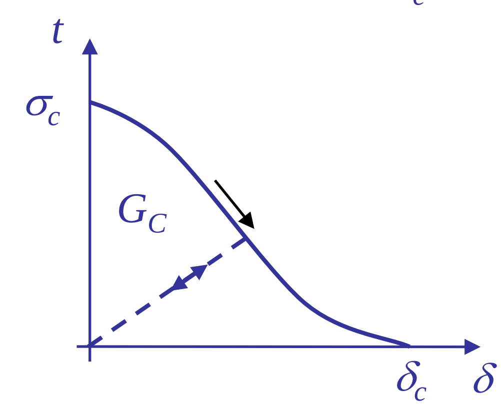
- Cohesive elements
  - Inserted between volume elements
    - Zero-thickness → no triaxiality accounted for



- Intrinsic Cohesive Law (ICL)
  - Cohesive elements inserted from the beginning
  - Efficient if a priori knowledge of the crack path
  - Mesh dependency [Xu & Needelman, 1994]
  - Initial slope modifies the effective elastic modulus
  - This slope should tend to infinity [Klein et al. 2001]:
    - Alteration of a wave propagation
    - Critical time step is reduced



- Extrinsic Cohesive Law (ECL)
  - Cohesive elements inserted on the fly when the failure criterion is verified [Ortiz & Pandolfi 1999]
  - Complex implementation in 3D (parallelization)



State of art: Discontinuous approaches

- Hybrid framework [Radovitzky et al. 2011]
  - Discontinuous Galerkin (DG) framework
    - Test and shape functions discontinuous
    - **Consistency**, **convergence rate**, **uniqueness** recovered though interface terms

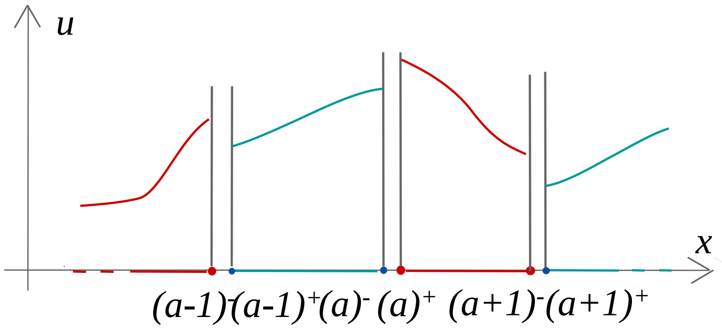
$$\int_{\Omega_0} \mathbf{P} : \nabla_0 \boldsymbol{\delta u} d\Omega +$$

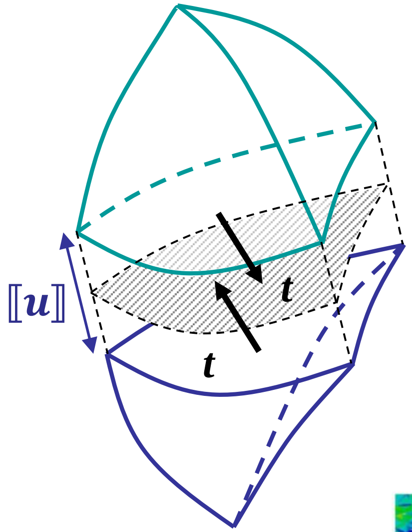
$$\int_{\partial_I \Omega_0} [\![\boldsymbol{\delta u}]\!] \cdot \langle \mathbf{P} \rangle \cdot \boldsymbol{N}^- d\partial\Omega +$$

$$\int_{\partial_I \Omega_0} [\![\boldsymbol{u}]\!] \cdot \langle \boldsymbol{C}^{\mathrm{el}} : \nabla_0 \boldsymbol{\delta u} \rangle \cdot \boldsymbol{N}^- d\partial\Omega +$$

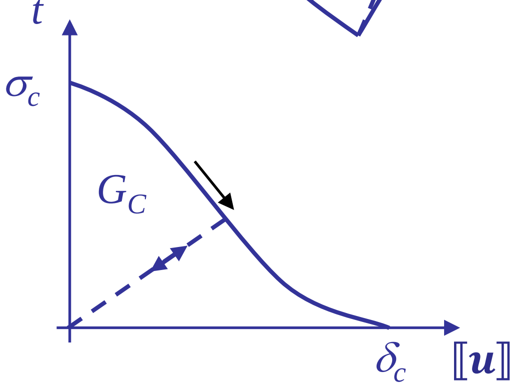
$$\int_{\partial_I \Omega_0} [\![\boldsymbol{u}]\!] \otimes \boldsymbol{N}^- : \langle \frac{\beta_s \boldsymbol{C}^{\mathrm{el}}}{h^s} \rangle : [\![\boldsymbol{\delta u}]\!] \otimes \boldsymbol{N}^- d\partial\Omega = 0$$

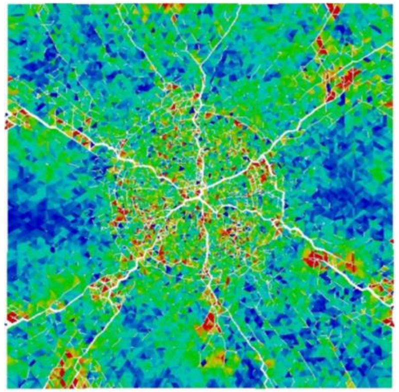
    - Interface terms integrated on interface elements





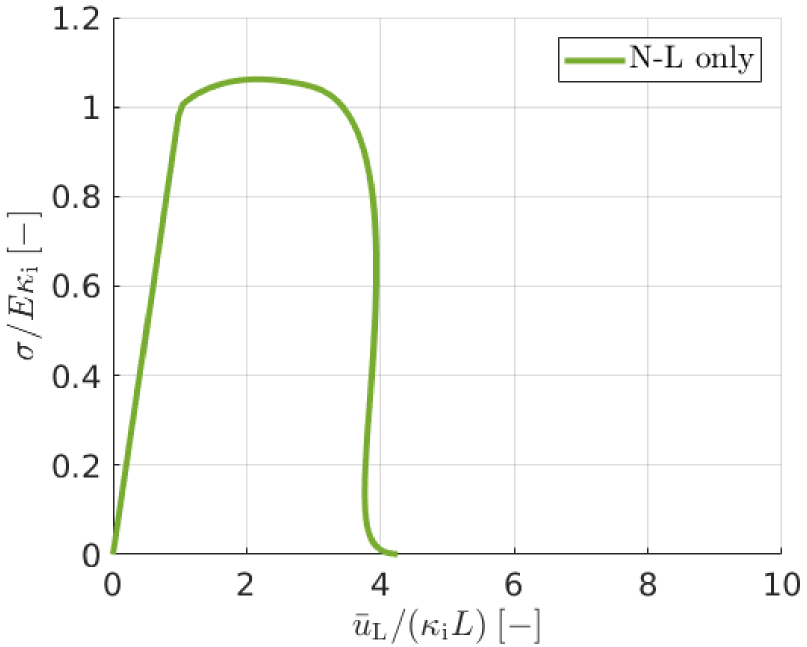
  - Combination with extrinsic cohesive laws
    - Interface elements already there
    - Switch to traction separation law
    - Efficient for fragmentation simulations

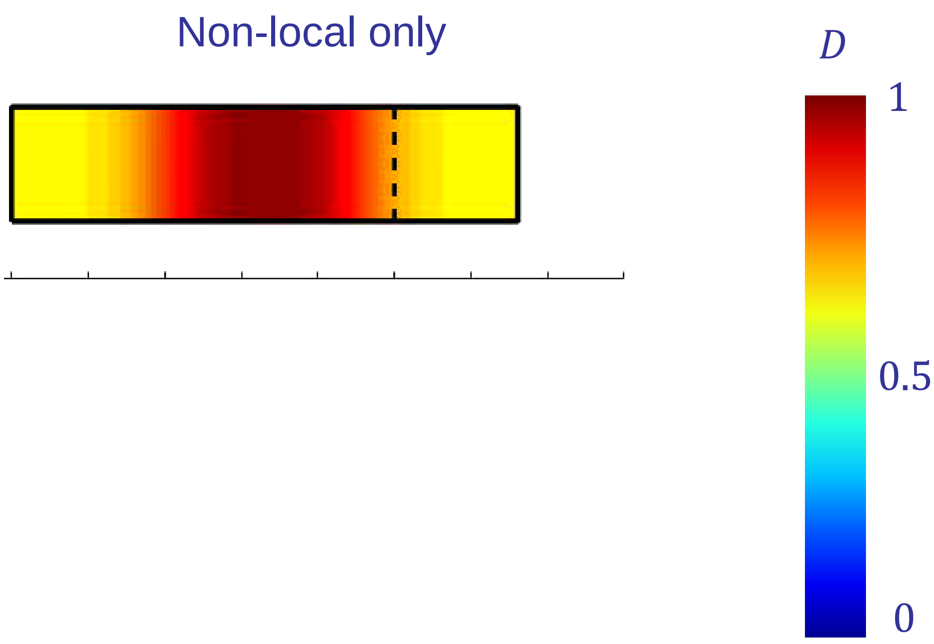




Damage to crack transition for elastic damage – Proof of concept

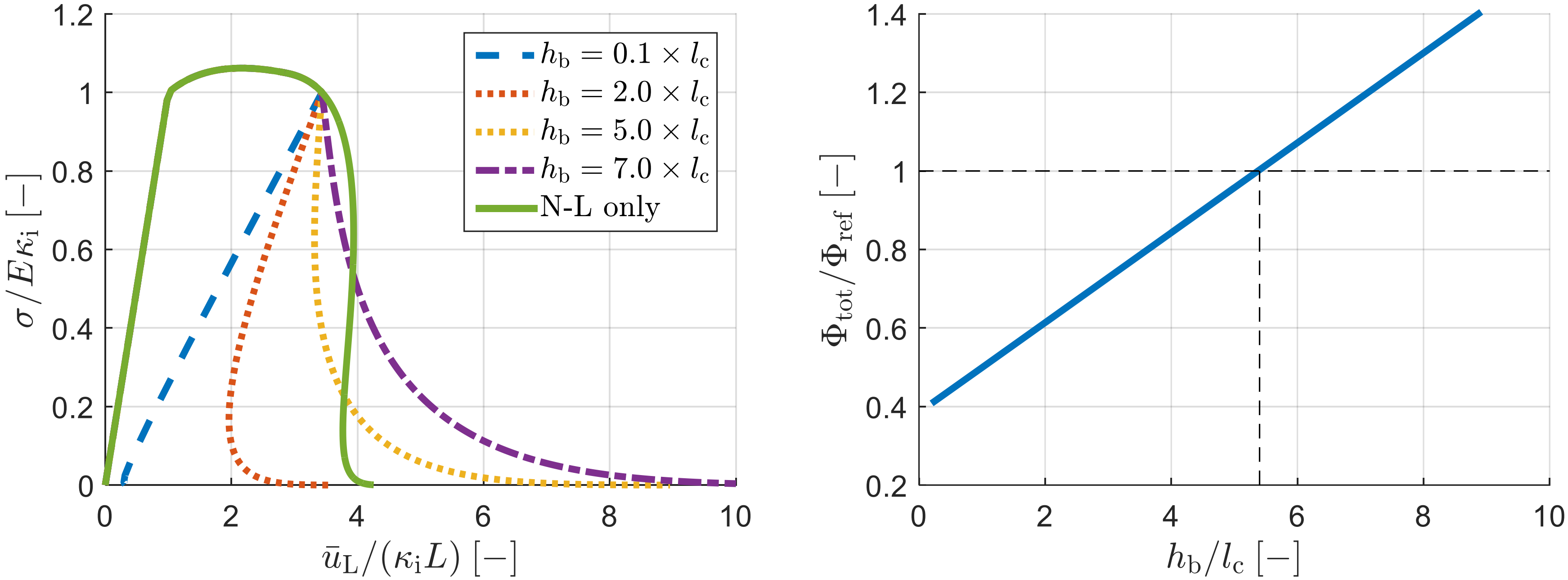
- Elastic damage material model
  - Constitutive equations
    - Helmholtz energy: $\rho\psi(\boldsymbol{\varepsilon}, D) = \frac{1}{2}(1-D)\boldsymbol{\varepsilon}: H: \boldsymbol{\varepsilon}$
    - Non-local maximum principal strain: $\tilde{e} - l_c^2 \Delta\tilde{e} = e$
    - Damage evolution $\dot{D}(\kappa) = (1-D)\left(\frac{\beta}{\kappa} + \frac{\alpha}{\kappa_c - \kappa}\right)\dot{\kappa}$ with $\kappa = \max_{t'} \tilde{e}(t')$
  - 1D non-local test



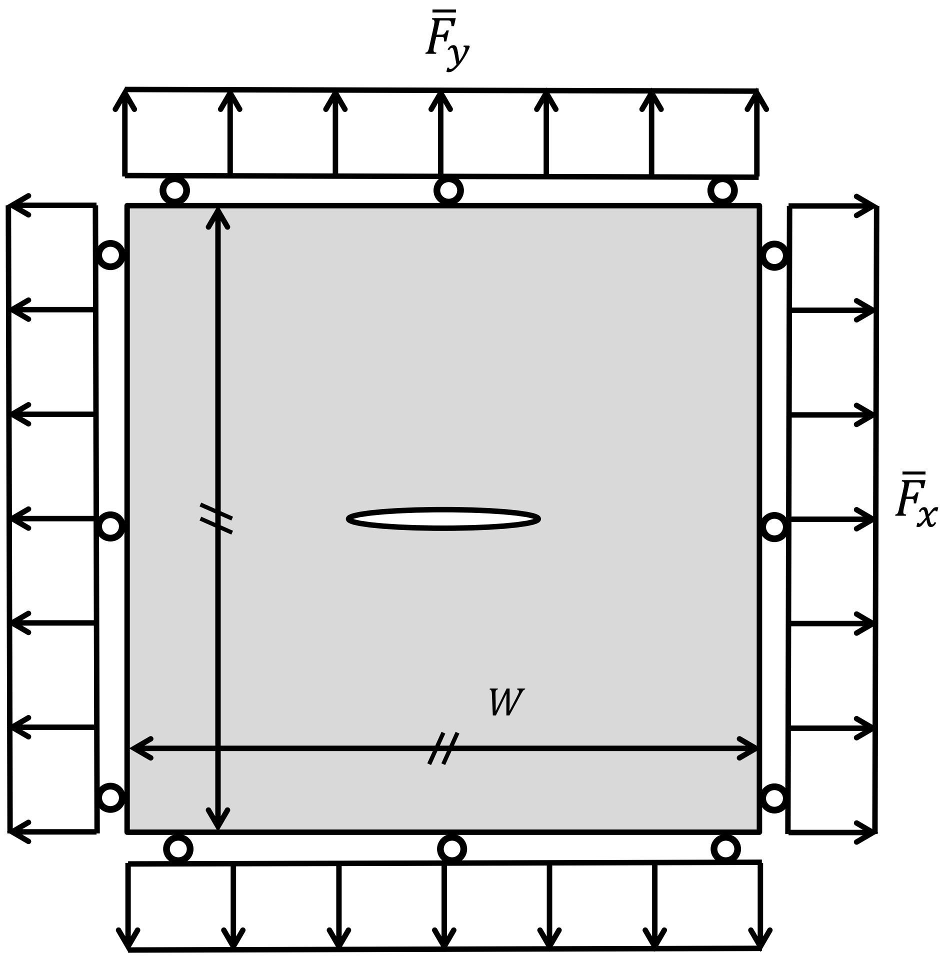


Damage to crack transition for elastic damage – Proof of concept

- Influence of h_{b} (for a given l_{c}) on response in a 1D elastic case [Leclerc et al. 2018]
  - Comparison with the pure non-local case
  - Has effect on the totally dissipated energy Φ
  - Could be chosen to conserve energy dissipation (physically based)
  - For elastic damage: $h_{\mathrm{b}} \simeq 5.4\, l_c$



- 2D elastic plate with a defect
  - Biaxial loading
    - Ratio $\bar{F}_x/\bar{F}_y$ constant during a test
  - In plane strain
  - Comparison between:
    - Pure non-local
    - Non-local + cohesive zone (CZM)
    - Non-local + cohesive band (CBM)

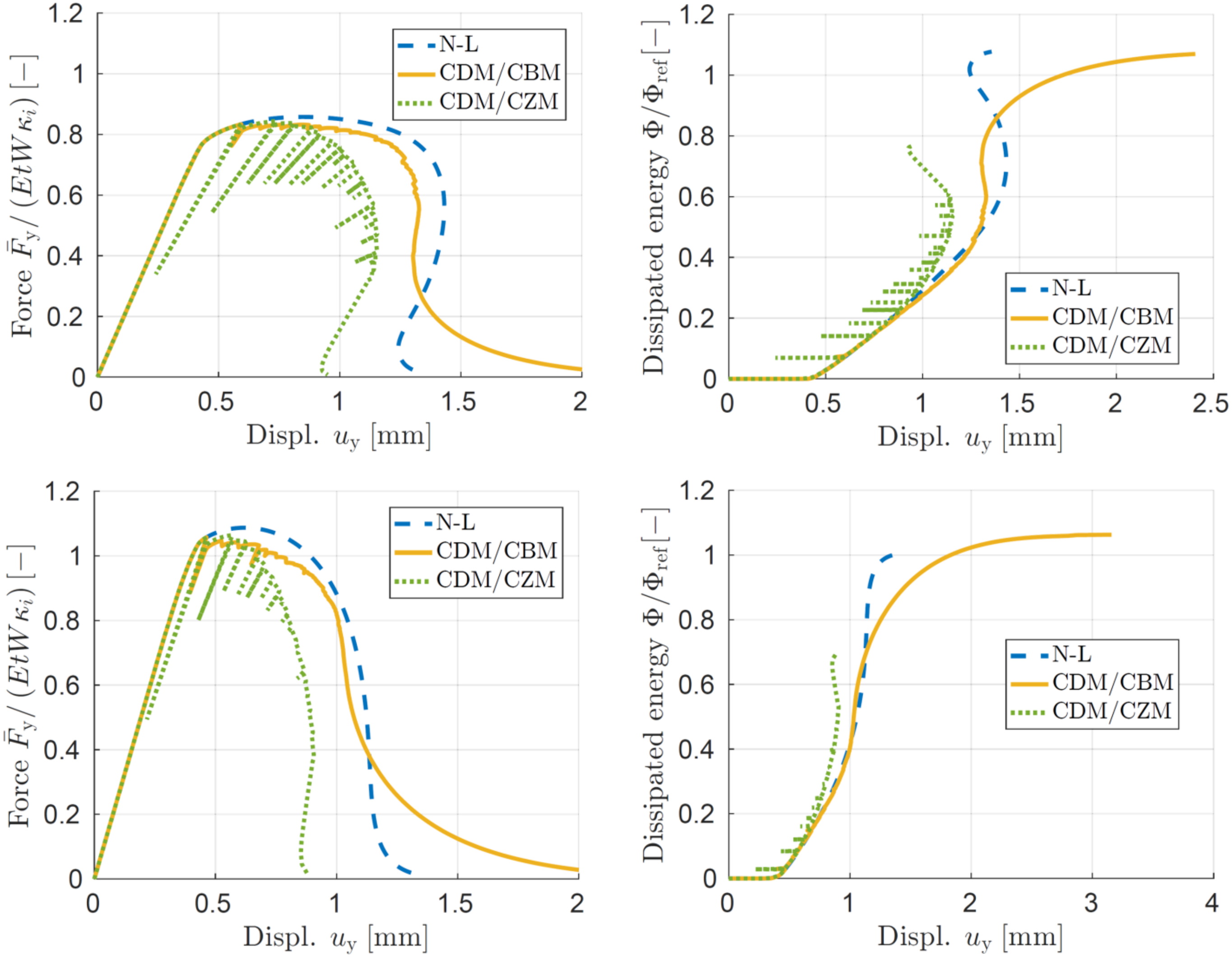


Damage to crack transition for elastic damage – Proof of concept

- Study of triaxiality effect on a slit-plate
  - Reference dissipated energy Φ_{ref} for non-local with $\bar{F}_x/\bar{F}_y = 0$

$\frac{\bar{F}_x}{\bar{F}_y} = -\frac{1}{2}$

$\frac{\bar{F}_x}{\bar{F}_y} = \frac{1}{2}$



Non-Local only

Non-Local - CZM

Non-Local - CBM

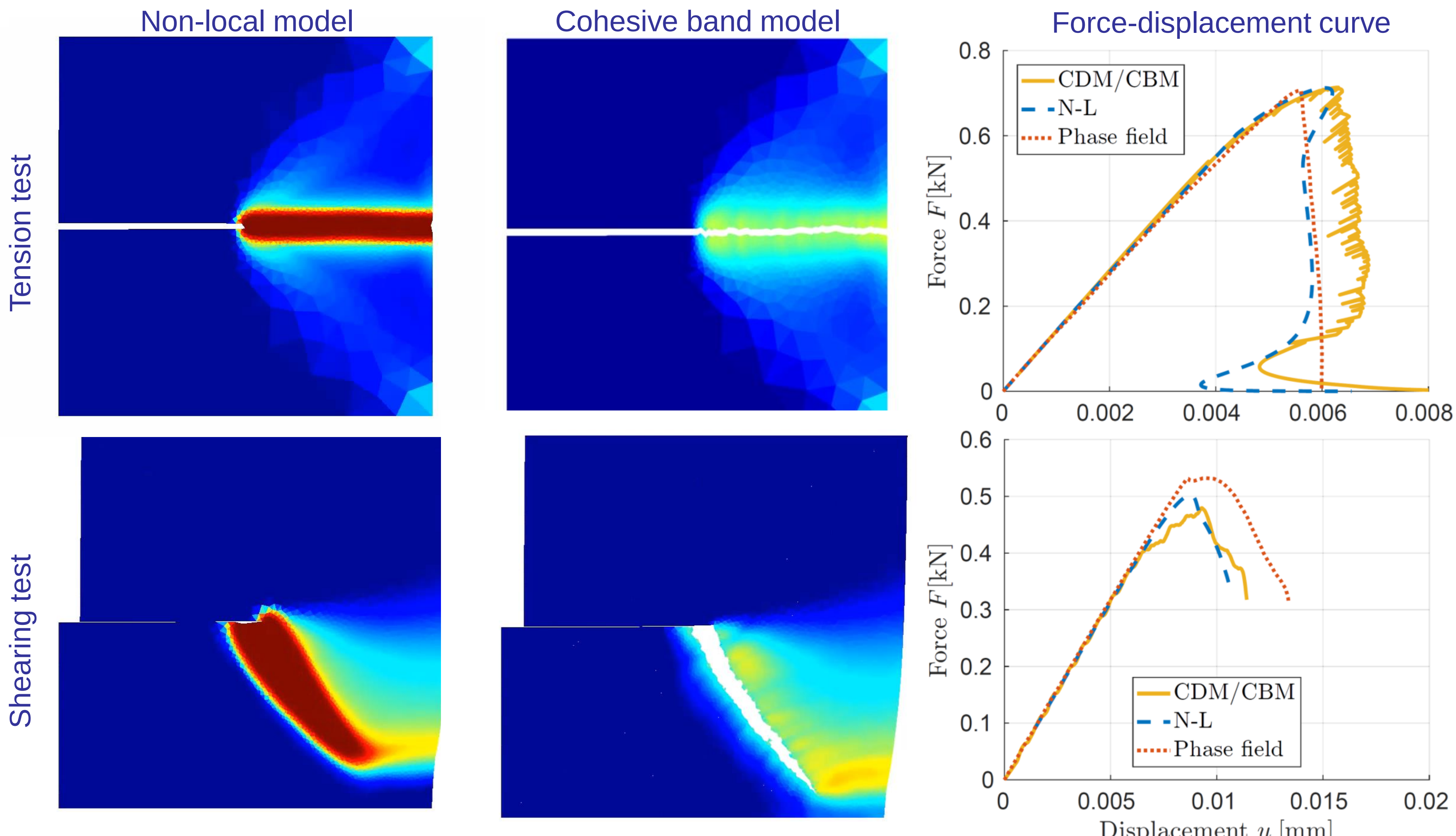
Error on total diss. energy

CZM: ~30%

CBM: ~3%

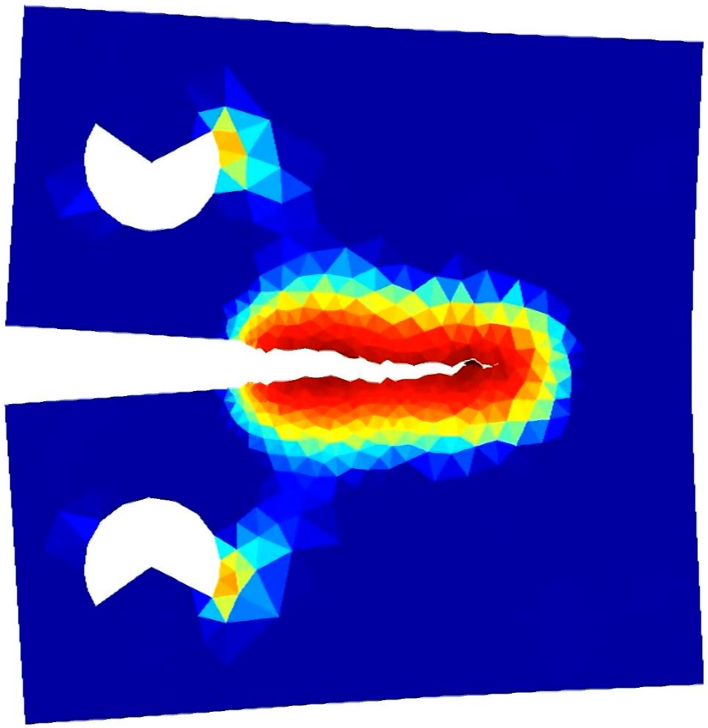
Damage to crack transition for elastic damage – Proof of concept

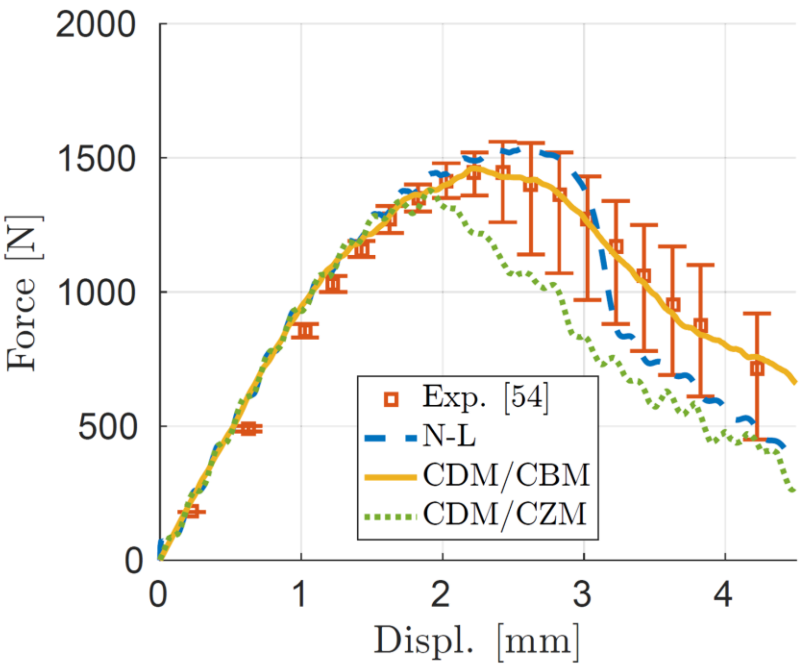
- Comparison with phase field
  - Single edge notched specimen [Miehe et al. 2010]
    - Calibration of damage and CBM parameters with 1D case [Leclerc et al. 2018]



Damage to crack transition for elastic damage – Proof of concept

- Validation with Compact Tension Specimen [Geers 1997]
  - Better agreement with the cohesive band model than the cohesive zone model or the non-local model alone [Leclerc et al. 2018]

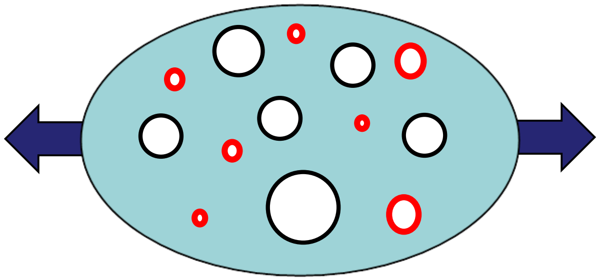




Porous plasticity – Voids nucleation

- Evolution of local porosity

$$\dot{f}_V = (1 - f_V)\mathrm{tr}(\mathbf{D}^\mathrm{p}) + \dot{f}_\mathrm{nucl} + \dot{f}_\mathrm{shear}$$



  - Voids nucleation $\dot{f}_\mathrm{nucl}$ modifies porosity growth rate
    - Linear strain-controlled growth

$$\dot{f}_\mathrm{nucl} = A_\mathrm{N}\dot{\hat{p}} \quad \text{with} \begin{cases} A_\mathrm{N} \neq 0 & \text{if } f_V > f_\mathrm{N} \\ A_\mathrm{N} = 0 & \text{if } f_V \leq f_\mathrm{N} \end{cases}$$

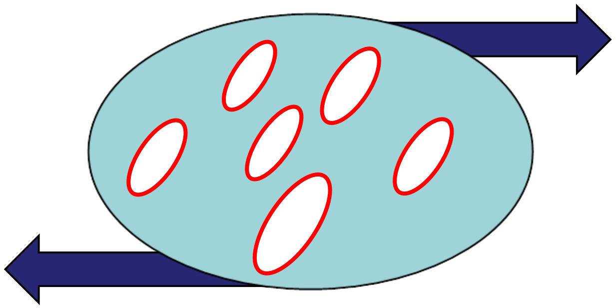
    - Gaussian strain-controlled growth

$$\dot{f}_\mathrm{nucl} = \frac{f_\mathrm{N}}{\sqrt{\{2\pi s_\mathrm{N}^2\}}} \exp\left(-\frac{(\hat{p} - \epsilon_\mathrm{N})^2}{2 s_\mathrm{N}^2}\right)\dot{\hat{p}}$$

    - where A_N, f_N, ϵ_N, s_N are material parameters

- Evolution of local porosity

$$\dot{f}_V = (1 - f_V)\mathrm{tr}(\mathbf{D}^{\mathrm{p}}) + \dot{f}_{\mathrm{nucl}} + \dot{f}_{\mathrm{shear}}$$



  – Shearing affect voids nucleation: $\dot{f}_{\mathrm{shear}}$

    - Includes Lode variable effect $\zeta(\boldsymbol{\tau}) = -\frac{27\,\det(\boldsymbol{\tau}^{\mathrm{dev}})}{2\,\tau_{\mathrm{eq}}^3}$

$$\dot{f}_{\mathrm{shear}} = f_V k_{\mathrm{w}} \left(1 - \zeta^2(\boldsymbol{\tau})\right) \frac{\boldsymbol{\tau}^{\mathrm{dev}} : \mathbf{D}^{\mathrm{p}}}{\tau_{\mathrm{eq}}}$$

    - where k_{w} is a material parameter

Non-local porous plasticity model

- Hyperelastic-based formulation
  - Multiplicative decomposition

    $\mathbf{F} = \mathbf{F}^{\mathrm{e}} \cdot \mathbf{F}^{\mathrm{p}},\ \ \mathbf{C}^{\mathrm{e}} = \mathbf{F}^{\mathrm{e}^T} \cdot \mathbf{F}^{\mathrm{e}},\ \ \ J^{e} = \det(\mathbf{F}^{\mathrm{e}})$
  - Stress tensor definition
    - Elastic potential $\psi(\mathbf{C}^{\mathrm{e}})$
    - First Piola-Kirchhoff stress tensor

      $$\mathbf{P} = 2\mathbf{F}^{\mathrm{e}} \cdot \frac{\partial \psi(\mathbf{C}^{\mathrm{e}})}{\partial \mathbf{C}^{\mathrm{e}}} \cdot \mathbf{F}^{\mathrm{p}^{-T}}$$
    - Kirchhoff stress tensors
      - In current configuration

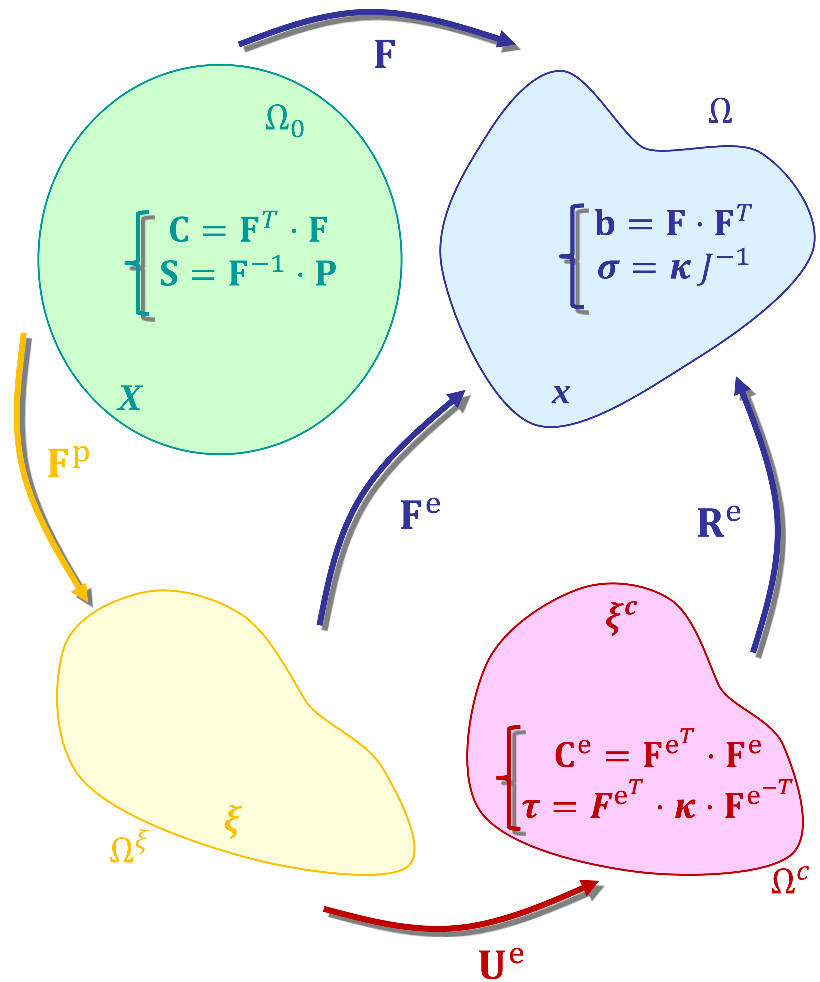
        $$\boldsymbol{\kappa} = \mathbf{P} \cdot \mathbf{F}^{T} = 2\mathbf{F}^{\mathrm{e}} \cdot \frac{\partial \psi(\mathbf{C}^{\mathrm{e}})}{\partial \mathbf{C}^{\mathrm{e}}} \cdot \mathbf{F}^{\mathrm{e}^T}$$
      - In co-rotational space

        $$\boldsymbol{\tau} = \mathbf{C}^{\mathrm{e}} \cdot \mathbf{F}^{\mathrm{e}^{-1}} \cdot \boldsymbol{\kappa} \cdot \mathbf{F}^{\mathrm{e}^{-T}} = 2\mathbf{C}^{\mathrm{e}} \cdot \frac{\partial \psi(\mathbf{C}^{\mathrm{e}})}{\partial \mathbf{C}^{\mathrm{e}}}$$

- Logarithmic deformation
  - Elastic potential ψ:

    $$\psi(\mathbf{C}^{\mathrm{e}}) = \frac{K}{2} \ln^2(J^{e}) + \frac{G}{4} (\ln(\mathbf{C}^{\mathrm{e}}))^{\mathrm{dev}} : (\ln(\mathbf{C}^{\mathrm{e}}))^{\mathrm{dev}}$$
  - Stress tensor in co-rotational space

    $$\boldsymbol{\tau} = \underbrace{K \ln(J^{e})}_{p}\ \mathbf{I} + G(\ln(\mathbf{C}^{\mathrm{e}}))^{\mathrm{dev}}$$



Integration algorithm

- Predictor-corrector procedure
  - Elastic predictor

$$\mathbf{F}^{\mathrm{e}^{\mathbf{pr}}} = \mathbf{F} \cdot \mathbf{F}_{\boldsymbol{n}}^{\mathrm{p}^{-1}}$$

  - Plastic corrector (radial return-like algorithm)
    - 3 equations
      - Consistency equation: $f\left(\tau_{\mathrm{eq}}, p; \tau_{\mathrm{Y}}, \boldsymbol{Z}(t'), \tilde{f}_V(t')\right) = 0$
      - Plastic flow rule: $\mathbf{D}^{\mathrm{p}} = \dot{\mathbf{F}}^{\mathrm{p}} \cdot \mathbf{F}^{\mathrm{p}^{-1}} = \dot{\gamma}\frac{\partial f}{\partial \boldsymbol{\tau}} = \dot{\hat{d}}\frac{\partial \tau_{\mathrm{eq}}}{\partial \boldsymbol{\tau}} + \dot{\hat{q}}\frac{\partial p}{\partial \boldsymbol{\tau}}$
      - Matrix plastic strain evolution: $\dot{\hat{p}} = \frac{\boldsymbol{\tau}:\mathbf{D}^{\mathrm{p}}}{(1-f_{V_0})\tau_{\mathrm{Y}}}$
    - 3 Unknowns $\Delta\hat{d}$, $\Delta\hat{q}$, $\Delta\hat{p}$
    - 3 linearized equations
      - Consistency equation: $f\left(\tau_{\mathrm{eq}}\left(\Delta\hat{d}\right), p(\Delta\hat{q}); \tau_{\mathrm{Y}}(\Delta\hat{p}), \boldsymbol{Z}\left(\Delta\hat{d}, \Delta\hat{q}, \Delta\hat{p}\right), \tilde{f}_V\right) = 0$
      - Plastic flow rule: $\Delta\hat{d}\frac{\partial f}{\partial p} - \Delta\hat{q}\frac{\partial f}{\partial \tau_{\mathrm{eq}}} = 0$
      - Matrix plastic strain evolution: $\left(1 - f_{V_0}\right)\tau_{\mathrm{Y}}\Delta\hat{p} = \tau_{\mathrm{eq}}\Delta\hat{d} + p\Delta\hat{q}$

Damage to crack transition in porous elasto-plasticity

- Porous plasticity (or Gurson) approach
  - Non-local form: $f\left(\tau_{\mathrm{eq}}, p, \tau_{\mathrm{y}}, \boldsymbol{Z}, \tilde{f}_{\mathrm{V}}\right) \leqslant 0$ with $\tilde{f}_{\mathrm{V}} - l_{\mathrm{c}}^2 \Delta \tilde{f}_{\mathrm{V}} = f_{\mathrm{V}}$
    - τ^{eq} is the von Mises equivalent Kirchhoff stress and p the pressure
    - $\tau_{\mathrm{Y}} = \tau_{\mathrm{Y}}(\hat{p}, \dot{\hat{p}})$ is the viscoplastic yield stress
    - f_{V} is the porosity and $\tilde{f}_{\mathrm{V}}$, its non-local counterpart
    - χ is the ligament ratio
    - $\boldsymbol{Z}$ is the vector of internal variables
    - l_{c} is the non-local length
  - Normal plastic flow
  - Hyperelastic formulation
  - Microstructure evolution (for spherical voids):
    - Eq. plastic strain of the matrix:

$$\dot{\hat{p}} = \frac{\boldsymbol{\tau} : \mathbf{D}^{\mathrm{p}}}{(1 - f_{\mathrm{V0}})\tau_{\mathrm{Y}}}$$

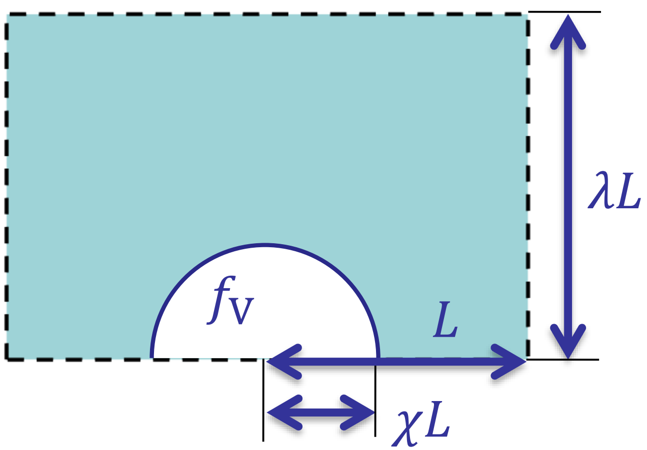
    - Porosity:

$$\dot{f}_{\mathrm{V}} = (1 - f_{\mathrm{V}})\mathrm{tr}\,\mathbf{D}^{\mathrm{p}} + \dot{f}_{\mathrm{nucl}} + \dot{f}_{\mathrm{shear}}$$

    - Ligament ratio:

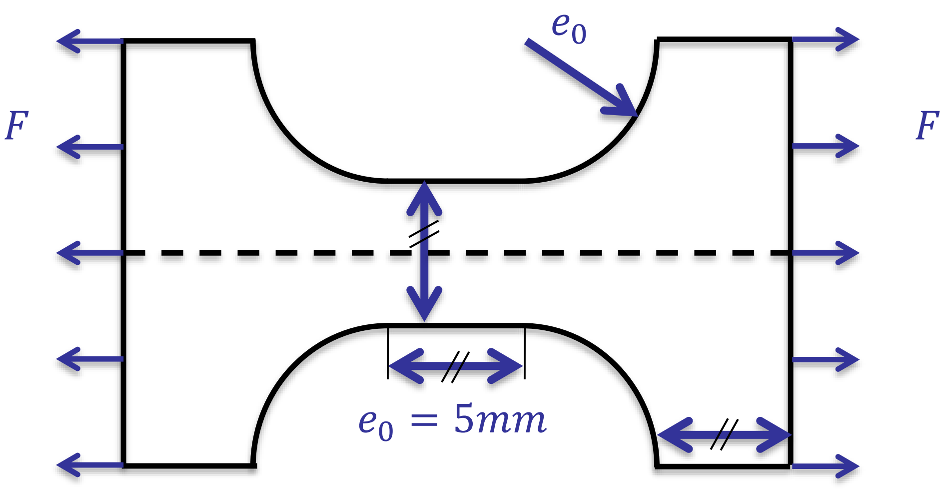
$$\dot{\chi} = \dot{\chi}\left(\chi, \tilde{f}_{V}, \kappa, \lambda, \mathbf{Z}\right)$$

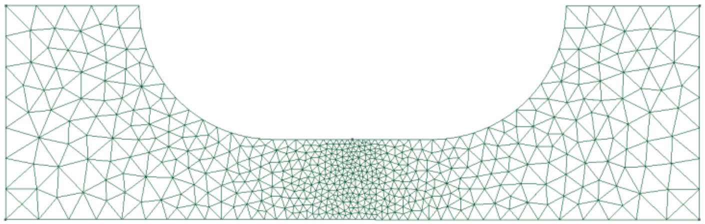
Microstructure parameters



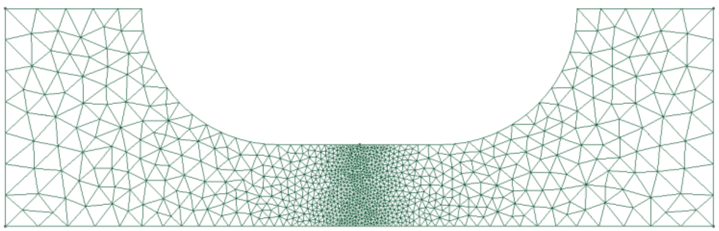
Non-local porous plasticity – Comparison with literature results

- Plane strain specimen [Besson et al. 2003]
  - Only half specimen is modelled
  - Three ≠ mesh sizes

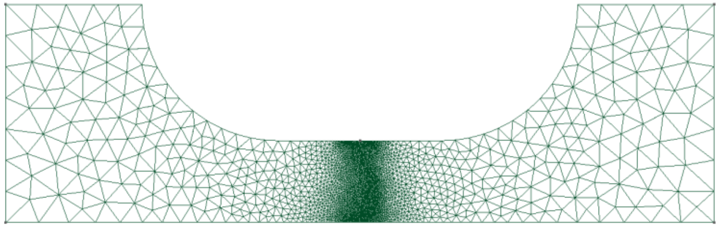




Coarse mesh
(~4600 elements, $l_m \cong 1.12\ l_c$)



Medium mesh
(~8100 elements, $l_m \cong 0.75\ l_c$)

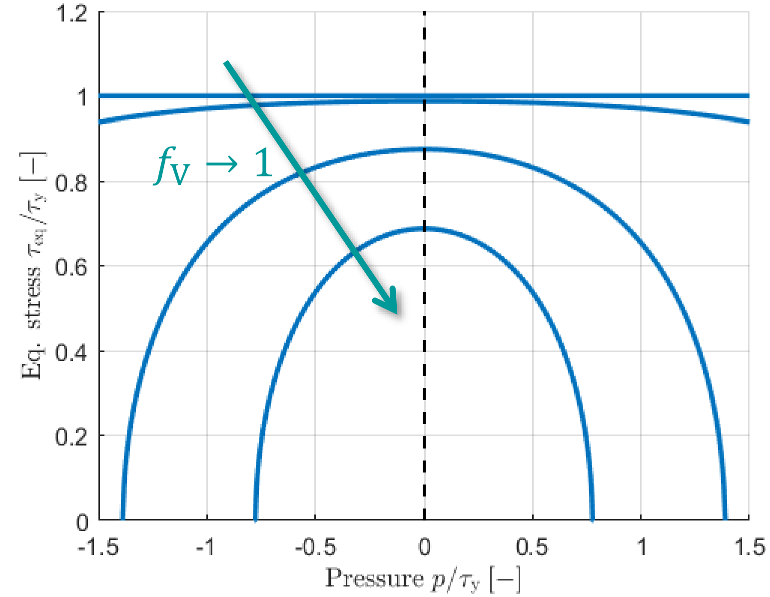


Fine mesh
(~15500 elements, $l_m \cong 0.5\ l_c$)

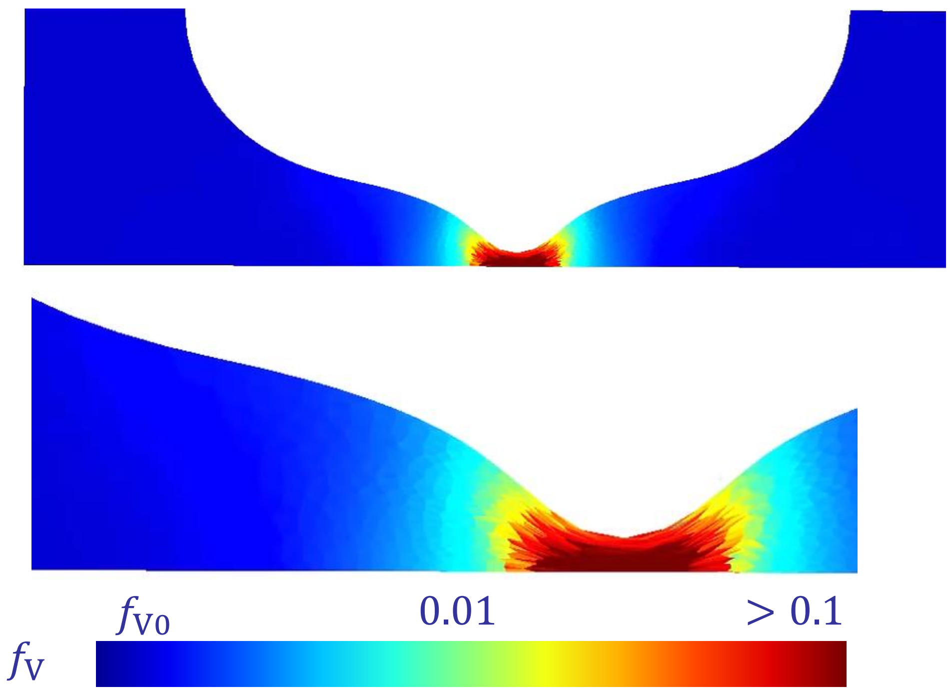
Non-local porous plasticity – void growth

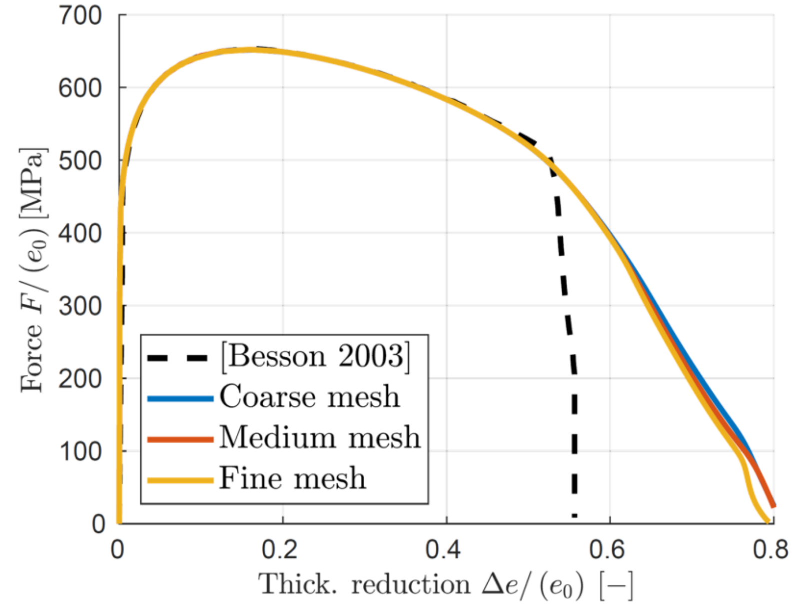
- Gurson model [Reush et al. 2003]
  - Particularized yield surface

$$f_G = \frac{\tau_{eq}^2}{\tau_Y^2} + 2 q_1 \tilde{f}_V \cosh\left(\frac{q_2 p}{2\tau_Y}\right) - 1 - q_3^2 \tilde{f}_V^2 \leq 0$$



- Verification of non-local model





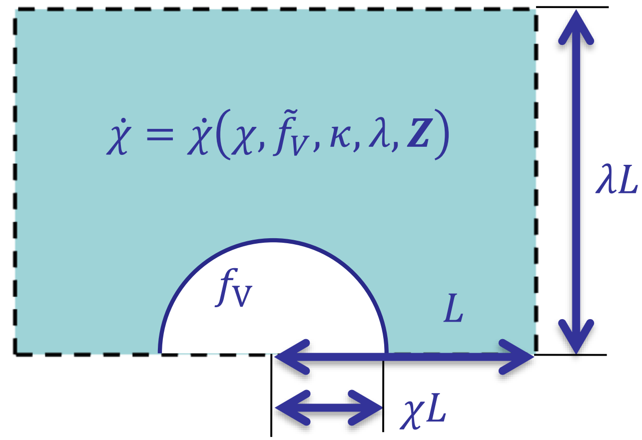
Non-local porous plasticity – void growth and coalescence

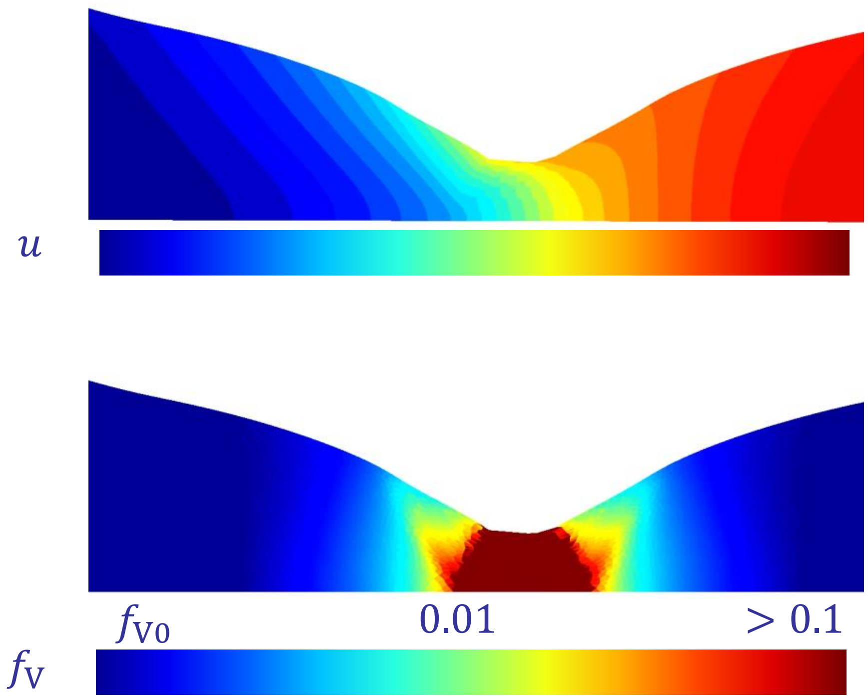
- Gurson model [Reush et al. 2003]
  - Phenomenological coalescence model:
    - Replace $\tilde{f}_V$ by an effective value $\tilde{f}_V^*$:

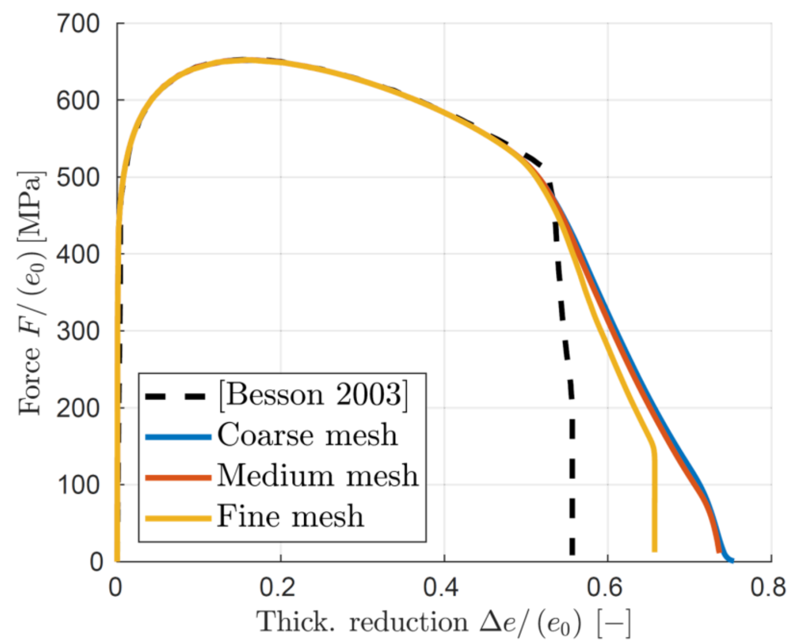
$$\tilde{f}_V^* = \begin{cases} \tilde{f}_V & \text{if } \tilde{f}_V \leq f_C \\ f_C + R\left(\tilde{f}_V - f_C\right) & \text{if } \tilde{f}_V > f_C \end{cases}$$

    - f_C from concentration factor $C_T^f(\chi)$ [Benzerga2014]

$$\max \operatorname{eig}(\boldsymbol{\tau}) - C_T^f(\chi)\tau_Y = 0$$





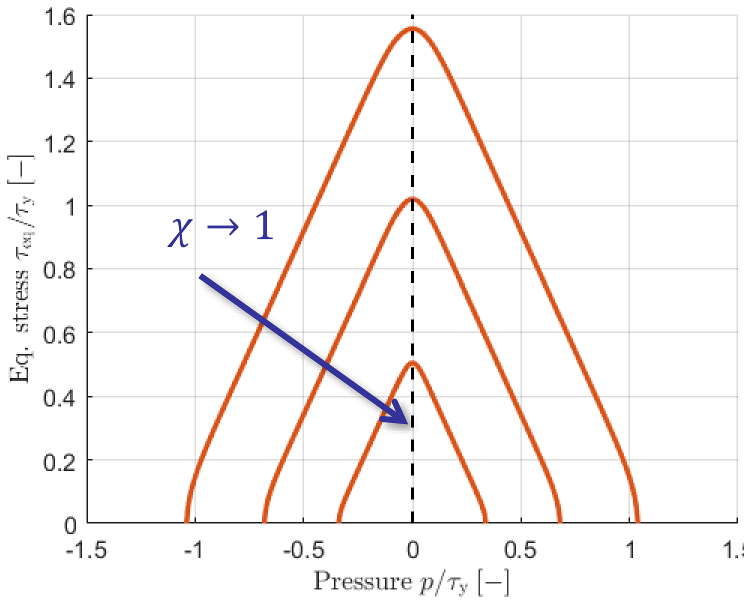


Non-local porous plasticity – void coalescence

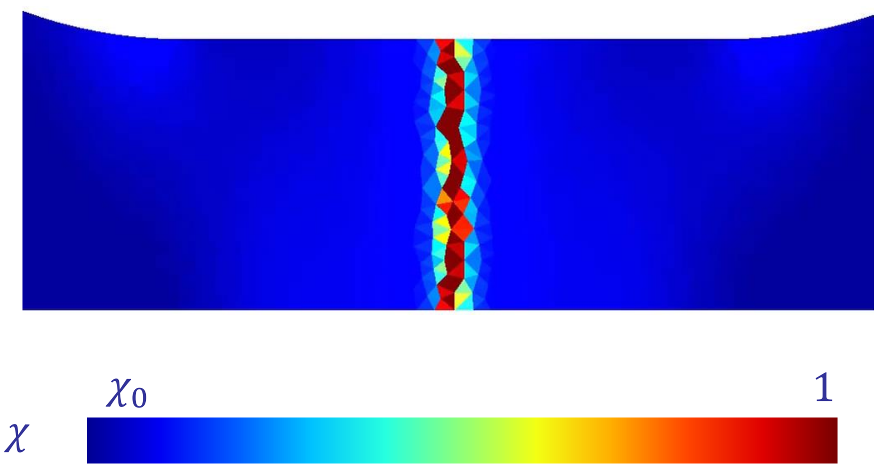
- Thomason model [Benzerga 2014, Besson 2009]
  - Particularized yield surface

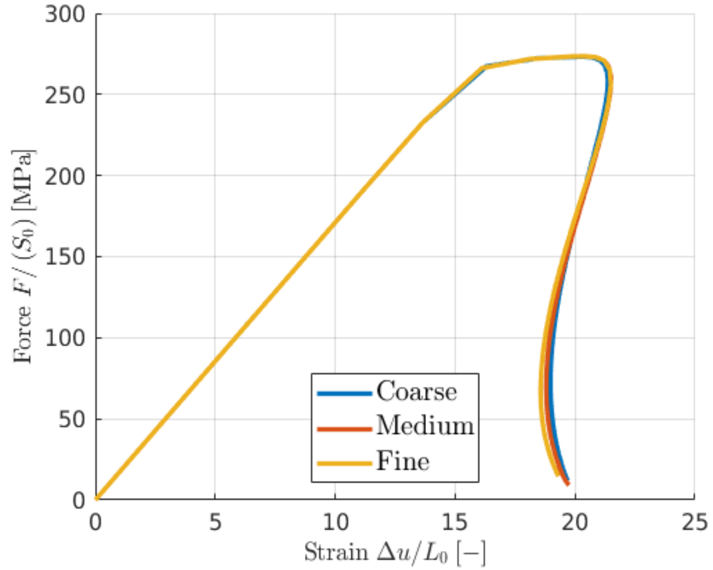
$$f_{\mathrm{T}} = \frac{2}{3}\tau_{\mathrm{eq}} + |p| - C_{\mathrm{T}}^{f}(\chi)\tau_{\mathrm{Y}} \leq 0$$

  - Higher porosity to trigger coalescence
  - No lateral contraction due to plasticity



- Verification of non-local model
  - For $\kappa = 0.5; \lambda = 0.5; l_c = 50\ \mu m$



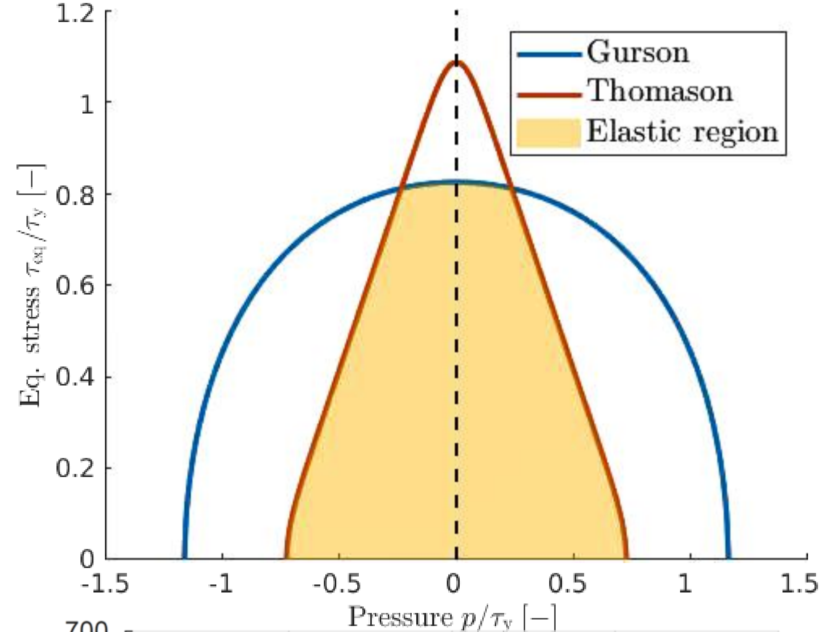


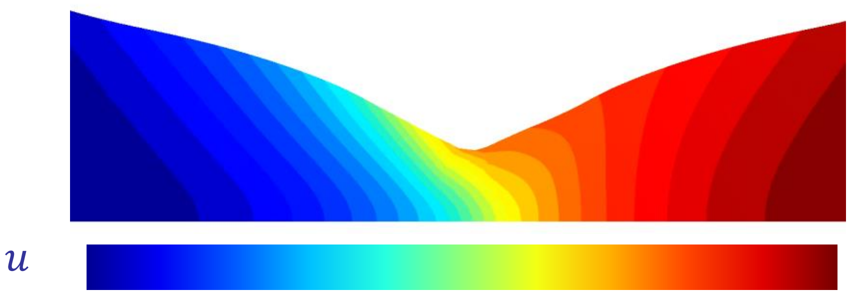
Non-local porous plasticity – void growth and coalescence

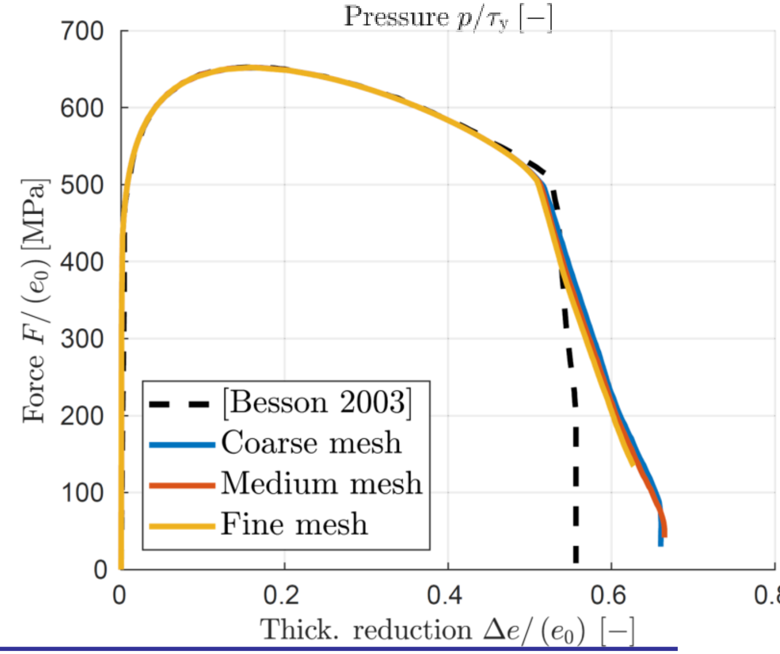
- Coupled non-local Gurson-Thomason
  - Competition between f_G and f_T

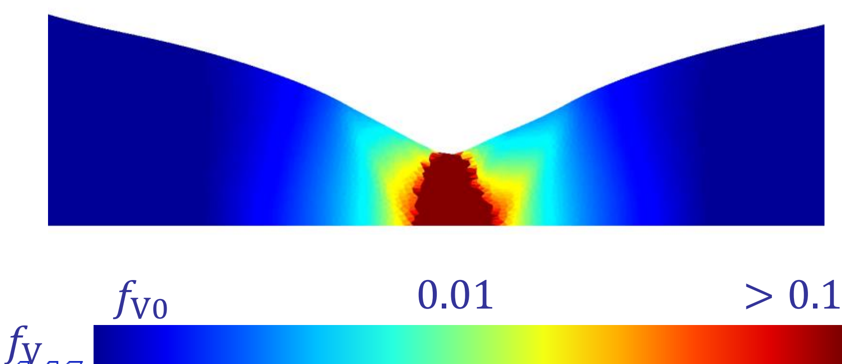
$$\begin{cases} f_G = \frac{\tau_{eq}^2}{\tau_Y^2} + 2q_1\tilde{f}_V \cosh\left(\frac{q_2 p}{2\tau_Y}\right) - 1 - q_3^2\tilde{f}_V^2 \le 0 \\ f_T = \frac{2}{3}\tau_{eq} + |p| - C_T^f(\chi)\tau_Y \le 0 \end{cases}$$

- For $\kappa = 0.5; \lambda = 0.5; l_c = 50\ \mu m$

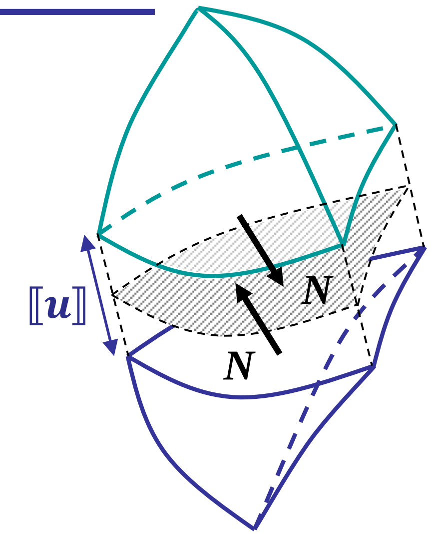




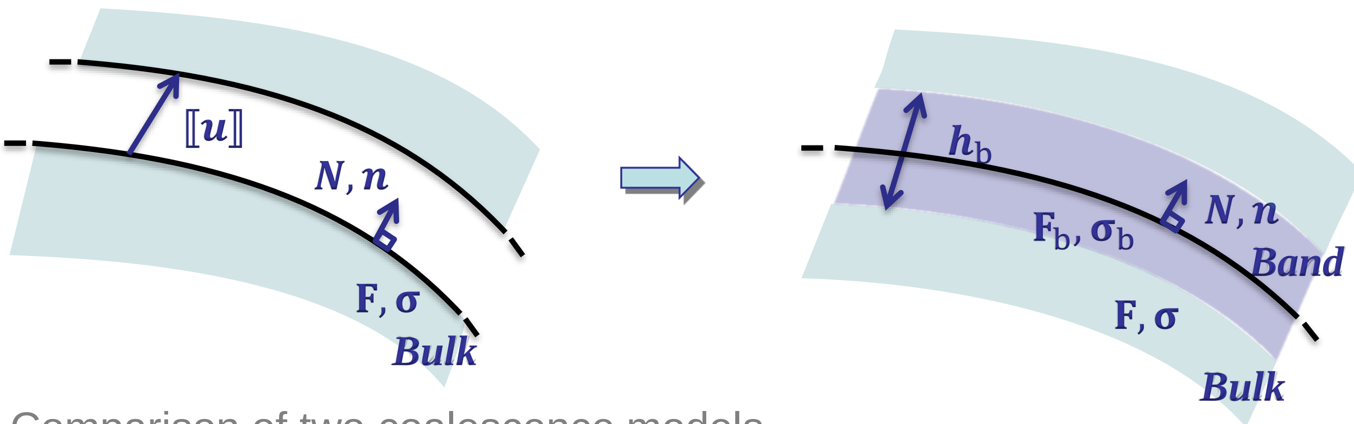




Damage to crack transition for porous plasticity



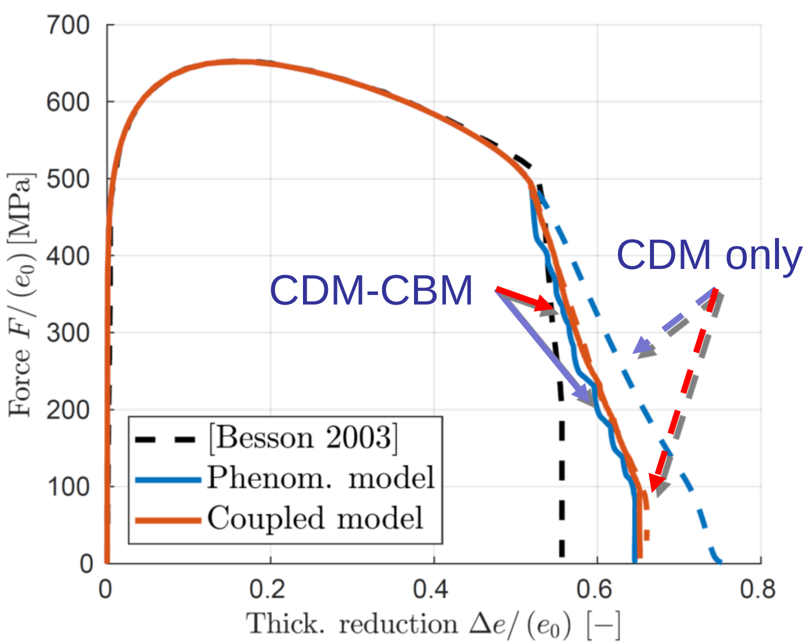
- Non-local Gurson model – CBM (arbitrary crack paths)
  - Gurson material model $f_{\mathrm{G}} = \frac{\tau_{\mathrm{eq}}^2}{\tau_{\mathrm{Y}}^2} + 2q_1 \tilde{f}_V \cosh\left(\frac{q_2 p}{2\tau_{\mathrm{Y}}}\right) - 1 - q_3^2 \tilde{f}_V^2 \leq 0$
  - Crack insertion at Thomasson criterion $\boldsymbol{N} \cdot \boldsymbol{\tau} \cdot \boldsymbol{N} - C_{\mathrm{T}}^f(\chi)\tau_{\mathrm{Y}} = 0$
  - At crack insertion: Cohesive Band Model



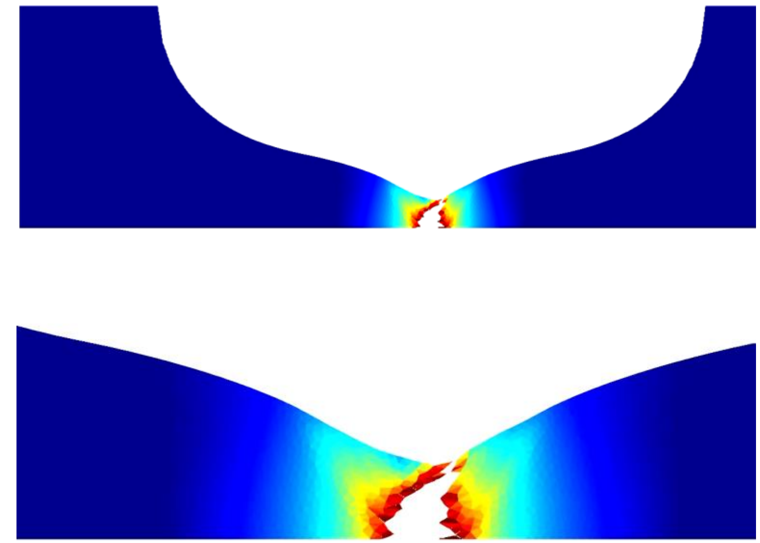
  - Comparison of two coalescence models
    - Phenomenological approach: $\tilde{f}_V^* = \begin{cases} \tilde{f}_V & \text{if } \tilde{f}_V \leq f_C \\ f_C + R(\tilde{f}_V - f_C) & \text{if } \tilde{f}_V > f_C \end{cases}$
    - Thomason model: $f_{\mathrm{T}} = \frac{2}{3}\tau_{\mathrm{eq}} + |p| - C_{\mathrm{T}}^f(\chi)\tau_{\mathrm{Y}} \leq 0$

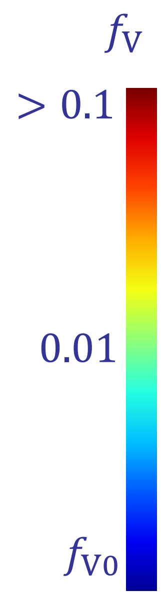
Damage to crack transition for porous plasticity

- Non-local Gurson model – CBM
  - CBM insertion at Thomason criterion
  - CBM with coalescence model
    - Comparison of 2 coalescence models
    - For $\kappa = 0.5; \lambda = 0.5; l_c = 50\ \mu m$



Thomason coalescence





Phenomenological coalescence